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Consumption Risk in Futures Markets

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I. Introduction

IN RECENT EMPIRICAL research examining commodity futures price changes in the Sharpe (1964)-Lintner (1965) capital asset pricing model framework, Dusak (1973), Grauer (1977), and Bodie and Rosansky (1979) found that virtually all commodity futures price changes examined had no systematic risk. That is, each author found that commodity futures prices had market "betas" that were insignificantly different from zero when estimated from historical data. If a commodity futures contract has no real systematic risk, then its price should neither tend to increase nor decrease as it matures, according to the CAPM.[1] In a study of price changes in 25 commodity futures markets over the 1947–1965 period, Rockwell found that futures prices increased on an average by approximately 4% per year. Dusak found that price changes in 3 grains over the 1952–1967 period were near zero on average, which was consistent with the CAPM, given her risk estimates for the grains. Grauer found positive price changes on average from 1959–1974 in his 14-commodity study, but they were not statistically different from zero. Most recently, Bodie and Rosansky examined "excess returns" during the 1950–1976 period for a portfolio of 23 futures contracts. The excess returns on the futures portfolio were the same size as the excess returns on common stocks over the same period—approximately 9.5%. Both stocks and futures had average excess returns that were more than twice their standard errors. The Bodie-Rosansky results were not consistent with the CAPM, since the betas of most of their commodities and of their benchmark portfolio were negative when measured relative to the S&P 500.

The purpose of this paper is to theoretically and empirically examine the systematic risks of all contract maturities for 20 different commodities. The principal difference between this paper and those cited is in the measurement of systematic risk. This study considers risk in the context of the intertemporal asset pricing model of Breeden and Litzenberger (1978) and Breeden (1979). Their models derive equilibrium expected excess returns on assets for more general economies than those considered by Sharpe and Lintner. In particular, both papers are consistent with stochastic investment opportunities; more importantly

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[1] Since no investment is required for a futures contract, futures prices' changes reflect only risk premia, if they exist. See Grauer (1977) or Bodie and Rosansky (1979) for more discussion of this point.

for the present study, the Breeden continuous-time analysis derives equilibrium expected excess returns in a multi-good economy with uncertain commodity prices for individuals with general and diverse preferences for consumption bundles.[2] The intertemporal pricing model states that the equilibrium expected excess return on an asset should be proportional to its covariance of returns with changes in aggregate real consumption, i.e., to its "consumption-beta". Thus, although many commodity futures contracts may have price changes that are unrelated to those of stocks or to the market portfolio's return, they should earn risk premia if their real price changes are correlated with changes in aggregate real consumption, according to the intertemporal CAPM.

It is certainly plausible that certain commodity futures contracts have price changes that are correlated with changes in aggregate real consumption. For example, the prices of live cattle futures and live hog futures should be positively correlated with consumption, since increased real consumption expenditures should result in greater demands for beef and pork, with consequent increases in their prices. Similarly, it is plausible that the price of copper is related to the output of the economy and to aggregate consumption. For commodities with a large degree of production uncertainty (such as the grains), the relation of the production uncertainty to aggregate consumption must also be considered to determine the correlation of the commodity's price change with consumption. An example of this effect of quantity uncertainty on a commodity's consumption-beta is as follows: the amount of grain available for consumption (typically indirectly through other products) depends upon the weather through its effect on the harvests. A larger harvest than expected makes greater consumption than expected possible, but the large harvest tends to decrease grain prices. Consequently, grain futures may exhibit negative consumption-betas due to the correlation of production with aggregate consumption, although the positive income elasticity of demand for grains tend to offset this effect. Thus, economic theory provides predictions of the determinants of consumption-betas. The next section of the paper develops these arguments more formally in a simple economic model.

Following the theoretical analysis of the relation of commodity price changes to changes in aggregate consumption, estimates of actual futures market consumption-betas are made for 20 commodities for the 1960–1978 period. Annual data are utilized for the analysis of price changes over the entire period, and quarterly data are utilized for an examination of futures market betas over the 1969–1978 period. The time periods were chosen based upon the availability of "consensus" economic forecasts for the growth rate of aggregate real consumption. The use of such forecasts to estimate deviations of consumption from its mean is recognized as only one of many possible approaches, but it obviates the need to estimate the mean of a possibly nonstationary series—the growth rate of aggregate real consumption. The data on annual economic forecasts were obtained from the Conference Board's Economic Forum.[3] The quarterly forecast data were

[2] Grauer and Litzenberger (1979) derived equilibrium expected excess returns in terms of real market betas in a multi-good model, but their derivation was for a two-period economy, with individuals who had identical and homothetic preferences.

[3] See the Conference Board's *Business Outlook*.

obtained from the survey of economists prepared by the American Statistical Association and the National Bureau of Economic Research (ASA/NBER).

II. An Economic Model of Commodity Consumption-betas

For simplicity and for mathematical tractability, assume that the economy is composed of many identical individuals who are price takers. With identical individuals, there will be no trading and it suffices to do the analysis as if the economy were composed of a single price-taking individual. There are uncertainties in the supplies of goods available for consumption. Defining the vector $q = (q_1 \, q_2 \cdots q_N)'$ as the aggregate production rates of the N consumption-goods, it is assumed that q follows a vector Ito process:[4]

$$d \ln q = \mu \, dt + \sigma \, dz, \tag{1}$$

where μ is an $N \times 1$ vector of drift rates for q, σ is an $N \times N$ diagonal matrix of the instantaneous standard deviations of percentage changes in production of the various goods, and dz is an $N \times 1$ vector of correlated Wiener processes. The different elements of dz may be viewed as measures of the effects of fluctuations of exogenous forces, such as weather in crop growing areas, that affect production yields. The parameters μ and σ in (1) may be (seasonal) functions of time and of the state of the world. This is quite general, except for the continuity of production assumed by the Ito formulation. It should be noted, however, that production rates and production uncertainty can be arbitrarily close to zero over certain time periods and still be consistent with this assumption. Jumps or discontinuties in the production processes are not consistent with (1). Note that in the empirical estimates of Section V some stationarity assumptions are required for the consumption-beta estimates to be valid.

An individual in this model chooses his optimal consumption expenditure based upon his wealth and consumption and investment opportunities. Given a choice of the optimal nominal consumption expenditure by the individual, e, the individual's consumption rates of the N consumption-goods, denoted by the vector c, will be functions of the vector of consumption-goods prices, p, i.e., $c = c(e, p)$. Assuming that the individual's optimal nominal expenditure and the commodity price vector follow Ito processes, which they will if they are sufficiently well-behaved functions of other Ito processes (such as q), we may write their stochastic differential equations as:

$$d \ln e = \mu_e \, dt + \sigma_e \, dz_e \tag{2}$$

$$d \ln p = \mu_p \, dt + \sigma_p \, dz_p \tag{3}$$

From Ito's Lemma applied to the function $c(e, p)$, the changes in the logarithms of the various consumption rates for goods may be written in terms of the individual's income elasticities of demand for the various goods, $\eta = \{\eta_j\}$, and his matrix of own- and cross-price elasticities of demand, $E = \{\epsilon_{ij}\}$:

[4] See Gihman and Skorohod (1972) for a discussion of stochastic differential equations and of Ito's Lemma. Additional references are in Breeden (1979).

$$d \ln c = \{\mu_e \eta + E\mu_p + h\}\, dt + \eta \sigma_e\, dz_e + E\sigma_p\, dz_p, \tag{4}$$

where h is a vector of the box products of elasticities of η and E with respect to e and p with the variance-covariance matrix of $(e\, p)$ multiplied by .5.

The budget constraint implies that $e = p'c$. Thus, the stochastic differential equation for nominal expenditure may be written in terms of the stochastic differentials for prices and quantities consumed. In doing this, note that the elasticities of nominal expenditure with respect to the various consumption-goods prices are equal to the respective budget shares of the consumption goods, as are the elasticities of expenditure with respect to the various quantities consumed. Denoting the vector of budget shares for the individual as w, the statements are that: $\partial \ln e/\partial \ln p = w$ and $\partial \ln e/\partial \ln c = w$. Furthermore, the matrices of second partial derivatives of expenditure with respect to prices and quantities are (where W is a diagonal matrix of the budget shares, I is an $N \times N$ identity matrix and 1 is an $N \times 1$ vector of ones):

$$\begin{aligned}\partial^2 \ln e/(\partial \ln p)(\partial \ln p') &= W\{I - 1w'\}\\ &= \partial^2 \ln e/(\partial \ln c)(\partial \ln c')\\ &= \partial^2 \ln e/(\partial \ln p)(\partial \ln c)'\end{aligned} \tag{5}$$

Applying Ito's Lemma to the function $e = p'c$ and using the partial derivatives as stated gives:

$$d \ln e = w'(d \ln c + d \ln p) + k\, dt, \tag{6}$$

where

$$k = \tfrac{1}{2}\begin{Bmatrix} W(I - 1w')\,W(I - 1w') \\ W(I - 1w')\,W(I - 1w') \end{Bmatrix} \square\, V_{pc}$$

and $\square$ represents a box product of the two matrices. The matrix V_{pc} is the variance-covariance matrix of the logarithms of prices and quantities consumed.

Substituting the stochastic differential for the logarithm of nominal expenditure, (6), into equation 4 via equation 2 gives.

$$d \ln c = \eta w'(d \ln c) + \eta w'(d \ln p) + E(d \ln p) + (k\eta + h)\, dt \tag{7}$$

or, alternatively,

$$\begin{aligned}(I - \eta w')(d \ln c) &= (E + \eta w')(d \ln p) + (k\eta + h)\, dt\\ &= E^*(d \ln p) + (k\eta + h)\, dt\end{aligned} \tag{8}$$

where $E^* = E + \eta w'$ is the matrix of compensated own and cross-price elasticities of demand. At this point, it would be convenient to be able to invert the matrix of compensated price elasticities, E^*, to determine the stochastic movements in prices from the stochastic movements of quantities consumed. However, economic theory implies through the Slutsky and aggregation conditions that E^* is not invertible, since $w'E^* = 0$ and $E^*1 = 0$ (0 is an $N \times 1$ vector of zeroes).[5] Thus,

[5] See Henderson and Quandt, *Microeconomic Theory*, Chapter 2.

nominal price distributions cannot all be determined in this system; not surprisingly, only relative prices may be determined.

Letting the Nth good be the numeraire whose price does not change over time, letting p_- be the $(N-1) \times 1$ vector of the remaining prices, letting E^*_- denote the $N \times (N-1)$ matrix of compensated price elasticities excluding the Nth column, equation 7 may be rewritten as:

$$(I - \eta w')(d \ln c) = E^*_-(d \ln p_-) + (k\eta + h)\, dt \tag{9}$$

Since $E^{*\prime}_- E^*_-$ will be of full rank $(N-1)$, equation 9 may be multiplied by $E^{*\prime}_-$ and inverted to find:

$$d \ln p_- = (E^{*\prime}_- E^*_-)^{-1} E^{*\prime}_- \{[I - \eta w'](d \ln c) - (k\eta + h)\, dt\} \tag{10}$$

Defining $M = (E^{*\prime}_- E^*_-)^{-1} E^{*\prime}_-$, the stochastic differential equations for prices may be expressed as follows:

$$d \ln p_- = \{M(I - \eta w')\mu_c - M(k\eta + h)\}\, dt + M(I - \eta w')\sigma_c\, dz, \tag{11}$$

where μ_c and σ_c are the drift and diffusion parameters for c. The useful characteristic of (11) is that the stochastic part of the changes in relative prices, as given by the coefficient of dz, depends only upon the individual's compensated price elasticities (through M), upon the individual's expenditure ("income") elasticities of demand (η), upon his vector of budget shares (w) and upon the uncertain fluctuations in consumption rates ($\sigma_c\, dz$). Since the government has continually compiled budget share data and economists have long been estimating income and price elasticities of demand, and since consumption fluctuations are potentially measurable, the elements of (11) have some intuitive appeal as being those which should affect the translation of quantity uncertainties into price uncertainties and for which approximate sizes are known.

The variance-covariance matrix of commodity price changes may be obtained from (11) as:

$$\text{Cov}(\ln p, \ln p') = M(I - \eta w')V_{cc}(I - w\eta')M' \tag{12}$$

where V_{cc} is the variance-covariance matrix of fluctuations in the logarithms of consumption rates for the various goods.

For the purpose of this paper, the covariances of the various commodity prices with real consumption are of considerable interest. Recognizing that real consumption is simply a quantity index (as noted by Samuelson and Swamy (1974)), with local weights proportional to the individual's budget shares, w, the covariances of changes in the logarithms of commodity prices with real consumption are obtained from (11) and Ito's Lemma as:

$$\begin{aligned} \text{Cov}(\ln p, \ln C) &= M(I - \eta w')V_{cc}w \\ &= M(Vw) - M\eta(w'Vw) \\ &= M\,\text{Cov}(\ln c, \ln c) - M\eta\,\text{Var}(\ln C) \end{aligned} \tag{13}$$

where C denotes the real consumption index with stochastic changes given by: $d \ln C = w'(d \ln c)$. Dividing (13) by the variance of the logarithm of real consumption gives the spot commodity prices' betas relative to changes in real consumption:

$$\beta = \frac{M \operatorname{Cov}(\ln c, \ln C)}{\operatorname{Var}(\ln C)} - M\eta \tag{14}$$

Equation (14) states that the various commodity prices' consumption-betas can be expressed as linear combinations of the various consumption rates' covariances with real consumption and of the income elasticities of demand for the various goods. The coefficients in the linear combinations depend only upon the compensated cross-price elasticity matrix.

To gain further insight into (14), consider the implications of an assumption that compensated cross-price elasticities are all zeroes for a set of goods—only own-price elasticities and income elasticities are significant. This implies a block-diagonal compensated price elasticity matrix, E^*. Technically, this violates the Slutsky conditions, but the intuition is helpful. For such goods, the consumption-betas of the goods' prices would be given by:

$$\beta_i = -\left[\frac{-1}{\epsilon_{ii}^*}\right]\frac{\operatorname{Cov}(\ln c_i, \ln C)}{\operatorname{Var}(\ln C)} + \left[\frac{-1}{\epsilon_{ii}^*}\right]\eta_i \tag{15}$$

The intuition of (15) is that, since compensated own-price elasticities of demand are negative, a commodity's consumption-beta depends upon: (1) the income elasticity of demand for the good, with higher income elasticities implying higher consumption-betas, (2) the covariance of the consumption of the good with the real consumption index, with large covariances of consumption of the good with the index resulting in a lower (even negative) consumption-beta, and (3) the own-price elasticity of demand for the good, with large own-price elasticities being associated with consumption betas near zero. The income elasticity of demand effect was predictable, given the live cattle example in the Introduction. A positive association of output and consumption with the real consumption index, as would be expected for the grains, tends to offset the income elasticity effect and may result in negative consumption betas for goods with large output covariances relative to income elasticities. The result that a good with a large (in absolute value) own-price elasticity should tend to have a beta near zero, *ceteris paribus*, is explicable by noting that any increases in the price of the good due to increases in aggregate demand would require only a small price increase to result in the same quantity demanded as before the aggregate demand increase.

Given the analysis of the simplified (and unrealistic) case of zero cross-price elasticities, some intuitive results for the general case may be anticipated. A good with a large cross-elasticity of demand with a second good that has a high income elasticity of demand should tend to have a positive association with consumption, since unexpected increases in expenditure should result in increased demand for the good with a large income elasticity. The resulting increase in the relative price of that good will result in increased demand in the other good through the cross-elasticity relation (assuming that the goods are substitutes, rather than complements). A second example of the effect of cross-elasticities on goods' consumption-betas is for a good that is a substitute for a good that has highly uncertain production (and consumption) that is related to aggregate consumption. In this case, the substitute would tend to have a negative beta from the cross-elasticity effect, in that fluctuations in output of the one good would be transmitted to the substitute's market.

III. Supply Elasticities and Consumption-Betas

The analysis of consumption-betas of various commodities' spot prices in terms of demand elasticities and consumption fluctuations is consistent with the existence of production uncertainties, production responses, and with holdings of inventories of goods. Production uncertainties and production and inventory responses to price changes affect commodity prices' consumption-betas through their effects upon the consumption covariances in (14) and (15). The effects of production uncertainties and production and inventory adjustments can be separated by a slightly more detailed analysis of the supply side.

To do this, assume that the vector of rates at which consumption goods are supplied to consumers from producers, q^s, depend upon the prices of all goods, the inventories of goods, x, the current (stochastic) production rates of the various goods, q, and time. That is, supply functions for goods are: $q^s = q^s(p, x, q, t)$. The stochastic differentials for quantities supplied are:

$$\begin{aligned} d \ln q^s &= S(d \ln p) + \left(\frac{\partial \ln q^s}{\partial \ln x}\right)(d \ln x) + \frac{\partial \ln q^s}{\partial t} dt \\ &\quad + \frac{\partial \ln q^s}{\partial \ln q}(d \ln q) + f(dt) \\ &= S(d \ln p) + \left\{\frac{\partial \ln q^s}{\partial \ln x} X^{-1}(q - q^s) + \frac{\partial \ln q^s}{\partial t} + f\right\} dt \\ &\quad + \frac{\partial \ln q^s}{\partial \ln q} d \ln q \end{aligned} \tag{16}$$

where f is a vector of box products given by Ito's Lemma, and S is the $N \times N$ matrix of supply elasticities, i.e., $S = \partial \ln q^s/\partial \ln p$. The matrix X is a diagonal matrix with diagonal elements x_j.

In equilibrium, quantities supplied to consumers must equal quantities demanded by them: $q^s = c$. Letting $L = E^*_- - (I - \eta w')S_-$, where S_- is the supply elasticity matrix less its last column, and using the market clearing conditions and the technique used to derive (10), we find:

$$\begin{aligned} d \ln p_- = (L'L)^{-1}L'\bigg\{&(I - \eta w')\frac{\partial \ln q^s}{\partial \ln q}(d \ln q) \\ &+ [(I - \eta w')g - (k\eta + h)]\, dt\bigg\} \end{aligned} \tag{17}$$

From (16) and (17), the covariances of consumption rates, $c = q^s$, with aggregate real consumption are given by:

$$\text{Cov}(\ln c, \ln C) = \{S_{-}(L'L)^{-1}L'(I - \eta w') + I\} \frac{\partial \ln q^s}{\partial \ln q} \text{Cov}(\ln q, \ln C) \quad (18)$$

By substituting (18) into (14), commodities' consumption-betas may be expressed in terms of the demand elasticities previously discussed and in terms of commodities' supply elasticities and covariances of *production rates* with real consumption (rather than covariances of consumption rates with aggregate consumption).

The economic intuition for the role of quantity covariances (or "quantity betas") has been described, but the importance of supply elasticities has not. Intuitively, the greater the elasticity of a commodity's supply with respect to its price, the lower its consumption-beta. Fluctuations in real consumption that will increase a good's demand will result in a price increase and an increase in supply, with the elasticity of supply reducing the price increase that would otherwise be necessary. Of course, for the general case, cross-elasticities of supplies affect consumption-betas as indicated by (18) and (14). The intuitive explanation is similar to that for cross-elasticities of demand and will not be repeated.

IV. Time-Varying Supply and Demand Elasticities

The derivation of commodities' consumption-betas from quantity betas and supply and demand elasticities was for spot prices, rather than for futures prices. Due to the very high correlations between spot and futures prices for most commodities, the determinants of futures' consumption-betas are likely to be explained in large part by the same factors as the spots are.[6] In applying the theory of Sections II and III to the analysis of futures betas, an important consideration is of the dependence of elasticities upon the time to maturity. Most supply and demand responses are more elastic as the time horizon is lengthened. For most commodities, supply elasticities may be assumed to be near zero for short times to maturity and demand elasticities may be relatively small. As the time to the maturity of the futures contract is lengthened, supply responses will affect consumption-betas. A possible result, which is seen in some commodities in the subsequent empirical estimates of consumption-betas, is that consumption-betas decline as the time to maturity increases, due to supply responses.

It is possible to explicitly model the time-variance of supply and demand elasticities within the context of the model of Sections II and III. None of the equations are changed, but the elasticities should be interpreted as short-run (actually instantaneous) elasticities of supply and demand for the analysis of spot price changes. The time-varying elasticities may be derived by permitting demand equations, $c(e, p)$, to be a function of past prices as well as present prices. A specific form of such demand functions that gives time-varying demand elasticities is the replacement of p by a "non-anticipating functional", p^*, where

[6] The fact that futures contracts are "marked to market" daily with transfers of funds from the day's loser to the day's winner adds an additional element of risk and return that equals the interest earned or lost by such transfers. This element is not considered in this paper.

$$\ln p^* = \int_0^t \alpha_{t-\tau} \ln \; p_\tau \, d\tau \tag{19}$$

That is, individuals behave as if they are choosing optimal bundles based upon a weighted average of past prices. Considering a price shock that persists for a period, it is clear that the longer the time horizon, the greater the elasticities of demand, *ceteris paribus*. Time-varying elasticities may be modeled on the supply side by a similar replacement of p in the supply functions by a function of past prices such as p^*.[7]

To summarize, the point of this brief section is to illustrate the principal modification of the theory of the previous sections when considering futures' betas, rather than spot commodities' betas.[8] The time-variance of elasticities suggests that futures contracts with different times to maturity have different risk characteristics. This affects the estimation procedure for futures contract betas in the following section, as will be noted.

V. Empirical Estimates of Futures Markets' Consumption Betas

To estimate the consumption-beta for any asset, a series must be obtained or constructed for deviations of aggregate real consumption from its mean. As stated in the Introduction, forecasts of aggregate consumption by professional economists are used in this paper as proxies for expected consumption. There are two important virtues of this approach relative to autoregressive and moving average models. First, all of these professional economists have access to such models and presumably use them to the extent that they are helpful in forecasting.[9] Second, deviations of actual consumption from forecasts made in advance should more genuinely reflect forecast errors as they were committed. To the extent that asset prices and consumption plans and production plans reflected those expectations, deviations of actual consumption from forecasts should be associated with revaluations of assets and with modifications of production and future consumption plans. In contrast, a mechanical model of the consumption process might have standard errors that are similar to those of the economists forecasts, but might not be a true measure of economic surprises.[10]

The use of forecast data poses some problems, however, the largest being the unavailability of forecasts. Economists' annual forecasts of nominal GNP are available for most years after 1947, but estimates of inflation and real GNP were

[7] An example of consumption preferences that support an indirect utility function that depends upon past prices is of preferences that exhibit time-complementarity in the utility of consumption bundles. A justification of time-varying supply elasticities is provided by cost functions that depend upon the speed of production.

[8] See footnote 6.

[9] See Su and Su (1975) for evidence that forecasters in the ASA/NBER survey generally do better than Box-Jenkins forecasts.

[10] Some preliminary evidence on this conjecture is that stocks' excess returns (measured by the NYSE value-weighted index) had a correlation of .72 with the consumption forecast errors used in the annual part of this study, whereas the correlation of stocks' excess returns with deviations from an autoregressive model of aggregate real consumption growth was .36 over the 1960–1978 period.

not as often forecast until the 1960's. This lack of inflation estimates is mitigated somewhat by the very low levels of inflation risk relative to output risk during that period. Forecasts of personal consumption expenditures are less readily available, and forecasts of real consumption excluding consumer durables are more recent yet. In theory, only the part of consumer durables expenditures that gives current utility should be included in the consumption variable; in practice, this is difficult to determine and depends upon the time interval considered. The government's measurements of real consumption that are available have problems, too. First, they are available only on a quarterly basis for personal consumption expenditures, although retail sales are available monthly. Second, the measurements are not for the consumption rate at any point in time, but of the integral of those rates over the quarter. Third, there are certainly measurement errors associated with the National Income and Product Accounts (NIPA) statistics, as is shown by their frequent and substantial revisions.

The data that are used in this study are (1) annual consensus economic forecasts for 1960–1978 from the Conference Board's Economic Forum of six well-known economists, and (2) quarterly median forecasts of the 25–80 professional economists surveyed jointly by the American Statistical Association and the National Bureau of Economic Research (ASA/NBER) from the 4th quarter of 1968 to the 3rd quarter of 1979 (11 years). The Conference Board's forecast includes estimates of personal consumption expenditures, GNP and its deflator, and the wholesale price index and consumer price index. Their forecasts of aggregate real personal consumption expenditures are utilized as the annual consumption growth forecast. The ASA/NBER quarterly survey is a much broader survey of economists than the Conference Board's. However, it does have a drawback for the present study: the eight variables forecasted do not include personal consumption expenditures, although sales of consumer durables are forecasted. Since changes in real final sales (GNP less the change in business inventories) are highly correlated with changes in real consumption, deviations of real final sales from the ASA/NBER survey's median forecast are used as proxies for quarterly consumption deviations.

In the estimation of the futures market betas, there are two alternative assumptions about the stochastic processes governing futures prices that affect the definition of the dependent variable. One assumption, which is that used by Dusak (1973), is that a particular futures contract maturity has percentage price changes over time that come from one population that has a stable joint distribution with the dependent variable (aggregate real consumption here). In that case, the time series of percentage price changes for, say, July corn is defined as the dependent variable. A second assumption, a modification of which was utilized by Grauer (1977), is that percentage price changes for all contract maturities of a given commodity have the same probability distribution when they have the same time to maturity. That is, December corn in June and July corn futures in January are assumed to have percentage price changes from the same distribution, since both represent corn price fluctuations for a contract 6 months from maturity.

The time-to-maturity formulation is suggested by the time-varying supply and demand elasticities discussed in Section IV, which imply betas varying by TTM. However, the contract maturity formulation also has merit. The December corn future in June and the July corn future in January may well be quite different, since the December contract matures after the corn harvest and the July contract matures before the harvest. Thus, the choice of an appropriate dependent variable is ambiguous for harvested crops. For commodities that do not have large harvests once a year, the time-to-maturity formulation appears superior, since different times to maturity would capture differences in a shock's price impacts that are due to differences between short-run and longer-run supply and demand price and income elasticities. Since many commodities examined are not annual harvest commodities, the time-to-maturity formulation for the dependent variable was chosen for this study.

The time series of percentage changes in futures prices that were analyzed for a particular commodity, such as cattle, were constructed as follows: the first series is the monthly series of changes in the logarithm of the price of the cattle futures contract nearest to maturity in existence at the beginning of the month (usually 1–2 months to maturity for cattle), the second series was the set of logarithmic price changes in the 2nd cattle contract from maturity (usually 3–4 months to maturity at the beginning of the month), and so on. For most commodities, approximately 5 such series were continuously available over the periods examined. These series may be interpreted as approximately equal to the excess returns to 2-asset portfolios of the various futures contracts and a 1-month Treasury bill. Quarterly excess returns series and annual excess returns series were summations of the monthly series. To adjust for the fact that betas for real portfolio returns are desired, rather than for nominal returns, the deviation of actual percentage change in the wholesale price index from its forecast was subtracted from the nominal log price change to estimate the excess real return to the bill-futures portfolio. The rationale for this is that a nominal log price difference of .08 coupled with a .05 deviation of inflation from forecast implies only a .03 excess real return to the bill-futures portfolio.

The estimated consumption-betas for 20 commodities for a number of times to maturity were computed using annual real excess returns as described and using annual deviations of the growth rate of real consumption from Conference Board forecasts. They are presented in Table 1. The *t*-statistics that would be valid if these variables were jointly normally distributed (lognormal real prices) are presented, too. These should be interpreted with some caution since distributions were not well-approximated by normality.[11]

In general, the estimates of consumption-betas conform to the predictions of the theory of Section II. Meats (cattle, hogs, broilers and pork bellies) tend to have large positive consumption-betas, with most of them exceeding twice their

[11] See Dusak (1973) and Grauer (1977) for evidence of the non-normality of futures' price change distributions. Note that Grauer found evidence of skewness in futures price changes, in addition to "fat tails".

Table 1

Annual Consumption-Betas*
Futures Contracts 1960–1978

Commodity	N	Contract Number by Time to Maturity 1	2	3	4	5**	6	Quantity Beta
Livestock and Meat								
Broilers	10	11.38	11.18	5.03	1.41	2.87	—	
		(2.84)	(3.07)	(1.58)	(0.60)	(0.93)		
Cattle	14	9.38	9.58	7.46	5.07	4.57		
		(3.40)	(3.53)	(3.05)	(1.99)	(1.88)		
Hogs	12	13.09	12.05	9.50	7.34	5.31	—	
		(2.91)	(2.45)	(1.96)	(1.38)	(1.02)		
Pork Bellies	15	9.47	8.68	7.24	6.87	—	—	
		(3.00)	(2.74)	(2.24)	(2.10)			
Metals and Wood								
Copper	19	12.61	8.47	8.36	8.27	7.50	7.21	
		(3.14)	(2.09)	(2.29)	(2.51)	(2.43)	(2.53)	
Platinum	12	12.21	5.19	3.65	2.03	1.23	—	
		(1.81)	(0.82)	(0.59)	(0.32)	(0.19)	—	
Plywood	8	9.28	10.85	10.49	9.46	9.37	7.85	
		(1.34)	(1.61)	(2.09)	(2.25)	(2.36)	(2.06)	
Silver	15	−2.40	−3.72	−4.46	−4.54	−4.48	−4.45	
		(0.83)	(1.08)	(1.27)	(1.26)	(1.29)	(1.29)	
Other Foods and Fibers								
Cocoa	19	2.88	−1.52	−1.07	−1.02	0.54	—	2.16
		(0.52)	(0.27)	(0.21)	(0.21)	(0.12)		(2.20)
Cotton	19	3.78	5.14	4.64	3.82	3.32	2.18	2.29
		(1.01)	(1.00)	(0.92)	(0.82)	(0.75)	(0.53)	(2.26)
Eggs	19	−4.51	1.25	0.03	−0.41	—	—	
		(1.22)	(0.24)	(0.01)	(0.17)			
Orange Juice	12	10.54	15.31	11.89	9.68	8.32	—	−4.97
		(1.43)	(1.73)	(1.42)	(1.23)	(1.13)		(2.83)
Potatoes	19	2.12	8.04	4.32	6.29	—	—	0.01
		(0.35)	(1.11)	(0.59)	(1.20)			(0.01)
Sugar	17	−31.64	−27.70	−24.62	−22.24	−19.83	—	−0.46
		(3.66)	(3.13)	(2.92)	(2.68)	(2.49)		(0.67)
Grains and Their Derivatives								
Corn	19	−1.68	−3.93	−4.27	−3.88	−3.74	—	1.86
		(0.57)	(1.39)	(1.64)	(1.58)	(1.61)		(1.58)
Oats	19	0.88	0.27	−0.48	−1.53	—	—	1.13
		(0.30)	(0.12)	(0.25)	(0.77)			(1.13)
Soybeans	19	−0.28	−3.09	−3.12	−3.55	−2.48	−2.73	1.12
		(0.07)	(1.11)	(1.09)	(1.21)	(1.02)	(1.17)	(1.40)
Soybean Meal	19	3.12	0.11	−0.76	−0.74	−0.86	—	1.12
		(0.73)	(0.03)	(0.21)	(0.22)	(0.28)		(1.40)
Soybean Oil	19	−6.23	−10.75	−8.51	−8.20	−7.35	−6.80	1.12
		(1.30)	(1.92)	(1.87)	(1.86)	(1.81)	(1.81)	(1.40)
Wheat	19	−0.51	−2.02	−2.41	−1.97	−1.79	—	0.37
		(0.16)	(0.51)	(0.65)	(0.52)	(0.50)		(0.40)

* Numbers in parentheses are absolute values of "t-statistics".

** Note "—" implies that this contract series was missing several observations and that statistics were not calculated for it.

standard errors.[12] This is expected, due to the relatively high income elasticities of demand and the relatively low supply uncertainties for the meats. Copper, platinum, and plywood, being industrial materials with demands that are dependent upon the level of economic activity and consumption, are expected to have and do have larger consumption-betas than does silver, a precious metal.

The commodities that have annual harvests and that are consumed rather directly, i.e., cocoa, cotton, orange juice, potatoes, and sugar, have the potentially offsetting effects of (positive) income elasticities of demand and (possibly negative) covariances of quantities produced with aggregate consumption. Estimated consumption-betas for these commodities vary greatly, with sugar being a large negative, orange juice a large positive, and with cocoa, potatoes, and cotton having betas that are small relative to their standard errors.

In an attempt to sort out the possibly opposing effects of supply and demand elasticities from output covariances, the betas of yields of harvested commodities with respect to real consumption were computed. They too are presented in Table 1. These "quantity betas" for cocoa and cotton are in excess of 2, which results in relatively low consumption-betas for cocoa and cotton over the period. The quantity beta for orange juice was very negative (−4.97) over the 1967–1978 period, which resulted in relatively large consumption-betas for it. To the extent that these quantity covariances are expected to persist, the consumption-betas of Table 1 may be used as estimates of future betas. To the extent that these large quantity covariances were just fortuitous, betas for cocoa and cotton should be expected to be higher than those measured here, while that for orange juice should be lower. Note that the quantity beta for sugar is incapable of explaining the extremely negative consumption-beta for sugar.

The grains and their derivatives (corn, oats, soybeans, soybean meal, soybean oil, and wheat) also have potentially offsetting supply and demand effects. However, since most grains are used to feed animals, and since expansion of livestock herds typically takes several months, it can be argued that the short-run derived income elasticities of demand for grains are small. If so, the covariances of quantities produced with aggregate real consumption will be the predominant determinant in the betas of these relatively short-term contracts. In recent years, the covariances of real grain prices (quantities) with aggregate consumption have been negative (positive), which resulted in negative real consumption-betas for the grains. *Ceteris paribus*, the positive correlation of grain yields with aggregate consumption should persist over long periods of time, resulting in low or negative consumption-betas for the grains, although possibly less strongly than over the period examined here.

For the 1969–1979 quarterly analysis, using deviations of real final sales growth from the ASA/NBER median forecast as proxies for consumption deviations, real consumption-betas of the various futures by time to maturity are presented in

[12] For comparison purposes, the annual consumption-beta for the stock portfolio over the same period was 8.71 with a t-value of 4.24. Based upon quarterly real final sales data, the stock portfolio had a consumption-beta of 2.49 ($t = 0.98$). From Tables 1 and 2, most of the meats and metals appear to have more consumption risk than do stocks; this contrasts with their lower market betas than stocks in Table 3.

Table 2. The quarterly real consumption-beta estimates are similar to the annual estimates, in that the grains have significant negative betas and livestock futures have significant positive betas, as expected. The absolute sizes of the betas tend to be somewhat smaller when measured on a quarterly basis, but that may be explained by the fact that a 1% consumption deviation for one quarter does not necessarily have the same impact as a 1% deviation for an entire year. The t-statistics for the quarterly consumption-betas tend to be smaller than for the annual betas. This may be attributed to: (1) the fact that real final sales deviations were used for the quarterly, rather than the consumption deviations of the annual, (2) the timing of the quarterly forecasts being in mid-quarter (late January, April, July, and November), rather than the annual December forecasts for the following year's consumption, or (3) the relatively larger measurement errors in the quarterly NIPA estimates from that of the annual estimates.

The measures of futures contracts' betas relative to the NYSE value-weighted index are presented in Table 3. The quarterly market betas are quite different from the annual market beta estimates, in that many with positive market betas from annual data have negative market betas from quarterly data (e.g., cattle, hogs, broilers, pork bellies, orange juice, and soybean meal). The quarterly market beta estimates are similar to those estimated by Bodie and Rosansky (1979) from 1950–1976 with quarterly data. Thus, the market beta estimates do not appear to be stable with respect to the differencing interval used to calculate them; the consumption betas are generally of the same sign when estimated with quarterly data as when estimated with annual data. Note that, contrary to the generally negative quarterly market beta estimates obtained by Bodie-Rosansky and verified here for many contract maturities, the consumption beta estimates show that several futures contracts have positive systematic risks as defined by the intertemporal CAPM. For these contracts, normal backwordation is predicted by the intertemporal CAPM, whereas the single-period CAPM predicts contango. Since the actual log price changes and their standard deviations are of interest, they are presented in Table 4. A test of the relation (if any) between estimated consumption or market betas and average excess returns is beyond the scope of this paper. However, the finding of differential risks among futures should permit such a test if data are available for long enough time periods to estimate means.

VI. Conclusion

A continuous-time economic model of commodity prices' real consumption-betas was sketched in this paper. The results were that a commodity price's covariance with real consumption depends upon (1) its supply and demand own and cross elasticities with respect to expenditure and commodity prices, and (2) the covariances of goods' production rates with aggregate consumption.

Commodity futures' consumption betas and market betas were estimated for 4–6 portfolio strategies for each of 20 commodities, using annual data from 1960–1978 and quarterly data for 1969–1979. In general, the futures consumption-beta estimates agreed with the predictions of the theoretical model, although a statistical test of the theoretical model's predictions was not presented. Such a

Table 2

Quarterly Consumption Betas*
Futures Contracts 1969(1)-1979(2)

Commodity	*N*	Contract Number by Time to Maturity					
		1	2	3	4	5	6
Livestock and Meat							
Broilers	42	8.79	6.66	4.46	2.15	−0.02	—
		(3.39)	(3.16)	(2.36)	(1.10)	(−0.01)	
Cattle	42	4.19	4.34	3.25	2.15	1.08	—
		(1.72)	(2.05)	(1.89)	(1.21)	(0.62)	
Hogs	42	5.66	7.10	6.18	4.49	3.11	—
		(1.83)	(2.05)	(1.91)	(1.36)	(0.89)	—
Pork Bellies	42	4.81	3.86	1.90	1.34	—	—
		(1.90)	(1.76)	(1.42)	(1.31)		
Metals and Wood							
Copper	42	4.13	3.79	3.68	3.90	3.49	3.18
		(1.07)	(0.87)	(0.89)	(1.00)	(0.93)	(0.89)
Platinum	42	0.42	−0.24	−0.21	−0.44	−0.78	—
		(0.11)	(−0.06)	(−0.06)	(−0.12)	(−0.21)	
Plywood	38	4.25	2.77	2.96	2.79	2.66	2.52
		(0.88)	(0.63)	(0.80)	(0.83)	(0.87)	(0.84)
Silver	42	−0.35	−1.42	−1.56	−1.52	−1.49	−1.47
		(−0.11)	(−0.43)	(−0.47)	(−0.46)	(−0.45)	(−0.44)
Other Foods and Fibers							
Cocoa	42	−1.37	−4.87	−5.63	−4.57	−4.23	
		(−0.28)	(−0.98)	(−1.19)	(−1.06)	(−1.09)	
Cotton	42	2.85	5.05	4.16	3.64	3.18	2.29
		(0.90)	(1.28)	(1.10)	(1.08)	(1.02)	(0.83)
Eggs	42	0.32	−2.64	−2.32	−1.15	—	—
		(0.08)	(−0.64)	(−0.81)	(−0.45)		
Orange Juice	42	6.28	5.79	5.14	4.50	4.10	—
		(1.54)	(1.35)	(1.26)	(1.19)	(1.16)	
Potatoes	42	10.13	7.20	8.24	7.60	—	—
		(2.25)	(1.34)	(1.45)	(1.40)		
Sugar	42	−13.91	−13.59	−13.98	−12.68	−11.50	—
		(−1.95)	(−2.07)	(−2.37)	(−2.29)	(−2.21)	
Grains and Their Derivatives							
Corn	42	−5.71	−7.25	−7.25	−6.48	−5.87	—
		(−1.97)	(−2.27)	(−2.32)	(−2.13)	(−1.97)	
Oats	42	−0.53	−3.00	−4.00	−3.95	—	—
		(−0.18)	(−0.97)	(−1.44)	(−1.41)		
Soybeans	42	1.15	−5.53	−6.49	−5.81	−4.71	−4.53
		(0.24)	(−1.15)	(−1.40)	(−1.42)	(−1.31)	(−1.29)
Soybean Meal	42	1.70	−3.30	−5.53	−4.71	−4.04	—
		(0.35)	(−0.54)	(−0.98)	(−0.99)	(−0.96)	
Soybean Oil	42	−3.87	−6.48	−5.63	−5.81	−5.41	−5.38
		(−0.79)	(−1.24)	(−1.23)	(−1.28)	(−1.23)	(−1.29)
Wheat	42	−8.06	−8.47	−8.16	−7.61	−7.20	—
		(−1.93)	(−1.83)	(−1.85)	(−1.76)	(−1.72)	—

* Numbers in parentheses are "t-statistics".

Table 3

Market Betas for Futures Contracts*
Contract Number by Time to Maturity

	Annual 1960–1978			Quarterly 1969–1979		
Commodity	1	3	5	1	3	5
Livestock and Meat						
Broilers	0.51	0.31	0.14	−0.09	−0.07	−0.19
	(1.60)	(1.51)	(0.71)	(0.49)	(0.54)	(1.67)
Cattle	0.64	0.49	0.33	0.05	−0.05	0.09
	(2.68)	(2.32)	(1.72)	(0.32)	(0.47)	(0.81)
Hogs	0.45	0.26	0.04	−0.01	−0.27	−0.27
	(1.19)	(0.70)	(0.10)	(0.05)	(1.32)	(1.29)
Pork Bellies	0.37	0.33	—	−0.57	−0.28	
	(1.16)	(1.14)		(1.78)	(1.02)	
Metals and Woods						
Copper	0.59	0.33	0.30	0.28	0.17	0.15
	(1.51)	(0.98)	(1.05)	(1.20)	(0.68)	(0.66)
Platinum	0.98	0.51	0.44	0.15	0.00	−0.04
	(2.44)	(1.33)	(1.08)	(0.61)	(0.01)	(0.16)
Plywood	0.84	0.82	0.70	0.35	0.45	0.43
	(2.37)	(3.44)	(3.68)	(1.18)	(2.08)	(2.45)
Silver	0.11	0.01	−0.01	0.25	0.11	0.09
	(0.47)	(0.02)	(0.05)	(1.34)	(0.55)	(0.44)
Other Foods and Fibers						
Cocoa	0.28	−0.02	0.13	−0.04	−0.24	−0.19
	(0.60)	(0.05)	(0.36)	(0.13)	(0.83)	(0.79)
Cotton	0.52	0.73	0.61	0.06	0.00	0.00
	(1.76)	(1.90)	(1.77)	(0.30)	(0.01)	(0.01)
Eggs	−0.35	0.09	—	−0.43	−0.31	—
	(1.13)	(0.33)		(1.92)	(1.83)	
Orange Juice	0.12	0.30	0.16	−0.31	−0.19	−0.25
	(0.22)	(0.51)	(0.31)	(1.24)	(0.75)	(1.17)
Potatoes	−0.32	0.04	—	−0.14	−0.25	—
	(0.64)	(0.07)		(0.49)	(0.70)	
Sugar	−1.85	−1.51	−1.21	−1.10	−1.03	−0.89
	(2.18)	(1.96)	(1.70)	(2.81)	(2.93)	(2.91)
Grains and Their Derivatives						
Corn	−0.29	−0.47	−0.37	−0.31	−0.65	−0.58
	(1.26)	(2.35)	(2.00)	(1.75)	(3.67)	(3.48)
Oats	0.10	0.04	—	−0.06	−0.32	—
	(0.43)	(0.23)		(0.34)	(1.90)	—
Soybeans	0.23	−0.25	−0.20	−0.55	−0.89	0.74
	(0.69)	(1.04)	(1.01)	(1.91)	(3.47)	(3.82)
Soybean Meal	0.54	0.19	0.12	−0.31	−0.72	−0.59
	(1.60)	(0.65)	(0.49)	(1.05)	(2.19)	(2.43)
Soybean Oil	−0.96	−1.09	−0.94	−0.69	−0.95	−0.99
	(2.82)	(3.44)	(3.30)	(2.44)	(3.87)	(4.39)
Wheat	−0.08	−0.28	−0.21	−0.17	−0.33	−0.33
	(0.29)	(0.92)	(0.71)	(0.59)	(1.19)	(1.26)

* Numbers in parentheses are absolute values of "t-statistics".

Table 4

Average Annual Log Price Changes
20 Commodities: 1960–1978*

Commodity	N	Contract Number by Time to Maturity 1	3	5
Livestock and Meat				
Broilers	10	.310	.040	.074
		(.209)	(.149)	(.157)
Cattle	14	.132	.063	.058
		(.200)	(.177)	(.160)
Hogs	12	.292	.131	.118
		(.265)	(.263)	(.269)
Pork Bellies	15	.085	.099	
		(.248)	(.232)	
Metals and Wood				
Copper	19	.073	.098	.082
		(.341)	(.288)	(.247)
Platinum	12	.049	.029	.032
		(.348)	(.292)	(.304)
Plywood	8	.188	.125	.110
		(.283)	(.222)	(.171)
Silver	15	.086	.027	.039
		(.189)	(.246)	(.245)
Other Foods and Fibers				
Cocoa	19	.109	.106	.100
		(.383)	(.367)	(.311)
Cotton	19	.021	.067	.100
		(.254)	(.341)	(.298)
Eggs	19	.218	−.043	
		(.267)	(.221)	
Orange Juice	12	.090	.116	.126
		(.372)	(.420)	(.359)
Potatoes	19	−.031	.017	
		(.424)	(.506)	
Sugar	17	−.042	.042	.076
		(.794)	(.709)	(.636)
Grains and Their Derivatives				
Corn	19	.019	.005	.029
		(.221)	(.220)	(.197)
Oats	19	.026	.001	
		(.197)	(.145)	
Soybeans	19	.046	.102	.095
		(.284)	(.230)	(.201)
Soybean Meal	19	.129	.107	.128
		(.280)	(.246)	(.221)
Soybean Oil	19	.160	.139	.135
		(.373)	(.376)	(.339)
Wheat	19	.030	.007	.013
		(.231)	(.279)	(.265)

* Numbers in parentheses are standard deviations of annual log price changes.

** Numbers of years (ending with 1978) for which data are available are listed as *N*.

test will require estimates of commodities' short-run and long-run supply and demand elasticities, which is a task left to subsequent research.

A result of this paper that differs from previous empirical research in its economic predictions for futures is that some futures contracts have significant systematic risks that should result in risk premia in those markets. Perhaps a more important development is the presentation of some economic determinants of those systematic risks. The identification of those determinants of commodities' systematic risks aids the understanding of the risk estimates obtained and suggests revisions in them that may give more accurate estimates of subsequent futures market consumption-betas.

REFERENCES

Bodie, Z. and V. Rosansky, 1979, "Risk and Return in Commodity Futures," *Financial Analysts Journal*, forthcoming.

Breeden, D. T., 1979, "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities," *Journal of Financial Economics* (7), No. 3, September.

Breeden, D. T. and R. H. Litzenberger, 1978, "Prices of State-contingent Claims Implicit in Option Prices," *Journal of Business*, (51), October.

Conference Board, Information Bulletin entitled *Business Outlook*, currently published in July and December.

Dusak, K., 1973, "Futures Trading and Investor Returns: An Investigation of Commodity Market Risk Premiums," *Journal of Political Economy*, (81), October.

Gihman, I. and A. Skorohod, 1972, *Stochastic Differential Equations*. New York: Springer-Verlag.

Grauer, F. L. A., 1977, "A Test of the Hypothesis that Backwardation is a Function of the Real Social Risk of Commodity Futures Contracts," Stanford University Graduate School of Business dissertation, chapter IV.

Grauer, F. L. A. and R. H. Litzenberger, 1979, "The Pricing of Commodity Futures Contracts, Nominal Bonds and Other Risky Assets under Commodity Price Uncertainty," *Journal of Finance*, (34), March.

Lintner, J., 1965, "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets," *Review of Economics and Statistics*, (47), February.

National Bureau of Economic Research, "Business Outlook Survey" reported quarterly in *Explorations in Economic Research*.

Rockwell, C., 1967, "Normal Backwardation, Forecasting and the Returns to Commodity Futures Traders," *Food Research Institute Studies* Supplement to Volume VII.

Samuelson, P. A. and S. Swamy, 1974, "Economic Index Numbers and Canonical Duality: Survey and Synthesis," *American Economic Review*, (64), September.

Sharpe, W. F., 1964, "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk," *Journal of Finance*, (19), September.

Su, V. and J. Su, 1975, "An Evaluation of ASA/NBER Business Outlook Survey Forecasts," *Explorations in Economic Research* (2), Fall.

Zarnowitz, V., 1979, "An Analysis of Annual and Multiperiod Quarterly Forecasts of Aggregate Income, Output and the Price Level," *Journal of Business*, (52), January.