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Empirical Tests of the Consumption-Oriented CAPM

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ABSTRACT

The empirical implications of the consumption-oriented capital asset pricing model (CCAPM) are examined, and its performance is compared with a model based on the market portfolio. The CCAPM is estimated after adjusting for measurement problems associated with reported consumption data. The CCAPM is tested using betas based on both consumption and the portfolio having the maximum correlation with consumption. As predicted by the CCAPM, the market price of risk is significantly positive, and the estimate of the real interest rate is close to zero. The performances of the traditional CAPM and the CCAPM are about the same.

IN AN INTERTEMPORAL ECONOMY, Rubinstein (1976), Breeden and Litzenberger (1978), and Breeden (1979) demonstrate that equilibrium expected excess returns are proportional to their "consumption betas." This contrasts with the market-oriented capital asset pricing model (hereafter, CAPM) derived in a single-period economy by Sharpe (1964) and Lintner (1965). While tests of the CAPM by Black, Jensen, and Scholes (1972), Fama and MacBeth (1973), Gibbons (1982), and others find a positive association between average excess returns and betas using a proxy for the market portfolio, the relation is not proportional. This paper studies similar empirical issues for the consumption-oriented capital asset pricing model (hereafter, CCAPM).

Even though the relevant market portfolio includes all assets, most empirical research focuses on common stocks for which accurately measured data are available. In contrast, reported consumption data are estimates of the relevant consumption flows, and the data are subject to measurement problems not found with stock indexes. In this paper the tests of the CCAPM incorporate some adjustments for these measurement problems.

The outline of the paper is as follows. Section I provides an alternative derivation of the CCAPM. Section II examines four econometric problems associated with measured consumption: the durables problem, the problem of

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measured consumption as an integral of spot consumption rates, the problem that consumption data are reported infrequently, and the problem of pure sampling error in consumption measures. Time series properties of consumption measures are also discussed in Section II. Section III analyzes the empirical characteristics of estimated consumption betas for various stock and bond portfolios. The composition of the portfolio whose return has the highest correlation with the growth rate of real, per capita consumption is also discussed in Section III. This portfolio is used in some of the tests of the model. Section IV presents empirical tests of the consumption and market-oriented CAPMs based on their implications for unconditional moments. Section V concludes the paper with a review of the results obtained.

I. A Synthesis of the CCAPM Theory

The Rubinstein (1976) derivation of the CCAPM assumes that, over a discrete time interval, the joint distribution of all assets' returns with each individual's optimal consumption is normal. More generally, Breeden and Litzenberger (1978) derive the CCAPM in a discrete-time framework for the *subset of assets* whose returns are jointly lognormally distributed with aggregate consumption. Breeden's (1979) continuous-time derivation of the CCAPM applies instantaneously to all assets, based on the assumption that assets' returns and individuals' optimal consumption paths follow diffusion processes. In all these papers, utility functions are time additive.

Since the CCAPM is well known, a standard review is unnecessary. The following synthesis provides a theoretical basis that is more relevant for the subsequent empirical work. In particular, theoretical predictions are derived for easily estimated models which are based on unconditional moments of returns using discretely sampled data.

Let $\{\tilde{R}_{it}, i = 1, \cdots, M\}$ be the rates of return on risky assets from time $t-1$ to time t. M may be less than the number of all risky assets in the economy. Let $\tilde{R}_{zt}$ be the rates of return on a portfolio whose return is uncorrelated with the growth rate in aggregate consumption. All individuals are assumed to have time-additive, monotonically increasing, and strictly concave von Neumann-Morgenstern utility functions for lifetime consumption. Identical beliefs, a fixed population with infinite lifetimes, a single consumption good, and capital markets that permit an unconstrained Pareto-optimal allocation of consumption are also assumed. From the first-order conditions for individual k's optimal consumption and portfolio plan, it follows that

$$\mathscr{E}[(\tilde{R}_{it} - \tilde{R}_{zt})[U^{k'}(\tilde{C}_t^k)/U^{k'}(C_{t-1}^k)] \mid \phi_{t-1}] = 0, \forall\ i, k, \tag{1}$$

where ϕ_{t-1} describes the full information set at time $t-1$. This relation holds for any sampling interval. This is well known (e.g., see Lucas (1978)).

An individual achieves an optimal portfolio by adjusting the portfolio weights and consumption plans until relation (1) holds for all assets. Breeden and Litzenberger (1978) show that, in a capital market that permits an unconstrained Pareto-optimal allocation of consumption, each individual's consumption at a given date is an increasing function of *aggregate* consumption. Furthermore, each individual's optimal marginal utility of consumption at a given date t is equal to a scalar, a_k, times a monotonically decreasing function of aggregate consumption, $g(C_t,t)$, which is identical for all individuals. The assumption that all individuals have the same subjective rate of time preference implies that the time dependence of the aggregate marginal utility function is the same for all dates, so $g(C_t,t) = f(C_t)$. Thus, in equilibrium in a Pareto-efficient capital market, the growth rate in the marginal utility of consumption would be identical for all individuals and equal to the growth rate in the "aggregate marginal utility" of consumption. That is,

$$\frac{U'(C_t)}{U'(C_{t-1})} = \frac{f(C_t)}{f(C_{t-1})} \cong 1 - [-C_{t-1}f'(C_{t-1})/f(C_{t-1})]c_t^*, \tag{2}$$

where c_t^* is the growth rate in aggregate consumption per capita and where the approximation follows from a first-order Taylor series. The term in square brackets is aggregate relative risk aversion evaluated at C_{t-1}. If we take relative risk aversion as approximately constant and denote it as b, we can combine (1) with these approximations in (2) and find (ignoring the approximations)[1]

$$\mathscr{E}\{(\tilde{R}_{it} - \tilde{R}_{zt})(1 - b\tilde{c}_t^*)|\phi_{t-1}\} = 0. \tag{3}$$

Since (3) is zero conditional on any information, it also holds in terms of unconditional expectations:

$$\mathscr{E}\{(\tilde{R}_{it} - \tilde{R}_{zt})(1 - b\tilde{c}_t^*)\} = 0. \tag{4}$$

The return on an asset may be stated as a linear function of the growth rate in aggregate consumption per capita, c_t^*, plus a disturbance. This disturbance term is assumed to be uncorrelated with $\tilde{c}_t^*$ for a proper subset of assets ($i = 1, \cdots, M$). These conditions, combined with the assumption of constant unconditional consumption betas and alphas, imply

$$\begin{gathered}\tilde{R}_{it} = \alpha_{ci}^* + \beta_{ci}^*\tilde{c}_t^* + \tilde{u}_{it}^*, \quad \forall i = 1, \cdots, M, \\ \mathscr{E}\{\tilde{u}_{it}^*\} = 0 \qquad \text{and} \qquad \mathscr{E}\{\tilde{u}_{it}^*\tilde{c}_t^*\} = 0,\end{gathered} \tag{5}$$

where $\beta_{ci}^* \equiv \text{cov}(\tilde{R}_{it}, \tilde{c}_t^*)/\text{var}(\tilde{c}_t^*)$, $\alpha_{ci}^* \equiv \mu_i - \beta_{ci}^*\mathscr{E}\{\tilde{c}_t^*\}$, and $\mu_i \equiv \mathscr{E}\{\tilde{R}_{it}\}$. Asterisks indicate parameters in relation to true consumption growth. Later asterisks are removed to indicate parameters in relation to measured consumption growth.

For a zero consumption beta portfolio consisting of just the M assets,

$$\begin{gathered}\tilde{R}_{zt} = \gamma_0 + \tilde{\mu}_{zt}^*, \\ \mathscr{E}\{\tilde{u}_{zt}\} = 0, \\ \mathscr{E}\{\tilde{u}_{zt}\tilde{c}_t^*\} = 0.\end{gathered} \tag{6}$$

Substituting the right-hand side (hereafter, RHS) of (5) and (6) into relation (4) gives the CCAPM:

$$\mu_i - \gamma_0 = \gamma_1^*\beta_{ci}^*, \qquad \forall\ i = 1, \cdots, M, \tag{7}$$

[1] The approximation can be avoided by making an additional distributional assumption that $\text{cov}(\tilde{u}_{it}^*, \tilde{X}_t) = 0$, where $X_t \equiv f(\tilde{c}_t)/f(\tilde{c}_{t-1})$ and $\tilde{u}_{it}^*$ is defined in (5) below. All the following results go through, and $\gamma_1^* \equiv \text{cov}(\tilde{c}_{t1} - \tilde{X}_t)/E(\tilde{X}_t)$. The market price of consumption beta risk, γ_1^*, appears in equation (7) below).

where $\gamma_1^* \equiv b\ \mathrm{var}(\tilde{c}_t^*)/[1 - b\mathscr{E}(\tilde{c}_t^*)]$. The market price of consumption beta risk, γ_1^*, increases as the variability of consumption increases. If $[1 - b\mathscr{E}(\tilde{c}_t^*)] > 0$ and $\mathscr{E}(\tilde{c}_t^*) > 0$, then γ_1^* also increases as relative risk aversion increases.

This model only gives the CCAPM for a proper subset of assets—those assets that have a conditionally linear relation with c_t^* over the measurement interval. Assets which do not satisfy (5) still are priced according to their joint distributions of payoffs with consumption, but higher order co-moments with consumption are required for pricing over discrete intervals. Since in the continuous-time model all assets' returns and consumption are locally jointly normally distributed, the CCAPM applies to all assets as long as returns can be measured over instantaneous intervals. However, since the available data are measured discretely, the CCAPM in (7) is more useful for empirical tests.

In continuous time with time-additive utility, Breeden (1979) demonstrates that Merton's (1973) intertemporal multi-beta asset pricing model is equivalent to a single-beta CCAPM. However, Cornell (1981) emphasizes that the conditional consumption beta in such a representation need not be constant. The tests presented in this paper are tests of restrictions on the unconditional co-moments of assets returns and consumption growth. As Grossman and Shiller (1982) point out, such tests do not ignore Cornell's (1981) concerns about changes in the conditional moments. An advantage of tests based on unconditional moments is that a specification of the changes in conditional moments is not required. To the extent that changes in the conditional moments could be modeled, the resulting tests may be more powerful. For examples of such tests see Gibbons and Ferson (1985), Hansen and Singleton (1983), and Litzenberger and Ronn (1986). Since the CCAPM has predictions for conditional and unconditional expectations, failure to reject the "unconditional CCAPM" is a necessary, but not sufficient, condition for acceptance of the model.

II. Econometric Problems Associated with Measured Consumption

In this section, a distinction is made between the appropriate theoretical definition of aggregate consumption per capita and the consumption reported by the Department of Commerce. Four measurement problems are examined: 1) the reporting of expenditures, rather than consumption, 2) the reporting of an integral of consumption rates, rather than the consumption rate at a point in time, 3) infrequent reporting of consumption data relative to stock returns, and 4) reporting aggregate consumption with sampling error since only a subset of the total population of consumption transactions is measured.

The CCAPM prices assets with respect to changes in aggregate consumption between two points in time. In contrast, the available data on aggregate "consumption" provide total expenditures on goods and services over a period of time. These differences between consumption in theory and its measured counterpart suggest the first two problems. First, goods and services need not be consumed in the same period that they are purchased. Second, measured aggregate consumption is closer to an integral of consumption over a period of time than to "spot" consumption (at a point in time). This second problem creates a "summation bias."

While returns on stocks are available on an intraday basis, corresponding consumption data are not available. Currently, only quarterly data are provided back to 1939, and monthly reporting begins in 1959. Infrequent reporting of aggregate expenditures on consumption is the measurement problem analyzed in the third subsection. The fourth subsection demonstrates that sampling error in aggregate consumption does not bias the statistical tests.

A. Description of the Consumption Data

Exploring the empirical implications of the CCAPM for a long sample period requires aggregate consumption data from different sources. The tables in Sections III and IV focus on a time series for consumption that requires "splicing" the data at two points. Each of these three data sources is discussed in turn.

As is discussed later, powerful tests of any asset pricing model require precise estimations for the relevant betas. Precision of the estimators improves if the variability of the consumption measure increases, holding everything else constant. Since consumption was quite variable in the 1930s, we want to include this time period in our empirical work.[2] Unfortunately, aggregate consumption data are not available, except for annual sampling intervals, from 1929 to 1939. However, nominal personal income less transfer payments is available on a monthly basis from the U.S. Department of Commerce,[3] and these income numbers are used to approximate quarterly consumption for this decade.

From 1929 to 1939 a regression of annual consumption data on personal income yields

$$\underset{}{z_{1t}} = \underset{(0.39)}{0.00186} + \underset{(11.51)}{0.56z_{2t}} + \hat{v}_t, \qquad R^2 = 0.94, \tag{8}$$

where $z_{1t} \equiv$ annual growth of real nondurables and services consumption per capita, $z_{2t} \equiv$ annual growth of real personal income less transfer payments per capita, and t-statistics are in parentheses. The data for the above regression are deflated by the average level of the Consumer Price Index (CPI) from the U.S. Bureau of Labor Statistics. The population numbers, which are used to calculate per capita values, are from the *Statistical Abstract of the United States* and reflect the resident population of the U.S. The monthly numbers on personal income less transfer payments are used to infer the consumption numbers based on the above regression equation. From these monthly estimates of consumption, quarterly growth rates are constructed.

From 1939 through 1958 the spliced consumption data rely on quarterly expenditures on nondurable goods and services based on national income accounting. From 1939 through 1946, the data are deflated by the average level of the monthly CPI for the relevant quarter. From 1947 through 1958, real consumption data are available from the Commerce Department. Only seasonally

[2] There is no doubt that part of the unusual volatility of consumption during the 1930s is due to data construction, not variation in true consumption.

[3] For both the annual and monthly data, see *National Income and Product Statistics of the United States, 1929–46*. This appeared as a supplement to the *Survey of Current Business*, July 1947.

adjusted numbers for consumption are available.[4] Average total U.S. population during a quarter as reported by the Commerce Department is used to calculate the per capita numbers. Various issues of *Business Conditions Digest*, *Business Statistics*, and *The National Income and Product Accounts of the United States* report the relevant data.

The consumption data from 1959 to 1982 are constructed in essentially the same manner as that from 1947 to 1958.[5] However, since the government started publishing monthly numbers during this latter period, these monthly numbers are used to compute growth in real consumption per capita over a quarter. For example, growth in a first quarter is based on expenditures during March, relative to expenditures during the prior December.

In later sections, the term "spliced" consumption data refers to the data base constructed in the above manner, which combines the quarterly observations on monthly income data from 1929 to 1938, the quarterly consumption expenditures from 1939 to 1958, and the quarterly observations on the monthly consumption expenditures from 1959 to 1982.

For the whole time period (1929–1982), the consumption data are based on expenditures on nondurables plus services, following Hall (1978). This is an attempt to minimize the measurement problem associated with expenditures versus current consumption of goods and services. No attempt is made to extract the consumption flow from durable goods.[6] While monthly sampling of consumption data is available after 1958, most of the tables do not rely on this information. As the sampling interval decreases, "nondurables" become more durable. However, some of the calculations have been repeated using monthly sampling intervals, and these results are summarized in the text and footnotes.

B. Interval versus Spot Consumption (the Summation Bias)

Ignoring other measurement problems, the reported ("interval") consumption rate for a quarter is the integral of the instantaneous ("spot") consumption rates during the quarter. The CCAPM relates expected quarterly returns on assets (e.g., from January 1 to March 31) and the covariances of those returns with the change in the spot consumption rate from the beginning of the quarter to the end of the quarter. This subsection derives the relation between the desired population covariances (and betas) of assets' returns relative to spot consumption changes and the population covariances (and betas) of assets' returns relative to changes in interval consumption. The variance of interval consumption changes is shown to have only two thirds the variance of spot consumption changes, while the autocorrelation of interval consumption is 0.25 due to the integration of spot

[4] Since the seasonal adjustment smoothes expenditures, such an adjustment may be desirable if the transformed expenditures better resemble actual consumption. Of course, seasonal adjustment is inappropriate if it removes seasonals in true consumption.

[5] The only exception to this occurs for the population number for December 1978. This number is adjusted from the published tables because there is an obvious typographical error.

[6] Alternative treatments for this measurement problem exist in the literature. For example, Marsh (1981) postulates a latent variable model to estimate the parameters of the CCAPM. A more recent attempt is made by Dunn and Singleton (1986), using an econometric approach that relies on the specification of preferences for the representative economic agent.

rates. These latter results are reported by Working (1960) and generalized by Tiao (1972). Similar results on time aggregation have been used in studies of stock prices and corporate earnings (Lambert (1978) and Beaver, Lambert, and Morse (1980)). In an independent and contemporaneous paper, Grossman, Melino, and Shiller (1987) derive maximum-likelihood estimates of CCAPM parameters, explicitly accounting for time aggregation of consumption data. Our bias corrections are much simpler but give similar results.

Without loss of generality, consider a two-quarter period with $t = 0$ being the beginning of the first quarter and $t = T$ being the end of the first quarter. All discussion will analyze annualized consumption rates, so $T = 0.25$ for a quarter. Initially, let the change in the spot consumption rate over a quarter be the cumulative of n discrete changes, $\{\tilde{\Delta}_1^C, \tilde{\Delta}_2^C, \cdots, \tilde{\Delta}_n^C\}$ for the first quarter, and $\{\tilde{\Delta}_{n+1}^C, \tilde{\Delta}_{n+2}^C, \cdots, \tilde{\Delta}_{2n}^C\}$ for the second quarter. That is, $\tilde{C}_T = C_0 + \sum_1^n \tilde{\Delta}_i^C$. Similarly, let the wealth at time T from buying one share of an asset at time 0 (and reinvesting any dividends) equal its initial price plus n random increments $\{\tilde{\Delta}_i^a\} : P_T = P_0 + \sum_i^n \tilde{\Delta}_i^a$.[7]

Changes in consumption, $\tilde{\Delta}_i^C$, are assumed to be homoscedastic and serially uncorrelated. Similar assumptions are made for the asset's return, $\tilde{\Delta}_i^a$, with variance σ_a^2. The contemporaneous covariation of an asset's return with consumption changes is σ_{ac}, and noncontemporaneous covariances are assumed to be zero. The variance of the change in the spot consumption from the beginning of a quarter to the end of the quarter is $\text{var}(\tilde{C}_T - \tilde{C}_0) = \text{var}(\sum_1^n \tilde{\Delta}_i^C) = \sigma_C^2 T$.

The first quarter's *reported* annualized consumption, C_{Q1}, is a summation of the consumption during the quarter, annualized by multiplying by 4 (or $1/T$):

$$C_{Q1} = (1/T) \textstyle\sum_{j=1}^n C_j \Delta t = (1/T) \sum_{j=1}^n (C_0 + \sum_{i=1}^j \Delta_i^C) \Delta t. \tag{9}$$

The annualized consumption rate for the second quarter, C_{Q2}, is the same as (9), but with the first summation for j being $n + 1$ to $2n$.

Continuous movements in consumption and asset prices can be approximated by letting the number of discrete movements per quarter, n, go to infinity ($\Delta t \to 0$). Doing this, the change in reported consumption becomes[8]

$$C_{Q2} - C_{Q1} = \int_0^T (t/T)\Delta_t^C \, dt + \int_T^{2T} ((2T - t)/T)\Delta_t^C \, dt. \tag{10}$$

[7] The summation bias is developed for price changes and consumption changes, not rates of return and consumption growth rates. When the prior period's price and consumption are fixed, the results of this section apply. However, in tests involving unconditional moments, the prior period's price and consumption are random. Since it is difficult to derive a closed-form solution for the summation bias in terms of rates, the subsequent analysis ignores this distinction.

[8] To see this, represent C_{Q2} and C_{Q1} as in (9) and take the difference:

$$\begin{aligned} C_{Q2} - C_{Q1} &= (1/T) \left\{ \textstyle\sum_1^{n+1} \Delta_i^C + \sum_1^{n+2} \Delta_i^C + \cdots + \sum_1^{2n} \Delta_i^C \right\} \Delta t \\ &\quad -(1/T) \left\{ \textstyle\sum_1^1 \Delta_i^C + \sum_1^2 \Delta_i^C + \cdots + \sum_i^n \Delta_i^C \right\} \Delta t \\ &= n^{-1} \{\Delta_2^C + 2\Delta_3^C + \cdots + (n-1)\Delta_n^C + n\Delta_{n+1}^C + (n-1)\Delta_{n+2}^C + \cdots + \Delta_{2n}^C\}. \end{aligned}$$

Letting n become large gives equation (10).

Given the independence of spot consumption change over time, (10) implies that the variance of reported annualized consumption changes is

$$\operatorname{var}(\tilde{C}_{Q2} - \tilde{C}_{Q1}) = \int_0^T ((t/T)^2\sigma_C^2)\, dt$$

$$+ \int_T^{2T} ((2T - t)/T)^2\sigma_C^2\, dt = (2/3)\sigma_C^2 T. \qquad (11)$$

Thus, the population variance of reported (interval) consumption changes for a quarter is two thirds of the population variance for changes in the spot consumption from the beginning of a quarter to the end of the quarter. The averaging caused by the integration leads to the lower variance for reported consumption.

Next, consider the covariance of an asset's quarterly return with quarterly changes in the consumption. The covariance of the change in spot consumption from the beginning of a quarter to the end of the quarter with an asset's return over the same period is $\sigma_{ac}T$, given the i.i.d. assumption. With reported, interval consumption data, the covariance can be computed from (10):

$$\operatorname{cov}(\tilde{C}_{Q2} - \tilde{C}_{Q1}, \tilde{P}_{2T} - \tilde{P}_T) = T^{-1}\int_T^{2T} (2T - t)\sigma_{aC}\, dt = T\sigma_{aC}/2. \qquad (12)$$

Thus, from (12) the population covariance of an asset's quarterly return with reported (interval) consumption is half the population covariance of the asset's return with spot consumption changes.

Given (11) and (12), betas measured relative to reported quarterly consumption changes are ¾ times the corresponding betas with spot consumption:

$$\beta_{ac}^{sum} = \frac{(1/2)\sigma_{aC}T}{(2/3)\sigma_C^2 T} = (3/4)\beta_{ac}^{spot}. \qquad (13)$$

Since the CCAPM relates quarterly returns to "spot betas," the subsequent empirical tests multiply the mean-adjusted consumption growth rates by ¾ to obtain unbiased "spot betas." The ¾ relation of interval betas to spot betas in (17) is a special case of the multiperiod differencing relation: $\beta_{ac}^{sum} = \beta_{ac}^{spot}\,[K - (1/2)]/[K - (1/3)]$, where K is the differencing interval. Thus, monthly data sampled quarterly (i.e., $K = 3$) should give interval betas that are $(5/2)/(8/3) = 0.9375$ times the spot betas. When quarterly consumption growth rates are calculated from monthly data, the quarterly numbers are mean adjusted and multiplied by 0.9375.

Although changes in spot consumption are uncorrelated, changes in reported, interval consumption rates have positive autocorrelation. To see this, use (10) to compute the covariance of the reported consumption change from $Q1$ to $Q2$ with the reported change from $Q2$ to $Q3$, noting that all covariance arises from the time overlap from T to 2 T:

$$\operatorname{cov}(\tilde{C}_{Q3} - \tilde{C}_{Q2}, \tilde{C}_{Q2} - \tilde{C}_{Q1}) = \int_T^{2T} ((t - T)(2T - t)/T^2)\sigma_C^2\, dt = (1/6)\sigma_C^2 T. \qquad (14)$$

The first-order autocorrelation in reported consumption is 0.25 since

$$\rho_1 = \operatorname{cov}(\tilde{C}_{Q3} - \tilde{C}_{Q2}, \tilde{C}_{Q2} - \tilde{C}_{Q1})/\operatorname{var}(\tilde{C}_{Q2} - \tilde{C}_{Q1}) = \frac{(1/6)\sigma_C^2 T}{(2/3)\sigma_C^2 T} = 0.25. \tag{15}$$

By similar calculations, higher order autocorrelation is zero. Table I presents the time series properties of reported *unspliced* quarterly consumption growth rates. First-order autocorrelation of quarterly real consumption growth for the entire 1939–1982 period is estimated to be 0.29, which is insignificantly different from the theoretical value of 0.25 at usual levels of significance. Higher order autocorrelations are not significantly different from zero. Thus, the model for reported consumption is not rejected by the sample autocorrelations.

Monthly growth rates of real consumption from 1959 to 1982 exhibit negative autocorrelation of −0.28, which is significantly different from zero and from the hypothesized 0.25. This may be caused by vagaries such as bad weather and strikes in major industries, which cut current consumption temporarily but are followed by catch-up purchases. Quarterly growth rates in consumption computed from the monthly series again have positive autocorrelation of 0.13, more closely in line with the value 0.0625 (or 1/16) predicted by the summation bias.[9] The longer the differencing interval, the less affected the data are by temporary fluctuations and measurement errors in consumption.

Chen, Roll, and Ross (1986) and Hansen and Singleton (1983) use monthly data on unadjusted consumption growth. Since those data's autocorrelation statistics suggest significant departures from the random-walk assumption, the statistics they present warrant re-examination. The use of larger differencing intervals should be fruitful.

C. Infrequent Reporting of Consumption: The Maximum Correlation Portfolio

Since the returns on many assets are available for a longer time and are reported more frequently than consumption, more precise evidence on the CCAPM can be provided if only returns were needed to test the theory. Fortunately, Breeden's (1979, footnote 8) derivation of the CCAPM justifies the use of betas measured relative to a portfolio that has maximum correlation with growth in aggregate consumption, in place of betas measured relative to aggregate consumption. This result is amplified below, as it is shown that securities' betas measured relative to this maximum correlation portfolio (hereafter, MCP) are equal to their consumption betas divided by the consumption beta of the MCP. If a riskless asset exists, then the consumption beta of the MCP can be changed by adjusting leverage. Our MCP excludes the riskless asset, resulting in a consumption beta of 2.9.

In the following, the first M assets have a linear relation with consumption as in equation (5). The CCAPM holds with respect to these M assets when betas are measured relative to the MCP obtained from these M assets. Second, for *any subset* N (where $N \leq M$) of these M assets, the CCAPM holds for that subset when betas are measured relative to the MCP obtained from these N assets.

[9] The derivation of this prediction is similar to the derivation of equation (15).

Table I

Time Series Properties of Percentage Changes in Real, Per Capita Consumption of Nondurable Goods and Services

Data are seasonally adjusted as reported by the Department of Commerce in the *Survey of Current Business*. T denotes the number of observations, while $\bar{c}$ and $\widehat{SD}(c)$ are the sample mean and standard deviation, respectively. Under the hypothesis that the observations are serially uncorrelated, the asymptotic standard errors for the sample autocorrelations are $1/\sqrt{T}$, as given by $SD^*(\hat{\rho}_k)$. Under the hypothesis that $\rho_1 = 0.25$ and $\rho_k = 0 \ \forall \ |_k| > 1$, $SD(\rho_1)$ and $SD(\rho_k)$ report the asymptotic standard errors using the results of Bartlett (1946). The test statistic for the joint hypothesis that all autocorrelations are zero for lags 1 through 12 is given by Q_{12}, the modified Box-Pierce Q-statistic. Q_{12} is asymptotically distributed as chi-square with 12 degrees of freedom. The p-value is the probability of drawing a Q_{12} statistic larger than the current value under the null hypothesis.

Time Period	T	$\bar{c}$	$\widehat{SD}(c)$	$\hat{\rho}_1$	$\hat{\rho}_2$	$\hat{\rho}_3$	$\hat{\rho}_4$	$\hat{\rho}_8$	SD^* $(\hat{\rho}_k)$	$\widehat{SD}$ $(\hat{\rho}_1)$	$\widehat{SD}$ $(\hat{\rho}_k)$	Q_{12}	p-Value
Panel A: Quarterly Consumption Data													
39Q2–82Q4	175	0.00543	0.00951	0.29	0.03	−0.00	0.07	0.02	0.08	0.07	0.08	23.93	0.02
39Q2–52Q4	55	0.00665	0.01517	0.30	0.03	−0.04	0.08	0.08	0.13	0.12	0.14	11.26	0.51
53Q1–67Q4	60	0.00463	0.00549	0.21	0.09	0.11	−0.01	−0.22	0.13	0.12	0.14	11.25	0.51
68Q1–82Q4	60	0.00511	0.00487	0.36	0.01	0.26	0.09	−0.31	0.13	0.12	0.14	25.95	0.01
Panel B: Monthly Consumption Data													
1959–1982	287	0.00178	0.00447	−0.28	−0.02	−0.14	−0.12	−0.19	0.06	0.05	0.06	43.09	0.00
1959–1970	143	0.00199	0.00467	−0.31	−0.11	0.18	−0.08	−0.17	0.08	0.08	0.09	33.49	0.00
1971–1982	144	0.00156	0.00427	−0.24	0.07	0.09	−0.16	−0.16	0.08	0.08	0.09	20.56	0.06
Panel C: Quarterly Sampling of Monthly Consumption Data													
59Q2–82Q4	95	0.00521	0.00568	0.13	−0.13	0.20	0.04	−0.17	0.10	0.09	0.11	13.42	0.34
59Q2–70Q4	47	0.00576	0.00506	0.13	−0.15	0.13	−0.03	−0.04	0.15	0.13	0.15	10.61	0.56
71Q1–82Q4	47	0.00468	0.00623	0.12	−0.07	0.22	−0.10	−0.26	0.14	0.13	0.15	11.40	0.50

The following notation will be used throughout the paper. Let μ be the $N \times 1$ vector unconditional expected returns, let $\mathbf{1}$ be an $N \times 1$ vector of ones, and let β_c^* be the $N \times 1$ vector of unconditional consumption betas. Let $\tilde{R}_{mcp}$ be the return on the MCP that excludes the riskless asset, let $\beta_{c,nb}^*$ be the unconditional consumption beta of this "no borrowing" MCP, and let β_{mcp} be the $N \times 1$ vector of unconditional MCP betas. The $N \times N$ unconditional covariance matrix for returns is $\mathbf{V}$, which is assumed to be nonsingular.

Consider the following portfolio problem: find the minimum-variance portfolio that has a consumption beta of $\beta_{c,nb}^*$ (i.e., with no borrowing). The consumption beta of a portfolio is the product of its correlation coefficient with consumption and the portfolio's standard deviation, divided by the standard deviation of consumption. By constraining the consumption beta to be fixed and then minimizing variance, the resulting portfolio has the maximum correlation with consumption, i.e., the MCP. Mathematically, the MCP solves

$$\min_{\{\mathbf{w}\}}: \mathbf{w}'\mathbf{V}\mathbf{w} + 2\lambda(\beta_{c,nb}^* - \mathbf{w}'\beta_{\mathbf{c}}^*), \tag{16}$$

where λ is a Lagrange multiplier. The weights (i.e., $\mathbf{w}$) in the MCP are not constrained to unity since the risky assets may be combined with a riskless asset without any effect on the correlation coefficient, the variance, or the consumption beta. Alternatively, if the weights obtained sum to the value S, those same weights multiplied by $1/S$ sum to unity and have the same correlation with consumption. Thus, the existence or nonexistence of a riskless asset does not affect the MCP analysis.

The first-order conditions imply

$$\mathbf{w}_{mcp} = \lambda\mathbf{V}^{-1}\beta_{\mathbf{c}}^*. \tag{17}$$

Since $\beta_{c,nb}^* = \mathbf{w}_{mcp}'\beta_{\mathbf{c}}^*$, $\lambda = \beta_{c,nb}^*/(\beta_{\mathbf{c}}^{*\prime}\mathbf{V}^{-1}\beta_{\mathbf{c}}^*)$. Pre-multiplying (17) by $\mathbf{w}_{mcp}'\mathbf{V}$ and simplifying implies $\mathbf{w}_{mcp}'\mathbf{V}\mathbf{w}_{mcp} = \lambda\beta_{c,nb}^*$. The MCP betas of risky assets are

$$\beta_{mcp} = \frac{\mathbf{V}\mathbf{w}_{mcp}}{\mathbf{w}_{mcp}'\,\mathbf{V}\mathbf{w}_{mcp}} = \frac{\lambda\beta_{\mathbf{c}}^*}{\lambda\beta_{c,nb}^*} = \beta_{\mathbf{c}}^*/\beta_{c,nb}^*, \tag{18}$$

using the facts just derived.

In words, (18) states that assets' betas measured relative to the MCP are proportional to their betas relative to true consumption. Substituting (18) into the zero-beta CCAPM and using the CCAPM to get the expected excess return on the MCP implies

$$\mu - \gamma_0\mathbf{1} = \beta_{mcp}(\mu_{mcp} - \gamma_0), \tag{19}$$

Where $\mu_{mcp} \equiv \mathscr{E}(\tilde{R}_{mcp,t})$. Thus, the CCAPM may be restated (and tested) in terms of the MCP, and the testable implication is that the MCP is ex ante mean-variance efficient. Obviously, any zero-consumption beta portfolio also has a zero beta relative to the MCP.

The above result also suggests an intuitive interpretation of the portfolio weights for the MCP. Equation (17) implies

$$\mathbf{w}_{mcp} = \lambda\mathbf{V}^{-1}\beta_{\mathbf{c}}^* = \theta\mathbf{V}^{-1}\mathbf{V}_{\mathbf{ac}}^*, \tag{20}$$

where $\theta \equiv \lambda/\text{var}(\tilde{c}) = \dfrac{\beta^*_{c,nb}}{(\beta^{*\prime}_c \mathbf{V}^{-1}\beta^*_c)\text{var}(\tilde{c}^*)}$ and $\mathbf{V}^*_{ac}$ is the $N \times 1$ vector of covariances of returns with consumption. From (20), the MCP's weights are proportional to the coefficients in a multiple regression of consumption on the various risky assets' returns, with θ being the factor of proportionality. Actually, θ equals $\beta^*_{c,nb}$ divided by the coefficient of determination (R^2) of the multiple regression just described. To see this, note that the weights in the multiple regression, $\mathbf{w}^*_c$, are $\mathbf{w}^*_c = \mathbf{V}^{-1}\mathbf{V}^*_{ac}$, and the R^2 in the regression is

$$R^2 = (\mathbf{w}^{*\prime}_c \mathbf{V} \mathbf{w}^*_c)/\text{var}(\tilde{c}^*) = (\beta^{*\prime}_c \mathbf{V}^{-1}\beta^*_c)\text{var}(\tilde{c}^*), \tag{21}$$

which shows that $\theta = \beta^*_{c,nb}/R^2$. If there is a riskless asset, a unit beta MCP has weights that equal the regression's coefficients divided by the R-squared value of the regression (with any residual wealth in the riskless asset). Betas with respect to such a unit-beta MCP equal the assets' direct consumption betas (see equation (18)).

The optimization problem of (16) does not involve a constraint on means, so a MCP is not tautologically a mean-variance efficient portfolio. However, the CCAPM does imply mean-variance efficiency of that MCP in equilibrium. This implication is tested later in our paper.

D. Sampling Error In Reported Consumption

In this section the problem of pure sampling error in reported consumption is examined. These errors are assumed to be random and uncorrelated with economic variables. Continue with $\tilde{c}^*_t$ as the true growth rate of real consumption from $t - 1$ to t, and let $\tilde{c}_t$ be the reported growth rate. The measurement error, $\tilde{\epsilon}_t$, is such that

$$\tilde{c}_t = \tilde{c}^*_t + \tilde{\epsilon}_t \tag{22}$$

$$\mathscr{E}(\tilde{\epsilon}_t) = 0, \qquad \text{cov}(\tilde{\epsilon}_t, \tilde{c}^*_t) = 0,$$

$$\text{cov}(\tilde{\epsilon}_t, \tilde{R}_{it}) = 0, \qquad \forall\ i = 1, \cdots, N. \tag{23}$$

Substituting (22) into the CCAPM of (7) gives

$$\begin{aligned}\mu_i - \gamma_0 = \gamma^*_1\ \beta^*_{ci} &= \gamma^*_1 \text{cov}(\tilde{R}_{it}, \tilde{c}_t - \tilde{\epsilon}_t)/\text{var}(\tilde{c}^*_t) \\ &= \gamma^*_1 \frac{\text{var}(\tilde{c}_t)\text{cov}(\tilde{R}_{it}, \tilde{c}_t)}{\text{var}(\tilde{c}^*_t)\text{var}(\tilde{c}_t)} = \gamma_1 \beta_{ci},\end{aligned} \tag{24}$$

where β_{ci} is the beta asset of i with respect to reported consumption, and $\gamma_1 \equiv \gamma^*_1[\text{var}(\tilde{c}_t)/\text{var}(\tilde{c}^*_t)]$. As long as the variance of the measurement error is positive, the variance of measured consumption exceeds the variance of true consumption. From (24), the slope coefficient, γ_1, in the relation between excess returns and betas with reported consumption is biased upward as an estimate of the price of risk, γ^*_1.

Sampling error in reported consumption does not cause a bias in the coefficients of a multiple regression of consumption growth on risky asset returns. However, the coefficient of determination for such a regression is downward biased. While the portfolio weights of the MCP are calculated by taking ratios of the regression coefficients divided by this R^2, the downward bias in R^2 affects all the weights in a proportional fashion. Thus, this has no effect on the subsequent tests.

Some other important measurement errors in aggregate consumption data involve interpolation (i.e., expenditures for all items are sampled every month), to which the analysis of this subsection is not applicable. This problem is similar to one faced by Fama and Schwert (1977, 1979) in their analysis of components of the CPI. Unfortunately, the interpolation problems with consumption are exacerbated by the summation bias, and it is difficult to disentangle the two effects. For example, either problem leads to serial correlation in consumption, noncontemporaneous correlation between aggregate consumption and returns, and more serious effects as sampling interval becomes shorter.[10] If the summation bias were not present, presumably an approach similar to that in Scholes and Williams (1977) would be appropriate. Perhaps the combination of interplation and summation bias explains the pattern of serial correlations in monthly data on consumption growth (see Panel B of Table I). Interpolation is yet another reason for avoiding monthly sampling intervals.

III. Empirical Characteristics of Consumption Betas and the MCP

Since existing empirical research on the CCAPM is not extensive, we summarize how consumption betas vary across different assets. Several types of assets will be studied, including government and corporate bonds and equities.

Monthly returns on individual securities are gathered from the Center for Research in Security Prices (CRSP) at the University of Chicago. Twelve portfolios of these stocks are formed by grouping firms using the first two digits of their SIC numbers. The grouping closely followed a classification used by Sharpe (1982), with the major exception being that Sharpe's "consumer goods" category is subdivided. This subdivision should increase the dispersion of consumption betas in the sample. While other groupings of stocks have been suggested (e.g., see Stambaugh (1982)), Sharpe's scheme is selected because the industry portfolios are reasonable and capture some important correlation patterns among stocks. Table II provides more details on the classification scheme. To represent the return on a "buy and hold" strategy, relative market values are used to weight the returns in a given portfolio. Every return on the CRSP tape from 1926 through 1982 is included, which should minimize problems with survivorship bias.[11]

[10] Interpolation should result in serial correlation in the residuals in regressions of returns on consumption growth. (Equation (26) below is such a regression.) Yet, when we examine the residuals, the autocorrelations are not striking. On the other hand, when we run a multiple regression of returns on a leading value of consumption, current consumption, and lagged consumption, we do get an interesting pattern. Generally, the coefficient on the lead value is insignificant, the coefficient on current consumption is significant and positive, and the coefficient on lagged consumption is significant and negative. Usually, the absolute value of the coefficient on the lagged value is about half the value of the coefficient on current consumption. However, as we note in the text, the significance of the coefficient on lagged consumption is predictable if only a summation bias is present.

[11] However, all firms with a SIC number of 39 (i.e., miscellaneous manufacturing industries) are excluded to avoid any possible problems with a singular covariance matrix when the CRSP value-weighted index is added to the sample.

Table II

Estimated Betas Relative to 1) Growth in Real, Per Capita Consumption[a], 2) Maximum-Correlation Portfolio for Consumption, and 3) CRSP Value-Weighted Index

NA denotes not available. The maximum correlation portfolio (MCP) is constructed from the seventeen assets given in Table III. The weights of the MCP are determined by maximizing the sample correlation between the return on the portfolio and the growth rate of real consumption; see Table III for more details.

Asset	Number of Firms			Spliced Consumption, Quarterly 1929–1982 ($T = 215$)			Max.-Correlation Cons. Portfolio, Monthly 1926–1982 ($T = 684$)			CRSP Value-Weighted Index Monthly 1926–1982 ($T = 684$)		
(SIC Codes)	1/26	6/54	12/82	$\hat{\beta}_c$	$t(\hat{\beta})$	R^2	$\hat{\beta}_{MCP}$	$t(\hat{\beta})$	R^2	$\hat{\beta}_{CRSP}$	$t(\hat{\beta})$	R^2
U.S. Treasury bills	—	—	—	−0.11	−1.27	0.01	0.03	3.86	0.02	0.01	2.04	0.01
Long-term govt. bonds	NA	NA	NA	−0.01	−0.02	0.00	0.07	2.53	0.01	0.07	4.93	0.03
Long-term corp. bonds	NA	NA	NA	0.24	0.91	0.00	0.07	2.52	0.01	0.08	6.62	0.06
Junk bond premium	NA	NA	NA	2.45	6.85	0.18	0.63	18.52	0.33	0.33	20.45	0.38
Petroleum (13, 29)	46	51	69	4.31	6.37	0.16	1.41	20.61	0.38	0.92	38.63	0.69
Finance & real estate (60–69)	16	43	234	5.85	6.30	0.16	1.50	18.81	0.34	1.19	75.95	0.89
Consumer durables (25, 30, 36, 37, 50, 55, 57)	69	157	180	6.86	6.80	0.18	1.79	22.03	0.42	1.29	80.79	0.91

Basic industries (10, 12, 14, 24, 26, 28, 33)	94	207	194	5.45	6.95	0.18	1.48	21.98	0.41	1.09	100.80	0.94
Food & tobacco (1, 20, 21, 54)	64	103	81	3.25	5.69	0.13	0.99	18.62	0.34	0.76	58.15	0.83
Construction (15–17, 32, 52)	5	28	53	7.36	7.06	0.19	1.57	19.16	0.35	1.20	61.22	0.85
Capital goods (34, 35, 38)	39	120	191	5.31	6.74	0.18	1.45	21.10	0.39	1.08	85.90	0.92
Transportation (40–42, 44, 45, 47)	78	85	46	5.15	4.97	0.10	1.27	13.52	0.21	1.19	49.04	0.78
Utilities (46, 48, 49)	24	102	176	3.73	6.10	0.15	1.04	19.40	0.35	0.75	46.34	0.76
Textiles & trade (22, 23, 31, 51, 53, 56, 59)	46	101	119	5.63	7.84	0.22	1.66	30.49	0.58	0.95	48.73	0.78
Services (72, 73, 75, 80, 82, 89)	3	4	57	4.21	4.18	0.08	1.65	12.97	0.20	0.80	12.82	0.19
Leisure (27, 58, 70, 78, 79)	12	31	59	7.35	6.95	0.18	1.85	23.03	0.44	1.22	49.82	0.78
CRSP value-weighted	NA	NA	NA	4.92	7.06	0.19	1.37	23.73	0.45	1.00	—	—

[a] The spliced consumption data are scaled to adjust for the summation bias problem. Real growth in per capita consumption is multiplied by 0.75 for observations between 1939Q2 and 1959Q1, and by 0.9375 otherwise.

While methods which handle data on individual securities rather than aggregate portfolios could be dveloped, this route was not followed.[12] The dimensionality of the parameter space is enormous when analyzing a large cross-section of securities, and conventional methods for statistical inference may become unreliable. Also, a grouping procedure by industry decreases the number of statistics to be reported—probably without a disastrous loss of information.

Several types of assets should be represented, for Stambaugh (1982) finds the statistical results are not robust to the assets under study. Short-term Treasury bills, long-term government bonds, and high-grade long-term corporate bonds are included using the data in Ibbotson and Sinquefield (1982). In addition, the recent work by Chen, Roll, and Ross (1986) suggests that the difference in returns is between low-grade long-term corporate bonds (or "junk" bonds) and long-term government bonds is useful in explaining expected returns, so these returns are included as well.[13] To capture the spread between junk bonds and government bonds, a return is calculated on a portfolio which buys junk bonds by shorting long-term government bonds and then invests in short-term T-bills.[14] This portfolio's return is referred to as the "junk bond premium." Returns on junk bonds relative to government bonds primarily reflect changes in investors' perceptions concerning the probability of default. This is related to their perceptions of current and future economic conditions, which should be related to consumption growth. In fact, our statistical analysis shows a strong relation between junk bond returns and real consumption growth.

Returns are expressed in real terms and on a simple basis without continuously compounding. Returns are deflated by the Consumer Price Index, as reported by the U.S. Bureau of Labor Statistics on a monthly basis for the entire sample period. For purposes of testing the CAPM, the CRSP value-weighted index is used as the proxy for the market portfolio.[15]

Table II reports estimated betas for various assets. (The construction of the MCP is described below.) The table reveals that different measures of risk are highly correlated. In fact, the correlation between the market betas and the consumption betas (or the MCP betas) is 0.96 (or 0.94). Of course, while the risk measures are highly correlated, the rankings of the risk measures for the various assets are not exactly the same.

As discussed by Breeden (1980), industries' consumption betas should be related to price and income elasticities of demand and to supply elasticities. Goods with high income elasticities of demand should have high consumption

[12] Using different econometric methods, Mankiw and Shapiro (1985) have analyzed a version of the CCAPM using individual securities. However, they only rely on quarterly consumption data from 1959 to 1982.

[13] We are grateful to Roger Ibbotson, who made these data available to us. Since Ibbotson's data ended in 1978, the data are extended through 1982 using the monthly return on a mutual fund which is managed by Vanguard. This portfolio, the High Yield Bond Fund, is based on an investment strategy very similar to the one used by Ibbotson in constructing his return series.

[14] The investment in short-term T-bills is convenient but not necessary. The asset pricing models are specified assuming that the assets are held with some net capital invested.

[15] See Roll (1977) for a discussion of the potential consequences of selecting a proxy for the true market portfolio. The reader should keep in mind that one usual form of the consumption-based theory includes the market portfolio as part of the statement. Nevertheless, the theoretical results hold when any security replaces the market portfolio (Breeden (1979)).

betas, ceteris paribus. This appears to be borne out in the data, for consumer durables, construction, and recreation and leisure all have high consumption betas. While the services portfolio may have a high income elasticity, it does not have a high consumption beta. However, the number of firms in that portfolio is quite low (<5) for the first thirty years, and the R-squared is also low (0.08). Goods with lower income elasticities of demand, such as utilities, petroleum, food and agriculture, and transportation, have the lowest consumption betas of the stock portfolios.

Section II. *C* discusses the usefulness of a maximum correlation portfolio. Equation (20) suggests a way to calculate the weights in an MCP. Table III reports the results of running a regression of consumption growth on the returns from the twelve industry portfolios, four bond portfolios, and the CRSP value-weighted index for the period 1929–1982. Consumption growth is adjusted so that the summation bias in the estimated covariances between consumption and the returns on assets is removed. Table III gives the coefficients after they are rescaled so that they sum to one hundred percent, for the MCP in Section IV does not use the riskless asset.

The composition of the MCP in Table III helps to explain why Chen, Roll, and Ross (1986) found such an unimportant role for aggregate consumption. The MCP gives large absolute weights to long-term government bonds (−31%), the

Table III

Estimated Weights for the Maximum-Correlation Portfolio for Consumption Based on Spliced Quarterly Data from 1929–1982

All data are in real terms. (Consumption growth is scaled to adjust for the summation bias). The coefficient of determination for the above regression is 0.25, and the F-statistic for testing the joint significance of all the coefficients is 3.93 with a p-value of 0.0001. Before running real consumption growth on the returns, the data are mean adjusted. Then consumption growth is multiplied by two for observations between 1939Q2 and 1959Q1, and by 1.2 otherwise.

Asset	Weight	t-Statistic
U.S. Treasury bills	0.01	0.02
Long-term government bonds	0.54	1.05
Long-term corporate bonds	−0.31	−0.64
Junk bond premium	0.59	2.71
Petroleum	0.27	1.13
Banking, finance and real estate	−0.17	0.38
Consumer durables	0.10	0.44
Basic industries	0.33	0.90
Agriculture, food, and tobacco	−0.35	−1.45
Construction	−0.11	−0.80
Capital goods	0.03	0.11
Transportation	−0.29	−2.25
Utilities	0.18	0.72
Textiles, retail stores, and wholesalers	0.49	2.69
Services	0.08	1.39
Recreation and leisure	0.13	1.17
CRSP value-weighted index	−0.51	−0.38
	1.00	

junk bond premium (59%), and the CRSP index (−51%). Since these three variables were included as factors in Chen, Roll, and Ross (1986), aggregate consumption may be dominated as an additional factor given multicollinearity and measurement error.

The weights reported in Table III seem extreme, for the MCP involves large short positions in assets. Placing restrictions on the estimated weights would eliminate the extreme positions but could sacrifice some consistency with the underlying theory. The collinearity among the assets makes it difficult to estimate any single weight with precision, but the fitted value from the regression may be useful for our purposes. To see how the MCP tracks consumption growth, the following regression is run using spliced quarterly data from 1929 to 1982 (again, consumption has been scaled so that the reported beta is free of the summation bias):[16]

$$R_{MCP,t} = \underset{(2.62)}{0.00828} + \underset{(10.19)}{2.90}c_t' + \hat{u}_{MCP,t},\ R^2 = 0.33, \tag{25}$$

where t-statistics are given in parentheses. Since the MCP places no funds in the minimum-variance zero-beta portfolio, it need not have a unit beta. Even though the correlation between the MCP and consumption growth is 0.57, the theory of Section II. C still predicts that the MCP should be mean-variance efficient relative to the assets that it contains. Furthermore, the estimated risk measures when using actual consumption growth versus the MCP give similar rankings, and the sample correlation between the two sets of betas in Table II is 0.98.

Unlike the CRSP value-weighted index, the MCP has fixed weights since the entire sample period is used in the estimation reported in Table III. Constant weights are appropriate for the empirical work focuses on unconditional moments.[17] Moreover, estimating the weights by subperiods is not practical since quarterly data limit the number of available observations.

To better understand the MCP, Table IV compares it with the CRSP value-weighted index, a portfolio that has been studied extensively. According to Table III, the CRSP index has a negative weight in the MCP (−51%), yet the two portfolios are positively correlated. For the overall period, the correlation is 0.67. Furthermore, the MCP has roughly half the mean and standard deviation as the proxy for the market. Risk aversion combined with the CAPM predicts that the

[16] For observations between 1939Q2 and 1959Q1, $c_t' = 0.75(c_t - \bar{c})$. Otherwise, $c_t' = 0.9375\ (c_{;t} - \bar{c})$, where $\bar{c}$ is the sample mean of c_t for the entire time period. By reducing the sizes of the consumption growth deviations, the slope coefficient is scaled up so as to be consistent (at least with regard to the summation bias).

[17] Even if second moments change conditional on predetermined information, working with a constant weight MCP is still appropriate for investigations involving unconditional moments. However, certain forms of heteroscedasticity may pose a problem for our statistical inference even in large samples. There is evidence of heteroscedasticity. We divided the overall period into four subperiods (1929Q2–1939Q1, 1939Q2–1947Q1, 1947Q2–1959Q1, and 1959Q2–1982Q4) and examined the constancy of the covariance matrix across all four periods. The covariance matrix is 18 × 18 involving the returns on seventeen assets in Table III and consumption growth. Using a likelihood-ratio test and an asymptotic approximation involving the F-distribution (Box (1949)), the F-statistic is 3.378 with degrees of freedom of 513 and 43165.7. The p-value is less than 0.001.

Table IV

Descriptive Statistics on Real Returns from Treasury Bills, the CRSP Value-Weighted Index, and the Maximum-Correlation Portfolio (MCP) for Consumption Based on Monthly Data, 1926–1982

The sample means are annualized by multiplying by 12. The sample standard deviations are annualized by multiplying by $\sqrt{12}$. (Since returns on T-bills are serially correlated, the annualized standard deviation is not the approximate standard deviation for annual holding periods.) Correlations between the CRSP return and the MCP return for the four periods are 0.67, 0.75, 0.59, and 0.41, respectively. The maximum-correlation portfolio (MCP) is constructed from the seventeen assets given in Table III. The weights of the MCP are determined by maximizing the sample correlation between the return on the portfolio and the growth rate of real consumption; see Table III for more details.

Date	Number of Observations	Mean of T-bills	t-Statistic for Mean of T-bills	Standard Deviation
1926–1982	684	0.0013	0.48	0.0204
1926–1945	240	0.0100	1.77	0.0253
1946–1965	240	−0.0082	−1.74	0.0211
1966–1982	204	0.0023	0.89	0.0106
Date	**Number of Observations**	**Mean of CRSP Return**	**t-Statistic for Mean of CRSP Return**	**Standard Deviation**
1926–1982	684	0.0767	2.88	0.2013
1926–1945	240	0.1002	1.61	0.2782
1946–1965	240	0.1039	3.70	0.1257
1966–1982	204	0.0172	0.44	0.1615
Date	**Number of Observations**	**Mean of MCP Return**	**t-Statistic for Mean of MCP Return**	**Standard Deviation**
1926–1982	684	0.0370	2.83	0.0987
1926–1945	240	0.0598	1.98	0.1351
1946–1965	240	0.0382	2.62	0.0651
1966–1982	204	0.0086	0.46	0.0786

mean of the market is positive, and the CCAPM makes the same prediction about the mean of the MCP. The point estimates for both portfolios are consistent with these predictions. However, when the standard deviation of the return is large in 1926–1945, the mean of the MCP is marginally significant while the market proxy is not.

IV. Testing the CCAPM and the CAPM

The usefulness of the risk measures in predicting expected returns is examined in this section. Two issues are studied. First, does expected return increase as the risk increases? Second, is the relation between risk and return linear? These two issues are synonymous with the question of mean-variance efficiency for a given portfolio. In addition, estimates of the expected real return on the zero-beta portfolio will be compared with the real return on a nominally riskless bill.

The empirical implications of the CCAPM in terms of aggregate consumption are examined first. Then the empirical results are extended by testing the mean-variance efficiency of the maximum-correlation portfolio. Finally, the CAPM is studied by testing the ex ante efficiency of the CRSP index.

Since the relevant econometric methodology is detailed by Gibbons (1982), only a brief development is provided here. In the case of the CCAPM, a regression similar to the market model is assumed to be a well-specified statistical model. That is, the joint distribution between the return on an asset and real growth in per capita consumption, $\tilde{c}_t$, is such that the disturbance term in the following regression has mean zero and is uncorrelated with $\tilde{c}_t$. Such an assumption justifies the following regression model:

$$\tilde{R}_{it} = \alpha_{ci} + \beta_{ci}\tilde{c}_t' + \tilde{u}_{it}, \qquad \forall\ i = 1, \cdots, N, \quad t = 1, \cdots, T. \tag{26}$$

Further, it is assumed that

$$\mathscr{E}(\tilde{u}_{is}\tilde{u}_{jt}) = \begin{cases} \sigma_{ij} & \forall\ s = t, \\ 0 & \text{otherwise.} \end{cases} \tag{27}$$

Since $\tilde{c}_t'$ has already been mean-adjusted, μ_i is equal to α_{ci}.[18] Also, $\tilde{c}_t'$ has been scaled so that the summation bias is avoided.

Using the CCAPM as modified in Section II. *D* to account for sampling error in consumption provides

$$\mu_i = \gamma_0 + \gamma_1\beta_{ci}. \tag{28}$$

The theoretical relation in (28) imposes a parameter restriction on (26) of the form:

$$\alpha_{ci} = \gamma_0 + \gamma_1\beta_{ci}. \tag{29}$$

Pooling the time-series regressions in (26) across all N assets and then imposing the parameter restriction given in (29) provides a framework in which to estimate the expected return on the zero-beta portfolio, γ_0, and the market price of beta risk, γ_1. In addition, the parameter restriction may be tested.

There are various econometric methods for estimating the above model. Many of these are asymptotically (as T approaches infinity) equivalent to a full maximum-likelihood procedure. In the past, these alternatives have been selected because of computational considerations. However, results by Kandel (1984) and extended by Shanken (1985) make full maximum likelihood easy to implement.[19]

[18] Consumption growth is adjusted by its sample mean, not the unknown population mean. Our statistical inference that follows is conditional on the sample mean equal to its population counterpart. We overstate the significance of our tests as a result.

[19] Shanken (1982) shows that the full maximum-likelihood estimator may have desirable properties as the number of assets, N, used in estimating the model becomes large.

Shanken establishes that the full maximum-likelihood estimators for γ_0 and γ_1 can be found by minimizing the following function:

$$L(\gamma_0, \gamma_1) = (1/(1 + (\gamma_1^2/s_c^2)))\mathbf{e}'(\gamma)\hat{\mathbf{\Sigma}}^{-1}\mathbf{e}(\gamma), \tag{30}$$

where

$\mathbf{e}(\gamma) \equiv \overline{\mathbf{R}} - \gamma_0 \mathbf{1}_N - \gamma_1 \hat{\beta}_c$,
$\hat{\mathbf{\Sigma}} \equiv T^{-1} \sum_{t=1}^{T} \hat{\mathbf{u}}_t \hat{\mathbf{u}}_t'$,
$s_c^2 \equiv T^{-1} \sum_{t=1}^{T} c_t'^2$;
$\hat{\beta}_c \equiv N \times 1$ vector with typical element $\hat{\beta}_{ci}$, where $\hat{\beta}_{ci}$ is the usual unrestricted ordinary least-squares estimator of β_{ci} in (26),
$\hat{\mathbf{u}}_t \equiv N \times 1$ vector with typical element $\hat{u}_{it}$, where $\hat{u}_{it}$ is the residual in (26) when ordinary least squares is performed, and
$\overline{\mathbf{R}} \equiv N \times 1$ vector with typical element $\overline{R}_i$, the sample mean of the return on asset i.

The concentrated-likelihood function is proportional to equation (30),[20] in which γ_0 and γ_1 are the only unknowns.

The first-order conditions for minimizing (30) involve a quadratic equation. The concentrated-likelihood function for the overall time period is graphed in Figure 1, a and b. Figure 1b suggests that, in the neighborhood of the maximum-likelihood estimates, γ_0 is estimated more precisely than γ_1 and there is negative correlation between the two estimates. Figure 1a has a coarser grid than Figure 1b. Figure 1a suggests that higher values for $\hat{\gamma}_1$ will not dramatically affect the maximized value of the likelihood function, but lower values for $\hat{\gamma}_1$ will have an impact.

Table V provides the point estimates of γ_0 and γ_1 along with the asymptotic standard errors. The subperiods in Table V correspond to the points where the data are spliced (see Section II. *A*). Unlike many studies on asset pricing models, the estimates of the expected return on the zero-beta asset are quite small. With the exception of the first subperiod, the point estimates are less than or equal to fifteen basis points (annualized), and in many cases the rate is negative (but only significant and negative in the second subperiod). This suggests that one implication of a riskless asset version of the CCAPM is consistent with the data. Table IV provides some information about the ex post real return on short-term Treasury bills during this time period, and in all cases the estimate of γ_0 is smaller than the sample mean in Table IV. Another implication of the CCAPM is that the market price of risk should be positive, for the expected return increases as the risk increases. This implication is verified for all periods, and

[20] Shanken derives this result by first maximizing the likelihood function with respect to β_{ci} and σ_{ij}. These estimators depend on γ_0 and γ_1. Shanken then substitutes the estimators for β_{ci} and σ_{ij} back into the original likelihood function, which then depends on only γ_0 and γ_1. This new function is the concentrated-likelihood function. After some algebra he discovers that maximizing the concentrated-likelihood function is equivalent to minimizing (30) above. Note that full maximum likelihood refers to maximizing the likelihood function with respect to σ_{ij} as well as γ_0, γ_1, and β_{ci}.

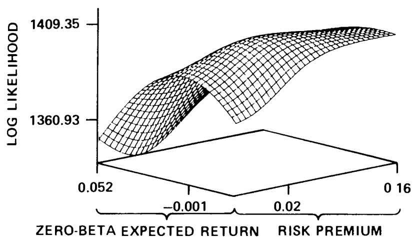

Figure 1a Likelihood surface for the CCAPM based on quarterly consumption data 1929-1982

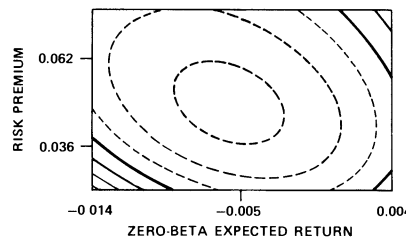

Figure 1b Contours of the likelihood surface for the CCAPM based on quarterly consumption data 1929-82

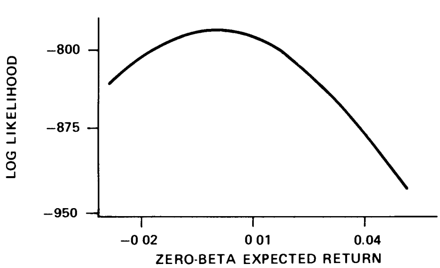

Figure 1c Likelihood surface for the CCAPM based on the MCP and monthly data 1926-82

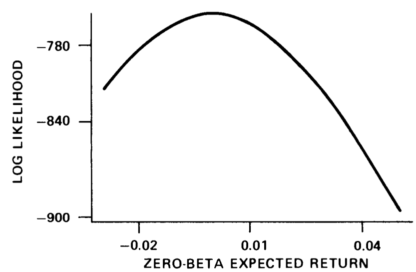

Figure 1d Likelihood surface for the CAPM based on the CRSP Index and monthly data 1926-82

Figure 1. The concentrated log likelihood functions, *l*, for the CCAPM and the CAPM. The relevant parameters of these functions are the expected annualized return on the "zero-beta" portfolio, γ_0, and the expected annualized premium for consumption-beta risk, γ_1.

the point estimate is statistically significant in most rows of Table V.[21] While the magnitude of the estimate of γ_1 seems large, Section II.*D* shows that γ_1 is biased upwards relative to γ_1^* by the variance of the sampling error in reported consumption. Reflecting the large standard errors in some subperiods, the variation in $\hat{\gamma}_1$ across subperiods is striking. Since γ_1 does reflect the variance in measurement error for consumption, the high value of $\hat{\gamma}_1$ in the earlier subperiods (except 1929–1939) may be the result.

The CCAPM also implies that the relation between expected returns and betas is linear, or the null hypothesis is the equality given in (29). This null hypothesis is tested against a vague alternative that the equality does not hold.

Gibbons (1982) suggests a likelihood ratio for testing hypotheses like (29). Such an approach relies on an asymptotic distribution as T becomes large. However, the methodology may have undesirable small sample properties, espe-

[21] In addition to the full maximum-likelihood procedure, the two step GLS estimator suggested in Gibbons (1982) was used. This estimator is not as desirable as the full maximum-likelihood approach as the number of securities approaches infinity, and it should be downward biased due to a phenomenon similar to errors-in-variables for simple regressions. Since consumption betas are measured less precisely than market betas, the difference between the full maximum likelihood and two-step GLS should be larger in this application than in past tests of the CAPM. In fact, the GLS estimate of γ_1 is usually half the value reported in Table V. On the other hand, the simulations by Amsler and Schmidt (1985) suggest that the finite sample behavior of the GLS estimator is better than the maximum-likelihood alternative. Since the sign of the estimate from either approach is the same across any row in Table V and since the significance from zero is the same across any row (except for one subperiod), the GLS results are not reported, but they are available on request to the authors.

Table V

Estimating and Testing the CCAPM Using Aggregate Consumption Data

All data are annualized and in real terms ($\tilde{R}_{it}$), and consumption growth ($\tilde{c}'_t$) is adjusted to correct for the summation bias. The model is fit to seventeen assets (twelve industry portfolios, four boud portfolios, and the CRSP value-weighted index). The econometric model is

$$\tilde{R}_{it} = \alpha_{ci} + \beta_{ci}\tilde{c}'_t + \tilde{u}_{it},$$
$$\mathscr{E}(\tilde{u}_t\tilde{u}'_s) = \Sigma \text{ if } t = s, 0 \text{ otherwise.}$$
$$H_0\text{: } \alpha_{ci} = \gamma_0 + \gamma_1\beta_{ci}, \ \forall\ i = 1, \cdots, 17.$$

The data are annualized by multiplying the quarterly returns by 4 and monthly returns by 12. $F(\beta_{ci} = \beta_{cj})$ is the F-statistic for testing the hypothesis that $\beta_{ci} = \beta_{cj}$ $\forall$ $i \neq j$, while $F(\beta_{ci} = 0)$ tests the hypothesis that $\beta_{ci} = 0$, $\forall$ $i = 1, \cdots, 17$. Both $\hat{\gamma}_0$ and $\hat{\gamma}_1$ are estimates from a full maximum-likelihood procedure, and their respective standard errors (given in parentheses below the estimates) are based on the inverse of the relevant information matrix. The likelihood ratio (LR) provides a test of the null hypothesis that expected returns are linear in consumption betas as implied by the CCAPM. The likelihood ratio is adjusted by Bartlett's (1938) correction. In all cases, p-value is the probability of seeing a higher statistic than the one reported under the null hypothesis. If the test statistics are independent across subperiods, then the last four rows can be aggregated into one summary measure. In the case of the likelihood-ratio test, the overall results yield a χ^2_{60} random variable with a realization equal to 69.06. (This yields a p-value equal to 0.198.) For the F-statistic, the overall results yield a standardized normal random variable with a realization equal to 0.68, which implies 0.25 as a p-value.

Date	Number of Observations	F-test: Betas Equal (p-Value)	F-test: Betas = Zero (p-Value)	$\hat{\gamma}_0$ (SE($\hat{\gamma}_0$))	$\hat{\gamma}_1$ (SE($\hat{\gamma}_1$))	LR Test of H_0 (p-Value)
Panel A: Spliced Quarterly Consumption Data, Adjusted for Summation Bias, 1929–1982						
1929Q2–	215	3.874	3.912	−0.0061	0.0478	28.03
1982Q4		(<0.001)	(<0.001)	(0.0044)	(0.0133)	(0.021)
1929Q2–	40	4.319	4.241	0.0484	0.0329	26.45
1939Q1		(0.001)	(0.001)	(0.0091)	(0.0189)	(0.034)
1939Q2–	32	0.502	1.410	−0.2558	0.5850	6.84
1947Q1		(0.908)	(0.261)	(0.0859)	(0.2507)	(0.962)
1947Q2–	48	1.006	1.182	−0.0699	0.2928	14.82
1959Q1		(0.476)	(0.334)	(0.0469)	(0.1865)	(0.464)
1959Q2–	95	2.257	2.277	0.0015	0.0187	20.95
1982Q4		(0.009)	(0.008)	(0.0028)	(0.0062)	(0.138)
Panel B: Unspliced Quarterly Consumption Data, Adjusted for Summation Bias, 1947–1982						
1947Q2–	144	1.342	1.695	−0.0325	0.2136	19.87
1982Q4		(0.182)	(0.052)	(0.0256)	(0.1430)	(0.177)
1959Q2–	96	1.398	1.450	−0.0007	0.0528	16.29
1982Q4		(0.165)	(0.137)	(0.0040)	(0.0179)	(0.363)
Panel C: Unspliced Monthly Consumption Data, Adjusted for Summation Bias, 1959–1982						
1959 Feb–	287	1.316	1.581	−0.0008	0.0804	10.47
1982 Dec		(0.186)	(0.069)	(0.0034)	(0.0263)	(0.789)

cially when the number of assets is large (Stambaugh (1982)). The simulation by Amsler and Schmidt (1985) indicates that Barlett's (1938) correction, which was suggested by Jobson and Korkie (1982), improves the small sample performance of the likelihood ratio even when the number of assets is large. This correction is applied in all of the following tables.[22]

For the overall period using spliced data, the linear equality between reward and risk implied by the CCAPM is rejected at the traditional levels of significance. The last column of Table V reports the statistic in Panel A. This rejection is confirmed by Shanken's (1985, 1986) lower bound statistic, which suggests that the inference is robust to the asymptotic approximation of the likelihood ratio.[23] However, as noted at the bottom of Table V, aggregation of the results for each subperiod fails to reject the CCAPM at traditional levels of significance. The subperiod of 1929–1939 is the most damaging to the model. Given the nature of the consumption data used for this time period (see Section II. *A*), such behavior is troubling, for the rejection of the CCAPM may be due to measurement problems. On the other hand, the *F*-statistics given in the third and fourth columns of Table V suggest another interpretation. These *F*-statistics examine the joint significance of the risk measures across the assets as well as the significance of the dispersion of the risk measures across the assets. If all the risk measures were equal, then tests of (29) would lack power, and γ_0 and γ_1 would not be identified. In the first subperiod the risk measures are estimated with the most precision, and as a result tests against the null are more powerful.

Panel A of Table V is based on spliced quarterly consumption data. That is, monthly income predicts consumption for 1929–1939, and monthly consumption forms the basis of the quarterly numbers from 1959 to 1982. The above statistics are also calculated using the unspliced quarterly data from 1947 to 1982 with quarterly sampling intervals and on unspliced monthly data from 1959 to 1982 with monthly sampling intervals. The spliced data are considered first because the time series is longer and the measurement problems are less severe for quarterly observations than for monthly.

However, the results based on the spliced data are the least favorable to the CCAPM. Panels B and C in Table V suggest that the linearity hypothesis is *never* rejected with the unspliced monthly numbers and the unspliced quarterly numbers. Shanken's upper bound test confirms this result except for the subperiod 1947Q2-1982Q4. Also, the market price of consumption beta risk is also higher (with one exception) for the unspliced results.[24]

[22] The Lagrange multiplier test (suggested by Stambaugh (1982)) and the CSR test (suggested by Shanken (1986)) were also computed for all time periods without dramatically different results and not reported here. Both tests are monotonic transforms of the likelihood ratio (Shanken (1985)). The choice of which statistic to report is somewhat arbitrary. Since the geometric interpretation of the likelihood ratio follows, this statistic is reported in the tables.

[23] For all results we confirmed the inferences with Shanken's (1985, 1986) tests which have upper or lower bounds based on *finite sample* distributions. If the null hypothesis is not rejected with the upper bound, then it would not be rejected using an exact distribution. Similarly, if one rejects with the lower bound, such a result holds with a finite sample distribution.

[24] These results are consistent with those of Wheatley (1986), who re-examined Hansen and Singleton's (1983) tests of the CCAPM. Using 1959–1981 data, Wheatley showed by simulation that measurement error in consumption biased their test statistics. After correcting for that bias, he was unable to reject the CCAPM.

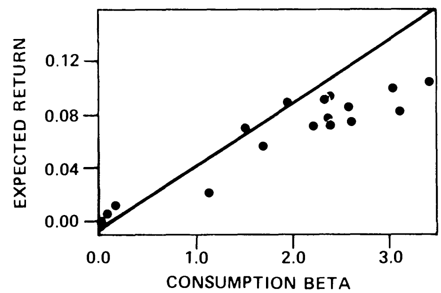

Figure 2a 1929Q2 1982Q4

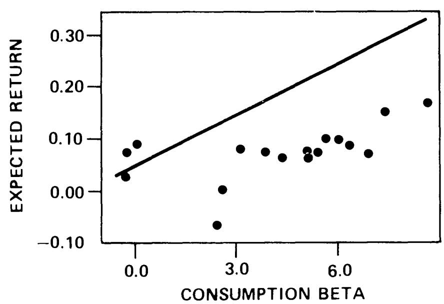

Figure 2b 1929Q2 1939Q1

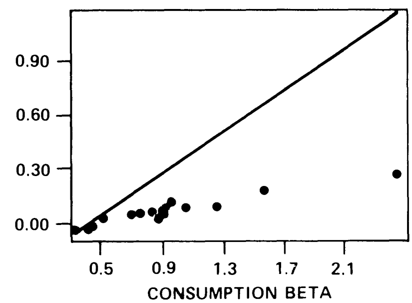

Figure 2c 1939Q2 1947Q1

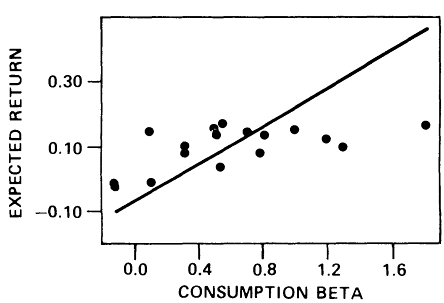

Figure 2d 1947Q2-1959Q1

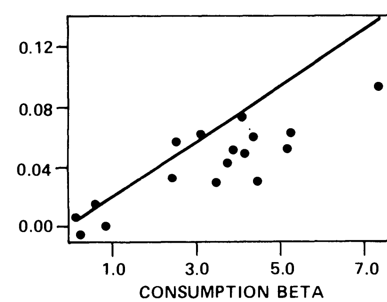

Figure 2e 1959Q2 1982Q4

Figure 2. Scatter plots of parameter estimates with and without CCAPM restriction. All data are annualized and in real terms, and consumption growth is adjusted to correct for summation bias. Seventeen assets (twelve industry portfolios, four bond portfolios, and the CRSP value-weighted index) are used. The intercept and slope of the solid straight line in each plot are determined by the maximum-likelihood estimates for the expected return on the "zero-beta" asset and premium for consumption-beta risk, respectively (*not* the ordinary least squares fit of the points). All points should fall on this line if the CCAPM is true. The seventeen points on each plot represent unrestricted estimates of expected return, $E(\tilde{R}_{it})$, and consumption beta, β_{ci}. (Note that the scale varies across the scatter plots.)

Except for the subperiod 1929–1939 (and its effect on the results for the overall time period), Table V provides positive support for the CCAPM. To provide a more intuitive interpretation of the empirical results, Figure 2 informally examines the deviations for the null hypothesis. Figure 2 plots the unrestricted mean returns against the unrestricted estimates of betas. The straight line represents the relation estimated by maximum likelihood. Despite the rejection of the theory by formal tests, the relation between expected returns and betas is reasonably linear[25]—perhaps more than could have been anticipated given the poor quality of the consumption data. In some of the plots (e.g., Figure 2d), a straight line fit to the points would be flat. Given the measurement error in the consumption betas, this flatness is expected. To better understand the "empirical validity" of the CCAPM, the efficiency of the maximum-correlation portfolio will now be considered.

Section II. *C* demonstrates that the MCP is ex ante mean-variance efficient under the CCAPM. This result is derived when the covariance matrix among returns on securities and consumption growth is known, which is not the case here. Thus, all the statistical inference concerning the ex ante efficiency of the MCP is conditional on the portfolio being the desired theoretical construct. Estimation error in the portfolio weights is ignored.

Following Gibbons (1982), consider testing the efficiency of any portfolio p when the riskless asset is not observed. Assume that the following regression is well specified in the sense that the error term has a zero mean and is uncorrelated with $\tilde{R}_{pt}$:

$$\tilde{R}_{it} = \alpha_{pi} + \beta_{pi}\tilde{R}_{pt} + \tilde{u}_{it}. \tag{31}$$

If portfolio p is efficient, then the following parameter restriction holds:

$$\alpha_{pi} = \gamma(1 - \beta_{pi}), \tag{32}$$

where γ is the expected return on the portfolio which is uncorrelated with p. Similar to the econometric model of (26) and (29) above, (31) and (32) are combined and then estimated by a full maximum-likelihood procedure. Furthermore, when (32) is treated as a null hypothesis, both a likelihood ratio and an asymptotic F are calculated. In the tests that follow, the maximum-correlation portfolio or the CRSP index is used as portfolio p.[26]

Figure 1, c and d, graphs the concentrated-likelihood function relative to possible estimates of the expected return on the zero-beta portfolio in the case of the MCP and CRSP index, respectively. Table VI summarizes the statistical results for both portfolios as well. Like Table V, the third column of Table VI indicates a small expected return on the zero-beta asset. Further, the point estimate when using the MCP never exceeds that when using the CRSP index. Also, the overall period rejects the efficiency of either the maximum-correlation portfolio or the CRSP index, as indicated by the last column of the table. (This rejection would occur even without relying on asymptotic theory to approximate the sampling distribution, for the lower bound test also rejects.) Unlike Table V, the rejection of the model does not stem from just the first subperiod. These

[25] Like beauty, perceived linearity is in the eyes of the beholder. One reviewer of this paper thought the graphs in Figure 2 revealed remarkable nonlinearities.

[26] Panels A and B of Table VI are based on sixteen assets, not seventeen as in Table V. The regressions using the CRSP index as the dependent variable have been excluded because otherwise the covariance matrix of the residuals would be singular.

Table VI

Estimating and Testing the Mean-Variance Efficiency of the Maximum-Correlation Portfolio (MCP) and the CRSP Value-Weighted Index, 1926–1982

All returns ($\tilde{R}_{it}$) are annualized and in real terms. The model is fit to sixteen assets (twelve industry and four bond portfolios). The econometric model is

$$\tilde{R}_{it} = \alpha_{pi} + \beta_{pi}\tilde{R}_{pt} + \tilde{u}_{it},$$
$$\mathscr{E}(\tilde{u}_t\tilde{u}'_s) = \Sigma \text{ if } t = s, 0 \text{ otherwise.}$$
$$H_0: \alpha_{pi} = \gamma(1 - \beta_{pi}),\ \forall\ i = 1, \cdots, 16.$$

$\tilde{R}_{pt}$ is either the return on MCP or a CRSP index. The maximum correlation portfolio (MCP) is constructed from the seventeen assets given in Table III. The weights of the MCP are determined by maximizing the sample correlation between the return on the portfolio and the growth rate of real consumption; see Table III for more details. The data are annualized by multiplying the monthly returns by 12. $\tilde{\gamma}$ is an estimate from a full maximum-likelihood procedure, and the standard errors (given in parentheses below the estimates) are based on the inverse of the relevant information matrix. The likelihood ratio (LR) provides a test of the null hypothesis that a given portfolio is efficient. The ratio is adjusted by Bartlett's (1938) correction. The p-value is the probability of seeing a higher statistic than the one reported under the null hypothesis. If the tests are independent across subperiods, then the last three rows in each panel can be aggregated into one summary measure based on either the likelihood ratio or the F-test. These aggregate test statistics always have p-values less than 0.0001.

Date	Number of Observations	$\hat{\gamma}$ (SE($\hat{\gamma}$))	LR Test (p-Value)
Panel A: Mean-Variance Efficiency Tests on the MCP			
1926–1982	684	−0.0009	26.86
		(0.0027)	(0.029)
1926–1945	240	0.0064	49.21
		(0.0054)	(<0.001)
1946–1965	240	−0.0151	40.96
		(0.0049)	(<0.001)
1966–1982	204	0.0016	19.25
		(0.0024)	(0.203)
Panel B: Mean-Variance Efficiency Tests on CRSP Index			
1926–1982	684	0.0000	26.77
		(0.0027)	(0.031)
1926–1945	240	0.0076	49.98
		(0.0053)	(<0.001)
1946–1965	240	−0.0125	36.62
		(0.0047)	(0.001)
1966–1982	204	0.0016	19.23
		(0.0024)	(0.204)

stronger rejections are probably due to the increased number of observations, which provides more precision. The joint significance of the betas across assets, as well as the significance of the dispersion of the betas across assets, is unrelated in Table VI, but it is much higher than the comparable F-statistics reported in Table V. Unfortunately, the test of efficiency of the MCP assumes that the portfolio weights are estimated without error, which is obviously not the case. If this measurement error were taken into account, the p-value would increase (see Kandel and Stambaugh (1988)).

The likelihood ratio test in Table VI can be given a geometrical interpretation based on the position of either the MCP or the CRSP index relative to the ex post efficiency frontier (Kandel 1984)). The mean-variance frontier is a parabola. A line joining the points corresponding to any given frontier portfolio and the minimum-variance portfolio intersects the mean axis at a point corresponding to the expected return of all portfolios having a zero beta relative to the frontier portfolio. When graphed with the variance on the horizontal axis, the slope of this line is equal to half the slope of the tangent at the point corresponding to the frontier portfolio (Gonzales-Gaviria (1973), pp. 58–61).

Building on this geometric relation, Figure 3 presents a graphical interpretation of the test statistic based on the ex post frontier. The maximum-likelihood estimates of the expected return on a portfolio having a zero beta relative to a test portfolio p (either MCP or CRSP in Figure 3) is denoted as γ. A line joining the mean axis at $\hat{\gamma}$ and the ex post minimum variance portfolio intersects the ex post frontier at a point (A or B in Figure 3) corresponding to the frontier portfolio having ex post zero-beta portfolios whose mean returns are equal to $\hat{\gamma}$. Let x equal the slope of this line. This portfolio would be the test portfolio, p, if and only if the test portfolio were ex post mean-variance efficient. Now consider a line joining the point corresponding to the test portfolio, p, and $\hat{\gamma}$. Denote the slope of this line by y. The LRT is equal to $T\ln(x/y)$ and is directly testing whether the slope of the second line is significantly less than the slope of the first line. A significantly lower slope for the second line implies rejection of the null hypothesis that the test portfolio is ex ante mean-variance efficient. The results of Table IV suggest that the two lines in either Figure 3a or 3b do have statistically different slopes.

Figure 3 also provides a comparison of the inefficiency of the MCP versus the CRSP index. For example, Figure 3a provides the unconstrained ex post frontier as well as a parabola which represents the maximum-likelihood estimate of the frontier assuming that the MCP is efficient. Figure 3b provides similar information in the case of the CRSP index. The scales of Figure 3a and 3b are equal, and there is little difference between the frontier constrained so that MCP is efficient versus a case where the CRSP index is efficient. Figure 3c, which has a very fine grid, is provided to see the difference between the two constrained frontiers.

Based on Table VI and Figure 3, the relative merits of the CCAPM versus the CAPM are difficult to discern. The inefficiency of either the MCP or the CRSP index is about the same. The two models are hard to compare because they are inherently non-nested hypotheses, which makes formal inference difficult. How-

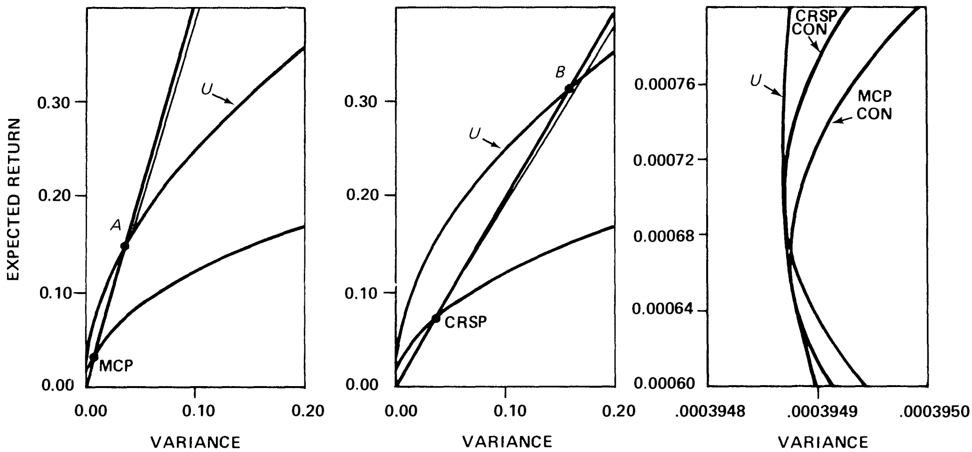

Figure 3a. Unconstrained *ex post* efficient frontier (U) versus frontier constrained so that MCP is efficient.

Figure 3b. Unconstrained *ex post* efficient frontier (U) versus frontier constrained so that CRSP Index is efficient.

Figure 3c. Unconstrained *ex post* efficient frontier (U) versus frontier constrained so that CRSP Index is efficient (CRSP CON.) versus frontier constrained so that MCP is efficient (MCP CON.). Note the grid is very fine.

Figure 3. A geometrical interpretation of the likelihood-ratio test, LRT, of ex ante efficiency for the MCP and CRSP value-weighted index based on monthly real returns, 1926–1982. The sample means and variances are annualized by multiplying by twelve. The LRT equals $T \ln(x/y)$. x is the slope of the straight line that passes through the maximum-likelihood estimate of the expected return on the zero-beta portfolio, $\hat{\gamma}$, and the global minimum variance point of the ex post frontier. y is the slope of the straight line that passes through $\hat{\gamma}$ and the test portfolio (either the MCP or CRSP index). The ex post frontier is based on sixteen assets (twelve industry portfolios and four bond portfolios) and either the MCP or CRSP index.

ever, the apparent inefficiency of the MCP is overstated since the portfolio weights are estimated with error.

V. Conclusion

This paper tests the consumption-oriented CAPM and compares the model with the market-oriented CAPM. Two econometric problems peculiar to consumption data are analyzed. First, real consumption reported for a quarter is an integral of instantaneous consumption rates during the quarter, rather than the consumption rate on the last day of the quarter. This "summation bias" lowers the variance of measured consumption growth and creates positive autocorrelation, even when the true consumption rate has no autocorrelation. This summation bias also underestimates the covariance between measured consumption and asset returns by half the true values, with the result that measured consumption betas are ¾ of their true values. The empirical work accounts of these problems.

A second major econometric problem is the paucity of data points for consumption growth rates. Some tests use the consumption data (adjusted for the summation bias). However, alternative tests are based on the returns of the portfolio of assets (the "MCP") that is most highly correlated with the growth rate of real consumption. The CCAPM implies that expected returns should be linearly related to betas calculated with respect to the MCP. Interestingly, the MCP has a correlation of 0.67 with the CRSP value-weighted index. Apart from stocks, a major component of the MCP is the return on a "junk bond" portfolio. Thus, the correlation between average returns and the sensitivity of returns on various assets to junk bond returns, which has been discussed by Chen, Roll, and Ross (1983), may be attributed to the correlation between junk bond returns and real growth in consumption.

A number of tests of the consumption-oriented CAPM are examined. Unlike past studies on asset pricing, the estimated return on the zero-beta asset is quite small. Except for one subperiod, all the estimates are less than or equal to fifteen basis points (annualized). This suggests that some of the implications of a riskless real asset version of the CCAPM are consistent with the data. Another implication of the CCAPM is that the market price of risk should be positive; in other words, the expected return increases as the risk increases. This implication is verified for all periods, and the point estimate is statistically significant in most of the subperiods.

Based on the quarterly consumption data for the overall period, the linear equality between reward and risk implied by the CCAPM is rejected at the 0.05 level. However, a plot suggests that the relation is reasonably linear given the poor quality of the consumption data. Analysis by subperiods reveals that the time period from 1929 through 1939 seems to be the most damaging to the model. In fact, when the model is estimated by subperiods and then the results are aggregated across subperiods, no rejection occurs at the usual levels of significance. The first subperiod may be rejecting the model because the risk measures are estimated more precisely due to the large fluctuations in consumption and asset returns in the 1930s. The added precision should increase the power of tests. On the other hand, the quality of the data for this time period is particularly suspicious. While the CCAPM is by no means a perfect description of the data, we found the fit better than we anticipated.

For the overall period (1926–1982), the mean-variance efficiency is rejected for both the CRSP value-weighted index and the portfolio with maximum correlation with consumption (the MCP). This rejection occurs in a number of time periods, not just the 1929–1939 subperiod. Given that the estimated risk measures for both models are highly correlated, this similarity in the performances by the CAPM and the CCAPM is predictable. Since these tests permit the use of monthly, not quarterly, data, the rejection could be attributed to the increased power of the tests due to additional observations. On the other hand, the statistical significance of the rejection of the efficiency of the MCP is overstated since the portfolio weights are unknown and had to be estimated.

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