

WHY DO FIRMS HEDGE? AN ASYMMETRIC INFORMATION MODEL

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ABSTRACT

We present an asymmetric information model of hedging that has the intuition that hedging is undertaken by higher ability managers who wish to “lock-in” the higher profits that result from their higher ability. Thus, hedging is an attempt to improve the informativeness of the learning process by the higher ability manager. We analyze two models. We first analyze a model where managers care only about their reputations. In this case, we show that an intuitive equilibrium that involves hedging by higher ability managers always exists. Lower ability managers also hedge when differences in abilities are low but do not hedge when differences in abilities are high. We consider a second model where managers hold equity in the firm in addition to caring for their managerial reputations. The presence of FDIC insurance or pre-existing debt makes hedging costly to equityholders as it is a variance reducing activity. However, this cost of hedging is lower for higher ability managers. This leads to both kinds of managers not hedging when the difference in ability is low. At higher differences in ability, the intuitive equilibrium in which the higher ability manager hedges exists. In this equilibrium, greater separation occurs relative to the case where managers were only concerned about their reputations.

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In recent years, volatility in interest rates and currencies has led corporate financial managers to consider and undertake hedging on a scale that is unprecedented. This awareness of hedging as a risk management tool is driven by a number of factors. First, there is the pervasive belief that disasters like the savings and loan disaster of 1980s could have been avoided if the institutions were properly hedged against interest rate risk (this view is expressed in the Wall Street Journal on August 17 1993). Similarly, numerous anecdotal examples of firms having large losses or going bankrupt due to their failure to hedge exchange rate movements exist. One famous example is the Laker Airways example where Laker's costs of borrowing (on airplanes) were in dollars while its revenues were split evenly between dollars and pounds (see Shapiro's book, *International Financial Management*, Third Edition, pages 275-76 for more details on this example). Second and equally important, there has been an enormous increase in the liquidity of markets (whether traded or over the counter) that provide instruments for hedging corporate risk.

Given this attention to hedging at a corporate level and the proliferation of complex hedging techniques (largely driven by academic research on option and futures pricing models), the paucity of academic research on the fundamental question of why and when firms should hedge is surprising. In fact, the traditional full information perfect capital markets model of the firm says very little about why firms hedge and implies that whether firms hedge or not is irrelevant as investors can undertake the necessary hedging activities by themselves. This view is expressed in a recent article by Culp and Miller (1995) where they state that "most value maximizing firms do not, in fact, hedge".

While the full information perfect capital markets paradigm has little to say about why firms hedge, other important paradigms like the option pricing paradigm

imply that firms will probably undertake risky activities if they are run by equityholders as the option value of equity is increased by such variance increasing activities. Hence, the main ideas of corporate finance have little to say on why firms hedge and seem to indicate that (if anything) there are strong incentives against hedging.

Our paper presents an explanation as to why firms hedge based on a theory of managerial responses to asymmetric information. The key intuition is that managers who have superior abilities with respect to some risks or uncertainties will try to ensure that their superior abilities are quickly discovered by the market. To ensure this, they will try to hedge those risks that are not under their control and where they have no exceptional abilities. Thus hedging reduces the noise in the learning process by “locking-in” the superior ability. In contrast, managers with inferior abilities have incentives to reduce the efficiency of the learning process. They would prefer that all managers undertake risky variance increasing activities. However, given that superior managers undertake hedging activities, lower ability managers may or may not hedge. As we will show, their decision to hedge or not to hedge depends on how much lower their ability is relative to managers of superior ability.

The above intuition assumes that managers are mainly concerned about their managerial reputations. A strong argument can be made that managerial compensation is related to equity performance. Hedging activities are inherently risk reducing and reduce the value of the equity when there is preexisting debt or when there is insurance like FDIC insurance to banks or government loan guarantees. If managers hold some fraction of equity and are also concerned about equity values, higher ability managers will not hedge unless the ability difference is substantial. When they do hedge, the equity option value foregone acts like a signaling cost in a traditional signaling model. The higher ability manager knows that his performance is going to be better. As a consequence, the probability of bankruptcy is much less, making the

equity option less valuable at the margin. The lower ability manager has a greater probability of bankruptcy and hence has a more valuable equity option at the margin. Consequently, the equity option increases the incentives of the higher ability manager to hedge.

Our model thus provides a rationale for why firms hedge and when firms will undertake hedging activities. In particular, our model indicates that if costs of hedging like the reduction in the equity option value are present, the firm managers will undertake hedging activities only when they believe that they have superior abilities and when these superior abilities result in performance substantially higher than that of other managers in the industry of lesser ability so as to compensate for these costs of hedging. In addition, our model indicates a relationship between managerial compensation and hedging policy. The greater the fraction of equity that the manager owns, the higher is the implicit cost of hedging and hence hedging occurs only when the manager has a substantial performance differential over managers in other related firms.

These results are consistent with a recent empirical study by Tufano (1995) on the gold mining industry. Tufano finds that younger managers are more likely to hedge than older managers. Since there is likely to be greater uncertainty about the ability of younger managers, this is consistent with the implication of our model that greater dispersion of abilities leads to more separation, i.e., higher ability young managers hedge while lower ability young managers do not hedge. With a cost of hedging, the model delivers the conclusion that a high difference in abilities leads to a “separating” equilibrium while a low difference in abilities leads to a “pooling” equilibrium (where managers of higher and lower abilities do not hedge). Again, this is consistent with Tufano’s results.

A second testable implication of our model is as follows. When the costs of hedging are low, less separation occurs. In contrast, when the costs of hedging are high, more separation occurs. Therefore, the value of firms that hedge relative to firms that do not hedge is much higher when the cost of hedging is higher. This is an implication of the model that is potentially testable. This implication is consistent with Tufano's result that managers with stock options do not hedge as such managers face high costs of hedging. However, a test of our model would also require that managers with stock options who hedge must be of substantially higher ability than managers without stock options who hedge (with the usual *ceteris paribus* assumption).¹

While hedging is not observable, subsequent to the event, one sometimes knows whether a firm hedges or not. This is because ex-post firms voluntarily disclose hedging activity in the notes to the balance sheet. If our story of hedging is correct, the profits (earnings) of firms in the quarter or year which they hedge should be higher and the volatility of cash flows should be lower than usual. Such a test differs from tests of signaling models where profits (earnings) are higher for firms that signal as no volatility implications are present. Because hedging reduces volatility, the volatility reduction is an additional implication. Some preliminary empirical work by DeGeorge, Moselle and Zeckhauser (1995) using Compustat data is supportive of this implication of the hedging hypothesis.

Our explanation differs from the other previous explanations that have been provided. In an early paper on this topic, Smith and Stulz (1985) propose risk aversion and taxes as rationales for firm hedging. Campbell and Kracaw (1987) use the ideas in Holmstrom (1979) to propose hedging as a method for ameliorating agency problems.

¹ Tufano (1995) also finds that the greater the equity holdings of the manager, the more likely one will observe hedging. It is difficult to use this observation directly as what matters in our model is the cost of hedging and not the fraction of equity held. A proxy for the cost of hedging must simultaneously account for the fraction of equity held and the value of the equity option or FDIC insurance that hedging destroys.

Froot, Stein and Scharfstein (1992) use that fact that outside capital is costly to raise as a rationale for hedging. In a paper that is closer to our ideas, DeMarzo and Duffie (1993) present a model with risk aversion and symmetric information where hedging is optimal. None of these papers presents what to us seems to be the fundamental rationale for hedging: the idea of “locking-in” performance. We believe that this is an important reason why hedging occurs and thus emphasize it in our model. None of the other papers in the literature consider what the costs of hedging are and how these costs for hedging are balanced against the incentives to hedge. The closest paper in the literature is that due to Ljungqvist (1994) who shows in a model with asymmetric information that there is an incentive for firms with low output to speculate. This is somewhat similar to some of our results on the model without hedging costs.²

Our emphasis on hedging as a decision by managers to “lock-in” performance where they have an advantage and eliminate risks that are not under their control is important. Hedging is not elimination of all risk but the management of risk.³ Alternative explanations like bankruptcy costs or the higher external costs of funding would imply corner solutions where managers try to eliminate all risks; this distinction between risks where managers have an advantage and risks which are not controllable by the manager is less important. We do not believe that hedging involves the elimination of all risk but rather only risks where the manager or firm does not have any special advantage. In particular, our model implies that firms hedge their risks only when they are sufficiently different from other similar firms in their abilities with respect to that risk. Only under these circumstances is it worthwhile to pay the im-

² In Ljungqvist’s setup, absent speculation, there is perfect discrimination between the good and bad types of managers. In our setup, this is not true as output is not perfectly revealing about the ability of the manager.

³ It is interesting to note that Culp and Miller (1995) state that “absent superior information, value-maximizing firms may not only avoid hedging, but may well shun the underlying activity itself.”

PLICIT cost of hedging, the reduced equity option value or the reduced FDIC insurance option value.

Finally, we emphasize that our model is not a signaling model as hedging is an unobservable activity. This unobservability of hedging makes it closer to the models of learning due to Holmstrom (1982), Campbell and Marino (1994), Fudenberg and Tirole (1986), Narayanan (1985), Palfrey and Spatt (1985), Scharfstein and Stein (1990) and Stein (1989). In these models agents do not have private information but undertake unobservable actions that affect the ability of the market or other agents to learn about the agent's underlying talent. Our model differs from these models in that the agent undertaking the unobservable action has some private information about his ability; this plays an important role in determining his optimal action. The other papers that we are aware of with this idea that agents have differing incentives to manipulate the measurement process are due to Allen and Gale (1990), Prendergast and Stole (1996) and Zweibel (1995).⁴ Finally, DeGeorge, Moselle and Zeckhauser (1996) present a similar idea.⁵

In Allen and Gale, the incentives of agents to manipulate the learning process is used in the context of a model that tries to explain why contracts do not contain all contingencies. In Prendergast and Stole (1996), managers exaggerate their own information to appear as fast learners when they are young but eventually become conservative and are unwilling to use new information. In Zweibel's (1995) work on corporate conservatism, managers who are behind tend to take efficient decisions

⁴ Ljungqvist (1994) also allows for manipulation of the measurement process. Matthews and Mirman (1983) is also closer to our model with costs of hedging. The model with costs of hedging can be viewed a noisy signaling as is the case of the model of Matthews and Mirman (1983).

⁵ DeGeorge, Moselle and Zeckhauser (1996) assume that firms with higher means have lower costs of variance reduction. In contrast, we make no such assumptions. Additionally, we model a cost for hedging that we believe to be of importance and one that yields the differential costs of hedging for firms with different mean cash flows.

while those ahead of the game tend to be conservative in their decision making. While all of these papers also model the idea that there are managerial incentives to distort the learning process, we believe that the unique contribution of our paper is to identify the costs and benefits of hedging. In particular, while hedging improves the informativeness of the learning process (the benefit), it has the associated cost of giving up some of the equity option value. This benefit-cost trade-off is important in delivering the results that we obtain that hedging should be undertaken by higher ability managers only when the benefits (the greater informativeness of the learning process) exceed the costs (the equity option value reduction).

The paper is organized as follows. Section I and II present the model and results when managers care only about their reputations. Sections III and IV consider the extension where managers hold equity and are concerned about the effect of hedging decisions on equity value. Section V concludes. All proofs are in the Appendix.

I. THE MODEL WITHOUT HEDGING COSTS

We consider a model where managers run a firm for two periods. In the first period the manager can invest \$1 in a project. The project yields a random amount $1 + y - e\Delta r$ at the end of the second period. The random payoff y is that part of profits that is under the control of the manager and can take one of two values y_h or y_l where $y_h > y_l$. The probability of the payoff y yielding the high state (y_h) is p_i which depends on the ability of the manager.

In contrast, the random factor Δr is not under the control of the manager. This random factor Δr can take the values δ or $-\delta$ with equal probability. The realization of the random factor is observable to the market. However, the firm's exposure to this factor e is not known to the market and can take one of three values, h , 0 or $-h$. The ex-ante probability of zero exposure is s while the probability of exposures h or $-h$ is $0.5(1 - s)$. We also assume that the $y_h - y_l = h\delta$. In this discrete space model, this assumption is needed to ensure that stochastic process due to the product of the exposure e conditional on the realization of the factor Δr is noise for the learning undertaken by the market.

There are a number of interpretations that one can give to the above setup. One is that of a bank where the payoff y represents credit risks where different banks differ in their ability while the payoff Δr represents interest rate risks for which banks (or class of banks) do not have any competitive advantage in prediction. While interest rate changes are observable, the exposure to interest rate risks, e , is not known by the market. Often, we will use this interpretation to illustrate our ideas. Another interpretation is that of a multinational firm where the process y represents that part of the profits that is related to the ability of managers while the process Δr represents exchange rate risks and e is the unknown exposure to exchange rate risks.

Given the assumptions that we have made on the discrete state space, there are four possible states of the world that can occur; we will label them z_1 , z_2 , z_3 and z_4 . The probability of each state occurring, conditioning on the interest rate factor realization, depends on whether the manager chooses to hedge or not hedge the cash flow and is shown in the following table:

TABLE I

State	NO HEDGING Conditional on Δr	Probability Of Occurrence $p(z_j NH, i)$	HEDGING	Probability Of Occurrence $p(z_j H, i)$
z_1	$y_h + h\delta$	$0.5(1 - s)p_i$		0
z_2	$y_h, y_l + h\delta$	$p_i s + 0.5(1 - s)(1 - p_i)$	y_h	p_i
z_3	$y_h - h\delta, y_l$	$0.5(1 - s)p_i + (1 - p_i)s$	y_l	$1 - p_i$
z_4	$y_l - h\delta$	$0.5(1 - s)(1 - p_i)$		0

where the index i refers to the manager's specific ability (we will discuss this further below) and the indexes NH and H represent the actions not hedging and hedging respectively.

The probability distribution over the outcomes z_1 to z_4 is different when the manager hedges and when the manager does not hedge. When the manager does not hedge the market cannot distinguish whether profits are high because the manager has good ability (skill) or because of luck (conditional on the factor Δr , the exposure e was in the right direction). For example, with no hedging (in either interest rate factor realization) the market does not know whether the outcome $z_2=y_h$ has occurred or whether the outcome $z_2 = y_l + h\delta$ has occurred. This inability to decompose profits perfectly is crucial for hedging to be valuable. If the market were able to know the exposures to interest rate risk, there would no role for managerial hedging.

In the example we presented of the bank manager, this corresponds to an inability to distinguish whether the manager is skillful at credit analysis or whether the manager had a positive exposure to interest rate risk and the interest rate movement was favorable. We note that in our set up, the symmetry assumption ensures that

the ex-post realization of the factor Δr is uninformative about the exposure e and the posterior probabilities of the outcomes z_1 to z_4 conditional on the realization Δr are just the prior probabilities. As a consequence, we suppress the dependence of the learning process on the realization of the factor Δr .

Finally, when the manager does hedge, we assume he completely hedges the risk away by setting his exposure to zero.⁶ In the example we gave of the bank, this corresponds to making sure that the hedged bank is neutral to interest rate risk. In the example of the multinational firm, it corresponds to locking in revenues or costs so that they do not depend on the fluctuations in exchange rates. When the manager hedges, the outcomes z_1 and z_4 do not occur. The profit process is less noisy and more informative about the abilities of the agents (if both managers were not to hedge).

The exact probability with which the various states occur (given the manager decides to hedge or not hedge) depends on the ability of the manager. To capture this differential ability of managers, we assume that there are two types of managers, the high ability manager (manager 1) and the low ability manager (manager 2). The market's prior probability that the manager is of higher ability is 0.5. We also assume that $p_2 = 1 - p_1$; this symmetry assumption ensures that none of our results are driven by asymmetric profit structures. We define $d = p_1 - p_2$. This is a measure of the difference in abilities and plays an important role in our results. Finally, $s > 1/3$, an assumption that is needed to ensure that when both firms do not hedge the posterior probabilities are monotone in the outcome z_j . Thus $m = s - 0.5(1 - s)$ is always positive. Low s values imply greater noise due to fluctuations in the exogenous noise process.

⁶ In this discrete state model, it is difficult to model partial hedging. Since we consider mixed strategies, one could view mixing between hedging and not hedging as partial hedging.

Given this basic structure we specify the objective of the manager. We follow Holmstrom (1982) and Holmstrom and Ricart i Costa (1986) and assume that managers receive the expected value for that period up front as the wage payment. However, contracts are renegotiated at the end of each period. If the ability of the manager, p_i , were known manager i would receive as wages value $p_i y_h + (1 - p_i) y_l - r$ as wage payments. Setting $\beta = y_h - y_l$ and $\kappa = y_l - r$, under full information the manager receives a wage $\beta p_i + \kappa$.

However p_i is not known, and hence the manager obtains $\beta(0.5p_1 + 0.5p_2) + \kappa = \beta(0.5) + \kappa$ (using the fact that $p_2 = 1 - p_1$) as the wage for the first period. The renegotiated wage for the second period is then given by:⁷

$$\beta [P(1|z_j)p_1 + P(2|z_j)p_2] + \kappa.$$

Here $P(1|z_j)$ is the posterior probability of the manager having higher ability given that outcome z_j is observed (a similar definition holds for $P(2|z_j)$). We suppress the constants β and κ they are scaling variables and play no role in the results of this section. Since the current wage is fixed, the manager maximizes the expected value of his future expected wage in the next period. This implies that the manager cares only about his reputation and not about the equity value of the firm. In Section 2 we relax this assumption by allowing managers to hold equity.

Hedging is an unobservable activity. Thus the market cannot condition its posterior beliefs about the ability of the manager on whether a firm hedges or not; beliefs are functions only of observed profits. Given our specification of the manager's objective we define the Bayesian Nash equilibrium of the updating game as:⁸

⁷ We note that the wage does not depend on the realization of the interest rate factor, Δr .

⁸ Again, we note that the updated wage does not depend on the realization of the interest rate factor, Δr .

(a) a market belief function $m^*: \{z_1, z_2, z_3, z_4\} \mapsto [0,1]$, where $m^*(z_j)$ is the posterior probability, given the observation z_j , that the manager is of higher ability.

(b) a firm action function $a^*: \{1,2\} \mapsto [0,1]$, where $a^*(i)$ is the probability with which the manager hedges (we are allowing for mixed strategies here).

We require that the pair $\{m^*(\cdot), a^*(\cdot)\}$ satisfy the conditions that:

(i) given $m^*(\cdot)$, manager i chooses

$$a^*(i) \in \arg \max_{a(i)} U(m^*(\cdot), a(i), i)$$

where

$$U(m^*(\cdot), a(i), i) = a(i) \left(\sum_{j=1}^4 p(z_j|H, i) m^*(z_j) (p_1 - p_2) + p_2 \right) \\ + (1 - a(i)) \left(\sum_{j=1}^4 p(z_j|NH, i) m^*(z_j) (p_1 - p_2) + p_2 \right)$$

(ii) given $a^*(i)$, $i = 1, 2$, the market belief function $i = 1, 2$, the market belief function $m^*(\cdot)$ satisfies

$$m^*(z_j) = \frac{p(z_j|a^*(1))}{p(z_j|a^*(1)) + p(z_j|a^*(2))}$$

for all z_j observed in equilibrium. Here $m^*(z_j)$ is the equilibrium posterior probability that the manager is of higher ability given the profit level z_j . By definition,

$$p(z_j|a^*(i)) = a^*(i)p(z_j|H, i) + (1 - a^*(i))p(z_j|NH, i).$$

The definition of $m^*(z_j)$ requires Bayes consistency over all profit levels observed in equilibrium but does not restrict market beliefs when a profit level is not observed. This occurs only when both firms hedge. In this case profit levels z_1 and z_4

are not observed and there is no restriction on market beliefs conditional on observing these states.

Finally, we note that our definition of the strategies chosen by the manager allows for randomization. In the context of our discrete space model, randomization can be interpreted as partial hedging.

II. EQUILIBRIUM IN THE MODEL WITHOUT HEDGING COSTS

A key idea behind our approach to hedging is the notion that managers of higher abilities will hedge so as to “lock-in” their profits and improve the informativeness of profits about their superior abilities. Therefore, we first investigate an intuitive equilibrium where the higher ability manager always hedges.

THEOREM 1: An equilibrium involving the high ability manager hedging always exists. The lower ability manager’s decision depends on the difference in abilities between the managers, d , in the following way: manner:

(i) For $0 < d < \delta_H$, $a^*(2) = 1$.

(ii) For $\delta_H < d < \delta_S$, $0 < a^*(2) < 1$.

(iii) For $\delta_S < d < 1$, $a^*(2) = 0$.

We refer to this equilibrium as a Type A equilibrium.

Proof: See Appendix.

We label this equilibrium a Type A equilibrium. A Type A equilibrium occurs for all possible values of parameter s ; this corresponds to all possible variances for the noise process r . Thus the Type A equilibrium requires no restrictions on the parameter space.

The intuition behind the Type A equilibrium is as follows. It is in the interest of the higher ability manager to hedge as hedging leads to a more informative learning

process. However, when the difference in the abilities of the two managers is low, it pays for the lower ability manager to follow suit and also hedge. A low difference in abilities implies that the learning process is not informative even when both managers hedge (although more informative than when both managers do not hedge); by not hedging the low ability manager runs the risk of the extreme states z_1 and z_4 occurring and revealing his ability. On the other hand when the difference in abilities is high, the learning process is very informative when both firms hedge; by not hedging the lower ability manager has some chance of being mistaken as the higher ability manager when the realizations of the process r are favorable.

While Type A equilibrium is most consistent with our intuition and exists no matter what the parameter configuration is, we need to investigate other equilibria. We first show that no equilibrium exists where the lower ability manager hedges and higher ability manager does not hedge or randomizes between hedging and not hedging.

THEOREM 2: The lower ability manager hedging and the higher ability manager not hedging or randomizing between hedging and not hedging can never constitute an equilibrium.

Proof: See Appendix.

The above analysis considers the possibility of equilibria where either the higher ability manager hedges or the lower ability manager hedges. We consider now equilibria where the higher ability manager randomizes between hedging and not hedging. These are characterized below.

THEOREM 3: The other Bayesian Nash equilibria of the model are:⁹

⁹ There is also an equilibrium that we do not discuss as it only exists for $s = 0.5$. In this case, we can have an equilibrium where $a(1) = a(2) = 0$.

Type C equilibrium: This equilibrium may occur only when $s < 0.5$ (high exogenous noise variance). The equilibrium strategies of the managers of different abilities is:

$$(i.) a^*(1) = a^*(2) = 1/(2(1 - s)).$$

Type D equilibrium: This equilibrium occurs when $s < 0.5$ (high exogenous noise variance). The equilibrium strategies of the managers of different abilities is:

$$(i.) a^*(1) = 0;$$

$$(ii.) 0 < a^*(2) < 1.$$

Proof: See Appendix.

A Type C equilibrium involves higher ability managers and lower ability managers randomizing between hedging and not hedging exactly to the same degree. This equilibrium is knife-edged as the values $a(1) = a(2)$ are chosen so that managers of any ability p_i are indifferent between hedging and not hedging. Such equilibria may occur when the noise in the interest rate process is high (low s).

A Type D equilibrium leads to the unintuitive outcome that higher ability managers do not hedge while lower ability manager randomize. The intuition is as follows. When the parameter s takes low values, fluctuations in the r process add a lot of noise to the inference process. If the higher ability manager does not hedge, then beliefs when extreme outcomes occur place weight on the inference that the manager is good. If the lower ability manager mimics and also does not hedge, then extreme outcomes are informative: z_1 is more consistent with a higher ability manager while z_4 is more consistent with a lower ability manager. Intermediate outcomes have little information as the noise in the r process is high. Thus the lower ability manager has incentives not to hedge to achieve these intermediate outcomes where learning is low. Hence he randomizes between hedging and not hedging.

The presence of these unintuitive equilibria for some parameter values may seem odd at first glance. It is important to remember that in this model there are no costs associated with hedging or not hedging. As a consequence, the optimal strategies depend on what market beliefs exist about intermediate states and extreme states. In general, the intuition that hedging is good for the higher ability manager depends on the idea that it improves learning by the market. As a consequence, the market views extreme states as indicating a lower ability manager. In the counter intuitive equilibrium, extreme states are more viewed more informative than intermediate states because the noise in the interest rate process is high and the difference in abilities is low. The multiplicity of equilibria in models with no signaling costs is a well known fact (see Crawford and Sobel (1982) and the related literature on “cheap talk”). We will argue next that there are costs associated with hedging and that these costs are lower for managers of higher ability. As we will see, this cost differential is sufficient to eliminate these counter-intuitive equilibria.

In this section, we have demonstrated the existence of an intuitive equilibrium where the high ability manager hedges while the low ability manager does not hedge. This equilibrium exists without any parameter restrictions and embeds the idea that the it was in the interest of the high ability manager to hedge so as to ensure a profit process that is more informative. The lower ability manager in this equilibrium may or may not hedge; this decision depended on the difference in their abilities. If the difference in ability was large enough, the lower ability manager will not hedge (so to lower the informativeness of the profit process) in the hope that luck will turn in his favor.

III. THE MODEL WHEN MANAGERS HOLD EQUITY

In demonstrating the results in the previous section, we have ignored the possibility that hedging may have costs associated with it. The existence of one important

cost is a consequence of the intuition of the option pricing model. If the firm is being run in the interests of equityholders, the incentives of the equityholders are to undertake activities that increase variance, not reduce variance. If the firm undertakes hedging activities, it is reducing the exposure of the firm to certain risks and hence the variance of profits. In the example that we considered of the bank, such variance reducing activities lowers the value of FDIC insurance option. Consequently, hedging has an implicit cost associated with it from the perspective of the equityholders. We explore the implications of the costs associated with hedging in the next section.

It is reasonable to assume that managers will be sensitive to the effects of their decision making on equityholders and that the wages they receive may include an equity component (this would induce some alignment of their objectives with that of the equityholders). Hence, we assume that in addition to caring about the effects of their decisions on managerial reputation, managers hold a fraction α of the equity of the firm.

In addition to allowing the manager to hold a fraction of the firm's equity we allow for external claimants like debtholders or for insurance such as FDIC insurance. The effect of either of these two possibilities is similar and we choose to use FDIC insurance in our analysis.

Institutions like savings and loans and banks have access to deposit insurance whose value is reduced by hedging. Thus hedging has an associated cost. However, a key intuition that is akin to that in signaling models is the following. This implicit cost of hedging is different for managers of different abilities. Firms where managers have higher ability have a lower probability of going bankrupt. Hence the marginal cost of undertaking hedging activities, which is the reduced value of the FDIC insurance (or the reduced wealth transfer from debtholders) is lower for a firm that has an higher ability.

There are two conclusions from the above intuition that are important. First hedging will not occur unless the difference in abilities is large enough because there is an associated cost to hedging. However, when hedging occurs the marginal cost differential results in the higher ability manager hedging and lower ability manager not hedging. When we analyze the equilibria below we will see that these two intuitive ideas are substantiated.

To operationalize FDIC insurance we assume that $z_4 < 0$ while $z_1 > z_2 > z_3 > 0$. The bank goes bankrupt in state z_4 . In this case the FDIC pays $-z_4$ to the depositors.

Thus the expected value of firm i when it hedges and when it does not hedge is given by:

$$\begin{aligned}
 V(H, i) &= p_i z_2 + (1 - p_i) z_3 \\
 V(NH, i) &= 0.5(1 - s)p_i z_1 + [p_i m + 0.5(1 - s)] z_2 \\
 &\quad + [(1 - p_i)m + 0.5(1 - s)] z_3 \\
 V(NH, i) - V(H, i) &= -0.5(1 - s)(1 - p_i) z_4 > 0
 \end{aligned}$$

When the firm (bank) makes a loss, the FDIC steps in and pays depositors. Hence, the expected value while not hedging is always higher than that while hedging. More importantly, this difference in expected values is higher for the lower ability manager as the FDIC option is more valuable to the lower ability manager.

The definition of a Bayesian Nash equilibrium is the same as before except

that the manager's objective, given the optimal market belief function $m^*(\cdot)$ is now:

$$\begin{aligned} & U(m^*(\cdot), a(i), i) \\ &= a(i) \left(\sum_{j=1}^4 p(z_j|H, i) m^*(z_j) (p_1 - p_2) + p_2 + \frac{\alpha}{\beta} V(H, i) \right) \\ & \quad + (1 - a(i)) \left(\sum_{j=1}^4 p(z_j|NH, i) m^*(z_j) (p_1 - p_2) + p_2 + \frac{\alpha}{\beta} V(NH, i) \right). \end{aligned}$$

Here β is not $y_h - y_l$ but $\eta(y_h - y_l)$ where η is the fraction of output that the manager receives in the future. The above formulation is similar to that in Prendergast and Stole (1996) in that their paper also uses an objective function in which managers care for both profits and end of period reputations.¹⁰ The fraction α represents the extent to which the manager cares for the current value while the fraction β represents the extent to which the manager cares for the future value.

IV. EQUILIBRIA IN THE MODEL WITH HEDGING COSTS

We first investigate the existence of the Type A equilibrium (the intuitive equilibrium) where the higher ability manager hedges. As we have discussed, the presence of cost of hedging should deter existence of the Type A equilibrium when the difference in abilities is low. On the other hand when the Type A equilibrium does exist, lower ability managers should be less willing to hedge relative to the case where managers only care for their reputations because of the cost differential between the two kinds of managers. This intuition turns out to be validated.

THEOREM 4: Type A equilibria involve $a^*(1) = 1$ and exist for $d > \delta(s)$; $\delta(s)$ is always well defined. Thus Type A equilibria always exist for high enough differences in managerial abilities. The behavior of the lower ability manager is as follows:

¹⁰ The signaling models of Ross (1977), Bhattacharya (1979), Harris and Raviv (1985) and Brennan and Copeland (1987) have a related formulation of the managerial objective function

- i. $\delta(s) < d < \delta_l^0$ and $d > \delta_h^0$, the lower ability manager does not hedge, $a^*(2) = 1$.
- ii. $\delta_l^0 < d < \delta_l^1$ and $\delta_h^1 < d < \delta_h^0$, the lower ability manager randomizes, $0 < a^*(2) < 1$.
- iii. $\delta_l^1 < d < \delta_h^1$, the lower ability manager hedges, $a^*(2) = 1$.
- iv. As hedging costs increases from 0, the region (δ_l^1, δ_h^1) first vanishes leaving only the region (δ_l^0, δ_h^0) where randomization occurs. For large enough hedging costs, the region (δ_l^0, δ_h^0) vanishes and the lower ability manager plays $a^*(2) = 0$.
- v. The region where $a^*(2) = 1$ when managers hold equity is a strict subset of the region where $a^*(2) = 1$ when managers only care of their reputations.

Proof: See Appendix.

Figure 1 provides the exact regions characterized in Theorem 4 above for the values $s = 0.4$ (high exogenous noise variance) and $s = 0.75$ (low exogenous noise variance).

Type A equilibria do not exist for low differences in ability due to the positive costs of hedging. A sufficiently large difference in ability is required to make hedging a viable strategy for the higher ability manager. Depending on the cost of hedging, they may or may not involve a region where the lower ability manager hedges. However, when there is an region where the low ability manager hedges, the equilibrium takes the following form – for intermediate differences in ability the lower ability manager does not hedge. As the difference in ability increases, he randomizes and eventually hedges. For sufficiently high differences in ability, the manager randomizes again and eventually does not hedge.

FIGURE 1A

EQUILIBRIUM WHERE THE HIGHER ABILITY MANAGER HEDGES
(TYPE A EQUILIBRIUM) $s=0.4$

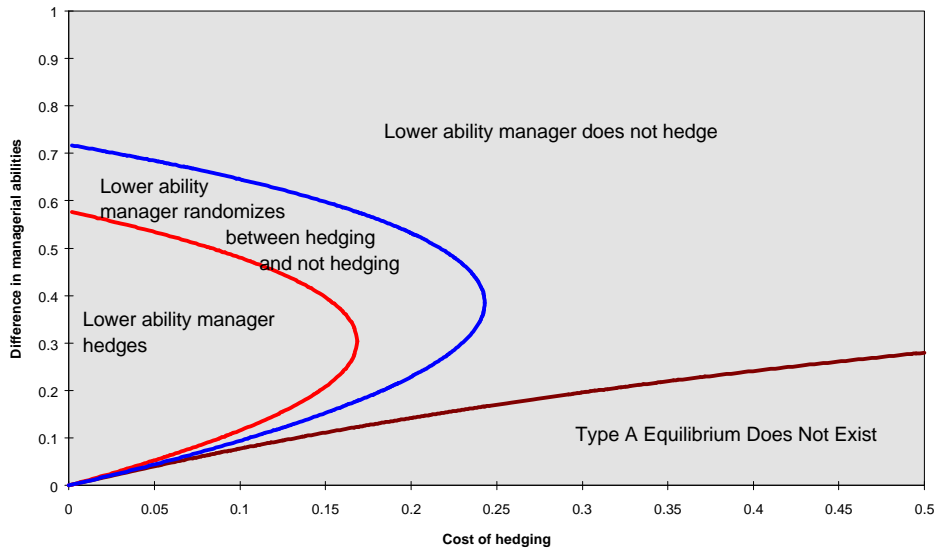
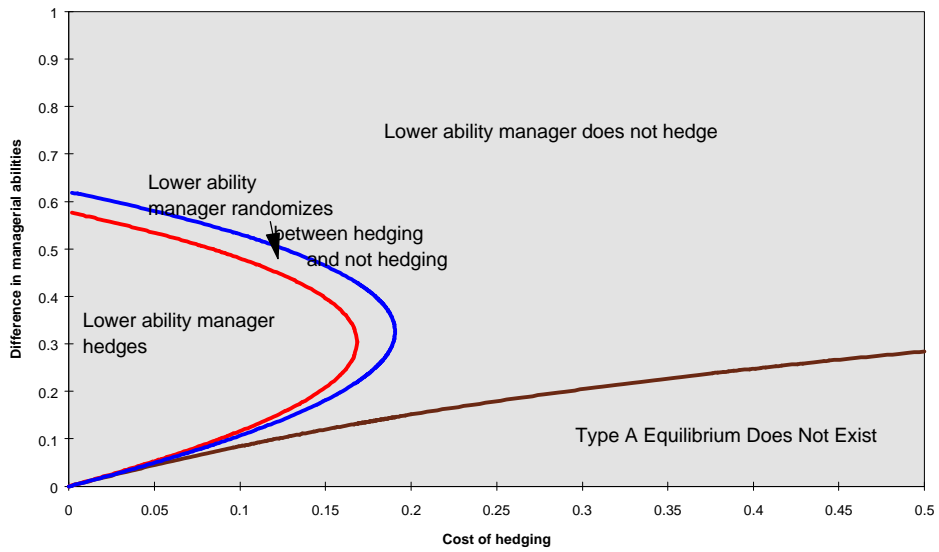


FIGURE 1B

EQUILIBRIUM WHERE THE HIGHER ABILITY MANAGER HEDGES
(TYPE A EQUILIBRIUM) $s=0.75$



The intuition for such behavior is as follows. When differences in ability are small and the higher ability manager finds it optimal to hedge, the lower ability manager does not as his cost is higher. As the difference in ability increases, so do the incentives to mimic the higher ability manager provided the learning process is not too informative. This leads to an intermediate region where the lower ability manager hedges. However, as the difference in ability becomes large the learning process when both managers hedge is very informative. In addition, the lower ability manager faces an increasingly higher hedging cost as the probability of going bankrupt increases. This leads to him to prefer not to hedge.

We next turn to regions where the differences in ability are very low. In such regions, the Type A equilibrium does not exist. Intuitively, both managers will not hedge. We thus investigate the Type B equilibrium where the higher ability manager does not hedge. This equilibrium is characterized in Theorem 5.

THEOREM 5: Type B equilibrium is an equilibrium where $a^*(2) = 0$ and $0 \leq a^*(1) < 1$. Thus the lower ability manager does not hedge and the higher ability manager does not hedge or randomizes between hedging and not hedging (note that we do not include the case where the higher ability manager hedges as this is a Type A equilibria). Type B equilibria have two subcases:

- i. *Equilibrium with $a^*(1) = 0$. These exist for $d \leq \gamma(s)$ where $\gamma(s) > \delta(s)$. $\gamma(s)$ is determined by the behavior of the higher ability manager when $s > 0.5$ and by the behavior of the lower ability manager when $s < 0.5$.*
- ii. *Equilibria with $0 < a^*(1) < 1$. This equilibrium exists for a region where $\delta(s) < d < \pi(s)$ where $\pi(s) \leq \gamma(s)$.*

Proof: See Appendix.

Type B equilibria contain the important equilibrium (Type B(i)) where both firms do not hedge. Since $\gamma(s) > \delta(s)$, we have constructively proved that an equilibrium always exists. In fact at $\delta(s)$, both the “pooling” equilibria (both firms not hedging) and the “separating” equilibria (higher ability firm hedges, lower ability firm does not) are equilibria. Figure 2 provides the exact regions where Type A and Type B(i) equilibria exist for the values $s = 0.4$ (high exogenous noise variance) and $s = 0.75$ (low exogenous noise variance).

To complete the characterization of the model we need to document what other equilibria, in addition to the above equilibria exist. Theorem 6 characterizes the other equilibria in this model.

THEOREM 6: In addition to the equilibria of Theorems 4 and 5, the only other Bayesian Nash equilibria of the model are:¹¹

Type D equilibria. This equilibria involves $a^(1) = 0$ and $0 < a^*(2) < 1$. It cannot occur for $s > 0.5$. It may occur for $s < 0.5$ depending on the parameters s and d .*

Proof: See Appendix.

The Type D equilibrium is the counter-intuitive equilibrium we have discussed. As discussed in Section I, this equilibrium requires high noise in the process r (low s). As previously mentioned, we think that the Type D equilibrium is less reasonable when there is a cost of hedging as the marginal cost of hedging is lower for the higher ability manager. We present a bound that eliminates this equilibrium. Essentially, this condition requires that the manager cares sufficiently for the equity value of the firm relative to his future career reputation.

¹¹ With positive costs of hedging the Type C equilibrium that involve randomization by managers of both higher and lower abilities does not exist.

FIGURE 2A

EQUILIBRIUM WHERE BOTH MANAGERS DO NOT HEDGE (TYPE B(i))
AND
EQUILIBRIUM WHERE THE HIGHER ABILITY MANAGER HEDGES (TYPE A)
s=0.4

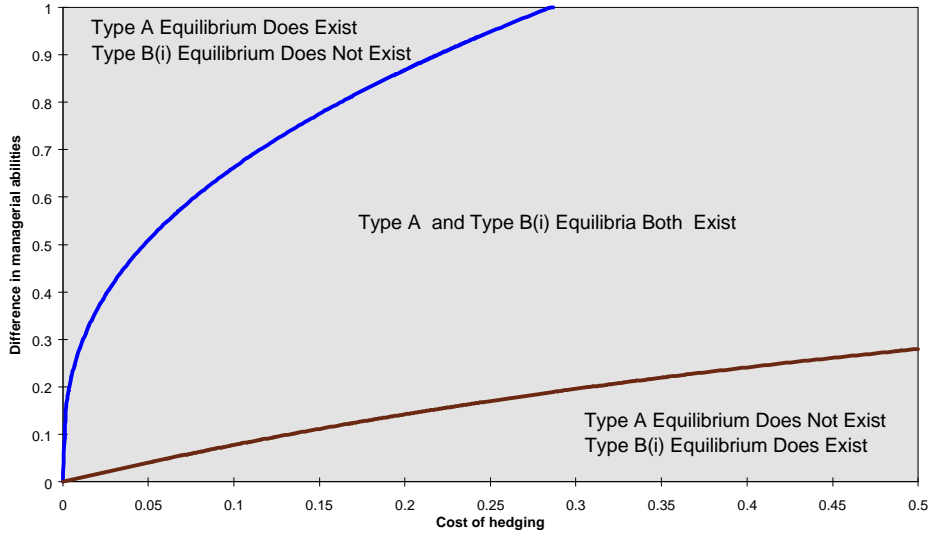
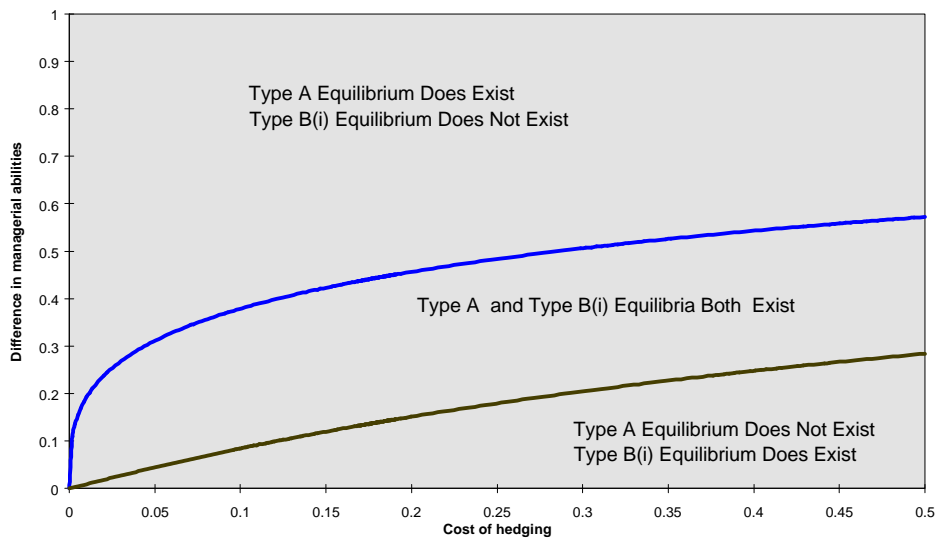


FIGURE 2B

EQUILIBRIUM WHERE BOTH MANAGERS DO NOT HEDGE (TYPE B(i))
AND
EQUILIBRIUM WHERE THE HIGHER ABILITY MANAGER HEDGES (TYPE A)
s=0.75



THEOREM 7: If

$$x(d) = -\frac{\alpha}{\beta}z_4 > \frac{d^3}{1+d} \text{ or}$$

$$\frac{(\alpha - x(d)\eta)}{x(d)\eta} \frac{|y_l - h\delta|}{|y_h - h\delta|} > 1,$$

then Type D equilibria do not exist.

Proof: See Appendix.

The bound given in Theorem 7 depends on the difference in managerial abilities. When the difference in managerial abilities is 1/3, the value of the bound is 1/36. When $d = 1$, the bound has its highest value of 1/2. In particular, suppose $\frac{|y_l - r_b|}{|y_h - r_b|}$ is 1/2 (high state profits are twice the absolute value of low state losses assuming the worst realization of the exogenous shock). Also, let $\eta = 0.1$ (the manager cares for 10% of future output). Then the bound when $d = 1/3$ yields $\alpha > (1/9)\eta$ or $\alpha > 0.0111$. The bound when $d = 1$ yields $\alpha > 1.5\eta$ or $\alpha > 0.15$. Hence in the worst possible case ($d = 1$), one needs the weight on current equity to be 1.5 times the weight on the share of future output. If the conditions of Theorem 7 are met, the cost differential between managers of differing abilities is such that behavior involving the lower ability manager hedging is implausible and does not occur.

We now discuss the empirical implications of our model. While hedging is not observable, subsequent to the event, one sometimes knows whether a firm hedges or not. This is because ex-post firms voluntarily disclose hedging activity in the notes to the balance sheet. If our story of hedging is correct, the profits (earnings) of firms in the quarter or year which they hedge should be higher and the volatility of cash flows should be lower than usual. Such a test differs from tests of signaling models where profits (earnings) are higher for firms that signal; no volatility implications are present. Because hedging reduces volatility, the volatility reduction is an additional implication. Some preliminary empirical work by DeGeorge, Moselle and Zeckhauser

(1995) using Compustat data is supportive of this implication of the hedging hypothesis.

A second empirical implication of our model is as follows. When the difference in abilities is high and there are costs to hedging, we are more likely to find hedging. In contrast, when the difference in abilities is low, the equilibrium that occurs involves no hedging by both kinds of managers. This implication is borne out in a recent empirical study by Tufano (1995) on the gold mining industry. First he finds that younger managers are more likely to hedge than older managers. Since there is likely to be greater (lesser) uncertainty about the ability of younger (older) managers, a higher (lower) difference in abilities is consistent with younger (older) managers. Hence, Tufano's work provides much support for our approach to hedging.¹²

A third empirical implication of our model is as follows. When the costs of hedging are low, less separation occurs. In contrast, when the costs of hedging are high, more separation occurs. Therefore, the value of firms that hedge relative to firms that do not hedge is much higher when the cost of hedging is higher. This is another implication of the model that is potentially testable. Tufano's finding the managers with stock options do not hedge is consistent with our idea that higher costs of hedging lead to less overall hedging. However, a test of our approach would require that if managers with stock options hedge, their firm value is much higher than that of managers without stock options who hedge.¹³

V. CONCLUSIONS

¹² We also note that Tufano (1995) finds no support for the costly bankruptcy approach to hedging in his study. However, Geczy, Minton and Schrand (1997) look at a large cross-section of firms and find evidence more consistent with the transactions cost of raising capital and presence of growth opportunities hypothesis put forward in Froot, Scharfstein and Stein (1993).

¹³ Tufano (1995) also finds that the greater the equity holdings of the manager, the more likely one will observe hedging. It is difficult to use this observation directly as what matters in our model is the cost of hedging and not the fraction of equity held. A proxy for the cost of hedging must simultaneously account for the fraction of equity held and the value of the equity option or FDIC insurance that hedging destroys.

We have presented a model wherein managers use hedging as an indirect vehicle to communicate their abilities. While hedging itself is not observable, the firm's decision to hedge or not to hedge and the market's beliefs about this decision affects the inferences that the market makes from profits. In a model where managers care only for their reputations, we show the existence of a class of equilibria where the higher ability manager hedges while the lower ability manager does not hedge when the ability difference is low but does not hedge when the ability difference is high. When differences in ability are low the learning process when both managers hedge is not very informative (though more informative than when both managers do not hedge) while not hedging allows the lower ability manager to be discovered when extreme states occur. Thus the lower ability manager prefers to hedge. When the difference in abilities is high, the learning process when both managers hedge is very informative and the lower ability manager prefers not to hedge as there is a chance that low interest rates realization leads to high profits.

We next allow managers to hold equity in their firms. If FDIC insurance exists (as in the case of banks) or the firm has pre-existing debt, hedging is costly. However, it is more costly for the lower ability manager as his probability of going bankrupt is higher. In such a case, not hedging is the equilibrium when the ability difference is low.

For a sufficiently high difference in abilities, there is an equilibrium where the higher ability manager hedges. In this equilibrium, the lower ability manager behaves as follows – for lower ability differences he does not hedge as his cost of hedging is higher. As the ability difference increases, there may be a region where he hedges. This occurs when reputation effects become more important and the difference in ability is still not too high. Finally, for large enough differences in ability he does not hedge as the learning process when both managers hedge is very informative and

his probability of going bankrupt is increasing. Relative to the model when manager care only for their reputations, more separation occurs in the case where they own equity.

Our results indicate that hedging occurs when higher ability managers are substantially different from lower ability managers or the costs of hedging are low. This substantiates the casual belief that hedging locks up higher profit opportunities in the same way that an arbitrageur locks up arbitrage opportunities.

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APPENDIX

Proof of Theorem 1: Given that $a(1) = 1$ and that $0 \leq a(2) \leq 1$, the market beliefs are

Profit	Market beliefs $m(\cdot)$	
z_1		0
z_2	$p_1/\{p_1 + a(2)p_2 + (1 - a(2))[p_2m + 0.5(1 - s)]\}$	(A1)
z_3	$p_2/\{p_2 + a(2)p_1 + (1 - a(2))[p_1m + 0.5(1 - s)]\}$	
z_4		0

The posterior beliefs are those consistent with Bayes rule except when $a(1) = 1$. In that case both firms hedge and thus z_1 and z_4 are off the equilibrium path. We assume the off equilibrium posterior for profit levels z_1 and z_4 is 0.

For the higher ability manager to find it optimal to hedge we must have,

$$\begin{aligned}
 U(m^*(\cdot), H, 1) &> U(m^*(\cdot), NH, 1) \\
 \iff p_1m(z_2) + p_2m(z_3) &> [p_1m + 0.5(1 - s)]m(z_2) + [p_2m + 0.5(1 - s)]m(z_3) \\
 \iff 0.5(1 - s)(3p_1 - 1)m(z_2) &+ 0.5(1 - s)(3p_2 - 1)m(z_3) > 0.
 \end{aligned}
 \tag{A2}$$

Note that $m(z_2) > m(z_3)$. Thus

$$\begin{aligned}
 &0.5(1 - s)(3p_1 - 1)m(z_2) + 0.5(1 - s)(3p_2 - 1)m(z_3) \\
 &> 0.5(1 - s)(3p_1 - 1)m(z_3) + 0.5(1 - s)(3p_2 - 1)m(z_3) \\
 &= 0.5(1 - s)m(z_3) \\
 &> 0
 \end{aligned}
 \tag{A3}$$

Thus under the above market beliefs manager B's best strategy is to hedge.

Next consider agent 2. If agent 2 randomizes between hedging and not hedging then

$$\begin{aligned}
U(m(\cdot), H, 2) &= U(m(\cdot), L, 2) \\
\iff p_2 m(z_2) + p_1 m(z_3) &= [p_2 m + 0.5(1-s)]m(z_2) + [p_1 m + 0.5(1-s)]m(z_3) \\
\iff 0.5(1-s)(3p_2 - 1) \frac{p_1}{1 - (1-a(2))(1-s)0.5(3p_2 - 1)} \\
&\quad + 0.5(1-s)(3p_1 - 1) \frac{p_2}{1 - (1-a(2))(1-s)0.5(3p_1 - 1)} = 0 \\
\iff (3p_1 - 1)p_2 [1 - (1-a(2))0.5(1-s)(2 - 3p_1)] \\
&\quad = -(2 - 3p_1)p_1 [1 - (1-a(2))0.5(1-s)(3p_1 - 1)] \\
\iff (1 - a(2)) &= \frac{(3p_1 - 1)(1 - p_1) + (2 - 3p_1)p_1}{(3p_1 - 1)(2 - 3p_1)0.5(1-s)}.
\end{aligned} \tag{A4}$$

The right hand side of the above expression is monotone in p_1 . Setting the right hand side of the last equation in (A4) equal to 0 yields

$$\begin{aligned}
(3p_1 - 1)(1 - p_1) + (2 - 3p_1)p_1 &= 0 \\
\iff 6p_1^2 - 6p_1 + 1 &= 0
\end{aligned} \tag{A5}$$

This yields the solution $\zeta_H = 0.5 + 0.5(1/\sqrt{3})$. Setting the right hand side of the last equation in (A4) to 1 yields

$$\begin{aligned}
3p_1 - 1 - 3p_1^2 + p_1 + 2p_1 - 3p_1^2 &= (6p_1 - 2 - 9p_1^2 + 3p_1)0.5(1-s) \\
\iff 6p_1 - 6p_1^2 - 1 &= (9p_1 - 9p_1^2 - 2)0.5(1-s) \\
\iff p_1 p_2 &= \frac{2s}{9s + 3}
\end{aligned} \tag{A6}$$

This defines $\zeta_S(s) = 0.5 + 0.5\sqrt{\frac{s+3}{9s+3}}$ which is decreasing in s . Randomized strategies are viable in the region $p_1 \in (\zeta_H, \zeta_S(s))$. Since the difference in abilities d is monotone in h , there is equivalently a region $(\delta_H, \delta_S(s))$ such that for $d \in (\delta_H, \delta_S(s))$ randomization is optimal for the lower ability manager. When $d < \delta_H$, the best strategy for

the lower ability manager is to hedge and when $d > \delta_S(s)$ the best response is to not hedge. ■

Proof of Theorem 2:

We show that no equilibria where the low ability manager hedges while the high ability manager randomizes can exist. In this case the posterior beliefs are

Profit	Market beliefs $m(\cdot)$	
z_1	1	
z_2	$\frac{a(1)p_1+(1-a(1))[p_1m+0.5(1-s)]}{a(1)p_1+(1-a(1))[p_1m+0.5(1-s)]+p_2}$	(A7)
z_3	$\frac{a(1)p_2+(1-a(1))[p_2m+0.5(1-s)]}{a(1)p_2+(1-a(1))[p_2m+0.5(1-s)]+p_1}$	
z_4	1	

The lower ability manager hedges. Thus

$$\begin{aligned}
& U(m(\cdot), H, 2) > U(m(\cdot), NH, 2) \\
\iff & 0.5(1-s)(3p_2-1)m(z_2) + 0.5(1-s)(3p_1-1)m(z_3) \\
& > 0.5(1-s)p_2m(z_1) + 0.5(1-s)p_1m(z_4) \\
\iff & (3p_2-1)m(z_2) + (3p_1-1)m(z_3) > p_2m(z_1) + p_1m(z_4) = 1
\end{aligned} \tag{A8}$$

where $m(z_1) = m(z_4) = 1$ is used. But

$$\begin{aligned}
& (3p_2-1)m(z_2) + (3p_1-1)m(z_3) \\
& = p_2m(z_2) + p_1m(z_3) + (2p_1-1)[m(z_3) - m(z_2)] \\
& < p_2m(z_2) + p_1m(z_3) \text{ using } m(z_2) > m(z_3) \\
& < p_2 + p_1 = 1.
\end{aligned} \tag{A9}$$

Thus $U(m(\cdot), H, 2) > U(m(\cdot), NH, 1)$ never holds. ■

Proof of Theorem 3: The proof is constructive and consists of three parts:

- a. A proof that strategies where $a(2) = 0$ and $0 < a(1) < 1$ are not equilibria.

b. a characterization of possible equilibria when $0 < a(1) < 1$ and $0 \leq a(2) \leq 1$.

c. a characterization of possible equilibria when $a(1) = 0$ and $0 \leq a(2) \leq 1$.

Part a. We first consider the case where the lower ability manager does not hedge while the higher ability manager randomizes between hedging and not hedging. The market beliefs in such a case are

Profit	Market beliefs $m(\cdot)$	
z_1	$[(1 - a(1))p_1] / [(1 - a(1))p_1 + p_2]$	
z_2	$\frac{a(1)p_1 + (1 - a(1))[p_1m + 0.5(1 - s)]}{a(1)p_1 + (1 - a(1))[p_1m + 0.5(1 - s)] + p_2m + 0.5(1 - s)}$	(A10)
z_3	$\frac{a(1)p_2 + (1 - a(1))[p_2m + 0.5(1 - s)]}{a(1)p_2 + (1 - a(1))[p_2m + 0.5(1 - s)] + p_1m + 0.5(1 - s)}$	
z_4	$[(1 - a(1))p_2] / [(1 - a(1))p_2 + p_1]$	

For this to be an equilibrium we need $U(m(\cdot), H, 1) = U(m(\cdot), NH, 1)$ and $U(m(\cdot), H, 2) < U(m(\cdot), NH, 2)$. Define the functions

$$\begin{aligned}
F_1(a(1)) &= (3p_1 - 1) \frac{a(1)0.5(1 - s)(3p_1 - 1) + p_1m + 0.5(1 - s)}{a(1)0.5(1 - s)(3p_1 - 1) + m + 1 - s} \\
&\quad + (3p_2 - 1) \frac{a(1)0.5(1 - s)(3p_2 - 1) + p_2m + 0.5(1 - s)}{a(1)0.5(1 - s)(3p_2 - 1) + m + 1 - s} \\
&\quad - p_1 \frac{(1 - a(1))p_1}{1 - a(1)p_1} - p_2 \frac{(1 - a(1))p_2}{1 - a(1)p_2} \\
F_2(a(1)) &= (3p_2 - 1) \frac{a(1)0.5(1 - s)(3p_1 - 1) + p_1m + 0.5(1 - s)}{a(1)0.5(1 - s)(3p_1 - 1) + m + 1 - s} \\
&\quad + (3p_1 - 1) \frac{a(1)0.5(1 - s)(3p_2 - 1) + p_2m + 0.5(1 - s)}{a(1)0.5(1 - s)(3p_2 - 1) + m + 1 - s} \\
&\quad - p_2 \frac{(1 - a(1))p_1}{1 - a(1)p_1} - p_1 \frac{(1 - a(1))p_2}{1 - a(1)p_2}
\end{aligned} \tag{A11}$$

We need to find parameter restrictions that imply $F_1(a(1)) = 0$ and $F_2(a(1)) < 0$

occurs. First note that

$$\begin{aligned}
F_1(0) (<) = (>) 0 \\
&\iff \\
(3p_1 - 1)\frac{p_1m + 0.5(1-s)}{m+1-s} + (3p_2 - 1)\frac{p_2m + 0.5(1-s)}{m+1-s} - p_1^2 - p_2^2 (<) = (>) 0 \\
&\iff (2s-1)2[p_1^2 + p_2^2] (<) = (>) (2s-1).
\end{aligned} \tag{A12}$$

When $s > 0.5$, $F_1(0) > 0$ and when $s < 0.5$, $F_1(0) < 0$. Second note that $dF_1(a(1)) / da(1) > 0$. Thus when $s > 0.5$, $F_1(a(1)) > 0$ for all $a(1)$ and no equilibrium exists.

Third note that

$$\begin{aligned}
F_1(1) &= (3p_1 - 1)\frac{0.5(1-s)(3p_1 - 1) + p_1m + 0.5(1-s)}{0.5(1-s)(3p_1 - 1) + m + 1 - s} \\
&\quad + (3p_2 - 1)\frac{0.5(1-s)(3p_2 - 1) + p_2m + 0.5(1-s)}{0.5(1-s)(3p_2 - 1) + m + 1 - s} \\
&= p_1m(z_2) + p_2m(z_3) + (2p_1 - 1)[m(z_2) - m(z_3)] > 0.
\end{aligned} \tag{A13}$$

as $m(z_2) > m(z_3)$. Thus a unique $0 < a(1) < 1$ exists when $s < 0.5$.

Whether equilibrium exists depends in this case on the behavior of $F_2(a(1))$.

First note that

$$\begin{aligned}
F_2(0) (<) = (>) 0 \\
&\iff (3p_2 - 1)\frac{p_1m + 0.5(1-s)}{m+1-s} + (3p_1 - 1)\frac{p_2m + 0.5(1-s)}{m+1-s} - 2p_1p_2 (<) = (>) 0 \\
&\iff 4p_1p_2(2s-1) (<) = (>) 2s-1.
\end{aligned} \tag{A14}$$

Thus $F_2(0) < 0$ when $s > 0.5$ and $F_2(0) > 0$ when $s < 0.5$. Second note that

$$\begin{aligned}
& \frac{dF_2(a(1))}{da(1)} \\
&= (3p_1 - 1)(3p_2 - 1)0.5(1 - s) \frac{p_2 m + 0.5(1 - s)}{[a(1)0.5(1 - s)(3p_1 - 1) + m + 1 - s]^2} \\
& \quad + (3p_2 - 1)(3p_1 - 1)0.5(1 - s) \frac{p_1 m + 0.5(1 - s)}{[a(1)0.5(1 - s)(3p_2 - 1) + m + 1 - s]^2} \\
& \quad + \frac{p_1 p_2^2}{(1 - a(1)p_1)^2} + \frac{p_1^2 p_2}{(1 - a(1)p_2)^2}.
\end{aligned} \tag{A15}$$

Up to $p_1 < 2/3$ this is surely positive. Since $F_2(0) > 0$, $F_2(a(1)) > 0$ for all $a(1)$ and no equilibrium exists. On the other hand if $p_1 > 2/3$, equilibrium may exist. However, a grid search over the region where $1/3 < s < 0.5$ and $2/3 < p_1 < 1$ shows so solution where $F_1(a(1)) = 0$ and $F_2(a(1)) < 0$.¹⁴ ■

Part b. We consider when strategies involving randomization by both managers are consistent with equilibrium. When both managers randomize, market beliefs satisfy,

Profit	Market beliefs $m(\cdot)$	
z_1	$(1 - a(1))p_1 / \{(1 - a(1))p_1 + (1 - a(2))p_2\}$	
z_2	$\frac{a(1)0.5(1-s)(3p_1-1)+p_1m+0.5(1-s)}{0.5(1-s)[a(1)(3p_1-1)+a(2)(3p_2-1)]+m+1-s}$	(A16)
z_3	$\frac{a(1)0.5(1-s)(3p_2-1)+p_2m+0.5(1-s)}{0.5(1-s)[a(1)(3p_2-1)+a(2)(3p_1-1)]+m+1-s}$	
z_4	$(1 - a(1))p_2 / \{(1 - a(1))p_2 + (1 - a(2))p_1\}$	

Randomization

requires that $U(m(\cdot), H, 1) = U(m(\cdot), NH, 1)$ and $U(m(\cdot), H, 2) = U(m(\cdot), NH, 2)$.

On simplifying these expressions we obtain that:

$$\begin{aligned}
& F_1(\lambda, \theta) \\
&= \frac{p_1 - \lambda 0.5(1 - s)(3p_1 - 1)}{1 - \lambda 0.5(1 - s) + 0.5(1 - s)\theta(3p_2 - 1)} - \frac{2}{3} \frac{\lambda p_1}{\lambda - \theta p_2} - \frac{1}{3} \frac{\lambda p_2}{\lambda - \theta p_1} \\
&= 0
\end{aligned} \tag{A17}$$

¹⁴ This computer program is available from the authors on request.

and

$$\begin{aligned}
& F_2(\lambda, \theta) \\
&= \frac{p_2 - \lambda 0.5(1-s)(3p_2 - 1)}{1 - \lambda 0.5(1-s) + 0.5(1-s)\theta(3p_1 - 1)} - \frac{1}{3} \frac{\lambda p_1}{\lambda - \theta p_2} - \frac{2}{3} \frac{\lambda p_2}{\lambda - \theta p_1} \quad (A18) \\
&= 0
\end{aligned}$$

where $0 < \lambda = a(1) < 1$ and $-\lambda < \theta = a(2) - a(1) < 1 - \lambda$.

If one sets $\theta = 0$ and $\lambda = 1/(2(1-s))$, the above equations are satisfied for any p_1 . Since $1/(2(1-s)) < 1$ only when $s < 0.5$, this equilibrium only exists if this condition is satisfied.

Surprisingly, there is no other equilibrium that involves randomization. This is proved by running a computer grid search over $1/3 < s < 1$ and $0.5 < p_1 < 1$ and searching whether there exist values of λ and θ satisfying equations (A17) and (A18) and satisfying the restrictions $0 < \lambda < 1$ and $-\lambda < \theta < 1 - \lambda$.¹⁵ ■

Part c. We now look at the case where the higher ability manager does not hedge and the lower ability manager randomizes between hedging and not hedging. The market beliefs are

Profit	Market beliefs $m(\cdot)$	
z_1	$p_1 / \{p_1 + (1 - a(2))p_2\}$	
z_2	$\frac{p_1 m + 0.5(1-s)}{m + 1 - s + 0.5a(2)(1-s)(3p_2 - 1)}$	(A19)
z_3	$\frac{p_2 m + 0.5(1-s)}{m + 1 - s + 0.5a(2)(1-s)(3p_1 - 1)}$	
z_4	$p_2 / \{p_2 + (1 - a(2))p_1\}$	

We need find if parameter values exist such that $U(m(\cdot), H, 1) <$

¹⁵ This computer program is available from the authors on request.

$U(m(\cdot), NH, 1)$ and $U(m(\cdot), H, 2) = U(m(\cdot), NH, 2)$. Define that functions

$$\begin{aligned}
& F_1(a(2)) \\
&= (3p_1 - 1) \frac{p_1 m + 0.5(1 - s)}{m + 1 - s + 0.5a(2)(1 - s)(3p_2 - 1)} \\
&+ (3p_2 - 1) \frac{p_2 m + 0.5(1 - s)}{m + 1 - s + 0.5a(2)(1 - s)(3p_1 - 1)} - p_1 \frac{p_1}{1 - a(2)p_2} - p_2 \frac{p_2}{1 - a(2)p_1} \\
& F_2(a(2)) \\
&= (3p_2 - 1) \frac{p_1 m + 0.5(1 - s)}{m + 1 - s + 0.5a(2)(1 - s)(3p_2 - 1)} \\
&+ (3p_1 - 1) \frac{p_2 m + 0.5(1 - s)}{m + 1 - s + 0.5a(2)(1 - s)(3p_1 - 1)} - p_2 \frac{p_1}{1 - a(2)p_2} - p_1 \frac{p_2}{1 - a(2)p_1}
\end{aligned} \tag{A20}$$

It suffices to find conditions under which $F_1(a(1)) < 0$ and $F_2(a(2)) = 0$.

First note that

$$\begin{aligned}
& F_1(0) (<) = (>) 0 \\
&\iff (3p_1 - 1) \frac{p_1 m + 0.5(1 - s)}{m + 1 - s} + (3p_2 - 1) \frac{p_2 m + 0.5(1 - s)}{m + 1 - s} \\
&\quad - p_1^2 - p_2^2 (<) = (>) 0 \\
&\iff 2[2s - 1](p_1^2 + p_2^2) (<) = (>) 2s - 1.
\end{aligned} \tag{A21}$$

Since $2(p_1^2 + p_2^2) > 1$, when $s > 0.5$, we have $F_1(0) > 0$. When $s < 0.5$ we have $F_1(0) < 0$.

Second note that

$$\begin{aligned}
& F_2(0) (<) = (>) 0 \\
&\iff (3p_2 - 1) \frac{p_1 m + 0.5(1 - s)}{m + 1 - s} + (3p_1 - 1) \frac{p_2 m + 0.5(1 - s)}{m + 1 - s} - 2p_1 p_2 (<) = (>) 0 \\
&\iff [2s - 1]4p_1 p_2 (<) = (>) 2s - 1.
\end{aligned} \tag{A22}$$

Since $4p_1 p_2 < 1$, when $s > 0.5$ we have $F_2(0) < 0$ and when $s < 0.5$ we have $F_2(0) > 0$.

Third note that $dF_1(a(2)) / da(2) < 0$.

When $s > 0.5$, $F_2(a(2)) < 0$ for all values of $a(2)$ and thus the equilibrium does not exist. In fact the equilibrium where both managers do not hedge fails to exist as $F_1(0) > 0$. The equilibrium where manager 1 does not hedge and manager 2 hedges does not exist as $F_2(1) < 0$.

When $s < 0.5$, the equilibrium where both managers do not hedge fails to exist as $F_2(0) > 0$. On the other hand

$$\begin{aligned}
F_2(1) &= (3p_2 - 1) \frac{p_1 m + 0.5(1 - s)}{m + 1 - s + 0.5(1 - s)(3p_2 - 1)} \\
&\quad + (3p_1 - 1) \frac{p_2 m + 0.5(1 - s)}{m + 1 - s + 0.5(1 - s)(3p_1 - 1)} - p_2 \frac{p_1}{1 - p_2} - p_1 \frac{p_2}{1 - p_1} < 0 \\
&\iff \frac{p_1 m + 0.5(1 - s)}{m + 1 - s + 0.5(1 - s)(3p_2 - 1)} - 1 < 0.
\end{aligned} \tag{A23}$$

Since the last statement is true $F_2(1) < 0$. Thus the equilibrium where manager 1 does not hedge and manager 2 hedges does not exist. However, there exists an unique $a(2)$ such that $F_2(a(2)) = 0$. If at this $a(2)$, $F_1(a(2)) < 0$ we have an equilibrium.

Now

$$\begin{aligned}
\frac{dF_1(a(2))}{da(2)} &= -(3p_1 - 1)(3p_2 - 1)0.5(1 - s) \frac{[p_1 m + 0.5(1 - s)]\{0.5(1 - s)(3p_2 - 1)\}}{[(m + 1 - s + 0.5(1 - s)a(2)(3p_2 - 1)]^2} \\
&\quad - (3p_2 - 1)(3p_1 - 1)0.5(1 - s) \frac{[p_2 m + 0.5(1 - s)]\{0.5(1 - s)(3p_1 - 1)\}}{[(m + 1 - s + 0.5(1 - s)a(2)(3p_1 - 1)]^2} \\
&\quad - \frac{p_1^2 p_2}{(1 - a(2)p_2)^2} - \frac{p_2^2 p_1}{(1 - a(2)p_1)^2}.
\end{aligned} \tag{A24}$$

When $p_1 \leq 2/3$, $dF_1(a(2)) / da(2) < 0$ and $F_1(0) < 0$ imply that $F_1(a(2)) < 0$ for all $a(2)$ and thus equilibrium exists. When $p_1 > 2/3$, a solution always exists. We verify

this using a computer grid search over the region $1/3 < s < 0.5$ and the $p_1 > 2/3$.¹⁶

■

Proof of Theorem 4: Type A equilibria require that $U(m(\cdot), H, 1) > U(m(\cdot), NH, 1)$ and $U(m(\cdot), H, 2) (<) = (>) U(m(\cdot), NH, 2)$. The inequality for the lower ability manager is:

$$\begin{aligned}
& (p_1 - p_2)[p_2 m(z_2) + p_1 m(z_3)] + p_2 \\
& (<) = (>) \quad (p_1 - p_2)[\{p_2 m + 0.5(1 - s)\}m(z_2) + \{p_1 m + 0.5(1 - s)\}m(z_3)] \\
& \quad - \frac{\alpha}{\beta} p_1 0.5(1 - s)z_4 \\
& \iff (p_1 - p_2) [(3p_2 - 1)p_1 [1 - (1 - a(2))0.5(1 - s)(3p_1 - 1)] \\
& \quad + (3p_1 - 1)p_2 [1 - (1 - a(2))0.5(1 - s)(3p_2 - 1)]] \\
& = (>) = (<) - \frac{\alpha}{\beta} p_1 0.5(1 - s)z_4 \times \\
& \quad [1 - (1 - a(2))0.5(1 - s)(3p_1 - 1)][1 - (1 - a(2))0.5(1 - s)(3p_2 - 1)] \\
& \iff (p_1 - p_2)[6p_1 p_2 - 1 - (1 - a(2))0.5(1 - s)(9p_1 p_2 - 2)] \\
& (>) = (<) - \frac{\alpha}{\beta} p_1 z_4 [1 - (1 - a(2))0.5(1 - s) + (1 - a(2))^2 0.25(1 - s)^2 (9p_1 p_2 - 2)]
\end{aligned} \tag{A25}$$

We first look at the case where $a(2) = 1$; the lower ability manager hedges.

We thus need that

$$(2p_1 - 1)(6p_1 - 6p_1^2 - 1) > -\frac{\alpha}{\beta} z_4 p_1 \tag{A26}$$

where $p_2 = 1 - p_1$ is used. Note that the left hand side is zero at $p_1 = 0.5$ while the right hand side is strictly positive. Also the left hand side has derivative $4(9p_1 p_2 - 2)$ and thus is a concave function with a maximum at $p_1 = 2/3$. The right hand side is a linear function in p_1 . Hence there are exactly two intersections or none. There

¹⁶ This computer program is available from the authors on request.

exists a region $p_1 \in (\zeta_l^1, \zeta_h^1)$ (or $d \in (\delta_l^1, \delta_h^1)$), possibly empty in which the lower ability manager finds it optimal to hedge.

Now consider the case where the lower ability manager does not hedge; $a(2) = 0$. The inequality for the lower ability manager simplifies to

$$\begin{aligned} & \left(2 - \frac{1}{p_1}\right)[6p_1p_2 - 1 - 0.5(1-s)(9p_1p_2 - 2)] \\ & < -\frac{\alpha}{\beta}z_4[1 - 0.5(1-s) + 0.25(1-s)^2(9p_1p_2 - 2)]. \end{aligned} \quad (A27)$$

The second derivative of the left hand side of equation (A27) is

$$\begin{aligned} & -\frac{2}{p_1^2}[6p_1p_2 - 1 - 0.5(1-s)(9p_1p_2 - 2)] + \frac{1}{p_1^2}[6 - \frac{9}{2}(1-s)](1 - 2p_1) \\ & + [6 - \frac{9}{2}(1-s)] \left\{ \frac{1}{p_1^2}(1 - 2p_1) - 2\left(2 - \frac{1}{p_1}\right) \right\} < 0. \end{aligned} \quad (A28)$$

Thus the left hand side is concave in p_1 . The right hand side has derivative $-\frac{\alpha}{\beta}z_4 \times 0.25(1-s)^2 \times 9(1-2p_1)$ and thus is a decreasing concave function. At $p_1 = 0.5$ and $p_1 = 1$, the left hand side is negative (or zero) while the right hand side is always positive; thus if an intersection occurs at least two intersections exist.

Equation (A27) is a cubic equation and has at most three real roots; we need to ensure that this third root is not between $p_1 = 0.5$ and $p_1 = 1$. But at $p_1 = 0$, the left hand side is s and the right hand side is 0. Thus a root exists between $p_1 = 0$ and $p_1 = 0.5$. Call the two intersection points between 0.5 and 1 ζ_l^0 and ζ_h^0 , $\zeta_l^0 < \zeta_h^0$. Next while considering randomized strategies, we show that $\zeta_l^0 < \zeta_l^1 < \zeta_h^1 < \zeta_h^0$. In the region outside (ζ_l^0, ζ_h^0) (the corresponding region for d is (δ_l^0, δ_h^0)), the lower ability manager does not hedge. In the region within (ζ_l^1, ζ_h^1) (the corresponding region for d is (δ_l^1, δ_h^1)), the lower ability manager hedges. In the intermediate region he randomizes.

Randomization is an optimal strategy if $U(m(\cdot), H, 2) = U(m(\cdot), NH, 2)$ or

$$\begin{aligned} & \left(2 - \frac{1}{p_1}\right)[6p_1p_2 - 1 - (1 - a(2))0.5(1 - s)(9p_1p_2 - 2)] \\ &= -\frac{\alpha}{\beta}z_4[1 - (1 - a(2))0.5(1 - s) + (1 - a(2))^2 0.25(1 - s)^2(9p_1p_2 - 2)]. \end{aligned} \tag{A29}$$

The left hand side is $1 - (1 - a(2))(1 - s) > 0$ at $p_1 = 0$ while the right hand side is 0. There is at least one root between 0 and 0.5. At $p_1 = 0.5$ the left hand side is 0 while the right hand side is a positive number. Finally for $p_1 = 1$, the left hand side is $-[1 - (1 - a(2))(1 - s)] < 0$ while the right hand side is still positive. Thus there are two intersections or no intersections at all between 0.5 and 1; more than two cannot occur as there are at most three roots to the equation and one root lies between 0 and 0.5. Call these intersection points $\zeta_l^{a(2)}$ and $\zeta_h^{a(2)}$.

The left hand side of (A29) is concave in p_1 while the right hand side is decreasing and concave in p_1 . Suppose $a(2)' > a(2)$. Then for $p_1 < 2/3$, $LHS(a(2)', p_1) > LHS(a(2), p_1)$ and for $p_1 > 2/3$, $LHS(a(2)', p_1) < LHS(a(2), p_1)$. Also $RHS(a(2)', p_1) > RHS(a(2), p_1)$.

Suppose $a(2)' > a(2)$. We first show that if $a(2)' < 2/3$, then $\zeta_l^{a(2)} < \zeta_l^{a(2)'}$. Since $\zeta_l^1 < 2/3$, the assumption that $\zeta_l^{a(2)'} < 2/3$ is justified by the very same argument. To show this claim note that at $p_1 = \zeta_l^{a(2)'}$,

$$\begin{aligned} & LHS(a(2)', p_1) = RHS(a(2)', p_1) \\ & \iff (p_1 - p_2)[6p_1p_2 - 1 - (1 - a(2)')0.5(1 - s)(9p_1p_2 - 2)] \\ &= -\frac{\alpha}{\beta}p_1z_4[1 - (1 - a(2)')0.5(1 - s) + (1 - a(2)')^2 0.25(1 - s)^2(9p_1p_2 - 2)]. \end{aligned} \tag{A30}$$

Then at $p_1 = \zeta_l^{a(2)'}$,

$$\begin{aligned}
& LHS(a(2), p_1) < RHS(a(2), p_1) \\
& \iff (p_1 - p_2)[6p_1p_2 - 1 - (1 - a(2))0.5(1 - s)(9p_1p_2 - 2)] \\
& \quad < -\frac{\alpha}{\beta}p_1z_4[1 - (1 - a(2))0.5(1 - s) + (1 - a(2))^20.25(1 - s)^2(9p_1p_2 - 2)] \\
& \iff (1 - 3p_1p_2) > (6p_1p_2 - 1)0.5(1 - s)(9p_1p_2 - 2)(2 - a(2)' - a(2)) \\
& \quad - 0.25(1 - s)^2(9p_1p_2 - 2)^2(1 - a(2))(1 - a(2)')
\end{aligned} \tag{A31}$$

where $LHS(a(2)', p_1) = RHS(a(2)', p_1)$ has been used. The left hand side of the last equation in (A31) is lowest when $p_1 = 0.5$ where its value is 0.25. The right hand side of the last equation in (A31) is highest at 0.5 provided $p_1 < 2/3$. At $p_1 = 0.5$, the right hand side of last equation in (A31) has value $0.125(1 - s)$ which is less than 0.25. Thus a root of the equation exists between 0.5 and $\zeta_l^{a(2)'}$. Hence $\zeta_l^{a(2)} < \zeta_l^{a(2)'}$.

To prove that $\zeta_h^{a(2)} > \zeta_h^{a(2)'}$, consider first the case where $\zeta_h^{a(2)'} > 2/3$. In this case $LHS(a(2), p_1) > LHS(a(2)', p_1) = RHS(a(2)', p_1) > RHS(a(2), p_1)$. Thus $\zeta_h^{a(2)} > \zeta_h^{a(2)'}$. On the other hand if $\zeta_h^{a(2)'} < 2/3$ an argument similar to that in the prior paragraph works and at $p_1 = \zeta_h^{a(2)'}$, $LHS(a(2), p_1) > RHS(a(2), p_1)$. Hence a second root exist at some p_1 value greater than $\zeta_h^{a(2)'}$. This proves the proposition.

Finally it is easy to show that as $-\frac{\alpha}{\beta}z_4$ increases the region (ζ_l^1, ζ_h^1) disappears leaving the region (ζ_l^0, ζ_h^0) within which randomization occurs. Eventually this region also disappears and for all p_1 values the lower ability manager finds it optimal not to hedge.

We now turn to the higher ability manager. He hedges as long as long as

$$\begin{aligned}
U(m(\cdot), H, 1) &> U(m(\cdot), NH, 1) \\
\iff (p_1 - p_2)[0.5(1 - s)(3p_1 - 1)m(z_2) + 0.5(1 - s)(3p_2 - 1)m(z_3)] \\
&> -\frac{\alpha}{\beta}p_2 0.5(1 - s)z_4 \\
\iff (p_1 - p_2) &\left[\frac{p_1(3p_1 - 1)}{1 - (1 - a(2))0.5(1 - s)(3p_2 - 1)} \right. \\
&\left. + \frac{p_2(3p_2 - 1)}{1 - (1 - a(2))0.5(1 - s)(3p_1 - 1)} \right] > -\frac{\alpha}{\beta}p_1 z_4
\end{aligned} \tag{A32}$$

After much simplification the above inequality reduces to

$$\begin{aligned}
\frac{p_1 - p_2}{p_2} [2 - 6p_1p_2 - (1 - a(2))0.5(1 - s)[4 - 15p_1p_2]] \\
> -\frac{\alpha}{\beta}z_4 \left[1 - (1 - a(2))0.5(1 - s) + 0.25(1 - a(2))^2(1 - s)^2(9p_1p_2 - 2) \right].
\end{aligned} \tag{A33}$$

The left hand side of the last equation in (A33) is strictly increasing in p_1 while the right hand side of the same equation is strictly decreasing in p_1 . Also the left hand side of the last equation in (A33) is zero at $p_1 = 0.5$ while the right hand side of the same equation is positive. At $p_1 = 1$, the left hand side of the last equation in (A33) is ∞ while the right hand side of the same equation is a finite number. Thus $\zeta(a(2))$ exists such that for $p_1 > \zeta(a(2))$, hedging is optimal.

Next, we show that $\zeta(a(2)) < \zeta_l^{a(2)}$. To prove this note that we need to compare equation (A29) with equation (A33). The right hand side of equation (A33) above is less than in equation (A29) as $p_2 < p_1$. The left hand side of equation (A33) is greater than the left hand side of equation (A29) provided that

$$\begin{aligned}
2 - 6p_1p_2 - (1 - a(2))0.5(1 - s)[4 - 15p_1p_2] \\
> 6p_1p_2 - 1 - (1 - a(2))0.5(1 - s)(9p_1p_2 - 2) \\
\iff 1 - 4p_1p_2 - (1 - a(2))(1 - s)(1 - 4p_1p_2) > 0
\end{aligned} \tag{A34}$$

Since this inequality holds $\zeta(a(2)) < \zeta_l^{a(2)}$. Define $\zeta(s) = \zeta(a(2))$. Then $\zeta(s) < \zeta_a^0$. The equilibrium constructed above then holds for $p_1 > \zeta(s)$ (similarly we have $d > \delta(s)$). In this region, given the lower ability manager's optimal action, the higher ability manager finds it optimal to hedge. ■

Proof of Theorem 5: Type B equilibria involve the lower ability manager not hedging [$U(m(\cdot), H, 2) < U(m(\cdot), NH, 2)$] and the higher ability manager not hedging or randomizing [$(U(m(\cdot), H, 1) < U(m(\cdot), NH, 1)$ or $U(m(\cdot), H, 1) = U(m(\cdot), NH, 2)$]. The inequality for the lower ability manager reduces to

$$(p_1 - p_2)F_2(a(1)) < -\frac{\alpha}{\beta}p_1z_4. \quad (A35)$$

The inequality for the higher ability manager

$$(p_1 - p_2)F_1(a(1)) = -\frac{\alpha}{\beta}p_2z_4 \quad (A36)$$

if he randomizes or

$$(p_1 - p_2)F_1(0) < -\frac{\alpha}{\beta}p_2z_4 \quad (A37)$$

if he does not hedge. $F_1(a(1))$ and $F_2(a(2))$ are as defined in part (b) of the proof of Theorem 3. From that proof we know that when $s > 0.5$, $F_1(0) > 0$ and $F_2(0) < 0$. Since $0 < -\frac{\alpha}{\beta}p_2z_4 < -\frac{\alpha}{\beta}p_1z_4$, an equilibrium where both managers do not hedge exists when

$$\begin{aligned} (p_1 - p_2)F_1(0) &< -\frac{\alpha}{\beta}p_1z_4 \\ \iff (p_1 - p_2) \times \\ &\left[(3p_1 - 1)\frac{p_1m + 0.5(1-s)}{m+1-s} + (3p_2 - 1)\frac{p_2m + 0.5(1-s)}{m+1-s} - p_1^2 - p_2^2 \right] \\ &< -\frac{\alpha}{\beta}p_2z_4 \\ \iff (p_1 - p_2)(2s - 1)[2(p_1^2 + p_2^2) - 1] &< -\frac{\alpha}{\beta}p_2z_4(m + 1 - s). \end{aligned} \quad (A38)$$

The left hand side of the last equation in (A38) is increasing in p_1 while the right hand side is decreasing. Also, $LHS(0.5) = 0 < RHS(0.5)$ while $LHS(1) > 0 = RHS(1)$. Thus a unique solution $\omega(s)$ exists such that for all $p_1 < \omega(s)$ (similarly $d < \gamma(s)$), equation (A38) holds.

To prove that $\omega(s) > \zeta(s)$ (or $\gamma(s) > \delta(s)$) argue as follows. At $\zeta(s)$ when the market believes that the higher ability manager hedges and the lower ability manager does not $U(m(\cdot), H, 1) = U(m(\cdot), NH, 1)$ and $U(m(\cdot), H, 2) < U(m(\cdot), NH, 2)$. But this is just

$$\begin{aligned} (p_1 - p_2)F_1(1) &= -\frac{\alpha}{\beta}p_2z_4 \\ (p_1 - p_2)F_2(1) &< -\frac{\alpha}{\beta}p_1z_4. \end{aligned} \tag{A39}$$

Since $dF_1(a(1)) / da(1) > 0$,

$$(p_1 - p_2)F_1(0) < (p_1 - p_2)F_1(1) = -\frac{\alpha}{\beta}p_2z_4 \tag{A40}$$

and

$$(p_1 - p_2)F_2(0) < 0 < -\frac{\alpha}{\beta}p_1z_4. \tag{A41}$$

Thus both managers not hedging is not an equilibrium and $\omega(s) > \zeta(s)$ (or $\gamma(s) > \delta(s)$).

When $s < 0.5$, $F_1(0) < 0$ and $F_2(0) > 0$. Now an equilibrium with both managers not hedging exist as long as

$$\begin{aligned} (p_1 - p_2)F_2(0) &< -\frac{\alpha}{\beta}p_1z_4 \\ \iff (p_1 - p_2)(2s - 1)(4p_1p_2 - 1) &< -\frac{\alpha}{\beta}p_1z_4(m + 1 - s) \end{aligned} \tag{A42}$$

The left hand side is a convex increasing function. The right hand side is linear function with slope $-\frac{\alpha}{\beta}z_4(m + 1 - s)$. Since $LHS(0.5) = 0 < RHS(0.5)$, one intersection or none exists. Thus $\omega(s)$ (and hence $\gamma(s)$) is well defined.

To prove that $\omega(s) > \zeta(s)$ (and $\gamma(s) > \delta(s)$) argue as follows. At $\zeta(s)$,

$$\begin{aligned}(p_1 - p_2)F_1(1) &= -\frac{\alpha}{\beta}p_2z_4 \\ (p_1 - p_2)F_2(1) &< -\frac{\alpha}{\beta}p_1z_4.\end{aligned}\tag{A43}$$

Now $F_1(1) > F_2(1)$. If we show that $F_2(0) < F_1(1)$ we are done as then

$$\begin{aligned}(p_1 - p_2)F_2(0) &< (p_1 - p_2)F_1(1) = -\frac{\alpha}{\beta}p_2z_4 < -\frac{\alpha}{\beta}p_1z_4 \\ (p_1 - p_2)F_1(0) &< 0 < -\frac{\alpha}{\beta}p_2z_4.\end{aligned}\tag{A44}$$

When $p_1 < 2/3$, $dF_2(a(1))/da(1) > 0$. Then $F_2(0) < F_2(1) < F_1(1)$ follows.

When $p_1 > 2/3$, the inequality $F_2(0) < F_1(1)$ is simplified as follows:

$$\begin{aligned}&\left[\frac{(1-2s)(1-4p_1p_2)}{m+1-s} \right] \\ &\quad \times \left[(m+1-s)^2 + 0.5(1-s)(m+1-s) + 0.25(1-s)^2(9p_1p_2-1) \right] \\ &< (3p_1-1) \\ &\quad \times [0.5(1-s)(3p_1-1) + p_1m + 0.5(1-s)][0.5(1-s)(3p_2-1) + m+1-s] \\ &+ (3p_2-1) \\ &\quad [0.5(1-s)(3p_2-1) + p_2m + 0.5(1-s)][0.5(1-s)(3p_1-1) + m+1-s] \\ &\iff \frac{(1-2s)(1-4p_1p_2)}{m+1-s} \times \\ &\quad \left[(m+1-s)^2 + 0.5(1-s)(m+1-s) + 0.25(1-s)^2(9p_1p_2-1) \right] \\ &< [(m+1-s)(1-s) - 0.5(1-s)](2-9p_1p_2) \\ &\quad + (m+1-s)2m(1-3p_1p_2) + (m+1-s)0.5(1-s)\end{aligned}\tag{A45}$$

On the left hand side the term associated with $9p_1p_2 - 2$ is negative as $p_1 > 2/3$. On the right hand side, $2 - 9p_1p_2$ and $(m + 0.5 - s)(1 - s)$ are positive. Thus it suffices

that

$$\begin{aligned}
& (1 - 2s)(1 - 4p_1p_2)[m + 1 - s + 0.5(1 - s)] \\
& < (m + 1 - s)2m(1 - 3p_1p_2) + (m + 1 - s)0.5(1 - s) \\
& \iff (1 - 2s)(1 - 4p_1p_2) < (m + 1 - s)2m(1 - 3p_1p_2) + (m + 1 - s)0.5(1 - s)
\end{aligned} \tag{A46}$$

where $m + 1 - s + 0.5(1 - s) = 1$ is used. This inequality is true as

$$\begin{aligned}
1 - 2s & < (m + 1 - s)2m\frac{1}{4} + (m + 1 - s)0.5(1 - s) \\
\iff 1 - 2s & < 0.5(1 + s)\left[\frac{s}{2} - \frac{1 - s}{4} + \frac{1 - s}{2}\right] \\
\iff 3 & < 9s.
\end{aligned} \tag{A47}$$

Since $s > 1/3$ this is true. Hence $\omega(s) > \zeta(s)$ (and $\gamma(s) > \delta(s)$).

We now turn to equilibria where the high ability manager randomizes and the low ability manager does not hedge. When $1/3 < s < 0.5$, we know that $F_1(0) < 0$ and that $dF_1(a(1))/da(1) > 0$. Hence there is a solution to the problem $(p_1 - p_2)F_1(1) = A_{max}p_2$. Thus a solution to the equation $(p_1 - p_2)F_1(a(1)) = Ap_2$ exists in the range $(0, A_{max})$. However at $A = 0$, $(p_1 - p_2)F_2(0) > 0$ and no solution exists. At $A = 1$, $(p_1 - p_2)F_2(0) < (p_1 - p_2)F_1(0) = Ap_2 < Ap_1$ and a solution exists. Let $G(A) = (p_1 - p_2)F_2(x(A)) - Ap_1$ where $x(A)$ solves $(p_1 - p_2)F_1(x(A)) = Ap_2$. Then

$$\begin{aligned}
\frac{dG}{dA} & = (p_1 - p_2)\frac{dF_2}{dx}\frac{dx}{dA} - p_1 \\
& = (p_1 - p_2)\frac{dF_2}{dx}\frac{p_2}{(p_1 - p_2)\frac{dF_1}{dx}} - p_1 \\
& = p_2\frac{\frac{dF_2}{dx}}{\frac{dF_1}{dx}} - p_1 \\
& < 0
\end{aligned} \tag{A48}$$

using the fact that $dF_1(x)/dx > dF_2(x)/dx$. Hence there is a unique region (A_{min}, A_{max}) where a solution exists. From this, given A , we can show that there

is an unique region $\delta(s) < d < \pi(s)$ where $\pi(s) < \gamma(s)$ (the bound $\delta(s)$ comes from noting that the equality $(p_1 - p_2)F_1(1) = Ap_2$ defines $\delta(s)$).

When $0.5 < s < 1$, the proof is a little different. At $A = 0$, we know that $F_2(0) < 0$. Hence the behavior of equilibrium depends on the higher ability agent. Since $F_1(0) > 0$, there is an A_{min} such that $(p_1 - p_2)F_1(0) = A_{min}p_2$. By the same argument as before, there is also an A_{max} such that $(p_1 - p_2)F_1(1) = A_{max}p_2$.

We claim that equilibrium exists in the region (A_{min}, A_{max}) . At A_{min} , we know that $(p_1 - p_2)F_2(0) < 0 < A_{min}p_1$. Hence, equilibrium exists. The fact that $dF_1(x)/dx > dF_2(x)/dx$ then implies that $(p_1 - p_2)F_2(x(A)) < Ap_1$ where $x = a(1)$ is the randomized strategy for the higher manager given A . From this we can deduce that given A , there is an unique region $\delta(s) < d < \pi(s)$ where $\pi(s) = \gamma(s)$. Again, the bound $\delta(s)$ comes from noting that the equality $(p_1 - p_2)F_1(1) = Ap_2$ defines $\delta(s)$. ■

Proof of Theorem 6: The theorem consists of three parts:

- a. a proof that strategies where $a(2) = 1$ and $0 < a(1) < 1$ are not equilibria.
- b. a proof that strategies where $0 < a(1) < 1$ and $0 < a(2) < 1$ are not equilibria.

Hence Type C equilibria do not exist.

- c. a characterization of equilibria where $a(1) = 0$ and $0 \leq a(2) \leq 1$. These are Type D equilibria.

Part a. When hedging was costless we have that $U(m(\cdot), H, 2) < U(m(\cdot), NH, 2)$.

With hedging costly, this inequality must still hold. Thus no equilibrium where $a(2) = 1$ and $0 < a(1) < 1$ exists. ■

Part b. By mimicking the proof of part (b) of Theorem 1, one can show that both

manager randomizing reduces to finding λ and θ such that

$$\begin{aligned}\hat{F}_1(\lambda, \theta) &= F_1(\lambda, \theta) - \frac{1 - \alpha 0.5(1 - s)z_4}{3 \beta(p_1 - p_2)} = 0 \\ \hat{F}_2(\lambda, \theta) &= F_2(\lambda, \theta) - \frac{2 - \alpha 0.5(1 - s)z_4}{3 \beta(p_1 - p_2)} = 0\end{aligned}\tag{A49}$$

where $F_1(\lambda, \theta)$ and $F_2(\lambda, \theta)$ are defined in Equations (A17) and (A18). One can show that the restriction above implies that $2F_1(\lambda, \theta) = F_2(\lambda, \theta)$. If we impose this restriction and any one of the restrictions in Equations (A49) above and search over the space $1/3 < s < 1$ and $1/2 < p_1 < 1$, there is no equilibrium where both kinds of managers randomize.¹⁷ ■

Part c. In this case, the higher ability manager does not hedge and hence $U(m(\cdot), H, 1) < U(m(\cdot), NH, 1)$. The lower ability manager hedges or randomizes between hedging and not hedging and thus $U(m(\cdot), H, 2) \geq U(m(\cdot), NH, 2)$. Equivalently, these inequalities can be stated as

$$(p_1 - p_2)F_1(a(2)) < -\frac{\alpha}{\beta}p_2z_4\tag{A50}$$

and

$$(p_1 - p_2)F_2(a(2)) \geq -\frac{\alpha}{\beta}p_1z_4\tag{A51}$$

where $F_1(a(2))$ and $F_2(a(2))$ are as defined in part (c) of the proof of Theorem 3 (Equation (A20)).

When $s > 0.5$, $F_1(0) > 0$ and $F_2(0) < 0$. Also $dF_2(a(2))/da(2) < 0$. Thus $(p_1 - p_2)F_2(a(2)) < -\frac{\alpha}{\beta}p_1z_4$ and no equilibrium where the higher ability manager does not hedge while the lower ability manager hedges or randomizes between hedging and not hedging is not possible.

¹⁷ Again, the computer program is available from the authors on request.

When $s < 0.5$, $F_1(0) < 0$ and $F_2(0) > 0$. Also $F_2(1) < 0$ and thus $(p_1 - p_2)F_2(1) < 0 < -\frac{\alpha}{\beta}p_1z_4$. Thus no equilibria exist where the higher ability manager does not hedge and lower ability manager hedges.

Equilibria involving randomization do exist if $(p_1 - p_2)F_2(0) > -\frac{\alpha}{\beta}p_1z_4$. In fact, such equilibria clearly exist at $d = \delta(s)$ as the lower ability manager is indifferent between hedging and not hedging at this point. They exist for the region $\delta(s) < d < 1$.¹⁸ ■

Proof of Theorem 7: To compute this bound, we use Equation (A51). If at $a(2) = 0$, we can show that Equation (A51) cannot hold we are done as $dF_2(a(2))/da(2) < 0$. When $a(2) = 0$, Equation (A51) reduces to:

$$(2s - 1)(4p_1p_2 - 1)\frac{p_1 - p_2}{m + 1 - s} < -\frac{\alpha}{\beta}p_1z_4 \quad (A52)$$

The highest value of the left hand side is reached when $s = 1/3$. Substituting this in, we obtain that:

$$\frac{1}{2}(p_1 - p_2)(1 - 4p_1p_2) < -\frac{\alpha}{\beta}p_1z_4 \quad (A53)$$

which is desired bound. The rest of the theorem follows from substituting $d = p_1 - p_2$ and $z_4 = y_l - r_b$ and simplifying. ■

¹⁸ The existence up to $d = 1$ is verified by a computer program that is available from the authors.