ON THE GRAVITATIONAL EFFECTS OF AN ELECTROMAGNTIC WAVE

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Consider a spherical electromagnetic wave of area $4\pi r^2$ originating at a point *P*.

The expanding wavefront produces stress and strain in the x, y and z planes and also in the T frame. These stresses and strains can be described in the form of a 4x4 matrix as shown in Appendix A below.

The stresses and strains in time and space as manifested within an electromagnetic wave can be described in a similar manner to the Einstein equations of general relativity. In the Einstein equations $R_{\mu\vartheta}$ is the Ricci tensor, $g_{\mu\vartheta}$ is the metric tensor and R is the curvature scalar. The λ term is the repulsive cosmological constant and *T* is the time component.

$$G_{\mu\vartheta} + \lambda g_{\mu\vartheta} = \frac{8\pi G}{c^4} T_{\mu\vartheta}$$

The Einstein tensor, $G_{\mu\vartheta}$ contains the Ricci tensor which describes the curvature and warping of spacetime and $g_{\mu\vartheta}$ describes the shape of the tensor in spacetime. $T_{\mu\vartheta}$ is the stress energy tensor and describes the mass, energy and time components of the tensor so that the whole expression can be expanded to read :-

$$R_{\mu\vartheta} - \frac{1}{2} g_{\mu\vartheta} R + \lambda g_{\mu\vartheta} = \frac{8\pi G}{c^4} T_{\mu\vartheta} \qquad Equ. 1.$$

To summarise; the LHS describes curved spacetime and the RHS describes the mass energy component of spacetime.

It is appropriate at this point to detail the derivation of the expression $\frac{1}{2}g_{\mu\vartheta}$. The Divergence theorem states that :-

$\int F\ dA = \int \nabla F\ dV$ (A=Area, V=Volume)

$$\therefore -4\pi G \int \rho\ dV = \int \nabla F\ dV \qquad \therefore \nabla F = -4\pi G\rho$$

Since $F = -\nabla\Phi$

$$\therefore \nabla F = -4\pi G\rho \qquad \therefore \nabla^2\Phi = 4\pi G\rho$$

The time component of the metric tensor $g_{000} = 2\Phi + const.$

$$\therefore \Phi = \frac{1}{2}g_{000} \qquad \therefore \nabla^2 \frac{1}{2}g_{000} = 4\pi G\rho$$

and multiplying both sides by 2 we get :-

$$\nabla^2 g_{000} = 8\pi G\rho.$$

In the case of the electromagnetic wave, the wave itself exerts a force over the quantum field.

In the spherical electromagnetic wave, if electromagnetic radiation of frequency f_1 is emitted at a point where its potential is low and which is symbolised by Φ_1 then frequency f_2 when measured where potential is high and is symbolised by Φ_2 is given by $f_2 = f_1\left(1 + \frac{\Delta\Phi}{c^2}\right)$ where $\Delta\Phi = \Phi_2 - \Phi_1$ and solving for $\Delta\Phi$ we can write :-

$\Delta\Phi = -c^2\left(1 - \frac{f_2}{f_1}\right)$.

Since $\Delta\Phi = \Phi_2 - \Phi_1$ represents a change in potential energy caused by the change in frequency of an electromagnetic wave, so we can describe this change as the work done by the change in frequency of the potential of the system as :- $W = \int_{\Phi_2}^{\Phi_1} F_x(x)\ dx$ thus $\Phi_2 - \Phi_1 = -\int_{\Phi_1}^{\Phi_2}\ F_x(x)\,dx$.

That is to say that the change in potential is the manifestation of a force $F_x(x)$ caused by the change in frequency of an electromagnetic wave and that $\Delta\Phi$ represents the change in frequency due to the distance *r* between areas of high and low potential within the spherical wave.

In a similar manner to an electromagnetic wave, if radiation of frequency f_1 is emitted at a point where gravitational potential is low and which is symbolised by Φ_1 then frequency f_2 will be observed when measured where gravitational potential is high and is symbolised by Φ_2. The expression here is

$f_2 = f_1\left(1+\frac{gr}{c^2}\right)$. Substituting $\Delta\Phi$ for *gr*, the same expression applies as for an electromagnetic wave and therefore we can write $\Delta\Phi = -\,c^2\left(1-\frac{f_2}{f_1}\right)$.

Again, this expression describes the manifestation of a force $F_x(x)$ caused by the change in frequency of an electromagnetic wave. The expression for this force is $F = K\,\frac{\mathrm{f_1 f_2}}{(\Delta\Phi)^2}$ where K is some constant depending on the physical environment of choice.

For reasons to become apparent later, it is worthwhile obtaining a further expression for force. In the case of gravitation, we have shown that the force $F_x(x)$ can be expressed as follows:-

$F = -\nabla\Phi \quad \therefore \quad \nabla^2\Phi = 4\pi G\rho$ or in the case of an electromagnetic wave $F_x(x)$ can be expressed as :- $\nabla^2\Phi = \frac{1}{4\pi\varepsilon_0}4\pi\rho \quad \therefore \quad \nabla^2\Phi = \frac{\rho}{e_0}$.

Again, in the case of the metric tensor $g_{000} = 2\Phi + const \quad \therefore \quad \Phi = \frac{1}{2}g_{0000}$

$\therefore\ \nabla^2\frac{1}{2}g_{000} = \frac{\rho}{\varepsilon_0}$ And multiplying both sides by 2 we get $\nabla^2 g_{000} = \frac{2\rho}{\varepsilon_0}$.

The force over the whole area of the sphere is :-

$$\int F\cdot dA = \int -\frac{1}{4\pi\epsilon_0}\frac{\rho}{r^2} = -\frac{1}{4\pi\epsilon_0}\frac{\rho}{r^2}\cdot 4\pi r^2 = -\frac{1}{4\pi\epsilon_0}\rho 4\pi = -\frac{\rho}{\varepsilon_0}$$

where ρ is the wave density or wave intensity and wavelength increases as

$\lambda \propto r^2$

The same result can be obtained by describing the spherical electromagnetic wave in relativistic terms. The Divergence theorem states that:-

$$\int F.dA = \int \nabla F.dV \;\; \therefore -4\pi \frac{1}{4\pi\varepsilon_0}\int \rho.dV = \int \nabla F.dV \therefore \nabla F = -\frac{\rho}{\varepsilon_0}$$

Since $F = -\nabla\emptyset$ where $\emptyset$ = potential and V is the volume of the wave

$$\therefore \nabla - \nabla\emptyset = -\frac{\rho}{\varepsilon_0}$$

The time component of the metric tensor can be written as –

$$g_{000} = 2\emptyset + const. \therefore \emptyset = \frac{1}{2} g_{000} \therefore \nabla^2 = \frac{1}{2} g_{000} = 4\pi \frac{1}{4\pi\varepsilon_0}\rho = \frac{\rho}{\varepsilon_0}$$

Multiplying both sides by 2 we get :-

$$\nabla^2 g_{000} = 8\pi \frac{1}{4\pi\varepsilon_0}\rho = \frac{2\rho}{\varepsilon_0}$$

Using the covariant derivative of the Ricci tensor (see Appendix B) we can write :-

$\nabla R_{\mu\vartheta} = \frac{1}{2}\nabla g_{\mu\vartheta} R$ and noting that $\nabla T_{\mu\vartheta} = 0$

and since $\nabla\left(R_{\mu\vartheta} - \frac{1}{2} g_{000} R\right) = 0$ we can write :-

$$R_{\mu\theta} - \frac{1}{2} g_{\mu\vartheta}\, R \ + \lambda g_{000} = \frac{2}{\varepsilon_0}\, T_{\mu\vartheta} \qquad \text{Equ. 2.}$$

Which describes the curvature and mass/energy components of spacetime within an electromagnetic wave.

Any segment of the wave can be described if we again consider a spherical electro-magnetic wave originating at a point *P* and having a surface area of $4\pi r^2$. Now consider a cuboid segment of that sphere bounded by field lines originating at point *P* and having six sides bounded by *A, B, C, D, E, F, G and H* as shown in Appendix B below.

And now consider a further, smaller cuboid bounded by the same field lines and six sides bounded by *a ,b, c, d, e, f, g* and *h* also as shown in Appendix B.

It is known that the energy carried by a spherical wavefront is spread out over spherical area $A = 4\pi r^2$, so for waves in three dimensions, energy per unit area decreases as ${}^{1}/_{r^2}$ and similarly wave intensity decreases $I \propto {}^{1}/_{r^2}$. On the other hand, wavelength increases as $\lambda \propto r^2$.

The sphere or any segment of the sphere is therefore a curved surface and so the cuboid as previously described is comprised of curved surfaces in all directions.

A cuboid section of an electro-magnetic field experiences stresses over *x, y* and *z* faces and also in the *T* dimension and therefore it is a multi-linear function and can be described by the same rank 4 tensor as previously described.

The cuboids are skewed forms in space in which no angle is equal to 90^0 and therefore the space contained in a cuboid section of an electro-magnetic wave exerts a stress over all its surfaces in the *x, y* and *z* directions. In any skewed form the stress at each face is described as the sum of all three vectors at each of the three faces of the cuboid. Thus the intensity of the wave at any point within the wave describes the geometry of spacetime at that point.

The λ term takes on an added significance when applied to an electromagnetic wave. In the case of gravitation and General Relativity, the λ factor is primarily effective at long distances from the point of origin. However, in the case of an electro magnetic wave the λ factor becomes effective at short distances within the electromagnetic wave and it can be shown that the λ factor is of equal value in the expressions for both General relativity an electromagnetism.

In the case of the expanding electromagnetic wave, both the frequency and the wavelength experience the well documented redshift effects at distances from the point of origin. Similar redshift effects are experienced by electromagnetic phenomena in the presence of a gravitational field and it follows that both redshift effects can be described in terms common to both phenomena.

Since the photon is a massless particle, we can define the dimensionless red shift parameter *z* of a photon as being $z = \frac{gr}{c^2}$ where *r* is the distance between two points *a* an *b* spatially separated within the sphere radially and solving for *g* we note that $g = \frac{c^2 z}{r}$ and that *z* is equivalent to the potential difference Δϕ. Since the gravitational force within the sphere is distributed uniformly over the surface of the sphere, the point of origin of the wave is lying in a symmetrical force field and consequently there is no tidal (i.e. directional) effect. Since the acceleration due to gravity is due entirely to the area $4\pi r^2 \rho$ thus

$$g = \lambda = 4\pi G\rho.$$

The expression already referred to relating to the change in frequency of an electromagnetic wave which is under the influence of a gravitational wave is $z = \frac{gr}{c^2}$. Now while *z* itself is generally presented as a dimensionless figure both *r* and *g* can be calculated and as the change in frequency is proportional to the change in gravitational potential of the field then a numerical value can be placed on the potential difference *z* and to achieve this we can resort to the familiar Planck units.

It will be recalled that Planck units (also known as natural units) represent the point at which relativistic space-time dimensions and quantum conditions share an interface and therefore can react with one another. The Planck units to be used are as follows :-

Planck time $t_p = \sqrt{\frac{G\hbar}{c^5}} = 5.4 \times 10^{-44}\ secs.$

Planck length $l_p = \sqrt{\frac{G\hbar}{c^3}} = 1.6 \times 10^{-35} mtrs.$

Planck mass $m_p = \sqrt{\frac{\hbar c}{G}} = 2.1 \times 10^{-5} gram.$

Planck energy $E_p = \sqrt{\frac{\hbar c^5}{G}} = 2.0 \times 10^{9} joul.$

And here we note that the constants $\hbar$, c and G are all normalised to 1.

With regard to the gravitational conditions which obtain inside a spherical electromagnetic wave and noting that the said gravitational field is repulsive, that is to say that the photons contained therein are moving apart from one another we can examine the expression for *z* as applied to the wave function and quantum state within the sphere. Taking the expression for *z* we can calculate the value for *g* (the acceleration due to gravity) within the sphere by writing $g = \frac{c-0}{t_p}$ i.e. the particles (photons) have been accelerated from rest to *c* and since the value for *c* is constant therefore :-

$$g = \frac{3\times10^{8}}{5.4\times10^{-44}} = 5.6 \times 10^{51}\ m.s^{-2}.$$

Subsequent to this a value can be placed on *z* and this is especially pertinent because *z* is an expression for the change in frequency of an electromagnetic wave under the influence of a gravitational field and similarly it is an expression for the change in potential in a gravitational field. In expressing a value for *z* at the quantum level we can again write $z = \frac{gr}{c^2}$ and knowing the value of g an substituting l_p for *r* we can write:-

$$z = \frac{(5.6\times10^{51}) \times (1.6\times10^{-35})}{9\times10^{16}} = \frac{9\times10^{16}}{9\times10^{16}} = 1$$

We have previously noted that the values of λ and g are related on the macroscopic scale via $g = \lambda = 4\pi G\rho$ so if both *G* an *z* are normalized to 1 we can write $\lambda = \frac{c^2}{r} = \frac{c^2}{l_p} = \frac{9\times10^{16}}{1.6\times10^{-3}} = 5.6\times10^{51} \therefore \lambda = g.$

The force required to initiate the expansion of the spherical wave at the quantum level can be calculated in the conventional way i.e:-

$F = m_p g = (2.1\times10^{-8})\times(5.6\times10^{51}) = 1.18\times10^{44}\,N.$ and the energy required is given by:-

$W = E = Fl_p = (1.2\times10^{44}) \times (1.6\times10^{-35}) \approx 2.0\times10^{9}\,joul.$ Which is equivalent to the Planck energy already described.

Therefore, both electromagnetism and General Relativity are included in the λ term.

$$\begin{vmatrix} \tau_{ttz} & \tau_{txz} & \tau_{tyz} & \tau_{tzz} \\ \tau_{xtz} & \tau_{xxz} & \tau_{xyz} & \tau_{xzz} \\ \tau_{ytz} & \tau_{yxz} & \tau_{yyz} & \tau_{yzz} \\ \tau_{ztz} & \tau_{zxz} & \tau_{zyz} & \delta_{zzz} \end{vmatrix}$$

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$$\begin{vmatrix} \tau_{tty} & \tau_{txy} & \tau_{tyy} & \tau_{tzy} \\ \tau_{xty} & \tau_{xxy} & \tau_{xyy} & \tau_{xzy} \\ \tau_{yty} & \tau_{yxy} & \tau_{yyy} & \tau_{yzy} \\ \tau_{zty} & \tau_{zxy} & \tau_{zyy} & \tau_{zzy} \end{vmatrix}$$

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$$\begin{vmatrix} \tau_{ttx} & \tau_{txx} & \tau_{tyx} & \tau_{tzx} \\ \tau_{xtx} & \tau_{xxx} & \tau_{xyx} & \tau_{xzx} \\ \tau_{ytx} & \tau_{yxx} & \tau_{yyx} & \tau_{yzx} \\ \tau_{ztx} & \tau_{zxx} & \tau_{zyx} & \tau_{zzx} \end{vmatrix}$$

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<u>Appendix A</u>

$$\begin{vmatrix} \delta_{ttt} & \tau_{txt} & \tau_{tyt} & \tau_{tzt} \\ \tau_{xtt} & \tau_{xxt} & \tau_{xyt} & \tau_{xzt} \\ \tau_{ytt} & \tau_{yxt} & \tau_{yyt} & \tau_{yzt} \\ \tau_{ztt} & \tau_{zxt} & \tau_{zyt} & \tau_{zzt} \end{vmatrix}$$

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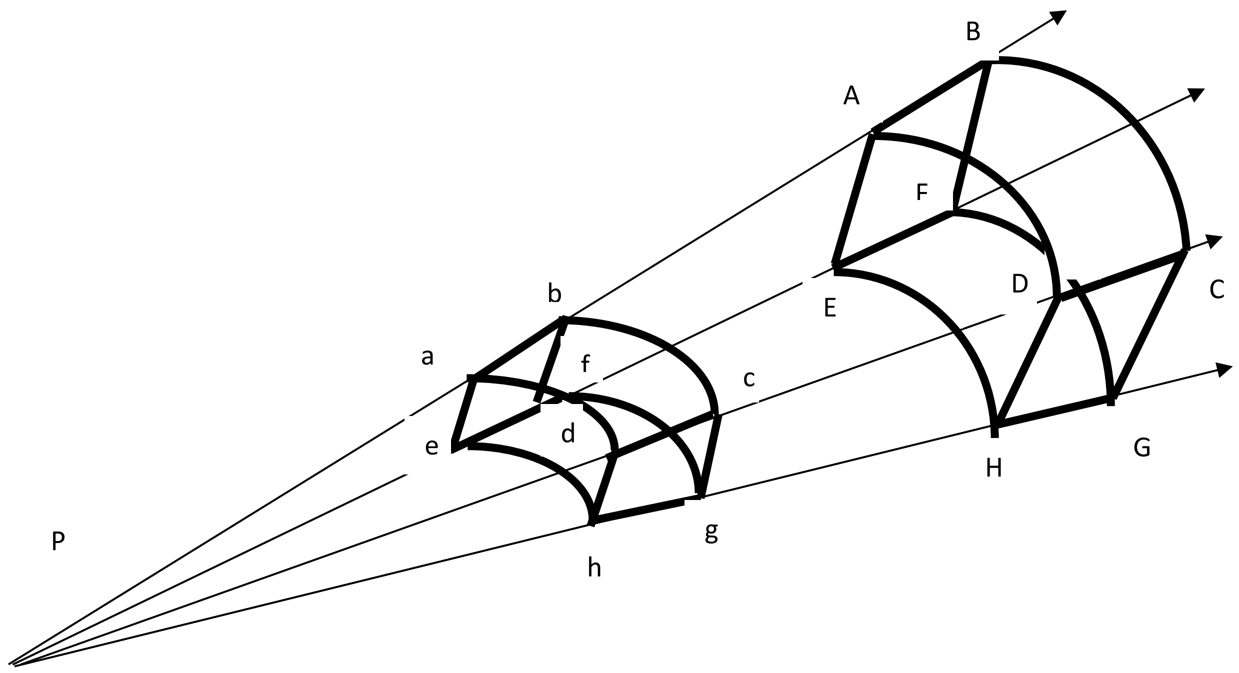
The matrix is shown in its expanded form in ‘slices’ umbered 1,2,3,4 from front to back.

Appendix B



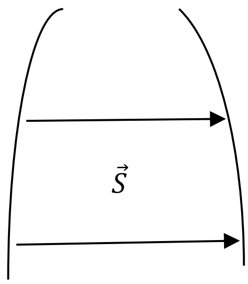
This appendix and the accompanying diagrams are designed to illustrate how it is that the expression $R_{\mu\vartheta}$ in General Relativity describes the curvature of spacetime and shows that the element of gravitation is not required in the description of spacetime.

Essentially the Ricci tensor is a summary of the Riemann tensor. The Ricci tensor describes volume changes along the geodesic lines causing objects within the geodesics either to increase or decrease in size. The Ricci scalar describes how the size of an object deviates from its standard flat space size. Both these phenomena are illustrated in the graphics below and thus we can conclude that the size an volume of an object are therefore dependent on the position of the observer relative to the position of the object within a spherical electromagnetic wave.

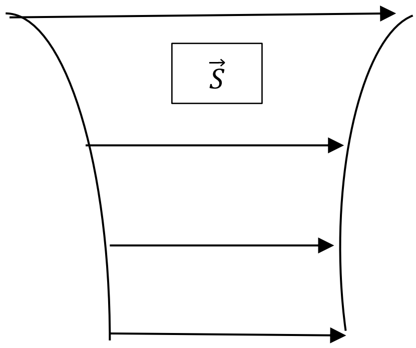
The second derivative of the Riemann tensor

describes the geodesic deviation in this case

the Riemann term is $\left[R\left(\vec{S}, \vec{V}\right)\vec{V}\right] \cdot \vec{S} > 0$



and the second derivative of the separation vector $\vec{S}$ is $\left[\nabla \vec{v}\ \nabla \vec{v}\, \vec{S}\right] \cdot \vec{S} < 0$ and is used because it describes the accelerating convergence of the geodesics due to the curvature of spacetime. Vectors on a curved surface spread apart at an accelerating rate as shown below.



Here, the Riemann tensor is summarised by the Ricci tensor and scalar which describes the volume change along geodesic. Again, the second derivative of the Riemann tensor describes the geodesic deviation but in this case the

Riemann term is $\left[R\left(\vec{S},\vec{V}\right)\vec{V}\right]\cdot\vec{S} < 0$ and the second derivative of the separation vector $\vec{S}$ is $\left[\nabla\,\vec{v}\,\nabla\,\vec{v}\,\vec{S}\right]\cdot > 0$

$[R(\vec{s}\,,\vec{v})\vec{V}]\cdot\vec{S}\ < 0$

Which is the same as saying in Ricci terms;-

$[\nabla\vec{v}\;\nabla\vec{v}\;\vec{S}\;]\cdot > 0$

(The second derivative is used in place of the first derivative because in flat space the first derivative $\nabla\vec{v}\vec{S}\ \neq 0$ and the geodesics accelerate at a constant velocity, whereas in curved space the geodesics accelerate.)

In conclusion, it appears that there is a principle of equivalence manifested here whereby the forces at work within the spherical electromagnetic wave are entirely equivalent to the usual gravitational effects which are products of the curvature of spacetime.

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