Linear and Angular Velocity

Linear Velocity: V, of a point can be found as the distance travelled, arc length, s, per unit time, t.

$$V = \frac{s}{t} \quad \text{where} \quad s = \text{arc length}, \; t = \text{time}$$

Angular Velocity: ω (omega) as a point moves along a circle of radius, r, its angular velocity ω, can be found as the angular rotation, θ (in rad) per unit time, t.

$$\omega = \frac{\theta}{t} \quad \text{where} \quad \theta = \text{angle in radians}, \; t = \text{time}$$

Arc length	Linear Velocity	Angular Velocity
$s = r\theta$	$V = \frac{s}{t}$	$\omega = \frac{\theta}{t}$

1) Isolate θ in angular velocity: $\omega = \frac{\theta}{t} \rightarrow \omega t = \theta$

2) Plug new expression into arc length formula: $s = r\theta \rightarrow s = r\omega t$

3) Plug new expression into linear velocity: $V = \frac{s}{t} \rightarrow V = \frac{r\omega t}{t}$

$$V = r\omega \qquad r = \frac{V}{\omega} \qquad \omega = \frac{V}{r} = V \cdot \frac{1}{r}$$

* linear velocity equals radius times angular velocity! *

Example 1

A truck with 32 in diameter wheels is traveling at 60 mi/hr. Find the angular speed of the wheels in rad/min. How many revolutions per minute do the wheels make?

$$\frac{60\ \text{mi}}{1\ \text{hr}} \cdot \frac{1}{16\text{in}} \cdot \frac{5280\ \text{ft}}{1\ \text{mi}} \cdot \frac{12\ \text{in}}{1\ \text{ft}} \cdot \frac{1\ \text{hr}}{60\ \text{min}} = \frac{3801600}{960\ \text{min}} = 3960\ (\text{rad})/\text{min}$$

$$\approx 3960\ (\text{rad})/\text{min}$$

$$\frac{3960\ (\text{rad})}{\text{min}} \cdot \frac{1\ \text{rev}}{2\pi\ (\text{rad})} \approx 630.25\ \text{rev/min}$$

Example 2

A wheel of radius 8in is rotating 15°/sec. What is the linear speed v, the angular speed in RPM and the angular speed in rad/sec?

$$15^\circ \cdot \frac{\pi}{180} = \frac{15\pi}{180} = \frac{3\pi}{36} = \frac{\pi}{12} \qquad \omega = \frac{\pi}{12}\ \text{rad/sec}$$

$$V = 8 \cdot \frac{\pi}{12} \rightarrow \frac{8\text{in}}{1} \cdot \frac{\pi}{12\ \text{sec}} = \frac{8\pi}{12}\ \text{in/sec} \quad \text{or} \quad 2.094\ \text{in/sec}$$

How long for mile?

$$5280\text{ft} \cdot 12\ \text{in} = \frac{63360\ \text{in}}{2.094\ \text{in}} = 30257.87\ \text{sec}$$

$$30257.87\ \text{sec} \cdot \frac{1\ \text{min}}{60\ \text{sec}} \cdot \frac{1\ \text{hr}}{60\ \text{min}} \qquad \frac{30257.87}{3600} \approx 8.4\ \text{hours}$$

$$\frac{\pi\ \text{rad}}{12\ \text{sec}} \cdot \frac{60\ \text{sec}}{1\ \text{min}} \cdot \frac{1\ \text{rev}}{2\pi} = \frac{60\ \text{rev}}{24\ \text{min}} \quad \text{or} \quad 2.5\ \text{RPM}$$