The Term Structure: Discussion
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Source: *The Journal of Finance*, Vol. 35, No. 2, Papers and Proceedings Thirty-Eighth Annual Meeting American Finance Association, Atlanta, Georgia, December 28-30, 1979 (May, 1980), pp. 435-438
Published by: Blackwell Publishing for the American Finance Association
Stable URL: http://www.jstor.org/stable/2327402
Accessed: 29/08/2010 10:57

Box-Jenkins [1976] cross-correlation analysis; (*ii*) cross-spectral estimation; and (*iii*) estimation of a regression for a linear approximation to the function. Since theory suggests i_t and C_t will be jointly endogenous, a simultaneous equation technique is required. Unfortunately, space does not permit a fuller explanation or results to be given. In general, the tests indicate (loosely) a "significant" relationship between i_t and C_t does exist.

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Discussion by

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MARSH'S PAPER ON equilibrium term structure models and tests of them has

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many diverse elements. First, there are some modest generalizations of recent interest rate solutions by Cox, Ingersoll, and Ross (1977), Vasicek (1977) and Ingersoll, Skelton and Weil (1978). A test is discussed for the predictions of those models that relative instantaneous expected excess rates of return on various bonds are functions solely of the times to maturity of the bonds compared. Real per capita consumption is used both in a competitive model of the term structure and in a complementary model with both the short rate and consumption as instrumental variables. Numerical integration is used to solve a partial differential equation for bond prices and a brief test of the expectations hypothesis is presented. Since each of these investigations could support a paper by itself, the explanations of the models, their assumptions, empirical implications and empirical methodology is very terse. My comments are intended to explain in more detail the economics of the models and to assess the significance of them.

In Section 2, a generalization of the solutions of CIR and Vasicek is presented. The generalization is that the instantaneous variance of the interest rate process is permitted to have a constant term in addition to a term proportional to the interest rate. The solution for discount bond prices' "betas" in these models exhibits separability into a function of the single state variable (or the interest rate) and a function of the time to maturity. This gives ratios of bond betas and ratios of their equilibrium expected excess returns as functions only of the times to maturity of the bonds. It is not shown (and may not be true) that the assumptions of logarithmic utility, the square root process and a single state variable are necessary assumptions for separability. The necessary conditions for separability are reserved for future research.

The discussion of a test methodology for the separability implications of the CIR and Vasicek models is quite insightful. Efficient estimators for the ratios of expected excess returns are discussed, both for the case of intertemporally constant ratios and for the case of stochastic ratios. An interesting theoretical issue is whether it is possible to have stochastic ratios of expected excess returns on bonds that are unrelated to opportunity set shifts. This issue must be addressed to determine whether a stochastic a_t in 2.17 is consistent with separability. An extension of the ISW model is presented in an attempt to show that stochastic relative risk premia are consistent with separability, but the conditions for such a conclusion, even in the model presented (2.22–2.24) are very restrictive. Even if those conditions held, risk premia would depend upon the random variable ξ, which generally affects individuals' expected utilities, consumption and portfolio rules. Thus, separability would hold only in the restricted sense that relative risk premia do not depend upon the level of the instantaneous rate; separability of bond betas with respect to the state vector and time would not hold in 2.22–2.24. A more detailed analysis that focuses upon real betas and nominal expected excess returns might give stochastic ratios when real betas and real excess returns are separable, but that is beyond the scope of the paper.

Marsh's use of per capita real consumption as an instrument in bond pricing functions such as $F(C^*, i, \tau)$ and $F(C^*, \tau)$ is promising, but requires some explanations beyond those given. There are two reasons why the stochastic process for consumption is of potential interest for bond pricing and bond returns.

First, in a continuous-time model that includes all of the economies discussed by Marsh, Breeden (1978) demonstrated that any asset's expected excess return is proportional to its covariance with aggregate consumption. Thus, in a general continuous-time model, separability of bonds' "consumption-betas" in the state vector and time will imply equation 2.4. Second, the equilibrium price of any discount bond should equal (for each individual) the expected marginal utility of consumption at the maturity date divided by the marginal utility of current consumption. If individuals have time-additive utility functions for consumption, then only today's consumption and the probability distribution of future consumption are needed to find discount bond prices from preferences.

For logarithmic and power utility functions, it can be more intuitively stated that a discount bond price is proportional to the expected value of a power of the ratio of current consumption to consumption at the maturity of the bond. Thus, for these utility functions in a general probabilistic model, the characteristics of the growth rate (its mean, variance, etc.) of consumption may be used to determine bond prices[1] If a simple Markov process is assumed for consumption (as in 3.8), then bond prices may be found in terms of a current consumption and time to maturity, $F(C^*, t, \tau)$, since this data describes the probability distribution of future consumption.

To the extent that the current consumption level describes its future growth rate well, the function $F(C^*, \tau)$ is economically justified. Alternatively, an interest rate is related to technological productivity (as shown by CIR) and may thereby describe the growth rate of consumption better than current consumption does. In that case, which is quite reasonable, a function $F(i, \tau)$ should better describe bond prices than $F(C^*, \tau)$. If the probability distribution of the growth of consumption is described by both C^* and i, then $F(C^*, i, \tau)$ is economically justified. For that case, the bond pricing solution presented by Marsh in 3.10, 3.11 and 3.12 is of considerable interest, despite its reliance upon the unrealistic assumption of constant absolute risk aversion. More work on closed-form solutions for vector Markov models (with more discussion of their economic properties and estimation of the model's parameters) is in order. The evaluation of these various models must ultimately be an empirical problem, to which the techniques mentioned in Section 2 may be applied.

Numerical methods are applied in Section 4 to estimate bond prices from consumption alone. As stated earlier, except for the unlikely case of a simple Markov process for consumption, this is not well justified. The pricing results, which are presented in Tables 1, 2, and 3, are not very impressive in accuracy. A simple model based upon a short-term interest rate should be estimated for comparison purposes. Of course, estimation of the multivariate model, $F(C^*, i, \tau)$ would be of even greater interest.

The final test mentioned, for which results were not given, can be viewed as a test of whether the consumption-beta of bond prices is nonzero. Marsh found "significant" nonzero estimates, from which he infers that the expectations

[1] See Rubinstein (1976) and Breeden (1977), Chapter VII, for derivations of interest rates from probability distributions of growth rates of per capita consumption for various time horizons.

hypothesis should not hold. A test of whether actual returns on bonds are related to their measured consumption risk is yet to be done.

In summary, many insights are presented by Marsh in his paper, but many questions remain. The results of the econometric tests described should be informative.

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