P vs NP

By

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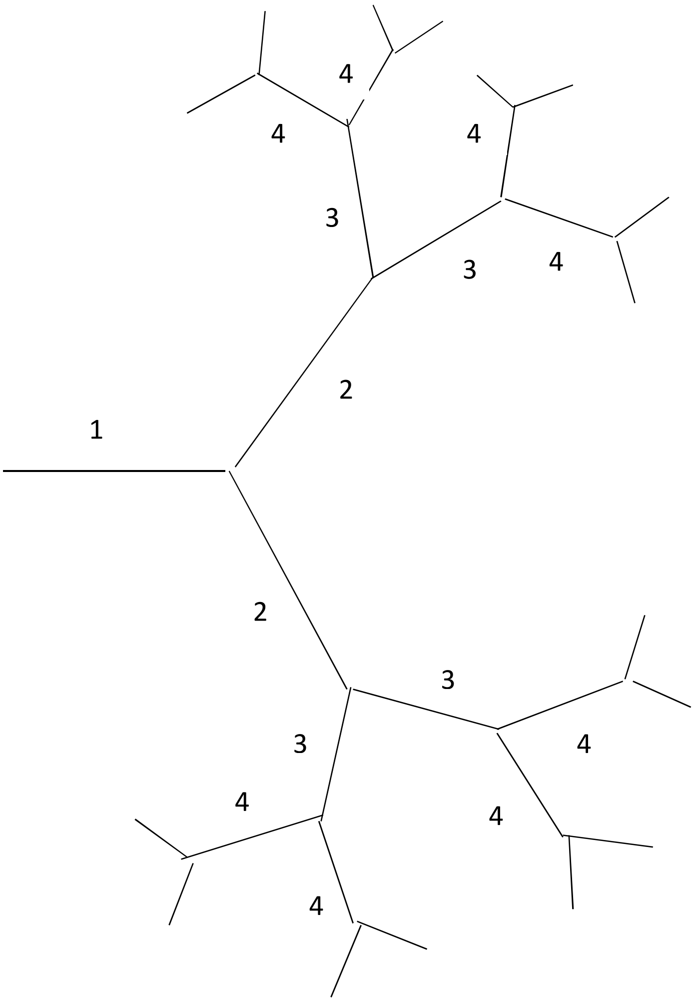

Fig. 1.

The total number of possible solutions to a problem can be represented by the branching diagram in Fig. 1.

Each line in the diagram represents at least one attempt at solving the equation. The length of the line represents the time involved in attempting a solution (obviously, the potential variation in time for each attempt cannot be represented in the diagram).

Each failed attempt leads to further attempts amounting to the total possibility space consumed in attempting a solution. The total possibility space potentially available is contained within the set *'x'* and the total number of possibilities available follows the rule:-

$$P_S = 2^N - 1$$

If this expression is to include the total time involved in obtaining a solution by writing :-

$T = 2^n - 1$

Here *'n'* is a figure which is less than *'N'* and constitutes the total time included in the set *'y'* which is the set of calculations constituting a solution to the set *'x'*.

At the first decision node there is a possibility space of 2. At the second node there is a possibility space of 4 and at the third node there is a possibility space of 8 and so on. For example, by junction 4, 30 possibilities will have been explored. Thus, if the solution will have been found say at the 16th node the time consumed will have been 30 times the sum of the length (distance) of all the branches up to and including number16.

Thus we can write:-

$$2_x^N - 1 \quad > \quad 2_y^n - 1$$

In the branching diagram, each branch represents a distance in time. The Euclidean distance between any two points on a time line depends on the length of the time segment connecting them (pq) and the distance between them is given by:-

$$\sqrt{(q-p)^2} = |\boldsymbol{q} - \boldsymbol{p}|$$

Thus in an n-dimensional space, the distance is :-

$$d(p-q) = \sqrt{(p_1 - q_2)^2 + (p_1 - q_2)^2 + \cdots + (p_i - q_i)^2 + (p_n - q_n)^2}$$

$$= \sqrt{\sum_{i-1}^{N} (p_i + q_i)^2}$$

The number $(p - q)$ is contained many times in the sum of all the time-lines of a branching time-line. Thus the time taken to solve any particular problem varies with the sum of the distances at which the solution in the branching diagram lies from the origin.

Thus creating a volume of possibility space, which we treat as a sphere P_S.

$$P_S = \frac{4}{3}\pi r^3$$

The 'sphere' of possibility space undergoes a fractional increase in volume γ per unit increase in distance *L*.

$$\gamma = \frac{dV}{V} \cdot \frac{1}{dL}$$

γ =The fractional increase of volume (i.e. dV/V) per unit rise of distance *L*.

A set 'x' can be defined as a set of un-resolved propositions requiring a solution. The un-resolved set 'x' contains all possible solutions and thus is itself a possible solution and can thus be said to be a set that contains itself. All possible solutions are contained in the branching diagram and can be expressed as a geometric progression of the form:-

$$\sum_{i}^{N} 1 + 2 + 4 + 8 \dots \dots . N$$

N is the total number of solutions within the set 'x'.

The branching diagram describes a volume of possibility space within which a solution can potentially be found, thus the set 'x' can be said to contain a Proper potential *'U'*.

The internal Resolution potential *'R'* of the system is given by:-

$$R_{U_{int}} = \sum R_{U_{int}} = R_{U_1} + R_{U_2} + R_{U_3} \dots \dots . N$$

Where the sub-scripts1,2,3…N coincide with the space contained within the branching diagram.

Finding a solution to the proposition set *'x'* requires the input of both time *'T'* and energy *'E'* to extract the algorithm which will serve as the solution to the proposition which is contained in *'x'*.

Time *T* we describe, for the sake of convenience, as kinetic time or the 'speed' in which a solution can be found. The term *'n'* we describe as the length or distance of each line of the branching diagram which is synonymous with the volume of possibility space occupied at that point and therefore the 'kinetic' element of the solution process can be described as:-

$$R_K = \sum ½\, n_i T_i^2 = ½ n_1 T^2 + ½ n_2 T^2 + ½ n_3 T^2 \ldots \ldots \ldots N$$

Thus the Proper potential of the set *'U'* is :-

$$P_u = R_{U_{int}} + R_K$$

The change in the Proper potential on a system (i.e. the set '*x*') is equal to the work *'W'* done on the system by external forces. Thus we can write :-

$\Delta P_U = W_{ext}$ and similarly we can write :-

$P = R_{U_{int}} + R_K + W_{ext}$ and $\Delta\left(R_K + R_{U_{int}}\right) = W_{ext}$

Until work is done on the set 'x', the set can be considered to be at rest relative to its external environment and therefore the set 'x' can be considered to have a total inertia, thus:-

$$\sum_i P_{U_i}(x) = 0$$

$$\therefore \frac{dP_U}{dt} = W_{ext}$$

If work is carried out on set 'x' by the external environment, then the algorithm which solves a particular proposition is "transferred" to set 'y'. obviously that algorithm contains all the values extracted from 'x' in terms of time, energy and potential and therefore we can write:-

$$y = W_{ext} + R_K$$

$\therefore R_{U_{int}} \neq W_{ext} + R_K$

$\therefore P \neq NP$

(N.B. note that $P \equiv x \; and \; NP \equiv y$)

Thus we can state that the possibility space is proportional time i.e. :-

$P_s \propto T$ Where P_s = possibility space and T = time.

In general the rate of change of the solution process *'Θ'* in respect of any proposition can be written as :-

$$\int_i^f d\Theta = \int_i^f \frac{dT}{T_{max}}$$

Where T is the elapsed time of the solution process to date and T_{max} is the total time available to the solution process in question and thus we can write :-

$$\Theta_f - \Theta_i = \int_i^f d\Theta$$

Where Θ_f and $\Theta_i$are the final and initial states respectively of the calculation process.

The multiplicative qualities of possibility space can be converted into the additive process of calculation by becoming a function of the natural logarithm of the possibility space as follows :-

$$\Theta_f - \Theta_i = \int_i^f \frac{dT}{T_{max}}$$

$$= const. ln \left(\frac{P_{S_f}}{P_{S_i}}\right)$$

That is to say, the possibility space increases over time.

For each failed attempt to find the algorithm which will solve the proposition, the process has to re-commence and thus a “half-life” is created which represents the time taken to abandon the attempt and re-commence or reverse the operation results in a change in the possibility space.

The change in the probability of when N attempts undergo a change in possibility space from P_{S_i} to P_{S_f} is :-

$$\Theta_f - \Theta_i = const. lnN\left(\frac{P_{S_f}}{P_{S_i}}\right)$$

Where Θ_f and Θ_i are the final and the initial states of the search and P_{S_i} and P_{S_f} are the initial and final volumes of possibility space respectively.

As attempts to continue to obtain a solution to any particular expression, we can say that the rate of reduction of possibility space at any given time is proportional to the number N of the total possibility spaces available at that time i.e. $P_S \propto N$ and we can write $-\frac{dN}{dT} \propto N$ and since $\frac{dN}{dT}$ is negative it follows that P_S decreases as T increases.

The introduction of a constant λ (which we will call the “space reduction” constant) gives:-

$$-\frac{dN}{dT} = \lambda N. \qquad \text{Equ. 2.}$$

Now if there are N_0 possibility spaces existing at $T = O$ and a lesser number N at a later time T then we can integrate Equ.1. to give :-

$$\int_{N_0}^{N} \frac{dN}{N_o} = -\lambda \int_{0}^{T} dT$$

$$\therefore (lnN)_{N_0}^{N} = -\lambda T$$

$$\therefore ln(N - N_0) = ln\left(\frac{N}{N_0}\right) = -\lambda T$$

$$\therefore N = N_0 e^{-\lambda}$$

The half-life of the process would be the inverse of the possibility space and is expressed as a geometric progression of the form :-

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \dots\dots\dots\dots.. = \sum_{r=0}^{N} (½)^r$$

In this case the first term is 1 or $(½)^0$ so that the first value that 'r' takes is zero.

In order to calculate the value of λ we relate its value to the number of attempts made to solve the expression at a time *T*.

Applying this to a model of possibility space we can say that the half-life of the calculation period is the time taken to reduce the number of original possibility spaces N_o to one half of their original value. Thus:-

$$N = N_o e^{-\lambda T}$$

$$\therefore \frac{N_o}{N} = e^{\lambda T}$$

When $N = \frac{N_o}{2}$ then $T = T_{\frac{1}{2}}$

$\therefore 2 = e^{T_{½}}$ and $ln2 = \lambda T_{½}$

$$\therefore T_{½} = \frac{0.693}{\lambda}$$

$$\therefore T_{total} = 2\left(\frac{0.693}{\lambda}\right)$$

Here we note that λ is a dimensionless number and finally we can conclude that $P \neq NP$.

END