Kaprekar's Constant

and its relation to the

Piyush Constant

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Kaprekar’s constant is a well-publicised anomaly in mathematics. It consists of a number of iterations of any four digit number of which at least two digits must be distinct. Each iteration consists of three distinct operations. These are

1/ Arrange the four digits of the chosen number into firstly another number in a descending series and secondly into another number in an ascending series. For example, the number 9328 can be arranged as 9832 and 2389.

2/ Then subtract the smaller number from the larger number i.e.

9328 - 2389 = 7443

3/ Then treat the resulting difference in the same way as in Step 2 i.e.

7443 – 3447 = 3996

4/ Then repeat the process i.e.

9963 – 3699 = 6642

and continue the process to obtain:

6642 – 2466 = 4176

4176 – 1467 = 6174

Iterating 6174 again and we obtain:

7641 – 1467 = 6174

and here we note that this iteration will continue to infinity.

It has been proven elsewhere (Thakar 2019. Uni. Of Rochester) that whatever four figure number is chosen to commence the series, that the series will always terminate within no more than seven iterations. Even more remarkable is the fact that whatever entirely diverse pairs of four digit numbers is chosen to commence the series (i.e. the series does not necessarily have to commence with just a single number), the end result will terminate in the number 6174 although the number of iterations is not constant.

Science Blog (April 2008) Published an expression described as the Piyush constant adopted with several slight amendments as follows.

Take any number of pairs of digits, in this case 25 and 32. These can be written in four ways;

25 x 32 = 800

23 x 23 = 575

52 x 23 = 1196

52 x 32 = 1664

Now subtract the lower figure from the larger figure in sequence;

1664 – 1196 = 468 = 4 + 6 + 8 = ***18*** = 1+ 8 = 9

1664 – 800 = 864 = 8 + 6 + 4 = ***18*** = 1+ 8 = 9

1196 – 575 = 621 = 6 + 2 + 1 = ***9***

1196 – 800 = 396 = 3 + 9 + 6 = ***18*** = 1 + 8 = 9

800 – 575 = 225 = 2 + 2 + 5 = ***18*** = 1 + 8 = 9

The figures in bold italics represent the sum of the digits in the preceding column of results.

Returning to the Kaprekar constant, Table 1 below illustrates several examples of the constant resulting from the iteration of a number of randomly chosen pairs numbers all of which result in producing the Kaprekar constant 6124. Here we note that the figures in bold type in the right hand column represent the sum of the preceding results of the iterations in the left hand column and here it should be noted that Table 1is representative of the fact that to date, no examples have been found where the Kaprekar constant does not apply to any randomly chosen four digit numbers.

<u>Table 1</u>

9631 – 4578 = 5053	= ***13*** = 1 + 3 = 4
5530 − 0355 = 5175	= ***18*** = 1 + 8 = 9 = 3^2
7551 – 1557 = 5994	= **27** = 2 + 7 = 9 = 3^2
9954 – 4599 = 5355	= ***18*** =1 + 8 = 9 = 3^2
5553 – 3555 = 1998	= ***27*** =2 + 7 = 9 = 3^2
9981 – 1899 = 8082	= **18** =1 + 8 = 9 = 3^2
8820 – 0288 = 8532	= ***18*** =1 + 8 = 9 = 3^2
8532 – 2358 = 6174	= ***18*** =1 + 8 = 9 = 3^2

8 Iterations

9872 – 1124 = 8748 = ***27*** = 2 + 7 = 9 = 3^2

8744 – 4478 = 4266 = ***18*** = 1 + 8 = 9 = 3^2

6642 – 2466 = 4176 = ***18*** = 1 + 8 = 9 = 3^2

7641 – 1467 = 6174 = ***18*** = 1 + 8 = 9 = 3^2

4 Iterations

9630 – 1345 = 8285 = ***23*** = 2 + 3 = 5

8852 – 2588 = 6264 = ***18*** = 1 + 8 = 9 = 3^2

6642 – 2466 = 4176 = ***18*** = 1 + 8 = 9 = 3^2

7641 – 1467 = 6174 = ***18*** = 1 + 8 = 9 = 3^2

4 Iterations

9521 – 4678 = 4843 = ***19*** = 1 + 9 = 10 = 1 + 0 = 1

8443 – 3448 = 4995 = ***27*** = 2 + 7 = 9 = 3^2

9954 – 4599 = 5355 = ***18*** = 1 + 8 = 9 = 3^2

5553 – 3555 = 1998 = ***27*** = 2 + 7 = 9 = 3^2

9981 – 1899 = 8082 = ***18*** = 1 + 8 = 9 = 3^2

8820 – 0288 = 8532 = ***18*** = 1 + 8 = 9 = 3^2

8532 – 2358 = 6174 = ***18*** = 1 + 8 = 9 = 3^2

7 Iterations

8764 – 1259 = 7505 = ***17*** = 1 + 7 = 8

7550 – 0557 = 6993 = ***27*** = 2 + 7 = 9 = 3^2

9963 – 3699 = 6264 = ***18*** = 1 + 8 = 9 = 3^2

6642 – 2466 = 4176 = ***18*** = 1 + 8 = 9 = 3^2

6741 – 1467 = 5274 = ***18*** = 1 + 8 = 9 = 3^2

7542 – 2457 = 5085 = ***18*** = 1 + 8 = 9 = 3^2

8550 – 0558 = 7992 = ***18*** = 1 + 8 = 9 = 3^2

9972 – 2799 = 7173 = ***18*** = 1 + 8 = 9 = 3^2

7731 – 1377 = 6354 = ***18*** = 1 + 8 = 9 = 3^2

6543 – 3456 = 3087 = ***18*** = 1 + 8 = 9 = 3^2

8730 – 0378 = 8352 = ***18*** = 1 + 8 = 9 = 3^2

8532 – 2358 = 6174 = ***18*** = 1 + 8 = 9 = 3^2

12 Iterations

<u>Table 2</u>

This Table represents the Piyush Constant extrapolated to include a further simplification to the prime factor 3 as already exemplified in Table1Kaprekar simplifications.

1664 – 1196 = 468 = ***18*** = 1+ 8 = 9 = 3^2

1664 – 800 = 864 = ***18*** = 1+ 8 = 9 = 3^2

1196 – 575 = 621 = ***9*** = 3^2

1196 – 800 = 396 = ***18*** = 1 + 8 = 9 = 3^2

800 – 575 = 225 = ***18*** = 1 +8 = 9 = 3^2

Continuing with this theme. It has been shown (SourceForce 2024) that the Kaprekar iterations can be applied to many other combinations of numbers, some of which are illustrated below in Table 3.

Table 3

No. of digits in the number	Kaprekar Constant	Iteration
3	495	= ***18*** = 1 + 8 = 9 = 3^2
6	631764	= ***27*** = 2 + 7 = 9 = 3^2
	549945	= ***36*** = 3 + 6 = 9 = 3^2
8	97508421	= ***36*** = 3 + 6 = 9 = 3^2
	63317664	= ***36*** = 3 + 6 = 9 = 3^2
9	864197532	= ***45*** = 4 + 5 = 9 = 3^2
	554999445	= ***54*** = 5 + 4 = 9 = 3^2
10	9753086421	= ***81*** = 8 + 1 = 9 = 3^2
	6333176664	= 4***5*** = 4 + 5 = 9 = 3^2
	9975084201	=***126*** =1 + 2 + 6 = 9 = 3^2

Similar results can be extrapolated forward and have been obtained and are available elsewhere. Thus we can hypothesise that both the Kaprekar and Piyush constants will apply into infinity, to the sets of numbers as described above.

It is noteworthy that a similar grammar applies to both Kaprekar’s constant and to the Piyush constant and it can be said that the Piyush expression is an extension of that of Kaprekar and this essay is an extension the of both of the constants resulting in the prime factor of 3.

What is remarkable is that the prime integer 3 appears to be common to all the iterations previously described and this begs the question--- Is there some underlying pattern to the structure of arithmetic and thus to mathematics generally which is somehow based on the prime factor 3? Perhaps further research would be informative.

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