# The Ontological Argument Part 2

Further notes on

St. Anselm’s Ontological argument

By

David J.M. Short

June 2022

# The Ontological Argument

PART 1

In 1970, the world became aware of Kurt Godel's proof of the existence of God (in fact the proof had been written but not published circa 1940). The proof bears a striking resemblance to St. Anselm's Ontological argument for the existence of God published circa 1077-1078.

Godel's proof takes the form of a proof in formal logic and is prefaced by a series of axioms and definitions which are to be applied when one is coming to an understanding of the proof. The intention here is to attempt to employ some of Godel's methodology directly to St. Anselm's argument and thereby hopefully adding to the understanding of the saint's outstanding contribution to philosophy.

St. Anselm's argument in Chapter 2 of The Proslogion (1077-78) can be summarised as follows;

Para 1. It is a conceptual truth that God is a being than which none greater can be imagined.

Para 2. God exists as an idea in the mind.

Para 3. A being that exists as an idea in the mind and in reality is greater than a being that exists only as an idea in the mind.

Para 4. Thus, if God exists only as an idea in the mind, then we can imagine something that is greater than God (that is, a greatest possible being that does exist).

Para 5. But we cannot imagine something that is greater than God (for it is a contradiction to suppose that we can imagine a being greater than the greatest possible being that can be imagined).

Para 6. Therefore, God exists.

In Chapter 3, Anselm presents a further argument in the same vein:

Para 1(a) By definition, God is a being that which none greater can be imagined.

Para 2(b) A being that necessarily exists in reality is greater than a being that does not necessarily exist.

Para 3(c) Thus, by definition , If God exists as an idea in the mind but does not necessarily exist in reality then we can imagine something that is greater than God.

Para 4(d) But we cannot imagine something that is greater than God.

Para 5(e) Thus, if God exists in the mind as an idea, then God necessarily exists in reality.

Para 6(f) God exists in the mind as an idea.

Para 7(g) Therefore, God necessarily exists in reality.

(Wikipedia 2022)

To summarise: A being that lacks real existence is not a being than which none greater can be conceived. A yet greater being would be one with a further attribute of existence. Thus the unsurpassably perfect being must exist; otherwise it would not be unsurpassably perfect.

(Encyclopaedia Britannica 2022)

This summary neatly encapsulates in a very few words the real essence of Anselm's argument and it is an argument with which it is very difficult to contest. Nevertheless, it is a fact that very many people do hold atheistic viewpoints so the purpose of this paper is to attempt to prove the existence of God using a similar format to that of St. Anselm's own argument. Here we describe a method by which the Ontological Argument can be expressed in formal logic and be given a meta-mathematical interpretation which can again in turn be further expressed in an arithmetical format of powers of prime numbers.

## The Axioms

### Axiom 1

If phi is a positive property and if for every x, x has the property psi, then psi is a positive property.

### Axiom 2

Not-phi is a positive property only if phi is not a positive property.

### Axiom 3

The property of being God-like is a positive property.

Axiom 4

If the property phi is a positive property, then it is provable that phi is a positive property.

Axiom 5

Provable existence is a positive property.

Axiom 6

Object z has the God-like property if and only if for every property phi, if phi is a positive property then z has the property phi.

(Axioms 1-6 see Kurt Godel (circa 1940))

NB. For the sake of clarity, the word ‘necessary’ has been interchanged to the word ‘provable’.

## The Definitions

Definition 1

God is a being than which none greater can be conceived. (The unsurpassably perfect being must exist otherwise it would not be unsurpassable). (see Para.1).

Encyclopaedia Britannica (2022)

Definition 2

A positive property is one of total perfection in whatever quality is in question. God possesses all positive properties and all positive properties ‘G’ are Godlike. (see Para.3)

## The Theorems

Theorem 1

$\exists x \exists y(\sim z = y \& \forall z(z = x \vee z = y))$

i.e. the first clause $\sim z = y$ asserts that z and y are two different things, z is an unsurpassable positive property and y is a positive property, while the second clause asserts that there is nothing other than z or y.

Theorem 2

$P(y) \cdot [(\exists z)P(z) \cdot Gr(z, y)]$ (see Para.1)

NB. P = an unsurpassable positive property. Gr = greater than

i.e. P is an unsurpassable positive property and there is at least one z such that z is an unsurpassable positive property and z is greater than y.

Theorem 3

$P(z) \cdot (y)[(P(y) \cdot \sim(z = y)Gr(z, y)]$ (see Para. 4)

i.e. z is an unsurpassably positive property and for every y, if y is not an unsurpassably positive property and therefore not the equivalent of z then z is greater than y.

Theorem 4

$(y)[P(y) \supset (\exists z)(P(z) \cdot Gr(z, y))]$ (see Para.5)

i.e. for every y if y is a positive property, there is at least one z such that z is a positive property and z is greater than y.

Theorem 5 (seePara.6)

$\Box\exists z G(z)$

In every respect z is greater than y therefore God exists.

Theorem 6

Paragraph 5 of the Ontological Argument states that "it is a contradiction to suppose that we can imagine a being greater than the greatest possible being that can be imagined". This being the case, the existence of the being that is the greatest possible that can be imagined can be proven by employing the Proof by Contradiction expressed in formal logic.

The expression $\forall P \vdash (P \vee \neg P)$ means that for all propositions P, either P or not-P is true. (Here P is an unsurpassable positive property)

An existence proof by contradiction assumes that some object doesn't exist, and then proves that this leads to a contradiction: thus such an object must exist.

The proposition to be proved, P is assumed to be false i.e. $\neg P$ is true:-

$P \equiv P \vee \bot \equiv \neg(\neg P) \vee \bot \equiv \neg P \to \bot$

If $\perp$ is reached from $\neg P$ via a valid logic, then $\neg P \rightarrow \perp$ is proved as true so P is proved as true.

$\Box \exists P G(P)$

Therefore God exists:

## Part 3

One of Godel's most notable pieces of work was his Incompleteness theorem. This is not an appropriate platform for discussion of that theorem but fortunately we can make use of Godel's methodology to further expand on our conclusions. Godel's breakthrough was his ability to give an arithmetical form to statements which have been made in meta-mathematics. This was achieved by allocating a unique number to each of the meta-mathematical symbols in any particular statement and then raising a series of prime numbers to the power allocated to the symbol (see appendix 1). For example the meta-mathematical statement $\Box \exists z G(z)$ can be arithmetised in Godelian terms as follows:

| | | | | | | |
|---|---|---|---|---|---|---|
| $\Box$ | $\exists$ | $z$ | $G$ | $($ | $z$ | $)$ |
| 13 | 11 | 8 | 10 | 1 | 8 | 2 |

$$2^{13} \; x \; 3^{11} \; x \; 5^{8} \; x \; 7^{10} \; x \; 11^{1} \; x \; 17^{8} \; x \; 19^{2} =$$

$$7.4613x10^{57}$$

Which expressed in prime factors is:

$2^{142}$, 5, 11, 677, 119687, 300301

Obviously, any meta-mathematical statement can generate a number unique to itself but of course some of the numbers generated are truly huge and are beyond the capacity of any domestic computer to handle.

If every conceivable positive property z is allocated a Godelian integer, then that set of integers must contain the total possible number of integers contained in the set of conceivable unsurpassably perfect properties. Any combination of integers representing y must be less than or equal to z. Therefore any combination of y integers which are used to prove z must be contained within the set z. But the set z being perfect in itself must have a proof which is itself perfect but that is not possible because the set z is unsurpassable in its perfection and therefore the proof must be the entire set of integers contained within z. Therefore the existence of the set z is proof of the properties contained in the set z.

If it is the case that $\exists!\, y \in z$ i.e. y is an essential element of the set z then the property y does not have the property of z because z contains all possible unsurpassably perfect properties and y does not. Therefore if y is contained in z then y is a property of z. Therefore $\exists!\,(z)$ i.e there exists exactly only one z therefore y is contained in z, therefore the set z says of itself that it is a proof or put another way:

$\exists y \forall z(y = z)$ i.e. there exists a set z to which all objects within the set are identical and that $\forall y \forall z(y = z)$ i.e. for any objects y and z, y is identical to z and so the universe contains at most just one set which is a state of perfect positivity and which is Godlike.

Therefore z contains all Godlike properties from which we infer that z is God and that God exists;

$\Box \exists z G(z)$

END

# Appendix 1

Symbols in Logic with accompanying Godel powers

x = 7

G = 10 = Godlike property

( = 1 = parentheses

) = 2 = parentheses

P = 9

E = 15

∀ = 12 = for all

¬ = 19 = not

∃ = 11 = there exists

≠ = 21 = not equal to

⌐ = 22

□ = 13 = it is necessary that or it is provable that

◊ = 18 = it is possible

↔ = 5 = if and only if

→ = 6 = implies

∧ = 16 = and

∨ = 20 = or

ess = 17 = essential property of

z = 8

∃!= 14

Ψ = 4

Φ = 3

⊥ = 23