Inverse Functions

- in simple terms, inverse functions "undo" one another

$y = 7x \quad ; \quad \frac{y}{7} = x$

Example 1

$C = \frac{5}{9}(F - 32)$ ← isolate the variable

$\frac{9}{5}C = F - 32$ ← multiply by 9/5

← add 32

$F = \frac{9}{5}C + 32$ ← inverse

Example 2

$f(x) = 5x + 7$ find $f^{-1}(x)$

$y = 5x + 7$ 1) replace $f(x)$ with y

$x = 5y + 7$ 2) switch x & y

$\frac{x-7}{5} = y$ 3) solve for y

$f^{-1}(x) = \frac{x-7}{5}$ 4) Replace y with $f^{-1}(x)$

Example 3

$f(x) = (x-5)^2 + 2$

$y = (x-5)^2 + 2$

$x = (y-5)^2 + 2$

$x - 2 = (y-5)^2$

$\sqrt{x-2} = y - 5$

$\sqrt{x-2} + 5 = y$

$f^{-1}(x) = \sqrt{x-2} + 5$

Check:

$f(2) = (2-5)^2 + 2$
$(-3)^2 + 2$
$9 + 2$
11

$f^{-1}(11) = \sqrt{11-2} + 5$
$= \sqrt{9} + 5$
$= 3 + 5$
$= 8$

Example 4

$f(x) = \frac{1}{2}(x-5)+3$; find $f^{-1}(x)$

$y = \frac{1}{2}(x-5)+3$

$x = \frac{1}{2}(y-5)+3$

$x-3 = \frac{1}{2}(y-5)$

$2x-6 = y-5$

$2x-1 = y$

$f^{-1}(x) = 2x-1$

$f(4) = \frac{1}{2}(4-5)+3$

$= \frac{1}{2}(-1)+3$

$-.5+3$

$f(4) = 2.5$

$f^{-1}(2.5) = 2(2.5)-1$

$5-1$

4

these are inverse functions

Example 5

$f(x) = 4x^3 - 7$ find $f^{-1}(x)$

$y = 4x^3-7$

$x = 4y^3-7$

$\frac{x+7}{4} = \frac{4y^3}{4}$

$\frac{x+7}{4} = y^3$

$\sqrt[3]{\frac{x+7}{4}} = y$

$f^{-1}(x) = \sqrt[3]{\frac{x+7}{4}}$

$f(2) = 4(2)^3-7$

$= 4(8)-7$

$32-7$

$\boxed{25}$

$f^{-1}(25) = \sqrt[3]{\frac{25+7}{4}}$

$= \sqrt[3]{\frac{32}{4}}$

$= \sqrt[3]{8}$

$= \boxed{2}$