Composition of Functions

- when the output of one function is used as the input of another, we call the entire operation a composition of functions.

- we write $f(g(x))$ or $(f \circ g)(x)$ and read this as "f of g of x"

composition operator.

Example 1 Suppose $f(x)$ gives miles that can be driven in x hours and $g(y)$ gives gallons of gas used in driving y miles which of these expressions is more meaningful $f(g(y))$ or $g(f(x))$?

- $g(f(x))$ = is more meaningful because the output of $f(x)$ gives us miles and miles are the input in the second function $g(x)$.

Example 2

Using the tables, evaluate $f(g(3))$ and $(g \circ f)(4)$

names table

x	f(x)
1	6
2	8
3	3
4	1

input output

x	g(x)
1	3
2	5
3	2
4	7

* always evaluate right to left.

$\boxed{f(g(3)) = 8}$

→ $g(3) = 2$ ← output/NEW input

$f(2) = 8$

$(g \circ f)(4) = 3$

$f(4) = 1$ ← output/NEW input

$g(1) = 3$

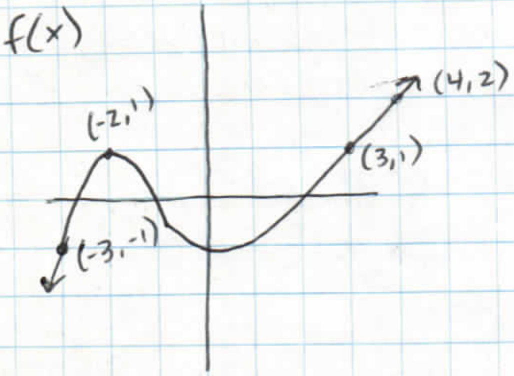
$\boxed{f(f(4)) = 6}$

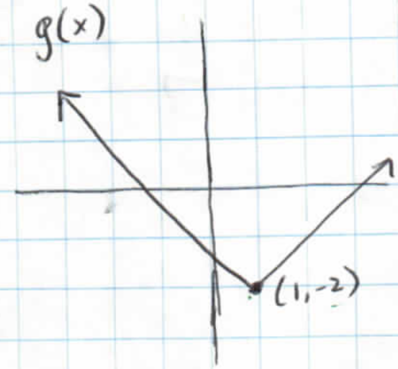
$f(4) = 1$

$f(1) = 6$

Example 3

Use the graph to evaluate $f(g(1))$ and $g(f(3))$





* use graph to find output, that output becomes input of next function

$f(g(1)) = 1$

$g(1) = -2$
$f(-2) = 1$

$g(f(3)) =$

$f(3) = 1$
$g(1) = -2$

$g(\boxed{f(3)})$
$g(1) = -2$

Example 4

use functions to evaluate

names function

$f(x) = x^2 + 5 \qquad g(x) = 4x - 2 \qquad h(x) = 2x^2 + x$

* use the above list of functions to evaluate different composition of functions

$f(g(h(2))) =$

$h(2) = 2x^2 + x$
$= 2(2)^2 + 2$
$= 2(4) + 2$
$= 8 + 2$
$= \underline{\underline{10}}$

output

$g(10) = 4x - 2$
$= 4(10) - 2$
$= 40 - 2$
$= \underline{\underline{38}}$

$f(38) = x^2 + 5$
$= 38^2 + 5$
$= 1444 + 5$
$= \underline{\underline{1449}}$

$f(g(h(2))) = 1449$

$(g \circ f \circ g)(-2) =$

$g(-2) = 4x - 2$
$= 4(-2) - 2$
$= -8 - 2$
$= \underline{\underline{-10}}$

$f(x) = x^2 + 5$
$f(-10) = (-10)^2 + 5$
$= 100 + 5$
$= \underline{\underline{105}}$

$g(105) = 4x - 2$
$= 4(105) - 2$
$= 420 - 2$
$= 418$

$g(h(3a)) = 72a^2 + 12a - 2$

$h(x) = 2x^2 + x$
$h(3a) = 2(3a)^2 + 3a$
$= 2(9a^2) + 3a$
$= \underline{18a^2 + 3a}$

$g(x) = 4x - 2$
$g(18a^2 + 3a) = 4(18a^2 + 3a) - 2$
$72a^2 + 12a - 2$

as good as it gets :)

Example 5

Functions into Functions :)

$p(x) = 2x^2 + 3 \qquad q(x) = x^2 - x \qquad s(x) = 4x - 1$

$p(s(x)) =$

$s(x) = \boxed{4x - 1}$

replace all x's in p with the function s(x)

$p(x) = 2x^2 + 3$

$p(4x-1) = 2(4x-1)^2 + 3$
$= 2(4x-1)(4x-1) + 3$
$= 2(16x^2 - 8x + 1) + 3$
$= 32x^2 - 16x + 2 + 3$

final answer $= 32x^2 - 16x + 5$

$(4x-1)(4x-1)$
$16x^2 - 4x - 4x + 1$

$q(p(x)) =$

$p(x) = \boxed{2x^2 + 3}$

replace both!

$q(x) = x^2 - x$

$q(2x^2+3) = (2x^2+3)^2 - (2x^2+3)$
$= (2x^2+3)(2x^2+3) - (2x^2+3)$
$= 4x^4 + 6x^2 + 6x^2 + 9 - 2x^2 - 3$

$= \boxed{4x^4} + 6x^2 + 6x^2 + 9 - 2x^2 - 3$

$= 4x^4 + 10x^2 + 6$ ← final answer