THE

"HAPPY ENDING"

CONJECTURE

By

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October 2024

The Happy Ending conjecture can be formally stated as:

Theorem: For any positive integer *N*, any sufficiently large finite set of points in the plane in general position has a subset of *n* points that form the vertices of a convex Polygon.

(Wikipedia 2024)

The happy ending conjecture was investigated by the mathematicians Erdos and Szekeres in the 1930's and was so-called because those two subsequently married each other.

A defining example of the conjecture can be demonstrated as follows; Place five dots at random positions on a flat surface. Connect any four of the dots and a convex quadrilateral will be formed as shown in Figure 1 below.

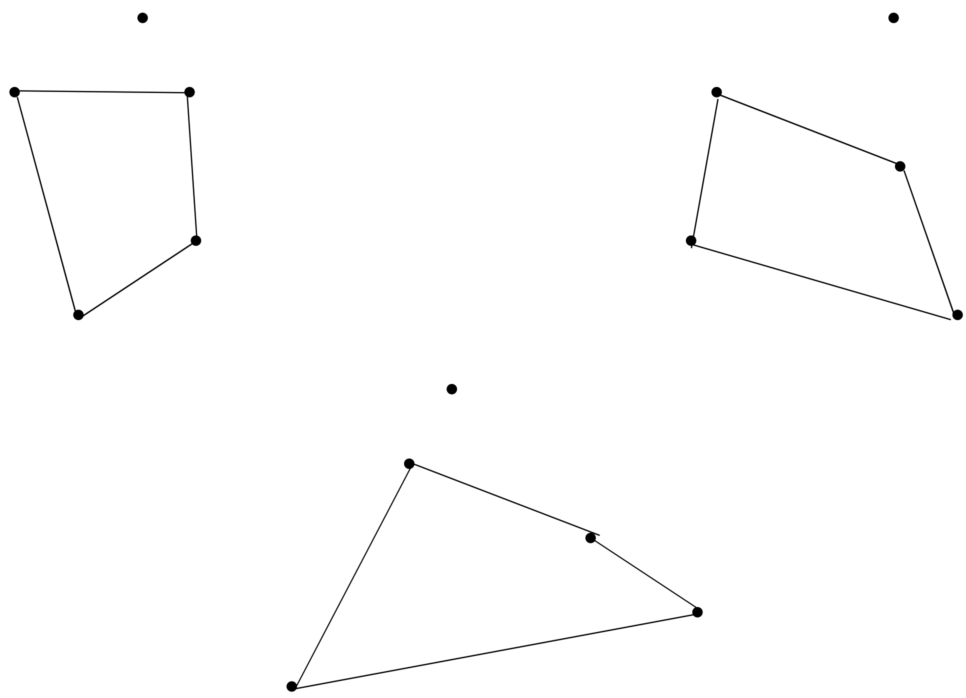

Fig. 1.

(Popular mechanics. G2816/5)

However, for other polygons the case is different and for a pentagon (5 sides) to be formed the convex polygon would require a source of 9 points and similarly for a hexagon (6 sides) to be formed, a source of 17 points would be required

Other than for those shapes described above, it is unknown how many points would be required to generate any further larger polygons.

It is generally considered that the equation:-

$$N = 1 + 2^{n-2} \qquad \text{Equ.1.}$$

Where N = the number of points and

n = the number of sides of the polygon.

will satisfy the conjecture, but to date the case for the equation is not proven.

Examination of Figure 2 below will show that the ratio between the points and the number of sides forms a rigid series making it possible to predict the ratio for any given polygon and it will show that Equation 1 does indeed satisfy all cases. Thus the conjecture will be proven.

Again consider Fig. 2. The figure demonstrates a clear sequence which is totally predictive of the ratio points to sides for all polygons of any number of sides to infinity.

In Figure 2, column A obviously defines each polygon. Column B describes the number of points defined as being the minimum number of points required to define each convex polygon. Column C describes the difference between the number of points relative to each successive polygon. Here we note that as previously stated, it has already been proven in the cases of the square i.e. 5points, the pentagon 9 points and the hexagon 17 points and here we state as a postulate that the difference is always twice the value plus1of that of the previous polygon in the sequence and conforms with expression:

$$N = 2^x + 1 \qquad \text{Equ. 3.}$$

where $x = 2,3,4....\infty$ *and also* $2^x \equiv 2 + x$ *being the no. of sides of the concomitant polygon.*

which is in accordance with column F.

A Polygon	B points	C diff'nce	D sides	E sides	F points
Square	5		4	$2^2 = 4$	$2^2 + 1 = 5$
		4			
Pentagon	9		5	$2^2 + 1 = 5$	$2^3 + 1 = 9$
		8			
Hexagon	17		6	$2^2 + 2 = 6$	$2^4 + 1 = 17$
		16			
Heptagon	33		7	$2^2 + 3 = 7$	$2^5 + 1 = 33$
		32			
Octagon	65		8	$2^2 + 4 = 8$	$2^6 + 1 = 65$
		64			
Nonagon	129		9	$2^2 + 5 = 9$	$2^7 + 1 = 129$
		128			
Decagon	257		10	$2^2 + 6 = 10$	$2^8 + 1 = 257$

Fig. 2.

Key to Fig 2:

A=Polygon description.

B=Number of points producing the polygon.

C= Difference between number of points per polygon.

D=Number of sides per polygon.

E=Expression for number of sides.

F=Expression for number of points.

Therefore it is submitted that the structure of all the series described I Figure 2 has the quality of being entirely provable for any polygon consisting of any number of sides.

This being the case, it is submitted that the expression:

$$N = 1 + 2^{n-2}$$

is proven.

END