Logarithmic Functions

A population of flies, 50 of them is expected to double each week, leading to the function $f(x) = 50(2)^x$. When will the population reach 500?

$\frac{500}{50} = \frac{50(2)^x}{50}$

$10 = 2^x$

$2^2 = 4$

$2^3 = 8$

$2^4 = 16$

approx between weeks 3 and 4

Logarithm:

The logarithm (base b) function written $\log_b(x)$ or $\log_b x$ is the inverse of the exponential function (base b) b^x

Logarithmic to Exponential $\longleftrightarrow$ Exponential to Logarithmic

$$\log_b C = a \longleftrightarrow b^a = C$$

1) $\log_6 36 = 2 \rightarrow 6^2 = 36$

2) $\log_3 215 = x \rightarrow 3^x = 215$

3) $\log_4 1/64 = -3 \rightarrow 4^{-3} = 1/64$

1) $5^x = 75 \rightarrow \log_5 75 = x$

2) $3^{-2} = 1/9 \rightarrow \log_3 1/9 = -2$

3) $4^5 = x \rightarrow \log_4 x = 5$

Common and Natural Logarithms

Common Log is the logarithm with base 10 and is typically written $\log(x)$ or $\log x$

Natural Log is the logarithm with base e, and is typically written $\ln(x)$ or $\ln x$.

Properties of Logs

** Change of Base **

$$\log_b a = \frac{\log_c a}{\log_c b}$$

*where C = 10, so it is the common log

Flies $\frac{500}{50} = \frac{50(2)^x}{50}$

$10 = 2^x$

1) $10 = 2^x$ — 1) Given

2) $\log_2 10 = x$ — 2) Convert to log form

3) $\frac{\log 10}{\log 2} = x$ — 3) Use change of base

4) $x = 3.3219$ — 4) convert weeks + days

3 weeks
2 days
6 hours
5 minutes
2 seconds

Example 2

Solve for x given $300 = 20(1.25)^x$

$\frac{300}{20} = \frac{20(1.25)^x}{20}$

$15 = 1.25^x$

$\log_{1.25} 15 = x$

$\frac{\log 15}{\log 1.25} = x$

$12.136 = x$

Example 3

If an island has a population of 300 birds and thanks to conservation efforts it is growing by 8% each year, when will the population surpass 500?

$f(x) = 300(1.08)^x$

$500 = 300(1.08)^x$

$5/3 = 1.08^x$

$\log_{1.08} 5/3 = x$

$\frac{\log(5/3)}{\log(1.08)} = x \qquad x = 6.6 \text{ yrs}$

Example 4

If \$5000 is deposited in an account compounded quarterly at 2.4%, how long before it reaches \$7500?

$f(t) = 5000\left(1 + \frac{.024}{4}\right)^{4t}$

$7500 = 5000(1.006)^{4t}$

$1.5 = 1.006^{4t}$

$\log_{1.006} 1.5 = 4t$

$\frac{\log 1.5}{\log 1.006} = 4t$

$67.78 = 4t \approx 16.95 \text{ or } 17 \text{ yrs}$

Example 5

If a population is decreasing by 14% per year and initially has 8000 people, when will there be less than 2000 people?

$2000 = 8000(.86)^x$

$.25 = .86^x$

$\log_{.86} .25 = x$

$\frac{\log .25}{\log .86} = x$

$9.19 \approx x$