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Consumption-Based Asset Pricing: Research and Applications

By

Douglas T. Breeden,* Robert H. Litzenberger,** and Tingyan Jia***

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*William W. Priest Professor of Finance, Fuqua School of Business, Duke University.
Contact: doug.breeden@duke.edu

**Edward Hopkinson Professor of Investment Banking Emeritus, The Wharton School, University of Pennsylvania.

***Guanghua School of Management, Peking University, and Visiting Scholar, Duke University Fuqua School of Business. Contact: jiatingyan2011@gmail.com

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Abstract

This article reviews the origins and development of the consumption-based asset pricing theory over the last four decades. Starting with the original CCAPM derivations by Rubinstein (1976), Breeden-Litzenberger (1978) and Breeden (1979), as well as related work by Lucas (1978), both theory and subsequent tests are reviewed, and some new applications are provided. While initial empirical tests such as those of Hansen-Singleton (1983) and Mehra-Prescott (1985) were largely negative, more recent tests are much more supportive of CCAPM theory. Empirical tests from several authors are presented, including those of Lettau-Ludvigson (2001), who use a consumption/wealth conditioning variable, Parker-Julliard (2005) who examine "ultimate consumption betas," and Jagannathan-Wang (2007) who examine intra-year decision making effects on tests. Important second generation consumption-based asset pricing advances of Campbell and Cochrane (1999) on external habit formation and Bansal and Yaron (2004) on long-run risk are also reviewed. These models develop utility functions that are consistent with both large cyclical changes in relative risk aversion and risk premiums, and lagged impacts of aggregate consumption changes on risk premiums. These second generation models have many free parameters and are able to fit the empirical data much better than the first generation CCAPM models.

I. Introduction.

Consumption-based asset pricing models have been among the leading multiperiod general equilibrium asset pricing models in financial economics research for the past 35 years. The Consumption Capital Asset Pricing Model (CCAPM) was first derived in the late 1970s in successively more general models by Rubinstein (1976), Breeden and Litzenberger (1978), and Breeden (1979).[1] While Lucas (1978) did not derive the CCAPM formula, his work on Euler equations was also helpful to many empiricists in subsequent consumption-based asset pricing tests. The CCAPM built on the classic single-period, market-based CAPM of Sharpe (1964) and Lintner (1965), as well as on the subsequent major work on the Intertemporal CAPM by Merton (1973). The Consumption CAPM links asset pricing with macroeconomic risk.

The CCAPM states that the expected excess return on any risky asset should be proportional to its "consumption beta." These authors showed that securities with higher sensitivities of returns to movements in real consumption spending have more systematic risk and should have proportionately higher excess returns. They pay off more when consumption is high and marginal utility is low, and pay less when consumption is low and marginal utility is high, so they are worth less in price and have higher equilibrium returns. This is different from the original market-oriented CAPM insights of Sharpe (1964) and Lintner (1965), as real consumption growth is not perfectly correlated with market returns. In a multiperiod model, market wealth can be high and still have marginal utility be high if the investment opportunity set is good, as shown by Merton (1973) and Breeden (1984).

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[1] See p. 412 of Rubinstein (1976) , eqs. 26 and 27 in Breeden-Litzenberger (1978) and eqs. 19, 19', and 35 in Breeden (1979) for the first CCAPM equations. Lucas's (1978) important paper has also been credited with CCAPM development, but the paper has no equations with consumption covariances or consumption betas and no CCAPM asset pricing formula. Lucas's development of the relation of asset prices to marginal utility is similar in economic intuition to the time-state preference literature of Hirshleifer (1970, Chapter 8), Rubinstein (1974, 1976), and Nielsen (1974). We view Lucas's most important contributions to be on the existence and stability of equilibrium and of an equilibrium pricing density with intuitive economic properties.

The first two decades of CCAPM tests produced mixed results. Tests of the special case of the CCAPM under constant relative risk aversion by Hansen and Singleton (1983), Mehra and Prescott (1985) and others rejected the model. Chen, Roll and Ross (1986) found no significant consumption factor priced when in the presence of other factors, including industrial production, junk bond returns, and inflation hedges. Grossman, Melino and Shiller (1987), Breeden, Gibbons and Litzenberger (BGL, 1989) and Wheatley (1988) examined measurement issues in consumption (such as time aggregation) and their biases on measures of volatility and consumption betas. BGL found a significant positive coefficient on consumption betas; and separately a significant positive coefficient on market betas; however, both the CCAPM and the CAPM were rejected based on stronger form tests of their respective implied first order conditions. BGL derived two useful results used by several subsequent authors: (1) biases in consumption betas due to time aggregation and how those biases are reduced with increased differencing intervals for consumption growth and (2) estimation of consumption betas relative to returns on a consumption mimicking portfolio, which allows greater number and frequency of observations and more precise estimates of consumption betas.

The very strong theory in support of the CCAPM, contrasted with weak early empirical support, motivated researchers to improve both their theoretical and empirical modeling. On the theoretical side, Pye (1972, 1973) and Greenig (1986) developed time-multiplicative utility functions, and then Sundaresan (1989), Constantinides (1990) and Abel (1990) modeled habit formation and then Epstein-Zin (1989) and Weil (1989) (often jointly referred to as EZ-W) developed preference structures that displayed time-complementarity in utility for consumption streams, allowing researchers to separate effects of different levels of intratemporal relative risk aversion (RRA) from levels of the elasticity of intertemporal substitution (EIS). Campbell and Cochrane (1999) later produced an empirically tractable model with the habit formation approach, using an "external habit." With a subsistence level of consumption for a "representative individual," their model allows for dramatic rises in relative risk aversion as surplus consumption (above habit) goes towards zero in severe recessions. With the flexibility afforded by this model, they were able to fit many aspects of empirical data on stock and bond returns as related to real consumption growth, especially the risk premium on the stock market, which Mehra and Prescott (1985) had found was substantially too high (the "equity premium puzzle"), given the low volatility of real consumption growth.

In the early 1990s, Mankiw and Zeldes (1991) considered that many households did not own stock at all or in significant amounts, a situation called "limited participation." They pointed out that there is no reason that the Euler equation should hold for households who are not investing. They found that for households who actually owned stocks, the implied estimates of relative risk aversion were much more reasonable than for households who did not own stocks. Heaton and Lucas (1992, 1996) examined "incomplete markets" that did not permit full hedging of labor income, thus causing consumers to have more volatile consumption streams. Brav, Constantinides and Geczy (2002) studied consumer spending data and found generally plausible estimates of relative risk aversion, given the high volatilities of individuals' consumption streams. Vissing-Jorgensen (2002) focused on estimating the "elasticity of intertemporal substitution," which determines how much consumers change their expected consumption growth rate when interest rates or expected returns on assets change. She finds the EIS to be quite different for stockholders than for nonstockholders, generally getting plausible estimates for those who chose to invest in stocks and bonds and based on trading off current consumption versus future consumption.

Also on the empirical side, advances were also made in examining changes in **conditional** means, variances and covariances and testing conditional versions of the CAPM and CCAPM, as in Harvey (1991), Ferson and Harvey (1991) Jagannathan and Wang (1996), and Ferson and Harvey (1999). Particularly insightful papers in testing the conditional CCAPM include Lettau and Ludvigson (2001a, b) and Jagannathan and Wang (2007). Lettau-Ludvigson use deviations of consumption from total wealth (which includes a human capital estimate in addition to stock market wealth) as a conditioning or "scaling" variable for changing mean returns. They find results quite compatible with Merton's (1973) and Breeden's (1984) intertemporal theories, in that high consumption versus wealth is a predictor of future investment returns, as consumers optimally smooth forward those changes in expected returns. Lettau and Ludvigson also find significant differences in the movements of consumption betas of value vs. growth stocks during recessions. They find that value stocks tend to have much larger increases in consumption betas during recessions, when risks and risk premiums are high, which helps to explain the Fama-French (1992) findings of higher returns on value stocks than predicted by the unconditional CCAPM betas, results which were viewed as anomalous. Jagannathan and Wang use recession and expansion periods identified by the NBER as a conditioning variable, and find that conditional consumption betas are excellent in describing conditional mean returns on the Fama-French portfolios.

More recently, Bansal and Yaron's (2004) article has had major impact by modeling the "long run risk" caused by small, persistent shocks in the drift and volatility of real consumption growth. They show that variance of real consumption growth grows more than proportionally with time, which is consistent with the persistence of growth shocks. Additionally, they provide evidence that shows that the conditional volatility of consumption is time-varying, which leads naturally to time-varying risk premia. Much subsequent research has been done on this long run risk model, most notably in the paper by Bansal, Dittmar, and Kiku (2009). Bansal, Dittmar and Kiku show that aggregate consumption and aggregate dividends have a cointegrating relation. They observe that "the deviation of the level of dividends from consumption (the error correction variable) is important for predicting dividend growth rates and returns at all horizons" (1, 5 and 10 years). Imposing cointegration allows them to predict 11.5% of the variation in 1-year returns, whereas only 7.5% of the variation is predicted without cointegration. Their conditional consumption betas account for about 75% of the cross-sectional variation in risk premia for the one-year horizon, and 85% for long horizons.

After Grossman, Melino and Shiller (1987) and Breeden, Gibbons and Litzenberger (1989) raised concerns about measuring consumption, in 2005 Parker and Julliard showed that it is important to measure "ultimate consumption betas," since consumption changes are slow-moving, and could take 2-3 years for the full effects to be observed. Using measures of these ultimate consumption betas, they were able to explain much of the Fama-French (1992) effects for size-related portfolios and value vs. growth (book/market) portfolios.

Jagannathan and Wang (2007) show that when consumption betas of stocks are computed using year-over-year consumption growth based upon the fourth quarter, the consumption-based

CAPM explains the cross-section of stock returns as well as the Fama and French (1992) three-factor model. Jagannathan-Wang argue that major investment and job decisions are most often made in the fourth quarter, as investors and firms plan for the coming year, so this is when the Euler equations should fit best. For estimation of consumption betas, they follow Breeden-Gibbons-Litzenberger (1989) in using a "consumption mimicking portfolio" (CMP) formed from the 6 Fama-French benchmark portfolios, using weights from an OLS regression of consumption growth on the benchmark portfolios. This allows them to substitute synchronous portfolio returns for (time aggregated) real consumption growth in the empirical tests, giving more precisely estimated consumption betas.

The plan of the paper is as follows: Section II derives the main theoretical results for aggregation of consumption in a discrete time-state preference model, and then the CCAPM in Merton's continuous-time model. Section III derives the term structure of interest rates from the term structure of expected growth of consumption, the term structure of volatility and the term structure of inflation, and then shows tests of this theory. Section IV discusses selected early 1980s tests of the CCAPM and focuses on the equity premium puzzle of Mehra and Prescott, given its large impact on subsequent research. Section V presents the Breeden, Gibbons, Litzenberger derivations for time aggregation and the consumption mimicking portfolio. Section VI presents the advances in modeling utility maximization with non-time-separable utility function. Section VII discusses significant research on limited participation and incomplete markets. Section VIII presents the 1990s empirical modeling of changing conditional risks and changing risk premiums. Section IX discusses the advances in empirical applications of models with habit formation, led by the Campbell-Cochrane model. Section X presents the 2001-2007 results on modeling changes in conditional consumption risk and changes in the investment and job opportunity sets, led by Lettau and Ludvigson. Section XI presents the long-run risk model and tests and developments based on the original work by Bansal and Yaron. Section XII presents research on cash flow betas (dividends and profits) on real consumption growth versus market returns. As the consumption-based asset pricing literature is so vast, Section XIII presents descriptions of additional selected works in the last decade, which has been dominated by additional extensions and tests of competing empirical models with habit formation, long-run risk and disaster risk. Section XIV makes some concluding remarks.

II. Review of Consumption CAPM Theory.

A. Individuals Are Different. Aggregation of Consumption

In reality, investment counselors know that individuals are often quite different in their preferences and behavior, having different levels of risk aversion, different tax brackets, and different preferences with regard to nonlinear risks, such as those causing positive and negative skewness (i.e., tail risks). Some prefer to lever up to get high returns and are willing to accept high risks, while others choose to hold a lot of Treasury securities and low return/low risk combinations. Some are willing to write insurance by taking credit risks or fixed rate mortgage prepayment risks, while others wish to purchase portfolio insurance, paying a price to limit downside risk while retaining much of the upside potential. Reactions to alternate possible consumption paths can also vary quite a lot, as some individuals may be comfortable taking the risk of having to reduce consumption significantly if markets fall sharply, while others may go to extreme lengths to smooth consumption or to protect their consumption from going below a certain subsistence threshold.

Dealing with heterogeneous preferences like these is a challenge analytically to asset pricing theorists. Papers have been written on the "aggregation problem" and how to allow individuals to have heterogeneous preferences and yet derive asset pricing results in terms of aggregate wealth or aggregate consumption. Yet it is surprising that many of the most well-known articles in consumption-based asset pricing simply assume either identical individuals or the existence of a "representative individual (e.g. Lucas (1978), Campbell-Cochrane (1999), Bansal-Yaron (2004)). This may lead students to believe that the aggregation problem is unsolvable and that we have to just assume it away. That is not true in some important cases, as shown by Merton (1973), Breeden and Litzenberger (1978) and Breeden (1979), which derive the Intertemporal CAPM (ICAPM) and the Consumption CAPM (CCAPM) and actually are able to aggregate fully diverse preferences in the class of time-additive utility functions. It is insightful to show how market price signals coordinate optimal consumption plans in such a way that legitimate aggregation results can be obtained.

Breeden and Litzenberger (1978, Theorem 1) provide the most general aggregation theorem that we are aware of to date. We use their basic time-state preference model to derive the aggregation result. Each individual, k, chooses planned consumption, c_{ts}^{k}, for each time-state ts, which maximizes the expected value of a time-additive utility function, $u^{k}(c_{ts}^{k}, t)$, subject to the usual budget constraint for initial wealth, W_0^{k}. Individuals are assumed to agree on the subjective probabilities for states $\{\pi_{ts}\}$, which sum to 1.0 for each date. Markets are assumed to be complete, and the Arrow security price of insurance that pays $1.00 if and only if state occuring at time t is ϕ_{ts}. To find the optimal contingent consumption plan, individuals maximize the Lagrangian:

$$\max_{\{c_{ts}^k\}} L = u_0^k(c_0^k) + \sum_t \sum_{s \in s_t} \pi_{ts} u^k(c_{ts}^k, t) + \lambda^k [W_0^k - c_0^k - \sum_t \sum_s \phi_{ts} c_{ts}^k] \tag{1}$$

First-order conditions for a maximum give:

$$\frac{\partial L}{\partial c_0^k} = u_0'^k - \lambda^k = 0 \Rightarrow \lambda^k = u_0'^k \qquad where: u_{ts}'^k \equiv \frac{\partial u^k(c_{ts}^k, t)}{\partial c_{ts}^k}, \tag{2}$$

$$\frac{\partial L}{\partial c_{ts}^k} = \pi_{ts}^k u_{ts}'^k - \lambda^k \phi_{ts} = 0 \qquad \Rightarrow \phi_{ts} = \frac{\pi_{ts} u_{ts}'^k}{u_0'^k} = \text{price of \$1 in time-state ts.} \tag{3}$$

$$\Rightarrow \frac{u_{ts}'^k}{u_0'^k} = \frac{\phi_{ts}}{\pi_{ts}^k} \qquad \Rightarrow \frac{1}{u_0'^k}\begin{pmatrix} u_{t_1 s_1}'^k \\ \vdots \\ u_{TS}'^k \end{pmatrix} = \begin{pmatrix} \dfrac{\phi_{t_1 s_1}}{\pi_{t_1 s_1}^k} \\ \vdots \\ \dfrac{\phi_{TS}}{\pi_{TS}^k} \end{pmatrix}, \qquad \boxed{\begin{array}{c}\text{High MU means low Consumption} \\ \text{(for all individuals, k)} \\ \text{Low MU means high Consumption}\end{array}} \tag{4}$$

So if ordered from high to low, state price/probability ratios at the optimum are positively and monotonically related to marginal utilities in different states, and negatively related to consumption across states. With homogeneous probability beliefs, the price/probability ratios computed for all are the same for each individual and the ordering of states from high to low by price/probability ratios will also order states by optimal consumption levels from low to high. And if every individual's optimal consumption level is ordered the same across states, then we can clearly see that the total of everyone's consumption, aggregate consumption, C, is also ordered in the same way. Given that, then there exists a positive, monotonic functional relationship of each individual's consumption to aggregate consumption, $c_{ts}^{k} = f^{k}(C_{ts}, t)$, where $f^{k\prime} > 0$. Optimal responses of individuals' consumption plans to price/probability ratios across states have coordinated everyone's consumption risks. Since every individual's marginal utility of consumption is the same for a given level of aggregate consumption and is the same monotonically decreasing function of aggregate consumption, an aggregate utility function that is monotonically increasing and strictly concave follows. Furthermore, if each individual's utility function has a positive third derivative (implied by decreasing absolute risk aversion), Kraus and Litzenberger (1983) have shown that the aggregate utility function would also have a positive third derivative and decreasing absolute risk aversion. The positive third derivative implies a preference for skewness, which implies, *ceteris paribus*, that assets having a convex relation to consumption would be preferred to those having a concave relation with consumption.

Understanding this consumption aggregation result in 1978 was key to Breeden's 1979 well known derivation of the Consumption CAPM in the very general framework of the continuous-time model of Merton (1969, 1971, 1973). Then at the University of Chicago, he brought together insights from the time-state preference models ("West Coast Models" developed at Berkeley, Stanford and UCLA) and the "East Coast" continuous-time models of Merton and Cox, Ingersoll and Ross (1985) developed at MIT, Yale and Penn. Breeden reasoned that if every individual's consumption was optimally a monotonically increasing function of aggregate consumption in a complete market, it must be the case that even in an incomplete market, movements in aggregate consumption would locally determine movements in marginal utilities for everyone, to the extent permitted by existing securities. He showed that randomness in individuals' constrained optimal consumption in an incomplete market would be uncorrelated with all assets' returns, for if any of the fluctuations were correlated with some asset's return, individuals' consumption changes were then not maximally correlated with aggregate consumption and the consumption plan was not optimal.

Substituting the fact that each individual's real consumption is a monotonic function of aggregate real consumption, we see that state prices depend only upon their probabilities, the level of real aggregate consumption in the state and the level of aggregate consumption today:

$$\Rightarrow \phi_{ts} = \frac{\pi_{ts} u_t'(C_{ts})}{u_0'(C_0)} \qquad and \qquad \frac{\phi_{ts}}{\pi_{ts}} = \frac{u_t'(C_{ts})}{u_0'(C_0)} \tag{5}$$

These state prices for aggregate consumption claims can be used to value any security's time-state contingent payoffs at time t in terms of its joint probability distribution with aggregate real consumption at that date, which gives consumption-based asset pricing for all assets.

If the cash flows to a security at different future dates and states are the set $\{X_{ts}\}$, then those cash flows can be replicated by purchasing Arrow securities and, to avoid arbitrage, must have a present value, V_0, that is the cost of the replicating portfolio, which is (substituting eq. 5):

$$V_0\{X_{ts}\} = \sum_t \sum_{s \in S(t)} \phi_{ts} X_{ts} = \sum_t \frac{E_0[\widetilde{X_t} u'(C_t,t)]}{u'(C_0,t_0)} \tag{6}$$

Dividing by the initial price to put the payoffs in return form, we get the Euler equation forms:

$$1 = \sum_t \frac{E_0[\widetilde{R_t} u'(C_t,t)]}{u'(C_0,t_0)} \qquad \text{for every asset} \tag{7}$$

$$and \qquad 0 = \sum_t \frac{E_0\left[(\widetilde{R_{i,t}} - \widetilde{R_{j,t}}) u'(C_t,t)\right]}{u'(C_0,t_0)}, \quad \text{for any two assets i and j.} \tag{8}$$

These Euler equations are often tested by econometricians, following Hansen and Singleton (1983). While the Euler conditions have frequently been tested assuming a representative investor with constant relative risk aversion, the above analysis is consistent with any monotonically increasing, strictly concave aggregate utility function that is based on diverse individual preferences and endowments. For example, an aggregate utility function displaying decreasing relative risk aversion is consistent with the above equations and would imply risk premiums that increase in economic contractions and decrease in economic expansions. The parameters of the aggregate utility function should be estimated by the econometrician, rather than restricted a priori without any theoretical justification.

B. Consumption CAPM in Continuous Time Model with Incomplete Markets

After Sharpe and Lintner's development of the single-period CAPM, Fama (1970), Hakansson (1970) and others recognized that multiperiod optimal portfolios (except for the special case of narrow log utility) would be different from the prescriptions of single-period models because individuals' indirect utility functions for wealth would depend on the investment opportunity set. In his continuous-time model, Merton (1969, 1971 and 1973) developed the most elegant solution to this problem. He first derived optimal consumption and portfolio rules, finding additional risk elements that make individuals' optimal portfolios differ from just holding the market portfolio, because as individuals desire to hedge or reverse hedge against changes in the investment and/or job market opportunity set. These new risks are priced in a multi-beta intertemporal CAPM, extending the Sharpe-Lintner model quite significantly. Merton's model was a path breaking contribution because of three key elements of generality that researchers in financial economics viewed as quite important and attractive:

(1) Individuals were allowed to be fully **diverse** in **preferences**, within the class of time-additive utility functions, which were the common assumption at the time. So, 1 billion individuals could have 1 billion different preference functions (and quite general and changeable risk aversions) for consumption. Note that nothing prevented relative risk aversions of individuals and market risk premium to get very large as consumption fell to low levels, for example, as in more recent models with external habit formation.

(2) Asset prices and consumption levels were allowed to follow very general diffusion processes, with **conditionally changing drifts, volatilities and correlations**, which can generate tremendously different probability distributions over discrete intervals (displaying non-normality, non-lognormality, and option-like features, for example). Thus, Merton's ICAPM (1973, p. 872) and Breeden's subsequent CCAPM (1979, p. 268) clearly were done in terms of conditionally expected returns and conditional consumption betas (a point that does not appear to be appreciated by some subsequent authors).

(3) **Markets** were not assumed to be **complete**, in the sense that Merton did not assume that there were assets or portfolios that would replicate the behavior of any or all economically important state variables. Thus, the allocation that was achieved was not necessarily "Pareto-optimal," or efficient, but was merely the best that could be attained with existing markets. Of course, the complete markets case is a special case of his model, so Merton's and Breeden's continuous time models apply to both complete and incomplete markets.

Merton's (1973) Intertemporal CAPM showed that, in equilibrium, the vector of instantaneous expected excess returns on risky assets, **μ-r**, is equal to the matrix of betas relative to the market portfolio and relative to the S state variables for the investment opportunity set, multiplied by the vector of risk premia for each of the S+1 risks:

$$\boldsymbol{\mu}_a - \boldsymbol{r} = \boldsymbol{\beta}_{\boldsymbol{a},\boldsymbol{Ms}} \begin{pmatrix} \mu_a - r \\ \boldsymbol{\mu}_s^* - \boldsymbol{r} \end{pmatrix} \qquad \text{(ICAPM) (9)}$$

Merton's ICAPM has empirical implications that are similar to those of Ross's (1976) Arbitrage Pricing Theory (APT). However, Ross uses statistical assumptions and merely the absence of arbitrage to derive his APT, whereas Merton uses arguably weaker statistical assumptions and stronger economic assumptions to derive the ICAPM. Testing of Merton's ICAPM and Ross's APT was inhibited by the fact that there were an unspecified number of opportunity set state variables (or factors) and that it was unclear whether their corresponding risk premiums would be positive or negative for different state variables. Many empirical articles focused on the weak prediction of these theories; i.e., the existence of several price factors, rather than predictions of which factors should be priced based on the a priori theory. Indeed, these models seemed to give those who tried to apply the model too much freedom to data mine in choosing state variables that had "statistically significant" risk premiums.

Breeden's (1979) article derived the Consumption CAPM in the exact same, very general intertemporal model that Merton used, showing that Merton's S+1 betas and risk premia could be replaced with a single beta relative to consumption and that risk premia for the state variables should all be proportional to their consumption betas. An outline of his derivation follows:

Following Merton (1973), at each instant, each individual k chooses dynamically an optimal consumption rate, c^k, and an optimal Ax1 vector of risky asset portfolio weights, $\boldsymbol{w}^k$, where the residual is invested in the riskless asset, $w_0=1-\Sigma_j w_j^k$. An optimal policy at every instant maximizes the sum of instantaneous utility of current consumption plus the expected change in remaining expected utility of lifetime consumption, $J^k(W^k,\mathbf{s},t)$, where $\mathbf{s}$ is an Sx1 vector of state variables that describe the investment and income opportunity sets. The Bellman equation says that the following maximum must be 0, or else the dynamic plan is not optimal:

$$0 = \max_{\{c^k,\boldsymbol{w}^k\}} u^k(c^k,t) + E_t\text{"}\left[\frac{dJ^k}{dt}\right]\text{"} \tag{10}$$

Merton then invokes Ito's Lemma to compute the expected change in J(W,**s,**t):

$$0 = \max_{\{c^k,\boldsymbol{w}^k\}} \left\{ \begin{array}{l} u^k(c^k,t) + \begin{pmatrix} J_W^k & \boldsymbol{J_s^k} & J_t^k \end{pmatrix} \begin{pmatrix} W^k\left[\boldsymbol{w}^{k\prime}(\boldsymbol{\mu}_a - \boldsymbol{r}) + r\right] + y^k - c^k \\ \boldsymbol{\mu_s} \\ 1 \end{pmatrix} \\ \\ \quad + \frac{1}{2}\left[\begin{pmatrix} J_{WW}^k & \boldsymbol{J_{Ws}^k} \\ \boldsymbol{J_{sW}^k} & \boldsymbol{J_{ss}^k} \end{pmatrix} \square \begin{pmatrix} (W^k)^2 \boldsymbol{w}^{k\prime}\boldsymbol{V}_{aa}\boldsymbol{w}^k & W^k\boldsymbol{w}^{k\prime}\boldsymbol{V_{as}} \\ \boldsymbol{V}_{sa}\boldsymbol{w}^k W^k & \boldsymbol{V_{ss}} \end{pmatrix}\right] \end{array} \right\}$$

(11)

where $\mathbf{V}_{aa}$ = AxA covariance matrix of risky asset returns, $\mathbf{V}_{as}$ = AxS covariance matrix with state variables, and $\mathbf{V}_{ss}$ = SxS covariance matrix for state variables. Mean vectors are $\boldsymbol{\mu}_a$ and $\boldsymbol{\mu}_s$, respectively. Subscripts on the J function indicate first and second partial derivatives. Setting derivatives of control variables = 0, and solving for the optimal portfolio and consumption gives:

$$\boldsymbol{w}^k W^k = T^k \boldsymbol{V}_{aa}^{-1}\left(\boldsymbol{\mu} - \boldsymbol{r}_f\right) + \boldsymbol{V}_{aa}^{-1}\boldsymbol{V}_{\boldsymbol{as}}\boldsymbol{H}_{\boldsymbol{s}}^k \tag{12}$$

and $$u_c^k(c^k,t) = J_W^k(W^k,\boldsymbol{s},t) \qquad \text{(Envelope condition)} \tag{13}$$

Where $T^k = -\frac{J_W^k}{J_{WW}^k} = k's\, risk\ tolerance\ for\ wealth$ and $\boldsymbol{H_s^k} = -\frac{\boldsymbol{J_{sW}^k}}{J_{WW}^k}$

$\boldsymbol{H_s^k}$ equals to individual K's $s \times 1$ vector of hedging or "reverse hedging" demands for the s portfolios that best hedge against changes in the investment and income opportunity set vector, $\boldsymbol{s}$.

The envelope condition shows that the marginal utility of \$1 consumed must equal to the marginal utility of \$1 invested. This means that in individual portfolio equilibrium the individual's indirect marginal utility of wealth, which depends on the investment opportunity set as well as her wealth, would equity the marginal utility of her consumption, which only depends on her consumption. Differentiating the envelope condition with respect to wealth W^k and then state variables and then substituting into $\boldsymbol{H}_s^k$ gives:

$$u_{cc}^k c_W^k = J_{WW}^k \qquad \text{and} \qquad u_{cc}^k \boldsymbol{c_s^k} = \boldsymbol{J_{sW}^k} \tag{14}$$

$$\text{So} \quad T^k = -\frac{J_W^k}{J_{WW}^k} = -\frac{u_c^k}{u_{cc}^k c_W^k} \qquad \text{and} \qquad \boldsymbol{H_s^k} = -\frac{\boldsymbol{J_{sW}^k}}{J_{WW}^k} = -\frac{\boldsymbol{c_s^k}}{c_W^k} \tag{15}$$

To gain insight into the optimal dynamic portfolios and consumption plans, assume the special case where individual investors have constant relative risk aversion, which differs across individuals. Let $\boldsymbol{\gamma_s^k}$ be the vector of percentage compensating variations in wealth that would hold lifetime utility constant for state variables' changes. Breeden (1984) showed that:

$$\boldsymbol{H_s^k} = W^k(1 - T^{*k})\boldsymbol{\gamma_s^k} \tag{16}$$

and that optimal consumptions sensitivities to state variables are:

$$\boldsymbol{c_s^k} = -c_W^k W^k(1 - T^{*k})\boldsymbol{\gamma_s^k} \tag{17}$$

Many authors have estimated typical relative risk aversion to be much greater than 1.0, so that the inverse of relative risk aversion, relative risk tolerance T^{*k} is much smaller than 1.0. Therefore, for most people we assume $(1 - T^{*k})$ is positive.

If a state variable j is a good one (in that higher s_j means higher expected lifetime utility), its compensating variation in wealth, γ_{s_j} will be negative. Eq. (16) shows that with normal relative risk aversion, H_s^k is then negative and the individual holds assets that hedge against adverse changes in the opportunity set by giving higher wealth when the good state variables decline. We see from eq. (17) that current consumption will increase when s_j increases, smoothing lifetime consumption, given normal relative risk aversion.

In contrast, denote a "speculator" as an individual who has a much higher tolerance for risk, with $T^{*k} > 1$. It follows from (17) that such a speculator would reduce current consumption to invest more when investment opportunities improve. The speculator would also invest in assets that give higher wealth when opportunities are good and lower wealth when opportunities are poor. Such consumption and portfolio strategies give the speculator a higher lifetime mean consumption stream, but with much higher volatility.

Substituting (14) and (15) into (12) and pre-multiplying by $c_W^k \boldsymbol{V}_{aa}$ and rearranging gives:

$$\boldsymbol{V}_{aW^k} c_W^k + \boldsymbol{V}_{as} \boldsymbol{c}_{\boldsymbol{s}}^k = T_c^k (\boldsymbol{\mu}_a - \boldsymbol{r}) \tag{18}$$

Using Ito's Lemma for $c^k(W^k, \boldsymbol{s}, t)$, the stochastic part of c^k is: $\widetilde{dc^k} = c_W^k \widetilde{dW^k} + \boldsymbol{c}_{\boldsymbol{s}}^{k\prime} \widetilde{d\boldsymbol{s}}$ so:

⇒Vector of covariances: $$\boldsymbol{V}_{\boldsymbol{a}c^k} = \boldsymbol{V}_{\boldsymbol{a}W^k} c_W^k + \boldsymbol{V}_{\boldsymbol{as}} \boldsymbol{c}_{\boldsymbol{s}}^k \tag{19}$$

Substituting (19) into (18) gives: $$\boldsymbol{V}_{ac^k} = T_c^k (\boldsymbol{\mu}_a - \boldsymbol{r}) \tag{20}$$

This says that, for each individual, assets are held in the portfolio in proportions that result in an optimal consumption rate that covaries with each asset in proportion to its expected excess return. Individuals do not influence $\boldsymbol{\mu}_a - \boldsymbol{r}$. They control W^k and, thus, $c^k(W^k, \boldsymbol{s}, t)$.

The aggregate consumption rate $\tilde{C} = \sum_k \widetilde{c^k} \Longrightarrow \sum_k \boldsymbol{V}_{ac^k} = \boldsymbol{V}_{aC} = (\sum_k T_c^k)(\boldsymbol{\mu}_a - \boldsymbol{r})$ (21)

Dividing by C: $\boldsymbol{\mu}_a - \boldsymbol{r} = (T_c^{*M})^{-\mathbf{1}} \boldsymbol{V}_{a,lnC}$ where $T_c^{*M} = \frac{\sum_k T_c^k}{C} = \frac{T_c^M}{C}$ (22)

For any portfolio M: $\mu_M - r = (T_c^{*M})^{-1} \sigma_{\boldsymbol{M,lnC}}$ $\Rightarrow (T_c^{*M})^{-\mathbf{1}} = \frac{\mu_M - r}{\sigma_{\boldsymbol{M,lnC}}}$, $\forall M$ (23)

Substituting (23) into (22) gives the Consumption CAPM:

$$\boldsymbol{\mu}_a - \boldsymbol{r} = \frac{\boldsymbol{\beta}_{\boldsymbol{a,lnC}}}{\beta_{\boldsymbol{M,lnC}}} [\mu_M - r] \qquad \text{(CCAPM) (24)}$$

This shows that the Ax1 vector of (conditional) expected excess returns on risky assets in equilibrium is proportional the the Ax1 vector of the betas with respect to percentage changes in real aggregate consumption. Merton's S+1 betas have been summarized in 1 consumption beta. Note that the CCAPM of (22) is identical in form to the original Sharpe-Lintner CAPM, but with their vector of market betas being replaced by a vector of "relative consumption betas," which for asset j is its consumption beta divided by the market portfolio's consumption beta.

An interesting issue is how the consumption CAPM replicates the risk premia from Merton's Intertemporal CAPM, both for World H where there is a predominance of hedgers and World RH where there is a predominance of speculators (or reverse hedgers). For World H, the world is dominated by people who want to short sell assets positively related to a good state variable, driving down their prices and giving a positive risk premium. In that world of hedgers, consumption increases with increases in a good state variable, so the consumption beta for the state variable is positive, causing the CCAPM to also give a positive risk premium.

In contrast, if the world is RH and is dominated by speculators (reverse hedgers), investors will want to go long assets correlated with a good state variable, pushing up their prices and giving a negative risk premium in the ICAPM. However, in this world, aggregate consumption is reduced when the good state variable is high (so as to invest more with good opportunities), which gives a negative consumption beta for the good asset and a negative risk premium according to the consumption CAPM, duplicating that of the Intertemporal CAPM. Thus, in both worlds dominated by hedgers and by speculators (reverse hedgers), the CCAPM properly identifies the same risk premium as Merton's ICAPM. Ultimately, to sign and estimate the risk for a given state variable only requires an estimate of its consumption beta.

The Consumption CAPM was extended to the global economy by Stulz (1981), who proved that the real expected excess return on a risky asset is proportional to the covariance of its return with changes in the world consumption rate. Additionally, as Backus and Smith (1993) proved, if there are no nontraded goods and markets are effectively complete, consumption in every country is optimally monotonically related to consumption in every other country, an extension of the aggregation result of Breeden and Litzenberger (1978). But as Stulz (1981) proved, if there are nontraded goods and consumption opportunity sets differ across countries, changes in real consumption rates will not generally be perfectly correlated across countries. Brandt, Cochrane and Santa-Clara (2006) show that real consumption growth correlations are statistically significant among the major economies of the United States, the United Kingdom, Germany and Japan, but far from 1.00. With quarterly data, U.S. consumption growth correlations are 0.31, 0.17 and 0.27 versus the UK, Germany and Japan respectively, and with annual changes, correlations are higher at 0.42, 0.24 and 0.35.

III. Term Structure of Interest Rates, Consumption Growth, Volatility, Inflation.

The previous section focused on the derivation of the Consumption CAPM, which provides equilibrium expected returns for **risky** assets in terms of their return sensitivities to aggregate real consumption. In this section, we examine the pricing of **riskless** bonds and the term structure of interest rates and the relation of the term structure of rates to the term structure of expected consumption growth and the term structure of volatility for consumption growth. While the general term structure results for the economies presented in the prior section are in Breeden (1986), those results end up being Taylor series approximations to the following simple model with constant relative risk aversion (power utility) and lognormally distributed consumption, the "CRRA-Lognormal model."[2] To use less space, while gaining greater understanding, riskless bond prices and implicit annualized interest rates are derived in the simple CRRA-LN model, with identical powers for all individuals. This combination of identical CRRA and lognormality allows the computation of closed form solutions for bond prices and interest rates.

We assume that the representative individual has the following power utility function, where γ is the constant relative risk aversion (CRRA) for the individual. γ is also the inverse of the "elasticity of intertemporal substitution."

$$Let \qquad u^k(c^k, t) = \frac{e^{-\rho t}(c_t^k)^{1-\gamma}}{1-\gamma} \tag{25}$$

$$\Rightarrow RRA = -\frac{u''}{u'}c_t = -\frac{-\gamma e^{-\rho t} c_t^{-\gamma-1}}{e^{-\rho t} c_t^{-\gamma}} c_t = \gamma \tag{26}$$

From the time-state preference valuation model, eq. 6, where the cash flows are $1.00 received for sure at T, we have that the riskless zero coupon bond price at time t equals expected marginal utility at time T divided by marginal utility at the present time, t. Given our aggregation result, this depends generally only upon the utility function and the distribution of aggregate consumption at T and t. With the power utility function, this simplifies to a dependence only upon the probability distribution of the percentage growth rate of aggregate consumption from t to T.

$$B_{t,T} = \frac{E_t[u_T'(c_T, T)]}{u_t'(c_t, T)} \qquad in\ general \tag{27}$$

$$B_{t,T} = \frac{E_t[e^{-\rho T} c_T^{-\gamma}]}{e^{-\rho t} c_t^{-\gamma}} = E_t\left[e^{-\rho(T-t)}\left(\frac{\widetilde{c_T}}{c_t}\right)^{-\gamma}\right] \qquad \text{CRRA} \tag{28}$$

[2] See Breeden (1977), Chapter 7.

Next we assume that aggregate per capita consumption is lognormally distributed, where the log's mean grows at a continuously compounded annual rate of $\mu^*_{t,T}$ and variance grows proportionally to time with annualized volatility of $\sigma_{t,T}$, i.e.:

$$c_t \sim lognormal\left[\ln c_0 + \mu^*_{t,T}(T-t), \sigma^{*2}_{t,T}(T-t)\right] \tag{29}$$

Then please note that for lognormals if $y \equiv e^x,\ and\ x\ is\ normal\ with\ E(x) = \mu, Var(x) = \sigma^2$, then $E(y) = e^{\mu + \frac{1}{2}\sigma^2}$. For our lognormal consumption growth, we have:

$$\frac{\widetilde{c_T}}{c_t} \equiv e^{\tilde{g}_{t,T}} \quad has \quad g_{t,T} \sim N\left(\mu_{t,T}, \sigma^2_{t,T}\right). \tag{30}$$

$$\Rightarrow \left(\frac{\widetilde{c_T}}{c_t}\right)^{-\gamma} = e^{-\gamma \tilde{g}_{t,T}} \quad has \quad -\gamma g_{t,T} \sim N\left(-\gamma\mu_{t,T},\ \gamma^2\sigma^2_{t,T}\right)$$

$$\Longrightarrow E_t\left[\left(\frac{\widetilde{c_T}}{c_t}\right)^{-\gamma}\right] = e^{-\gamma\mu_{t,T} + \frac{1}{2}\gamma^2\sigma^2_{t,T}} \quad which\ gives{:}\quad B_{t,T} = e^{-\rho(T-t) - \gamma\mu_{t,T} + \frac{1}{2}\gamma^2\sigma^2_{t,T}} \tag{31}$$

$$\Longrightarrow \begin{pmatrix} r_{t,1} \\ r_{t,2} \\ \vdots \\ r_{t,T} \end{pmatrix} = \rho \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} + \gamma \begin{pmatrix} \mu^*_{t,1} \\ \mu^*_{t,2} \\ \vdots \\ \mu^*_{t,T} \end{pmatrix} - \frac{1}{2}\gamma^2 \begin{pmatrix} \sigma^{*2}_{t,1} \\ \sigma^{*2}_{t,2} \\ \vdots \\ \sigma^{*2}_{t,T} \end{pmatrix} \tag{32}$$

Applying eq. 30 for different dates to plot the term structure of interest rates, we see that the term structure of interest rates reflects the term structure of expected growth rates for consumption over different time horizons and the term structure of volatility of consumption growth over those same horizons.

Breeden (1986) examined the term structure in an economy (with individuals who have time-additive utility functions) with a multigood model and derived corresponding term structure results in a world with inflation and deflation. He derived the nominal term structure of interest rates to have real terms as above, but now the term structure of inflation is added, along with a risk premium or risk deduction for the consumption risk of inflation that is imbedded in returns of nominally riskless bonds. The equation derived is:

$$r - \frac{\mu_I}{I} + \sigma^2_{\ln I} = \rho + \gamma\mu_{\ln e^*} - \frac{\gamma^2}{2}\sigma^2_{\ln e^*} + \gamma\sigma_{-\ln I, \ln e^*} \tag{33}$$

Note that in a model presented subsequently in Section VIII with external habit formation, Campbell and Cochrane (1999, eq. 8) found a corresponding equation for riskless rates to be:

$$r_t^f = -\ln(\delta) + \gamma g - \gamma(1-\phi)(s_t - \bar{s}) - \frac{\gamma^2\sigma^2}{2}[1 + \lambda(s_t)]^2 \tag{34}$$

In both models, we see the classic positive relation of the real rate to expected real consumption growth, g, as well as the negative relation to volatility of consumption, reflecting rational "flight to quality" responses of consumers. Additionally, in the external habit formation model, the riskless rate is affected by where surplus consumption is relative to its long-term mean and the speed of adjustment parameter, ϕ.

Harvey (1988, 1989, 1991) empirically tested whether or not the slope of the term structure of interest rates actually forecasted expected real growth of the whole economy (as consumption is 70% of GDP and is highly correlated). Both Breeden and Harvey observed that late in the economic cycle near an economic peak, when growth is expected to slow considerably and possibly enter a recession, the term structure should be negatively sloped, with lower real rates on longer maturities reflecting slower longer-term growth. Correspondingly, they argued that near the bottom of a recession, when consumers and investors usually expect that "things will likely get better over the longer term," longer-term growth forecasts should be much higher than shorter-term growth and the term structure should be strongly upward sloping.

Figure 1 below shows that upward sloping term structures are the norm, as the spread between 10-year yields and 3-month Treasury yields is normally positive. The yield curve slope was near zero or negative in 1970, 1974, 1980, 1981, 1989, 2000-2001 and in 2006-2007. Figure 1 also shows that in each of these periods the unemployment rate subsequently surged:

Figure 1

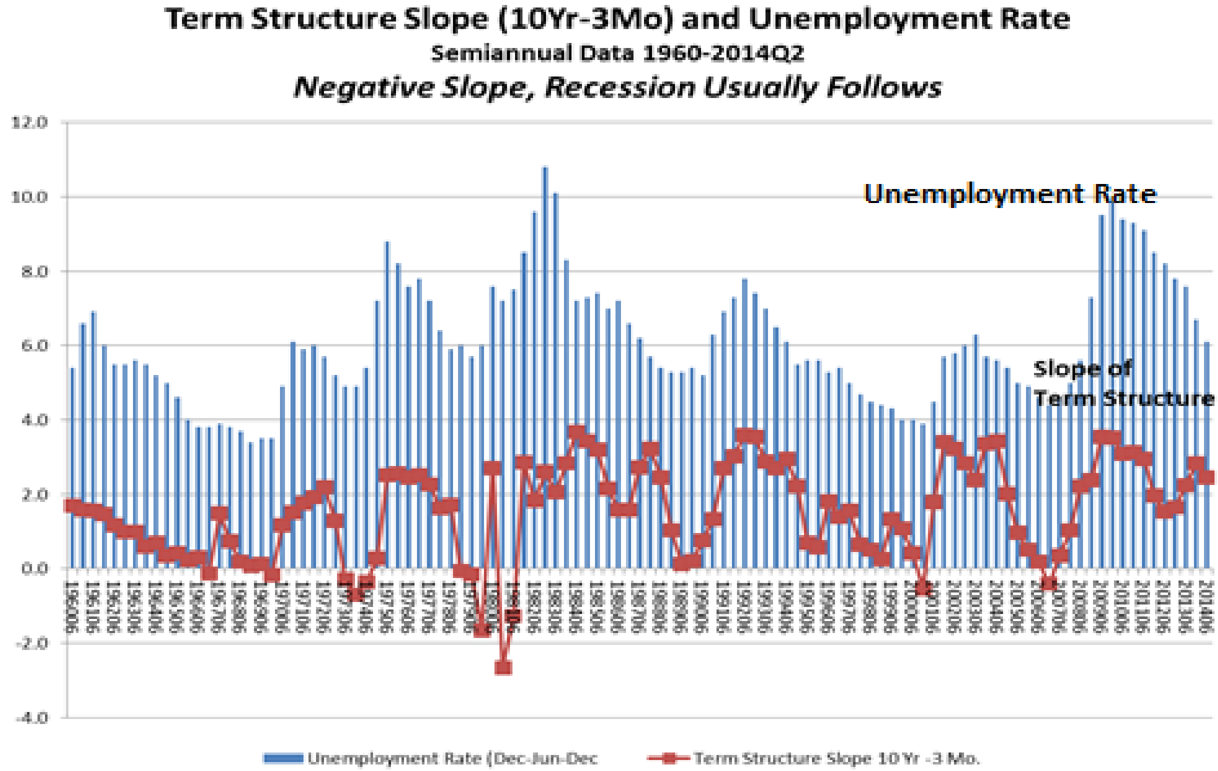
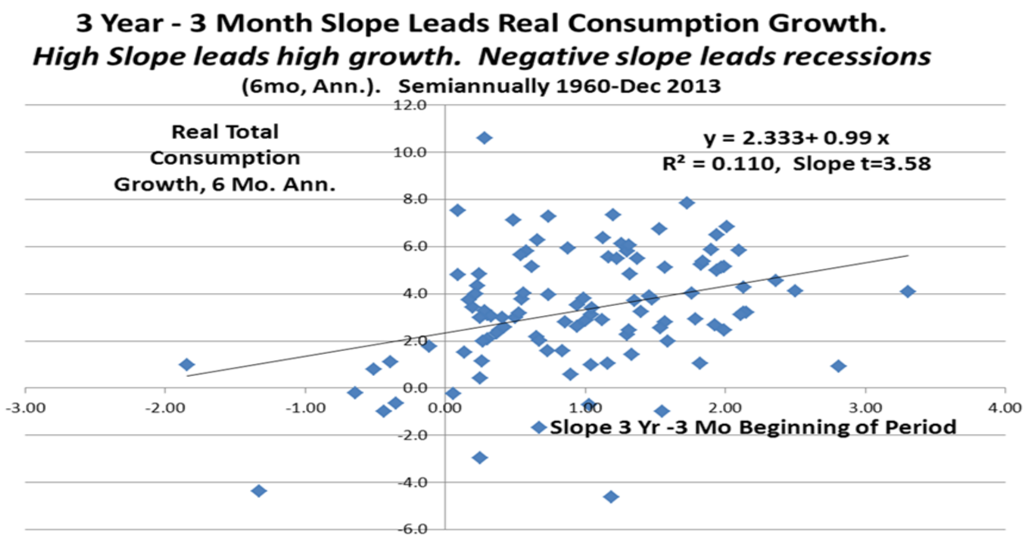


Figure 2 gives a scatterplot and regression results that show that the 3 year – 3 month Treasury term structure slope was positively related to subsequent (next 6 months, annualized) real consumption growth in the 1959-2013 period, with a t-statistic of 3.6, indicating a significant relationship. This was true also in subperiods. Although a straight line fit is shown, the relationship has intriguing nonlinearity, worthy of further study.

Figure 2



Harvey's tests demonstrated that the slope of the yield curve (defined as either the 5-year or 10-year Treasury yield minus the 3-month yield) had significant predictive ability with regard to the subsequent 4 quarters of GDP growth in his sample, both in-sample and in out of sample forecasts: In sample results are:

Table 1

Table The Forecasting Performance of the Yield Spread and Stock Market Return Models, 1953:2-1989:2*

Variable	a	b	$\bar{R}^2$
1953:2-1989:2 (140 observations)			
Five-Year Yield Spread	0.02	1.48	0.30
	[5.17]	[5.57]	
10-Year Yield Spread	0.02	1.29	0.32
	[5.36]	[5.76]	
One-Quarter Stock Return	0.03	0.10	0.05
	[7.45]	[2.46]	
Four-Quarter Stock Return	0.03	0.01	0.00
	[7.20]	[0.50]	

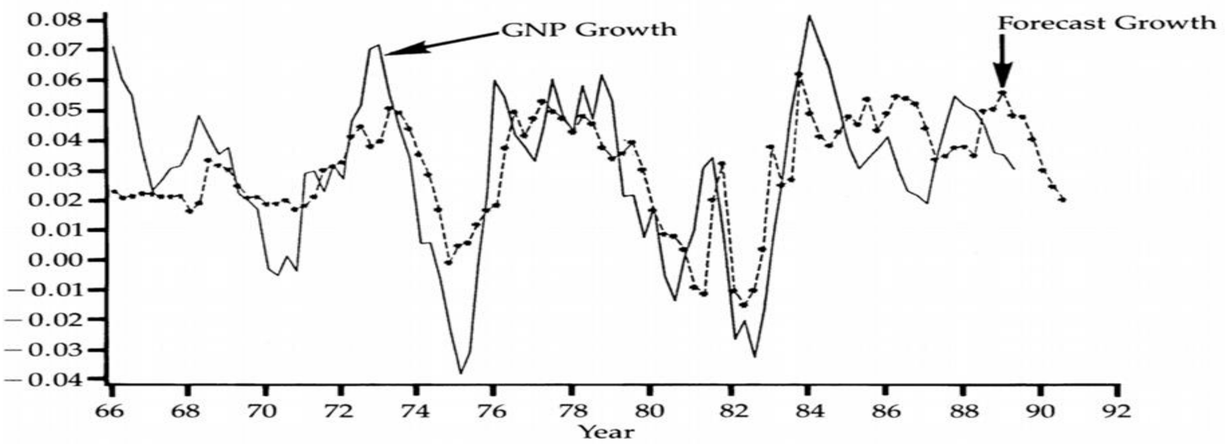
* The model estimated is $\Delta GNP_{t+1:t+5} = a + bX_t + u_{t+5}$. ΔGNP is the annual logarithmic growth in real Gross National Product. The data for the second quarter of 1989 are based on the first release available on July 27, 1989. X is one of: the logarithm of the ratio of one plus the five-year yield divided by the three-month Treasury bill rate, the logarithm of the ratio of one plus the 10-year yield divided by the three-month Treasury bill rate, the one-quarter return on the Standard & Poor's 500 stock index, or the four-quarter return on the Standard & Poor's 500; t-ratios are in brackets.

Source: Harvey (1989b), Table I.

Graphically, Harvey shows that the term structure is quite helpful in explaining GDP growth:

Figure 3

Annual GNP Growth and Forecast GNP Growth*



Source: Harvey (1989b).

Harvey (1989b) also showed that this simple 1-variable predictor had root mean squared forecast errors that were as low as those of most of the top professional forecasters over the periods examined, as in the table below:

Table 2

Table III Yield Spread Model Forecasting Performance vs. Other Econometric Models, 1976:1–1985:1*

Model	*Mean Absolute Error*	*Root Mean Squared Error*
Five-Year Yield Spread	1.7	2.1
10-Year Yield Spread	1.7	2.1
One-Quarter Stock Return	2.4	3.0
Four-Quarter Stock Return	2.5	3.0
BEA	1.7	2.4
BMARK	2.1	2.5
Chase	2.0	2.4
DRI	1.6	2.1
EFP	1.7	2.1
RSQE	1.7	2.1
WEFA	1.5	2.1

* The parameters of each model are reestimated at each point in the time series during 1975:4–1984:4. These parameters are used to forecast annualized percentage growth in the 1976:1–1985:1 period. The initial estimation period is 1953:2–1975:4. All figures are in annualized percentage growth. BEA is Bureau of Economic Analysis; BMARK is the Benchmark forecast from Charles R. Nelson Associates, Inc.; Chase is Chase Econometric Associates, Inc.; DRI represents Data Resources, Inc.; EFP is the Econometric Forecasting Project at Georgia State University; RSQE denotes the Research Seminar on Quantitative Economics at the University of Michigan; and WEFA represents Wharton Econometric Forecasting Associates, Inc. The forecast evaluation statistics for the seven models are from McNees (see footnote 12). The forecast evaluation statistics for the model are based on Gross National Product data available in mid-1985.

Source: Harvey (1989b), Table III.

Harvey (1991) also demonstrated that the relationship of the slope of the term structure to subsequent economic growth is true both for the USA and for several other G-7 countries.

Table 3

Predicting Economic Growth with Local Term Structure

Country	a	b	$\bar{R}^2$
1970:1-1989:4 (76 observations)			
Canada	0.03	1.11	0.48
	[6.22]	[4.53]	
France	0.03	0.52	0.13
	[6.11]	[2.14]	
Germany	0.01	0.75	0.29
	[2.60]	[4.50]	
Italy	0.04	0.71	0.26
	[7.19]	[5.15]	
Japan	0.04	0.23	0.01
	[10.58]	[1.38]	
United Kingdom	0.02	0.42	0.08
	[4.20]	[1.71]	
United States	0.02	1.27	0.47
	[3.20]	[5.71]	
World	0.02	1.42	0.54
	[5.66]	[6.8]	

Source: Harvey (1991).

Estrella and Hardouvelis (1991) published results quite similar to Harvey's, but with some additional tests. In Figure 4, they show estimated recession probabilities based upon the slope of the term structure 4 quarters earlier. The correlation is quite striking.

Figure 4

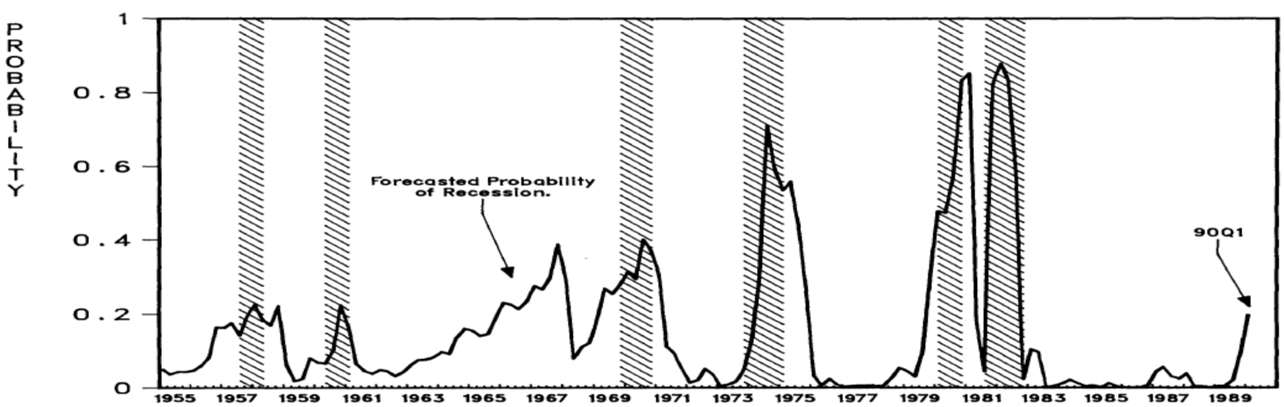


Figure 2. Forecasted probability of recession for current quarter based on the slope of the yield curve 4 quarters earlier. The shaded areas denote current NBER-dated recessions. The forecasted probability of recession denotes the within-sample fit of a probit model, estimated over the quarterly sample period from 1956:1 through 1988:4, in which the dependent variable is a binary variable denoting the current presence or absence of an NBER-dated recession, and the only explanatory variable is the slope of the yield curve observed 4 quarters earlier. The slope of the yield curve is the difference between the 10-year Treasury bond rate and the 3-month Treasury bill rate. Both rates are quarterly averages of annualized bond equivalent yields.

Source: Estrella and Hardouvelis (1991), Figure 2.

Estrella and Hardouvelis (1991) looked more broadly at the ability of the term structure slope to forecast the components of GDP – consumption, investment and government spending. As Table 4 shows, they find statistically significant predictability for approximately 2 years forward for consumption, both total and durables, and also for investment. However, the term structure slope does not have any explanatory power for government spending:

Table 4

Predicting Future Cumulative Changes of Real GNP Components Using the Slope of the Yield Curve

The sample is quarterly from 1955:2 through 1988:4. The estimated models are as follows:

$$\textit{Cumulative Change}: (400/k)(\log x_{t+k} - \log x_t) = \alpha_0 + \alpha_1\, SPREAD_t + \epsilon_t$$

x_{t+k} is the quarter $t+k$ value at constant prices of: consumption (consumer nondurables plus services), consumer durables, gross private domestic investment, and government spending. k is the forecasting horizon. $SPREAD_t$ is the difference between the 10-year T-bond and 3-month T-bill rates of quarter t. The interest rates are annualized quarterly average bond equivalent yields. Inside the parentheses are Newey and West (1987) corrected standard errors that take into account the moving average created by the overlapping of forecasting horizons as well as conditional heteroskedasticity. $\bar{R}^2$ is the coefficient of determination adjusted for degrees of freedom, and SEE is the regression standard error.

k Quarters Ahead	Consumption				Consumer Durables				Investment				Government Spending			
	α_0	α_1	$\bar{R}^2$	SEE	α_0	α_1	$\bar{R}^2$	SEE	α_0	α_1	$\bar{R}^2$	SEE	α_0	α_1	$\bar{R}^2$	SEE
1	2.57*	0.57*	0.10	1.94	0.28	4.16*	0.12	12.96	−2.15	5.27*	0.08	20.14	2.23*	0.06	−0.01	5.06
	(0.45)	(0.21)			(1.44)	(0.96)			(1.60)	(1.55)			(0.63)	(0.28)		
2	2.60*	0.54*	0.16	1.47	0.46	3.93*	0.23	8.51	−3.24*	6.34*	0.20	14.81	2.28*	0.02	−0.01	3.83
	(0.43)	(0.19)			(1.35)	(0.86)			(1.59)	(1.40)			(0.65)	(0.26)		
3	2.61*	0.53*	0.20	1.24	0.68	3.73*	0.29	6.84	−3.19*	6.33*	0.28	12.00	2.26*	0.06	−0.01	3.26
	(0.41)	(0.17)			(1.19)	(0.65)			(1.57)	(1.38)			(0.65)	(0.27)		
4	2.64*	0.50*	0.21	1.14	1.01	3.42*	0.33	5.81	−2.77	5.98*	0.32	10.29	2.24*	0.10	−0.01	2.97
	(0.39)	(0.15)			(1.12)	(0.51)			(1.48)	(1.30)			(0.66)	(0.27)		
5	2.68*	0.46*	0.20	1.08	1.47	3.02*	0.30	5.37	−2.27	5.57*	0.35	8.93	2.25*	0.12	0.00	2.78
	(0.38)	(0.13)			(1.03)	(0.40)			(1.35)	(1.11)			(0.66)	(0.27)		
6	2.73*	0.41*	0.18	1.01	1.98*	2.58*	0.26	5.11	−1.72	5.06*	0.36	7.82	2.23*	0.15	−0.00	2.66
	(0.38)	(0.12)			(0.96)	(0.25)			(1.24)	(0.98)			(0.66)	(0.28)		
7	2.78*	0.37*	0.16	0.96	2.44*	2.17*	0.20	4.98	−1.12	4.48*	0.35	7.08	2.21*	0.19	0.00	2.56
	(0.37)	(0.11)			(0.89)	(0.18)			(1.09)	(0.83)			(0.65)	(0.26)		
8	2.84*	0.30*	0.12	0.94	2.91*	1.69*	0.14	4.85	−0.44	3.78*	0.30	6.63	2.18*	0.23	0.00	2.46
	(0.36)	(0.11)			(0.84)	(0.17)			(0.98)	(0.65)			(0.64)	(0.23)		
12	3.01*	0.14	0.03	0.83	4.17*	0.51	0.01	3.96	1.75*	1.56*	0.11	5.22	2.12*	0.29*	0.02	2.12
	(0.33)	(0.11)			(0.68)	(0.30)			(0.73)	(0.32)			(0.58)	(0.14)		
16	3.10*	0.06	0.00	0.74	4.71*	0.06	−0.01	3.14	2.75*	0.70*	0.03	4.00	2.17*	0.23*	0.01	1.88
	(0.28)	(0.08)			(0.49)	(0.33)			(1.44)	(0.33)			(0.52)	(0.12)		
20	3.12*	0.03	−0.01	0.67	4.84*	−0.10	−0.01	2.65	3.02*	0.45	0.02	3.06	2.24*	0.11	−0.00	1.72
	(0.23)	(0.07)			(0.38)	(0.33)			(0.24)	(0.39)			(0.50)	(0.18)		

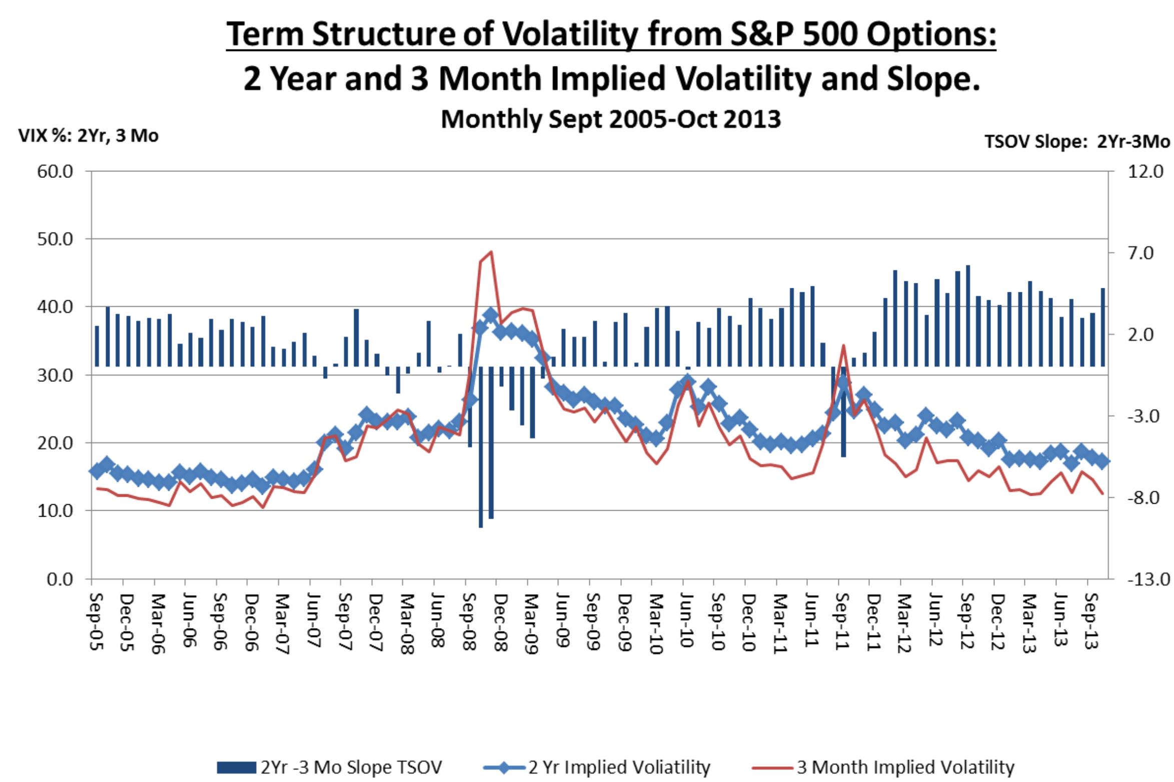
*Significantly different from zero at the 5% level in a two-tailed test.

Source: Estrella and Hardouvelis (1991), Table II.

In 1996, after Harvey's and Estrella and Hardouvelis empirical work, the slope of the term structure was added as a predictor variable in the Conference Board's Leading Economic Indicators series. In a 1998 study, Dotsey of the Federal Reserve Bank of Richmond found that a negative term structure slope gave 18 correct signals and 2 incorrect signals of recession in the 1955-1995 time period. Thus, the slope of the term structure of interest rates is a closely watched barometer of the likely future growth of the economy.

Do note that Harvey's tests focused on the theoretically positive relation between real rates and expected future growth. However, the term structure of volatility is also a factor in the term structure theory equation, (32). An upward sloping term structure of volatility would help explain a downward sloping term structure (not necessarily related to a decline in growth), a condition that occurred in 2005-2006. A downward sloping term structure of volatility, as in late 2008 and and early 2009, at the depths of the Great Recession) can explain a positive term structure slope at a time when growth may not have been projected to significantly increase quickly. Figure 6 displays the slope of the term structure of volatility from 2005-2013, including during the Great Recession, using Black-Scholes implied volatilities from option price data for 3-month and 2-year at-the-money options on the S&P 500 stock price index:

Figure 5



Note that the slope of the term structure of volatility swung from being sharply positive in 2005-2006 (when volatilities were historically low) to being sharply negative in 2008-2009, when stock market volatility was very high. Combining analysis of changes in the term structure of volatility and the term structure of inflation with changes in the term structure of expected real growth is a promising area for future research on the term structure of nominal interest rates.

IV. Early 1980s Tests of CCAPM: Mehra-Prescott (1985) Equity Premium Puzzle.

In this section, we review some of the early tests of consumption-based asset pricing and the CCAPM. The first two decades of CCAPM tests produced mixed relative results (vs. the original market-oriented CAPM) and mainly negative absolute results, with initial strong model rejections in tests of the CCAPM Euler conditions based on a representative individual with constant relative risk aversion. Most prominent were Hansen and Singleton (1983) for multiple assets, and Mehra and Prescott (1985) for the S&P 500. The Chen, Roll and Ross (1986) examination of prices of economic factors was also interpreted as rejecting the CCAPM. Grossman, Melino and Shiller (1987), Breeden, Gibbons and Litzenberger (BGL, 1989) and Wheatley (1988) examined measurement issues in consumption (such as time aggregation) and their biases on measures of volatility and consumption betas. BGL found that the risk premium on consumption betas (based on the consumption mimicking stock portfolio), was significantly positive, as was that for the risk premium on market index betas, but mean-variance portfolio efficiency of both the consumption mimicking portfolio and the stock market index portfolio were rejected. In subsequent years, empirical tests have become more sophisticated and have found much more positive results than in these earliest tests.

A. Commodity Futures: Consumption Betas vs. Market Betas.

In one of the first applications of CCAPM insights, Breeden (1980) presented comparisons of unconditional consumption and market beta estimates for commodity futures returns. Dusak's (1973) well-known study found S&P 500 betas of commodities to be near zero, which, based on the CAPM, predicted that risk premiums of commodity futures should be near zero and futures prices should be unbiased estimates of future spot prices. Breeden argued that due to different income elasticities of demand, and relatively fixed supplies in the short run for commodities such as beef, pork and chicken, these commodities should have positive consumption betas. In his model, positive elasticities of supply with respect to price dampen the consumption betas for more distant maturity commodity futures contracts. His statistical estimates support these predictions. The near contracts of meats had an average consumption beta of 10.9 (with an average t-statistic of 3.0), indicating that their prices rose almost 11% for every 1% that real consumption growth was above forecast. This is almost twice the consumption beta for the S&P 500, indicating significant systematic risk in near futures contracts for livestock and meat. Consistent with the ability to adapt supplies over time, the third and fifth contracts to maturity had consumption betas of 7.3 and 4.3, respectively, dampening down the risk of the nearest contract. This pattern was also demonstrated in both consumption and market betas by the industrial metals and wood (copper, platinum and plywood), since demand for these commodities is quite sensitive to the economic cycle, there is ability to adapt supplies over time. With these commodities, there is relative little supply uncertainty in the short run, so demand risks dominate.

In contrast, grains and their derivatives have substantial supply uncertainty that is sometimes big enough to affect economic growth. A major drought in 1974-1975 drove grain prices substantially higher at the same time oil prices jumped and economies fell into a serious recession. Thus, the supply effect is offsetting to demand risks and resulted in consumption betas for grains that were near zero or even slightly negative. For these commodities, the CCAPM predicts near zero risk premiums and that futures prices would be unbiased estimates of future spot prices. In contrast, the meats should have futures prices that are downward biased estimates of future spot prices to provide rewards to investors for their positive systematic risks. Other foods with major supply uncertainties that are not normally large enough to affect economic growth (e.g., sugar, orange juice) have betas that can be large and positive or negative, due to the coincidence of their crops with consumption growth during the sample period.

B. Euler Equation Tests and The Equity Premium Puzzle: Mehra-Prescott (1985)

In 1982, Grossman and Shiller showed that the Consumption CAPM could be tested and should hold even when there are heterogenous probability beliefs, if empiricists conditioned only on information that was "common knowledge" in investors' information sets. This should include tests based on past prices, earnings and dividends, for example, so it is a nontrivial extension. They also proved that the CCAPM holds with heterogeneous beliefs for non-tradable risky assets, as long as consumption and the prices of tradable assets form an Ito process.

Also in 1982, Hansen and Singleton provided a now widely used method for testing nonlinear stochastic Euler equation restrictions on the joint movements of asset returns and consumption (see eqs. 6'-6''). Using a narrow power utility function for a "representative investor", they obtained plausible estimates of constant relative risk aversion, but were able to reject the CRRA model when examining multiple asset returns simultaneously. In 1983, Hansen and Singleton used maximum likelihood estimation to test Euler equation restrictions with monthly consumption growth data and concluded that the CRRA-lognormal model is unable to fit the data and that more general preference specifications or probability distributions are required. Also in 1983, Kraus and Litzenberger followed up their well-known 1976 work on skewness preference and demonstrated that, with the quite plausible assumption of decreasing absolute risk aversion (less risk averse to constant dollar gambles as wealth increases), a three-moment Consumption CAPM could be derived for assets with quadratic characteristic lines. Assets with a convex relation with real consumption growth have lower exposures to economic contractions and higher exposure to economic expansion cet. par. have lower expected returns in equilibrium. Conversely, assets with a concave relation with real consumption have high exposure to economic contractions than to economic expansions and cet. par. need to have higher expected returns to make them attractive to hold.

In a groundbreaking, controversial article in 1985, Mehra and Prescott examined 89 years of annual data from 1889 to 1978 on real consumption growth (nondurables and services), the real riskless rate estimated from Treasury bills, and the real return on the Standard and Poor 500 Stock Price Index. They tested whether or not the average level of the estimated real riskless rate and the average level of the equity risk premium could be explained by a consumption-based asset pricing model based on a "representative individual" with constant relative risk aversion. Using what were believed to be plausible levels of constant relative risk aversion (1 to 10). Table 5 (below) has their summary statistics for the means and standard deviations of real, per capita nondurables and services consumption growth, of the estimated real riskless rate from Treasury bills, and the real S&P 500 return and its equity risk premium over Treasury bills. They found a mean real stock return of 7.0% over the entire period, while the real riskless rate averaged 0.8%, giving an equity risk premium of 6.2%. The standard deviation of the annual real stock return was 16.5%, while the volatility of real consumption growth was 3.6%. Note that while the volatility of stocks showed no obvious trend over the decades, the volatility of annual consumption growth declined quite dramatically from about 5.0% in the first two decades (1889-1908) to approximately 1.1% over the most recent three decades from 1949-1978, following the end of World War II. This very low level of volatility of consumption growth and the moderate (0.40) correlation of stocks with consumption growth leads to Consumption CAPM predicted risk premiums for stocks that are much lower than actually experienced by investors, which Mehra and Prescott named the "equity premium puzzle." They also found it difficult to fit what they viewed as the low level of the real riskless rate, 0.8%.

Table 5

Annualized Means and Volatilities of Real NDS Consumption Growth, the Real Riskless Return and Real Returns on Equities and the Equity Premium: 1889-1978 By Decade

Time periods	% growth rate of per capita real consumption		% real return on a relatively riskless security		% risk premium		% real return on S&P 500	
	Mean	Standard deviation	Mean	Standard deviation	Mean	Standard deviation	Mean	Standard deviation
1889–1978	1.83 (Std error = 0.38)	3.57	0.80 (Std error = 0.60)	5.67	6.18 (Std error = 1.76)	16.67	6.98 (Std error = 1.74)	16.54
1889–1898	2.30	4.90	5.80	3.23	1.78	11.57	7.58	10.02
1899–1908	2.55	5.31	2.62	2.59	5.08	16.86	7.71	17.21
1909–1918	0.44	3.07	−1.63	9.02	1.49	9.18	−0.14	12.81
1919–1928	3.00	3.97	4.30	6.61	14.64	15.94	18.94	16.18
1929–1938	−0.25	5.28	2.39	6.50	0.18	31.63	2.56	27.90
1939–1948	2.19	2.52	−5.82	4.05	8.89	14.23	3.07	14.67
1949–1958	1.48	1.00	−0.81	1.89	18.30	13.20	17.49	13.08
1959–1968	2.37	1.00	1.07	0.64	4.50	10.17	5.58	10.59
1969–1978	2.41	1.40	−0.72	2.06	0.75	11.64	0.03	13.11

Source: Mehra and Prescott (1985), p. 147.

Mehra and Prescott simulated economies that mimicked historic volatility properties of stocks and consumption growth, as well as the -0.14 autocorrelation in real consumption growth in that time period, and enforced constraints that (1) constant relative risk aversion could not exceed 10 and that (2) the real riskless rate had to be between 0% and 4.0%. With these constraints, they estimate the admissible region for the equity risk premium to be related to the real riskless rate estimate as shown in the graph below:

Figure 6

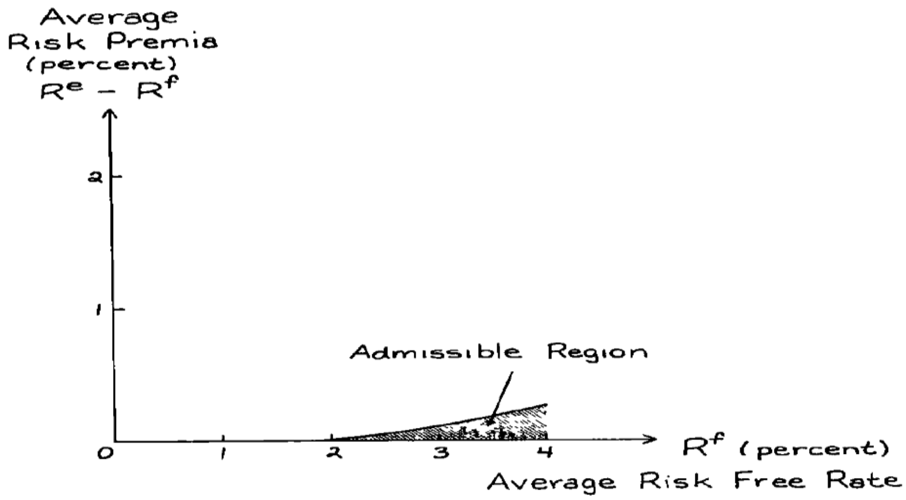


Fig. 4. Set of admissible average equity risk premia and real returns.

Source: Mehra and Prescott (1985), p. 155.

Quoting Mehra-Prescott (p. 156), "The largest premium obtainable with the model is 0.35%, which is not close to the observed value" (which was 6.18%). Given from Table 5 that the volatility of the risk premium is 1667 basis points annually, the Mehra-Prescott estimate that a risk premium of 35 basis points would properly reward investors for taking equity risk is highly implausible unless equity risk is uncorrelated with marginal utilities of investors (i.e, there is little consumption risk). With that risk premium and volatility, equity investors would have barely more than a 50/50 chance of having returns that exceeded Treasury bills!

Let us check on whether equities had much systematic consumption and marginal utility related risk over the sample period. We will look at the correlations from 1889-2013 (124 years) of their annual returns with real consumption growth and with changes in the unemployment rate, using data from the NBER MacroHistory database and the Department of Commerce's *Historical Statistics of the United States: Colonial Times to 1970* for the early data. First, we computed the correlations of annual real stock returns with annual real consumption growth and annual changes in the unemployment rate to be 0.40 and -0.40, respectively, a moderate, but significant correlation, as expected. Simple OLS regressions of contemporaneous stock returns on contemporaneous values of those variables had slopes with the predicted signs and t-statistics of 4.4 and -3.9, respectively. Thus, the stock market clearly had significant consumption risk and likely was highly related to changes in marginal utility of consumption and wealth.

Going further, since stocks have been shown to strongly lead changes in the unemployment rate and real growth (e.g., Breeden (2012)), we examined "bad economy" years when the unemployment rate increased by 1.25% (0.5 sigma) or more in year t and another 1.25% in year t+1, and we found that the real stock return in period t averaged -15.3% in those years, approximately 22% below the unconditional mean real return on stocks of 7.0%. On the upside (in a good economy), when the unemployment rate dropped by 1.25% in year t and again in year t+1, the real stock return in year t averaged a gain of +13.3%, or 6% above the unconditional mean return. These statistics are little different if we look at economies with back-to-back 0.5 standard deviation moves in real consumption growth, as Table 6 shows:

Table 6

Real Stock Returns in Good and Bad Economies

Economic State Based upon Real Consumption Growth and Unemployment Rate Changes

Years when economy is 0.5 standard deviations from the unconditional mean for 2 consecutive years

		1890-1978		1890-2013	
		Bad Economy	***Good Economy***	***Bad Economy***	***Good Economy***
Real NDS Consumption Growth	***# Obs***	*9*	*4*	*10*	*7*
	Mean Return	-15.5%	+14.0%	-15.7%	+15.0%
	Abnormal Return	-22.5%	+7.0%	-22.7%	+7.0%
Unemployment Rate Change	***# Obs***	*4*	*10*	*4*	*10*
	Mean Return	-15.3%	+13.3%	-15.3%	+13.3%
	Abnormal Return	-22.3%	+6.3%	-22.3%	+6.3%

Note the strong nonlinearity of the conditional means of stock returns to the economy: Good economies have real stock returns that are 6% to 7% above the unconditional mean, whereas bad economies have mean real stock returns that are approximately -22% below the unconditional mean. This is true whether one looks at real consumption growth or unemployment rate changes as the economic barometer for changes in individuals' marginal utilities. Table 6 and the statistics cited make clear that the stock market does have large, highly concave exposure to consumption growth and that investors would require risk premiums substantially in excess of the 35 basis points that the Mehra and Prescott analysis supports.

Furthermore, their assumption of constant relative risk aversion does not allow cyclical variations in risk premiums per unit of risk.

An influential early article by Rietz (1988) proposed to explain the high level of the equity premium and low riskless returns with a model of rare disasters, like stock market crashes. This is quite compatible with the Kraus-Litzenberger (1983) model that implies that a sufficiently concave characteristic line of equities with consumption growth could help explain the high risk premium on equities because of asymmetrically large exposure to contractions as compared to expansions. Rietz extended the Mehra-Prescott model to include a third state, which represented a very low probability of a major depression or "crash." Using a simple power utility function with constant relative risk aversion, Rietz was able to match both a low real riskless rate (less than 1.0%) and the historic equity market risk premium of 6% to 7% with a relatively moderate level of constant relative risk aversion (5 to 7), as shown in the following table:

Table 7

Example 1: $\lambda_3 = (1 + \mu)/2$.[a] **(Output falls to one-half of its normal expected value during a crash.)**

Parameter configurations that give risk-free returns and risk premia very near the economy's sample values				
Crash probability (η)	Risk aversion parameter (α)	Time preference parameter (β)	Corresponding risk-free return (annual %)	Corresponding risk premium (annual %)
0.0008	7.05	0.997	0.77	6.36
0.0008	7.00	0.999	0.83	6.18
0.0009	6.90	0.994	0.87	6.38
0.0009	6.90	0.995	0.77	6.38
0.0009	6.85	0.997	0.83	6.19
0.0009	6.85	0.998	0.73	6.19
0.0010	6.75	0.993	0.88	6.34
0.0010	6.75	0.994	0.78	6.33
0.0010	6.70	0.996	0.84	6.15
0.0010	6.70	0.997	0.74	6.14
0.0010	6.65	0.999	0.79	5.96
0.0020	5.75	0.989	0.83	5.92
0.0020	5.75	0.990	0.73	5.92
0.0030	5.30	0.980	0.89	6.15

[a] **Here λ_3 is the gross growth rate in output during a crash year and $(1 + \mu)$ is the average gross growth rate during 'normal' years.**

Source: Rietz (1988), Table 3.

Rietz's results have enduring interest, for as we shall see in Section XII, Barro (2006), Barro and Ursua (2008) and Wachter (2013) have followed up with data and have extended his model.

Mehra and Prescott's "equity premium puzzle" article stimulated a huge amount of additional research. In addition to the disaster risk analysis of Rietz, in the late 1980s theorists began intensive modeling of preferences that were not based on time-additive narrow power utility functions, but instead had a representative utility function displaying decreasing relative risk aversion, time-complementarity and habit formation, or were of a recursive, forward looking form. Several of the major articles in those areas are reviewed in Section VI.

Following that review, there is another strand of research spawned by this work. As Mehra and Prescott said in the concluding remarks to their article: "In the absence of such markets, there can be variability in individual consumptions, yet little variability in aggregate consumption." … "The fact that certain types of contracts may be non-enforceable is one reason for the non-existence of markets that would otherwise arise to share risk." … "To test such theories, it would probably be necessary to have consumption data by income or age groups." Since that article was written, the U.S. government has indeed collected and disseminated such data. One of the most important strands of research that has helped us figure out the equity premium puzzle is a group of papers on "incomplete markets" and "limited participation" of investors in the stock market. In Section VII, we will review some of the works in these areas, plus show some calculations of our own from the Consumer Expenditure Survey (CEX), which was used by some authors.

V. Measuring Consumption Risks: Breeden, Gibbons, Litzenberger (1989)

Stephen Ross, the distinguished financial economist and developer of the Arbitrage Pricing Theory (APT), in his book entitled *Neoclassical Finance* (2005, page 37), said that "The consumption beta model is marvelous theory but it surprises me that people take it as seriously as they do for empirical work." That's quite a challenging statement for CCAPM proponents. To understand the basis for his remark, let us examine the test of the Arbitrage Pricing Theory (APT) and Merton's Intertemporal CAPM done by Chen, Roll and Ross (CRR, 1986) and the result they found for their estimate of consumption risk. CRR did an interesting analysis of exposures of stock returns to "economic state variables," such as (1) fluctuations in monthly industrial production, a good monthly measure of economic activity, (2) CPI inflation, (3) changes in expected inflation, (4) credit risk, as measured by the "junk bond premium" and (5) the unanticipated change in the term structure, as measured by Ibbotson's long-term government bond return less the return on 1-month Treasury bills. The junk bond premium was computed as the return on bonds rated Baa or less (junk rated below Baa), less Ibbotson's long-term U.S. Government bond return. Chen, Roll and Ross's estimates of the risk premiums for exposures to the various fundamental risk factors (excluding consumption) are given in the next table, where t-statistics are in parentheses.

Plausibly, CRR find in Panel A that, standing alone, exposure to equity market beta carries a positive risk premium, where betas are estimated with 60 month rolling regressions with prior data. Panel B shows the results for their five economic factors, which are also plausible. In a multiple regression, the coefficient of the market return variable is negative, and CRR (p. 399) state that "… (their betas) do not explain cross-sectional differences in average returns after the betas of the economic state variables have been included." The key results found are that exposure to general economic activity (MP, industrial production) is rewarded with a significantly positive risk premium, as is exposure to credit risk (UPR), which is also very pro-cyclic, both quite sensible results. Exposure to both expected and unexpected inflation gives negative risk premiums, as inflation hedging assets appear to be priced higher and give lower average returns, which are also plausible results. Stocks that had returns that were inversely related to interest rates fell sharply received negative premiums for that exposure (UTS), perhaps as falling interest rates indicate weak economies, so such stocks have less real consumption risk. Thus, all of the risk premiums are plausible, with the exception of the market risk premium.

Table 8

Economic Variables and Pricing

Years	VWNY	MP	DEI	UI	UPR	UTS	Constant
	A						
1958-84	14.53 (2.36)	…	…	…	…	…	-5.83 (-0.96)
1958-67	5.01 (0.67)	…	…	…	…	…	6.85 (0.93)
1968-77	17.99 (1.46)	…	…	…	…	…	-15.03 (-1.25)
1978-84	23.19 (1.94)	…	…	…	…	…	-10.8 (-0.91）
	B						
1958-84	-9.99 (-2.01)	12.19 (3.15)	-0.15 (-1.82)	-0.91 (-2.59)	9.81 (3.36)	-5.45 (-1.61)	10.71 (2.76)
1958-67	-5.71 (-1.01)	13.02 (1.85)	0.004 (0.06)	-0.19 (-0.37)	6.1 (1.99)	-0.59 (-0.26)	9.53 (1.98)
1968-77	-17.4 (-1.82)	14.47 (2.21)	-0.29 (-3.39)	-1.61 (-3.3)	14.37 (3.13)	-9.23 (-1.78)	8.58 (1.17)
1978-84	-5.52 (-0.51)	7.73 (1.3)	-0.15 (-0.57)	-0.94 (-1.05)	8.6 (1.06)	-6.99 (-0.68)	15.45 (1.87)

NOTE— VWNY = return on the value-weighted NYSE index; EWNY = return on the equally weighted NYSE index; MP = monthly growth rate in industrial production; DEI = change in expected inflation; UI = unanticipated inflation; UPR = unanticipated change in the risk premium (Baa and under return – long-term government bond return); and UTS = unanticipated change in the term structure (long-term government bond return – Treasury-bill rate). *T*-statistics are in parentheses.

Source: Chen, Roll and Ross (1986), Table 5.

When Chen, Roll and Ross added a factor for 1-month percentage changes in real per capita consumption growth (like Hansen and Singleton (1983)), the results are in the next table:

Table 9

Pricing with Consumption							
Years	CG	MP	DEI	UI	UPR	UTS	Constant
1964-84	0.68	14.96	-0.17	-0.85	8.81	-6.92	2.29
	(0.11)	(3.8)	(-1.74)	(-2.25)	(2.58)	(-1.79)	(0.63)
1964-77	-0.49	18.15	0.17	-0.95	11.44	-9.19	-1.91
	(-0.66)	(3.54)	(-2.42)	(-2.49)	(3.29)	(-2.41)	(-0.44)
1978-84	1.17	8.59	-0.17	-0.65	3.56	-2.38	10.69
	(1.00)	(1.48)	(-0.66)	(-0.77)	(0.47)	(-0.27)	(1.61)

Source: Chen, Roll and Ross (1986), Table 6. t-statistics in parentheses.

Monthly consumption data in the U.S. is available starting in 1959, so using 60 prior months of data to estimate rolling consumption betas means that the first data point is in 1964. Over the entire sample period, the point estimate of the risk premium is slightly positive, but insignificant. For the 1964-77 subperiod, their consumption risk premium was actually negative! Signs and significance of the other economic state variables were essentially unchanged.

Why did Chen, Roll and Ross (1986) and Hansen and Singleton (1983) get such poor results for consumption risk premiums? Breeden, Gibbons and Litzenberger's (BGL, 1989) article gives some clues. First, they note that consumption is measured with a great deal of error and is affected by considerable "noise" in the monthly data that is related to weather effects on spending and to major strikes and significant tax changes that affect income and spending. In many cases, these are short-term effects that are followed by catch-up gains or sharp declines in spending in subsequent months. Thus, BGL found that the monthly data have significantly negative autocorrelation (-0.28), whereas the quarterly consumption data have positive first-order autocorrelation (+0.29), as shown in the following table:

Table 10

Time Series Properties of Percentage Changes in Real, Per Capita Consumption of Nondurable Goods and Services

Data are seasonally adjusted as reported by the Department of Commerce in the *Survey of Current Business.* T denotes the number of observations, while $\bar{c}$ and $\widehat{SD}(c)$ are the sample mean and standard deviation, respectively. Under the hypothesis that the observations are serially uncorrelated, the asymptotic standard errors for the sample autocorrelations are $1/\sqrt{T}$, as given by $SD^*(\hat{\rho}_k)$. Under the hypothesis that $\rho_1 = 0.25$ and $\rho_k = 0 \; \forall \; |_k| > 1$, $SD(\rho_1)$ and $SD(\rho_k)$ report the asymptotic standard errors using the results of Bartlett (1946). The test statistic for the joint hypothesis that all autocorrelations are zero for lags 1 through 12 is given by Q_{12}, the modified Box-Pierce Q-statistic. Q_{12} is asymptotically distributed as chi-square with 12 degrees of freedom. The p-value is the probability of drawing a Q_{12} statistic larger than the current value under the null hypothesis.

Time Period	T	$\bar{c}$	$\widehat{SD}(c)$	$\hat{\rho}_1$	$\hat{\rho}_2$	$\hat{\rho}_3$	$\hat{\rho}_4$	$\hat{\rho}_8$	$SD^*(\hat{\rho}_k)$	$\widehat{SD}(\hat{\rho}_1)$	$\widehat{SD}(\hat{\rho}_k)$	Q_{12}	p-Value
Panel A: Quarterly Consumption Data													
39Q2–82Q4	175	0.00543	0.00951	0.29	0.03	−0.00	0.07	0.02	0.08	0.07	0.08	23.93	0.02
39Q2–52Q4	55	0.00665	0.01517	0.30	0.03	−0.04	0.08	0.08	0.13	0.12	0.14	11.26	0.51
53Q1–67Q4	60	0.00463	0.00549	0.21	0.09	0.11	−0.01	−0.22	0.13	0.12	0.14	11.25	0.51
68Q1–82Q4	60	0.00511	0.00487	0.36	0.01	0.26	0.09	−0.31	0.13	0.12	0.14	25.95	0.01
Panel B: Monthly Consumption Data													
1959–1982	287	0.00178	0.00447	−0.28	−0.02	−0.14	−0.12	−0.19	0.06	0.05	0.06	43.09	0.00
1959–1970	143	0.00199	0.00467	−0.31	−0.11	0.18	−0.08	−0.17	0.08	0.08	0.09	33.49	0.00
1971–1982	144	0.00156	0.00427	−0.24	0.07	0.09	−0.16	−0.16	0.08	0.08	0.09	20.56	0.06
Panel C: Quarterly Sampling of Monthly Consumption Data													
59Q2–82Q4	95	0.00521	0.00568	0.13	−0.13	0.20	0.04	−0.17	0.10	0.09	0.11	13.42	0.34
59Q2–70Q4	47	0.00576	0.00506	0.13	−0.15	0.13	−0.03	−0.04	0.15	0.13	0.15	10.61	0.56
71Q1–82Q4	47	0.00468	0.00623	0.12	−0.07	0.22	−0.10	−0.26	0.14	0.13	0.15	11.40	0.50

Source: Breeden, Gibbons, Litzenberger (1989), Table I.

Thus, monthly growth rates of consumption likely are being dominated by the noise of weather, strikes, and tax changes, whereas the quarterly data (and semiannual and annual percentage changes) are much more likely to pick up real economic signals. Subsequent empirical tests by Parker and Julliard (2005) and Jagannathan and Wang (2007) appear to support this explanation; i.e., Parker-Julliard examine consumption betas measured by cumulating consumption growth over 11 quarters, and Jagannathan-Wang examine 4-quarter changes. To have more nonoverlapping data, we prefer the use of 6-month or 2-quarter percentage changes, as in Vissing-Jorgensen (2002) and Breeden (2012). Tests based upon monthly percentage changes in real consumption are simply not reliable, given the low signal-to-noise ratio and the presence of large reversals, which can statistically dominate the results.

To illustrate that real consumption growth measured with 6-month or 2-quarter differencing is picking up real economic risks and fluctuations in marginal utilities, consider the relation of contemporaneous changes in the unemployment rate versus real consumption growth and real stock market returns in the following graphs from Breeden (2013):

Figure 7

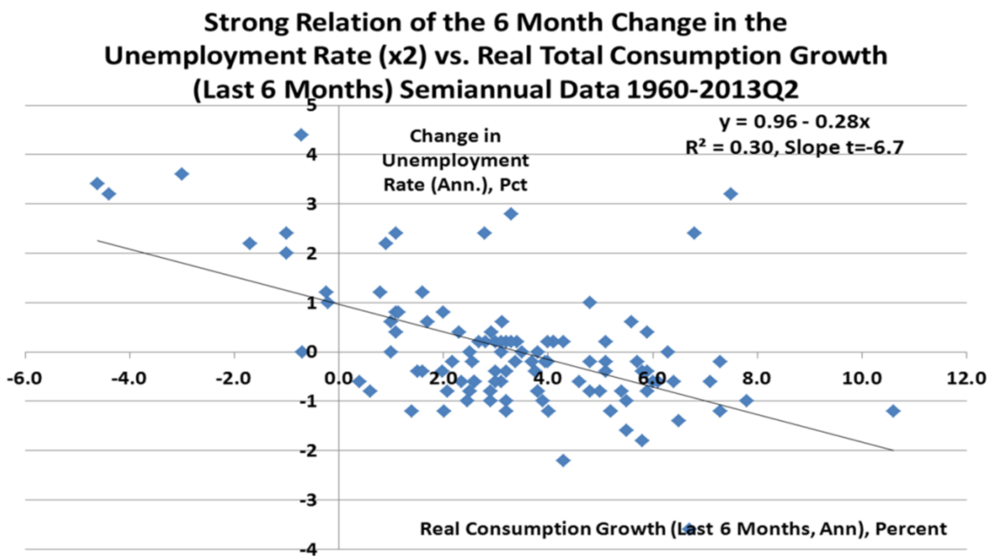
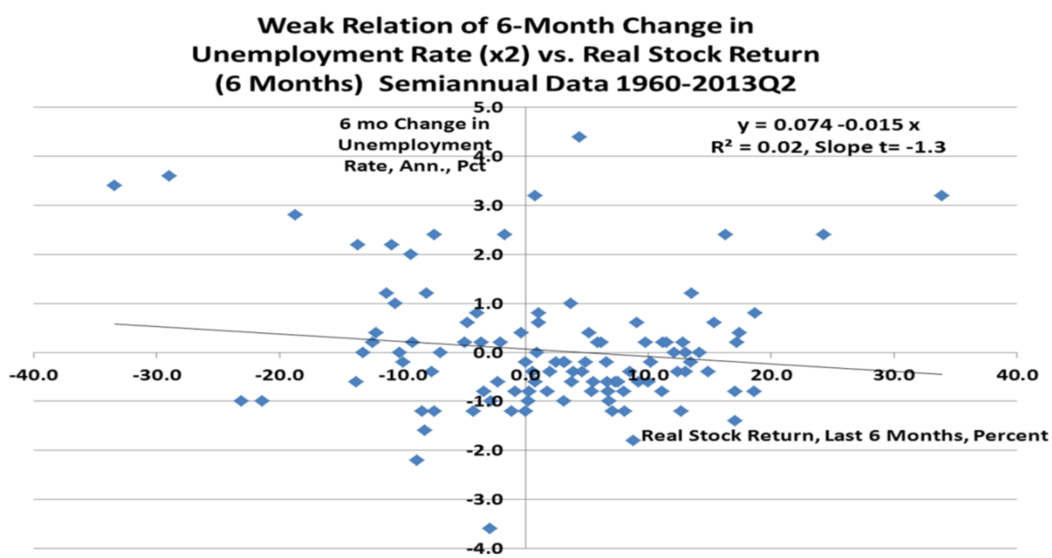


Figure 8



Changes in the unemployment rate are much more contemporaneously correlated with real consumption growth than with real returns on the stock market, with a slope t-statistic of -

6.7 versus -1.3, respectively, and corrected R^2 values of 0.30 versus 0.02 with nonoverlapping data from 1960-2013. Thus, even given the difficulties of measuring consumption growth, it appears to be strongly correlated with likely measures of marginal utility. Correlations of real consumption growth with total employment growth and loan chargeoffs show similar results:

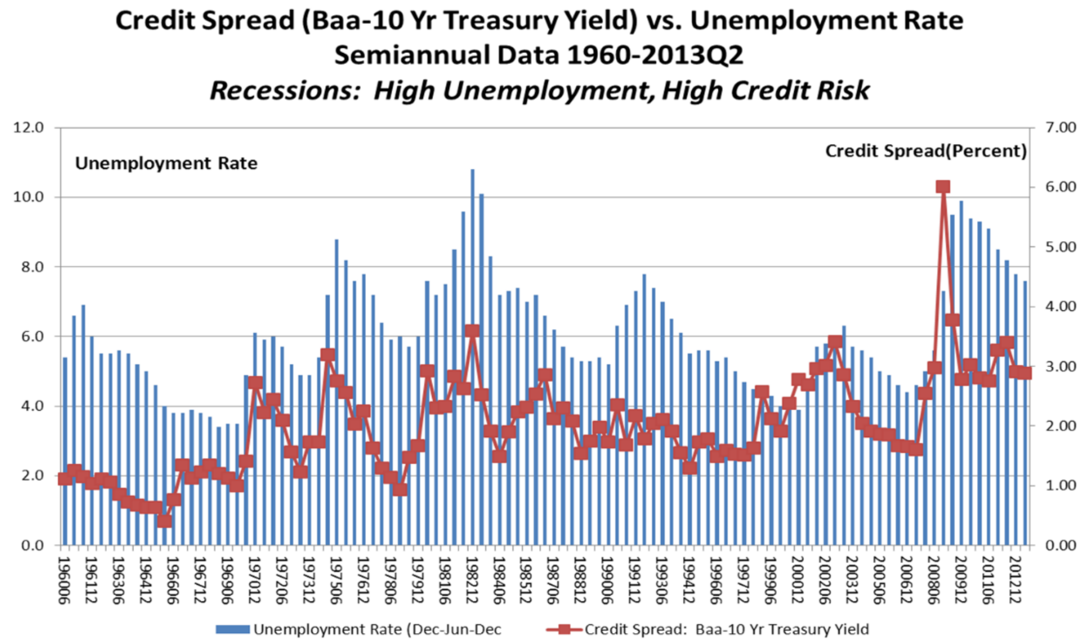
Table 11

Consumption and Marginal Utility:

Dependent Variable / Independent Variable	6 Month Unemployment Growth 1960-2011Q2			Total Loan Charge-offs 1985-2011Q2		
	Slope	t-statistics	CRSQ	Slope	t-statistics	CRSQ
S&P 500 *6 Month Real Return*	0.001	0.1	-0.01	-0.006	-0.7	-0.01
Total Consumption *6 Month Real Growth*	0.35	5.6	0.23	-0.16	-4	0.23
NDS Consumption *6 Month Real Growth*	0.52	6.2	0.27	-0.22	-4.5	0.27

Chen, Roll and Ross find the credit risk variable (junk bond premium) to be highly significant in pricing risks. Thus, the prior table, which shows that loan chargeoffs are closely correlated with real consumption growth (t=4.0 to 4.5 for total real consumption and NDS growth), shows that this is likely picking up part of the consumption risk of the assets, but with more precisely measured variables. Figure 10 shows the relation of the credit spread between Baa rated bond yields and 10-year Treasury yields and the unemployment rate. It is quite easy to see the close relation of credit spreads to the unemployment rate and economic recessions and growth periods.

Figure 9



As credit yield spreads are forward looking, much as the stock market is quite forward looking, they are contemporaneously more highly correlated with stock returns than are loan chargeoffs, changes in the unemployment rate and the growth in the total number of jobs.

Table 12

Consumption and Marginal Utility:
Loan Credit Risk: BP Change in Yield Spread of Baa-Treas **Corporate Bonds**: 1960-2011Q2(6m chg)

Independent Variable	Slope	t-statistic	CRSQ
S&P 500 6m Rl %chg	-1.96	-4.0	0.13
Total Real Consumption 6m Rl %chg	-8.50	-4.1	0.13
NDS Consumption 6m Rl %chg	-8.56	-2.8	0.06

The results give in the above tables suggests that the Chen, Roll and Ross economic factors may be viewed as instrumental variables for the unobservable true rate of growth in per capital consumption.

Breeden, Gibbons and Litzenberger (BGL, 1989)[3], Grossman, Melino and Shiller (1987) and Wheatley (1988) examined measurement issues in consumption (such as time aggregation) and their biases on measures of volatility and consumption betas. BGL derived two useful results used by many subsequent authors: (A) biases in consumption betas due to time aggregation, and how those biases are reduced with increased differencing intervals for consumption growth, and (B) estimation of consumption betas relative to returns on a consumption mimicking portfolio, which allows greater number and frequency of observations and more precise estimates of consumption betas. BGL's main results are derived next.

A. Time Aggregation Biases in Consumption Growth: Breeden, Gibbons, Litzenberger[4]

Ignoring other measurement problems, the reported ("interval") consumption rate for a quarter is the integral of the instantaneous ("spot") consumption rates during the quarter. The CCAPM relates expected quarterly returns on assets (e.g., from January 1 to March 31) and the covariances of those returns with the change in the spot consumption rate from the beginning of the quarter to the end of the quarter. This subsection derives the relation between the desired population covariances (and betas) of assets' returns relative to changes and the population covariances (and betas) of assets' returns relative to change in interval consumption. The variance of interval consumption changes is shown to have only two thirds the variance of spot consumption changes, while the autocorrelation of interval consumption is 0.25 due to the integration of spot rates.

Without loss of generality, consider a two-quarter period with t=0 being the beginning of the first quarter and t=T being the end of the first quarter. All discussion will analyze annualized consumption rates, so T=0.25 for a quarter. Initially, let the change in the spot consumption rate over a quarter be the cumulative of n discrete changes, $\left\{\widetilde{\Delta}_1^C, \widetilde{\Delta}_2^C, \ldots, \widetilde{\Delta}_n^C\right\}$ for the first quarter, and $\left\{\widetilde{\Delta}_{n+1}^C, \widetilde{\Delta}_{n+2}^C, \ldots, \widetilde{\Delta}_{2n}^C\right\}$ for the second quarter. That is $\widetilde{C_T} = C_0 + \sum_1^n \widetilde{\Delta}_i^C$. Similarly,

[3] Note that the BGL paper results in this section were complete by 1984 and were contemporaneous with or prior to Grossman, Melino and Shiller (1987).

[4] This subsection is taken from Breeden, Gibbons, Litzenberger (1989), Section II.B.

let the wealth at time T from buying one share of an asset at time 0 (and reinvesting any dividends) equal its initial price plus *n* random increments $\{\widetilde{\Delta}_i^a\}$: $P_T = P_0 + \sum_1^n \widetilde{\Delta}_i^a$.

Changes in consumption, $\widetilde{\Delta}_i^C$, the asset's return, $\widetilde{\Delta}_i^a$, are assumed to be homoscedastic and serially uncorrelated. The contemporaneous covariation of an asset's return with consumption changes is σ_{ac}. The variance of the change in the spot consumption from the beginning of a quarter to the end of the quarter is:

$$\mathrm{var}\left(\widetilde{C_T} - \widetilde{C_0}\right) = var(\textstyle\sum_1^n \widetilde{\Delta}_i^C) = \sigma_C^2 \,\mathrm{T}. \tag{35}$$

The first quarter's *reported* annualized consumption, C_{Q1}, is a summation of the consumption during the quarter, annualized by multiplying by 3 (or $\frac{1}{T}$):

$$C_{Q1} = \frac{1}{T}\sum_{j=1}^n C_j \Delta t = \frac{1}{T}\sum_{j=1}^n (C_0 + \sum_{i=1}^j \Delta_i^C)\Delta t \tag{36}$$

The annualized consumption rate for the second quarter, C_{Q2}, is the same as (36), but with the first summation for j being n+1 to 2n.

Continuous movements in consumption and asset prices can be approximated by letting the number of discrete movements per quarter, n, go to infinity ($\Delta t \to 0$). Doing this, the change in reported consumption becomes

$$C_{Q2} - C_{Q1} = \int_0^T \frac{t}{T}\Delta_t^C \, dt + \int_T^{2T} \frac{2T-t}{T}\Delta_t^C \, dt \tag{37}$$

Given the independence of spot consumption change over time, (37) implies that the variance of reported annualized consumption changes is

$$var(\tilde{C}_{Q2} - \tilde{C}_{Q1}) = \int_0^T \left(\left(\frac{t}{T}\right)^2 \sigma_C^2\right) dt + \int_T^{2T} \left(\left(\frac{2T-t}{T}\right)^2 \sigma_C^2\right) dt = \frac{2}{3}\sigma_C^2 T \qquad (38)$$

Thus, the population variance of reported (interval) consumption changes for a quarter is two thirds of the population variance for changes in the spot consumption from the beginning of a quarter to the end of the quarter. The averaging caused by the integration lead to the lower variance for reported consumption.

Next, consider the covariance of an asset's quarterly return with quarterly changes in the consumption. The covariance of the change in spot consumption from the beginning of a quarter to the end of the quarter with an asset's return over the same period is $\sigma_{ac}T$, given the i.i.d. assumption. With reported, interval consumption data, the covariance can be computed from (38):

$$cov(\tilde{C}_{Q2} - \tilde{C}_{Q1}, \tilde{P}_{2T} - \tilde{P}_T) = T^{-1} \int_T^{2T} (2T - t)\sigma_{aC}\, dt = \frac{1}{2} T\sigma_{aC} \qquad (39)$$

Thus, from (39) the population covariance of an asset's quarterly return with reported (interval) consumption is half the population covariance of the asset's return with spot consumption changes.

Given (38) and (39), betas measured relative to reported quarterly consumption changes are $\frac{3}{4}$ times the corresponding betas with spot consumption:

$$\beta_{ac}^{sum} = \frac{\frac{1}{2}T\sigma_{aC}}{\frac{2}{3}\sigma_C^2 T} = \frac{3}{4}\beta_{ac}^{spot} \qquad (40)$$

Since the CCAPM relates quarterly returns to "spot betas," the subsequent empirical tests multiply the mean-adjusted consumption growth rates by $\frac{3}{4}$ to obtain unbiased "spot betas." The $\frac{3}{4}$ relation of interval betas to spot betas in (40) is a special case of the multiperiod differentiating relation: $\beta_{ac}^{sum} = \beta_{ac}^{spot} \frac{K-\frac{1}{2}}{K-\frac{1}{3}}$, where K is the differencing interval. Thus, monthly data sampled quarterly (i.e., K=3) should give interval betas that are $\frac{\frac{5}{2}}{\frac{8}{3}} = 0.9375$ time the spot betas. When quarterly consumption growth rates are calculated from monthly data, the quarterly numbers are mean adjusted and multiplied by 0.9375.

Although changes in spot consumption are uncorrelated, changes in reported, interval consumption rates have positive autocorrelation. To see this, use (37) to compute the covariance of the reported consumption change from Q1 to Q2 with the reported change from Q2 to Q3, noting that all covariance arises from the time overlap from T to 2T:

$$cov(\tilde{C}_{Q3} - \tilde{C}_{Q2}, \tilde{C}_{Q2} - \tilde{C}_{Q1}) = \int_T^{2T} \frac{(t-T)(2T-t)}{T^2} \sigma_C^2 \, dt = \frac{1}{6} \sigma_C^2 T \qquad (41)$$

The first-order autocorrelation in reported consumption is 0.25 since

$$\rho_1 = \frac{cov(\tilde{C}_{Q3}-\tilde{C}_{Q2}, \tilde{C}_{Q2}-\tilde{C}_{Q1})}{var(\tilde{C}_{Q2}-\tilde{C}_{Q1})} = \frac{\frac{1}{6}\sigma_C^2 T}{\frac{2}{3}\sigma_C^2 T} = 0.25 \qquad (42)$$

By similar calculations, higher order autocorrelations are zero. BGL's Table I presents the time series properties of reported *unspliced* quarterly consumption growth rates. First-order autocorrelation of quarterly real consumption growth for the entire 1939-1982 period is estimated to be 0.29, which is insignificantly different from the theoretical value of 0.25 at usual levels of significance. Higher order autocorrelations are not significantly different from zero. Thus, the model for reported consumption is not rejected by the sample autocorrelations.

Monthly growth rates of real consumption from 1959 and 1982 exhibit negative autocorrelation of -0.28, which is significantly different from zero and from the hypothesized 0.25. This may be caused by vagaries such as bad weather and strikes in major industries, which cut current consumption temporarily but are followed by catch-up purchases. Quarterly growth rates in consumption computed from the monthly series again have positive autocorrelation of 0.13, more closely in line with the value 0.0625 (or $\frac{1}{16}$) predicted by the summation bias. The long the differencing interval, the less affected the data are by temporary fluctuations and measurement errors in consumption.

Chen, Roll and Ross (1986) and Hansen and Singleton (1983) use monthly data on unadjusted consumption growth. Since those data's autocorrelation statistics suggest significant departures from the random-walk assumption, the statistics they present warrant re-examination. The use of larger differencing intervals should be fruitful and were found to be in subsequent research (see Vissing-Jorgensen (2002), Parker-Julliard (2005) and Jagannathan-Wang (2007)).

Since even consumption goods classified as non-durables often have consumption flows in future periods, the marginal utility of consumption depends in part on past consumption expenditure, so current consumption expenditures impact the future marginal utility of consumption. Therefore, even under time-additive utility, past consumption should have an impact of current risk premiums. This suggests longer consumption horizons or distributed lags in empirical test of the time additive CAPM; e.g., the longer horizon used by Parker-Julliard (2005), which is discussed in Section X.

B. "Maximum Correlation Portfolio" or "Consumption Mimicking Portfolio." (CMP)[5]

Testing of the Consumption CAPM is hampered by the infrequent measurement of consumption relative to the frequency of measuring stock returns, as aggregate consumption in the USA has been measured only with annual totals back into the 1800s, quarterly only since 1939, and monthly only since 1959. Many other countries have even less frequent measurements of consumption than does the USA. In contrast, we have monthly indexes for the S&P 500 back to the 1800s, and daily data from the University of Chicago's CRSP return series that begins in 1926. The data is available on Dartmouth Professor Kenneth French's website. Thus, for example, we have **daily** data on stock returns during the very important economic time period

[5] This section is from Breeden, Gibbons, Litzenberger (1989), Section II.C, following Breeden (1979, footnote 8).

covering the Great Depression of 1929-1938, but only **annual** data on aggregate consumption. The USA is one of the few countries with monthly consumption data for the past 55 years. Most other countries in the world only have quarterly data on aggregate consumption, which contrasts with daily data on stock returns, giving stock measurements about 66 times the frequency of consumption measurements for most countries at present and for the past 50 years or more.

Fortunately for econometric testing, Breeden, Gibbons and Litzenberger (BGL, 1989), proved that if one would first find the portfolio that has maximum correlation of its return with real consumption growth, then the consumption CAPM should hold where betas are measured against the returns of that portfolio. Articles by BGL (1989), Jagannathan-Wang (JW, 2007) and Malloy, Moskowitz and Vissing-Jorgensen (MMV-J, 2009) use this technique to get more powerful tests of the CCAPM, especially as conditional consumption betas can be estimated more precisely using this much higher frequency data. This portfolio was called the "maximum correlation portfolio" (MCP) by BGL, the "consumption mimicking portfolio" (CMP) by J-W and the "consumption growth factor-mimicking portfolio" (CGF) by MMV-J. As it seems more descriptive, we will use the "consumption-mimicking portfolio" (CMP) nomenclature of Jagannathan-Wang in this paper.

A simplified version (where a riskless asset exists) of the BGL derivation of the Consumption CAPM in terms of the Consumption Mimicking Portfolio's return is as follows: First, given a riskless asset, let us choose the minimum variance Ax1 portfolio of risky assets, ***w*,** that is levered or unlevered to have a consumption beta of 1.00.

$$\beta_{pC} = \rho_{pC}\frac{\sigma_p}{\sigma_C} = 1.00 \quad implies\ that \quad \sigma_p = \frac{\sigma_C}{\rho_{pC}}, so\ minimizing\ \sigma_p\ maximizes\ \rho_{pC}.$$

$$\underset{\{w\}}{min}:\ w'Vw + \lambda(\mathbf{1} - w'\beta_C) \tag{43}$$

$$at\ the\ optimum: \qquad w_{cmp} = (\tfrac{1}{2})\lambda V^{-1}\beta_C \tag{44}$$

$$\beta_{cmp} = \frac{Vw_{cmp}}{\sigma_{cmp}^2} = \frac{\lambda}{2\sigma_{cmp}^2}\beta_C \tag{45}$$

Substituting (45) into (24) gives:

$$\boldsymbol{\mu} - \boldsymbol{r}_f = \frac{2\sigma_{cmp}^2}{\lambda}\left(\frac{\mu_M - r_f}{\beta_{MC}}\right)\beta_{cmp} \tag{46}$$

Pre-multiplying (46) by the row vector of market portfolio's weight gives:

$$\boldsymbol{w}'_M(\boldsymbol{\mu} - \boldsymbol{r}_f) = \mu_M - r_f = \frac{2\sigma_{cmp}^2}{\lambda}\left(\frac{\mu_M - r_f}{\beta_{MC}}\right)\boldsymbol{w}'_M\beta_{cmp} \tag{47}$$

$$\Rightarrow \frac{\lambda}{2\sigma_{cmp}^2} = \frac{\beta_{M,cmp}}{\beta_{MC}} \tag{48}$$

Substituting (48) into (46) gives:

$$\boldsymbol{\mu} - \boldsymbol{r}_f = \beta_{cmp}\frac{\mu_M - r_f}{\beta_{M,cmp}} \qquad \text{(CCAPM-CMP) (49)}$$

which is the Consumption CAPM, where the Ax1 vector of consumption betas are measured relative to the Consumption Mimicking Portfolio and the market price of risk is the risk premium of the market portfolio, divided by its beta relative to the CMP.

Breeden, Gibbons and Litzenberger used their quarterly data on industry stock returns and bond returns to estimate a consumption mimicking portfolio for the 1929-1982 period (using spliced consumption growth estimates for 1929-1939). Their portfolio weights are in the following table. Note that the junk bond premium is the strongest variable by t-statistic, confirming that the Chen-Roll-Ross statistics for that economic state variable may be attributed to its real consumption risk.

Table 13

Estimated Weights for the Maximum-Correlation Portfolio for Consumption Based on Spliced Quarterly Data from 1929–1982

All data are in real terms. (Consumption growth is scaled to adjust for the summation bias). The coefficient of determination for the above regression is 0.25, and the F-statistic for testing the joint significance of all the coefficients is 3.93 with a p-value of 0.0001. Before running real consumption growth on the returns, the data are mean adjusted. Then consumption growth is multiplied by two for observations between 1939Q2 and 1959Q1, and by 1.2 otherwise.

Asset	Weight	t-Statistic
U.S. Treasury bills	0.01	0.02
Long-term government bonds	0.54	1.05
Long-term corporate bonds	−0.31	−0.64
Junk bond premium	0.59	2.71
Petroleum	0.27	1.13
Banking, finance and real estate	−0.17	0.38
Consumer durables	0.10	0.44
Basic industries	0.33	0.90
Agriculture, food, and tobacco	−0.35	−1.45
Construction	−0.11	−0.80
Capital goods	0.03	0.11
Transportation	−0.29	−2.25
Utilities	0.18	0.72
Textiles, retail stores, and wholesalers	0.49	2.69
Services	0.08	1.39
Recreation and leisure	0.13	1.17
CRSP value-weighted index	−0.51	−0.38
	1.00	

Source: Breeden, Gibbons, Litzenberger (1989), Table II.

BGL then estimated consumption betas for the various industry stock portfolios and bonds using the data on real consumption growth, the returns on the consumption mimicking portfolio, and the value-weighted return on stocks from CRSP. Consumption betas estimated from 1929-1982 from quarterly consumption data have t-statistics that for stocks are primarily in the 6.0 to 7.5 range, whereas the estimates relative to the CMP have t-statistics that are approximately 19.0 to 23.5 and those for stock market betas relative to the stock market index are typically 45.0 to 60.0 or more. Thus, using the consumption mimicking portfolio appears to improve the estimation of consumption risk quite significantly. Subsequent articles by Jagannathan and Wang (2007) and Malloy, Moskowitz and Vissing-Jorgensen (2009) use the BGL technique in fitting CMPs using data for the Fama-French portfolios stratified by size and book to market.

Table 14

Estimated Betas Relative to 1) Growth in Real, Per Capita Consumption[a], 2) Maximum-Correlation Portfolio for Consumption, and 3) CRSP Value-Weighted Index

NA denotes not available. The maximum correlation portfolio (MCP) is constructed from the seventeen assets given in Table III. The weights of the MCP are determined by maximizing the sample correlation between the return on the portfolio and the growth rate of real consumption; see Table III for more details.

Asset (SIC Codes)	Number of Firms			Spliced Consumption, Quarterly 1929–1982 ($T = 215$)			Max.-Correlation Cons. Portfolio, Monthly 1926–1982 ($T = 684$)			CRSP Value-Weighted Index Monthly 1926–1982 ($T = 684$)		
	1/26	6/54	12/82	$\hat{\beta}_c$	$t(\hat{\beta})$	R^2	$\hat{\beta}_{MCP}$	$t(\hat{\beta})$	R^2	$\hat{\beta}_{CRSP}$	$t(\hat{\beta})$	R^2
U.S. Treasury bills	—	—	—	−0.11	−1.27	0.01	0.03	3.86	0.02	0.01	2.04	0.01
Long-term govt. bonds	NA	NA	NA	−0.01	−0.02	0.00	0.07	2.53	0.01	0.07	4.93	0.03
Long-term corp. bonds	NA	NA	NA	0.24	0.91	0.00	0.07	2.52	0.01	0.08	6.62	0.06
Junk bond premium	NA	NA	NA	2.45	6.85	0.18	0.63	18.52	0.33	0.33	20.45	0.38
Petroleum (13, 29)	46	51	69	4.31	6.37	0.16	1.41	20.61	0.38	0.92	38.63	0.69
Finance & real estate (60–69)	16	43	234	5.85	6.30	0.16	1.50	18.81	0.34	1.19	75.95	0.89
Consumer durables (25, 30, 36, 37, 50, 55, 57)	69	157	180	6.86	6.80	0.18	1.79	22.03	0.42	1.29	80.79	0.91
Basic industries (10, 12, 14, 24, 26, 28, 33)	94	207	194	5.45	6.95	0.18	1.48	21.98	0.41	1.09	100.80	0.94
Food & tobacco (1, 20, 21, 54)	64	103	81	3.25	5.69	0.13	0.99	18.62	0.34	0.76	58.15	0.83
Construction (15–17, 32, 52)	5	28	53	7.36	7.06	0.19	1.57	19.16	0.35	1.20	61.22	0.85
Capital goods (34, 35, 38)	39	120	191	5.31	6.74	0.18	1.45	21.10	0.39	1.08	85.90	0.92
Transportation (40–42, 44, 45, 47)	78	85	46	5.15	4.97	0.10	1.27	13.52	0.21	1.19	49.04	0.78
Utilities (46, 48, 49)	24	102	176	3.73	6.10	0.15	1.04	19.40	0.35	0.75	46.34	0.76
Textiles & trade (22, 23, 31, 51, 53, 56, 59)	46	101	119	5.63	7.84	0.22	1.66	30.49	0.58	0.95	48.73	0.78
Services (72, 73, 75, 80, 82, 89)	3	4	57	4.21	4.18	0.08	1.65	12.97	0.20	0.80	12.82	0.19
Leisure (27, 58, 70, 78, 79)	12	31	59	7.35	6.95	0.18	1.85	23.03	0.44	1.22	49.82	0.78
CRSP value-weighted	NA	NA	NA	4.92	7.06	0.19	1.37	23.73	0.45	1.00	—	—

[a] The spliced consumption data are scaled to adjust for the summation bias problem. Real growth in per capita consumption is multiplied by 0.75 for observations between 1939Q2 and 1959Q1, and by 0.9375 otherwise.

Source: Breeden, Gibbons, Litzenberger (1989)

Breeden (2005) illustrated a simple, 3-variable consumption mimicking portfolio, which likely has more robust coefficients than the original BGL portfolio, which estimated weights for 17 portfolios. He found that a portfolio of just the S&P 500 stock index, the credit spread of Baa versus 10-year bonds, and the 3-year vs. 3-month Treasury yield spread had a corrected $R^2 = 0.24$ for real nondurables and services consumption growth and 0.29 for real total consumption growth, both statistics quite comparable to the BGL fit with an $R^2 = 0.25$. All of these variables can be measured daily and intraday, so they hold promise for better estimates of a good consumption mimicking portfolio that can be used to estimate consumption betas more precisely.

Table 15

Maximum Correlation Portfolio

Semiannual Data (Dec-Jun) 1960-2004

	PCETot Slope	PCETot t-stat	PCENDS Slope	PCENDS t-stat
S&P 500	.0685	3.30	.0549	3.55
dBaa -10 Yr Treas (bp sprd)	-1.54	-3.06	-0.85	-2.27
Lagged 3Yr-3Mo TS Slope	0.70	2.74 RSQ=.29	0.39	2.03 RSQ=.24

Source: Breeden (2005)

VI. Non-time-separable Utility Functions. Habit Formation and Recursive Utility

One of the weak links in the theory of the 1970s and 1980s that was used to derive the Consumption CAPM was the assumption that the utility of a lifetime consumption plan is additive over time (as in eq. 1). Thus, the utility of consumption expenditure at time t depended only upon the real amount consumed at that time. The utility function could be quite general and nonlinear, but utility was not allowed to depend upon the prior history or the expected future path of consumption for the individual. This assumption was made primarily for mathematical tractability, as behavioral researchers have known for some time (and most people know introspectively) that people really do not like to reduce consumption very significantly, once a standard of living is established. Researchers now say that an individual establishes a "habit" of consuming a certain set of goods costing a certain amount, and is very averse to falling much below that level. When individuals are fortunate and consume significantly above a well-established habit level, perhaps they are not too risk averse to falling back in consumption somewhat, as long as they do not fall below a baseline habit level. Habit levels presumably evolve over time and are established gradually as a household's standard of living improves and is maintained at a higher level. If one moves from spending $30,000 per year for 10 years up to spending $60,000 per year for several years, the habit or "subsistence" level of consumption will have moved up towards the new level of $60,000. Consumption flows from purchases of durables such as houses and autos are costly to reverse, which tends to reinforce this effect.

In attempts to solve Mehra and Prescott's equity premium puzzle, researchers developed utility functions with "time complementarity," wherein utility for consumption at one point in time is affected by consumption levels at other points in time. Leaders in deriving and using utility functions with time complementarity were very early articles by Pye (1972, 1973), Kreps and Porteus (1978), and then by Bergman (1985), Greenig's Princeton dissertation (1986), Sundaresan (1989), Epstein and Zin (1989), Weil (1989), Constantinides (1990) and Abel (1990). Pye (1972, 1973) and Greenig (1986) modeled maximization of the expected utility of lifetime consumption with a multiplicative function of consumption at different dates, raised to various powers. Pye's time-multiplicative lifetime utility function is:

$$U = \delta \prod_0^T C_i^{\gamma\alpha^i} \qquad \delta = \begin{cases} +1 & \gamma > 0 \\ -1 & \gamma < 0 \end{cases} \quad \gamma \neq 0 \,. \tag{50}$$

This allowed Pye to model relative risk aversion that is age-dependent, as RRA=q, where q is:

$$q_t = 1 - \lambda_{t+1} = 1 - \gamma \sum_{t+1}^{T} \alpha^i \tag{51}$$

Thus, Pye found that relative risk aversion increases with age for those more tolerant of risk than with log utility, and decreases with age for those less tolerant than log utility.

In 1989-1990, there was a flurry of five significant papers published with non-time-separable utility functions: Sundaresan (1989) and Constantinides (1990) model "internal habit formation," whereby the utility that consumers get from a certain consumption level today depends upon how that level compares to their own past levels of consumption, which forms their habit. Abel (1990) proposes "catching up with the Joneses," a model of "external" habit formation. It is external, as the habit is not a choice variable for the individual, as utility from consumption is modeled as a function of the person's consumption relative to that of lagged aggregate per capita consumption. Epstein and Zin (1989) and Weil (1989), (often collectively referred to as "EZ-W"), following Bergman (1985), develop "recursive utility" models that consider anticipated future consumption levels in determining the utility of alternative consumption levels today. These utility functions have been used in several empirical tests.

Both Bergman (1985), using recursive preferences, and Sundaresan (1989), using habit formation, found that with these preferences that display time complementarity "Merton's multi-beta Intertemporal CAPM is still valid, but it can no longer be collapsed to Breeden's (1979) single consumption beta model." (Bergman, 1985, Abstract). Sundaresan added "Nor are these models based on time-separable utility able to explain the remarkably stable behavior of the per capita consumption series, despite the tremendous volatility of the wealth series…" Sundaresan shows that while higher consumption increases current utility, the utility increment is diminished due to the negative utility effect of having a higher consumption standard (internal habit) going forward. Knowing this causes consumers with nonseparable utility to optimally dampen their consumption responses to wealth shocks (both up and down). Thus, with nonseparable preferences, any given shock in the system must cause greater wealth fluctuations in order to have a given impact on consumption. On the portfolio policy side, Sundaresan (1989, p.85) showed that with his nonseparable preferences, "the optimal investment policy is to invest (in the risky asset) a constant proportion of wealth in excess of the capitalized value of the consumption standard." This justifies a portfolio insurance creation strategy, as in Black and Perold (1987).

Constantinides' (1990) work was especially influential. Like Sundaresan (1989), he had a model of consumers maximizing expected utility with an "internal habit," meaning one that is established by the consumer's own history of past consumption. This is more intuitive than an "external habit," but is mathematically more complex. In contrast, with an external habit, consumers gauge their satisfaction by comparison with consumption of others or comparison with average consumption per capita (see Abel (1990) and Campbell-Cochrane (1999)). In the latter case, the consumer's current decisions do not affect the habit that is developed, so the mathematical solutions are simplified. Constantinides (1990) assumes consumers maximize the expected value of the following utility function:

$$E_0 \int_0^\infty e^{-\rho t} \gamma^{-1} [c(t) - x(t)]^\gamma dt, \tag{52}$$

where

$$\mathrm{x(t)} = e^{-at} x_0 + b \int_0^t e^{a(s-t)} c(s) ds. \tag{53}$$

Thus, Constantinides' models habit as an exponentially decaying weighted average of past consumption rates, quite a sensible mathematical model for an internal habit. As consumption drops down towards the habit level, it is as if consumption approaches zero in prior CRRA models, and marginal utilities approach infinity, which made it optimal to never go to zero. While habit formation could be interpreted as a kinked utility function with the marginal utility of consumption having a large upward jump as consumption falls below the habit. In contrast, the above formultation is an extreme version of habit formation that implies a Duesenberry type racheting consumption demand that prevents consumption from falling below the exponentially weighted average of past consumption.

With these models of time complementarity, Constantinides (1990) pointed out that a wedge is driven between the elasticity of intertemporal substitution and the relative risk aversion for an individual, as later emphasized by Vissing-Jorgensen (2002) and others. Constantinides demonstrated that habit persistence can generate the sample mean and variance of the historic consumption growth rate with a low exponent on the excess consumption term (c(t) –x(t)). Table 16 from Constantinides describes economies that can be generated with his model of habit persistence:

Table 16

MEAN AND VARIANCE OF THE CONSUMPTION GROWTH RATE GENERATED BY THE MODEL WITH HABIT PERSISTENCE

Parameter a, per year	.1	.2	.3	.4	.5	.6
Parameter b	.093	.172	.250	.328	.405	.492
Mode ($\hat{z}$) of the state variable z	.86	.82	.81	.80	.79	.81
Mean annual growth rate in consumption:						
Unconditional mean	.018	.019	.018	.018	.018	.018
At $z = \hat{z}$	.011	.013	.014	.014	.014	.014
Standard deviation of the annual growth rate in consumption:						
Unconditional mean	.036	.036	.036	.036	.036	.034
At $z = \hat{z}$	.023	.029	.032	.033	.034	.032
RRA coefficient:						
Unconditional mean	8.67	4.37	3.47	3.09	2.88	2.81
At $z = \hat{z}$	7.03	4.09	3.36	3.03	2.84	2.78
Elasticity of substitution (s) at $z = \hat{z}$	.06	.08	.09	.09	.09	.09
$s \cdot$ RRA at $z = \hat{z}$	.42	.33	.30	.27	.26	.25

NOTE.—The assumed parameter values are $r = .01$, the annual rate of return of the riskless technology; $\mu - r = .06$, the difference between the mean annual rate of return of the risky technology and the annual rate of return of the riskless technology; $\sigma = .165$, the standard deviation of the annual rate of return of the risky technology; $\gamma = -1.2$, the power in the utility function; and $\rho = .037$, the rate of time preference in units $(\text{year})^{-1}$.

Source: Constantinides (1990)

where a is the exponential decay rate in weighting past consumption levels in the habit and b is the multiplier for the past consumption in the utility function.

The recursive preferences developed by Epstein-Zin (1989) and Weil (1989), (EZ-W), who built upon fundamental preference modeling by Kreps and Porteus (1978), are frequently used in modern financial models, such as the long-run risk model of Bansal and Yaron (2004). Epstein and Zin's recursive preferences allow the elasticity of intertemporal substitution (EIS) to be disentangled from the coefficieint of relative risk aversion (RRA). In Boguth and Kuehn's (2013) notation, the agent with EZ-W preferences maximizes recursive utility over consumption, using the formula:

$$U_t = \left\{(1-\beta)C_t^\rho + \beta\left(\mathbb{E}_t\left[U_{t+1}^{1-\gamma}\right]\right)^{\rho/(1-\gamma)}\right\}^{1/\rho}, \tag{54}$$

where C_t denotes consumption, $\beta \in (0,1)$ the rate of time preference, $\rho = 1 - 1/\psi$ and ψ the EIS, and γ RRA.

In a representative agent model, Epstein and Zin (1989, p. 958) find that "Thus, both consumption and the market return enter into the covariance that defines systematic risk." And they find that an asset's "price equals the discounted value of future dividends where the discount factors involve both consumption and market returns."

Weil (1989), in a follow-up article to his 1987 paper that independently derived recursive preferences similar to Epstein and Zin (1987), focused on what he saw as the "riskfree rate puzzle," due to the historically low level of the riskless rate in comparison with his model results. However, as Weil (1989, p. 416) states "… introducing heterogeneity between agents in the form of undiversifiable individual consumption risk goes a long way towards explaining both the equity premium and risk-free rate puzzles. If individual consumption is more risky than aggregate consumption, one can explain why the risk premium is large even though agents are only moderately risk-averse in the aggregate. At the same time, the price a consumer will be willing to pay for a safe unit of consumption tomorrow will rise – i.e., the risk-free rate will decrease. Therefore, the existence of heterogeneity and of market imperfections is likely to hold center stage in the explanation of the equity premium and risk-free rate puzzles." This leads nicely into our next section on limited participation, incomplete markets and much larger individual consumption risks than aggregate per capita consumption risks.

VII. Limited Participation and Incomplete Markets

Mankiw and Zeldes (1991) were first to consider the implications of **limited participation** in asset markets. Their article, using the Panel Study of Income Dynamics (PSID), separates and analyzes the consumption of stockholder and non-stockholder households to explain the equity risk premium, given that stockholders only accounted for 25% of the whole US households in 1984. The authors argue that: "One group is involved in the stock market and is at an interior solution with respect to the holding of stocks, and the other group holds no stocks at all. The relationship between aggregate consumption and the stock market considered above is no longer valid, because aggregate consumption includes the consumption of both the individuals who satisfy the first-order conditions and those who do not."

Mankiw and Zeldes's find that stockholder consumption growth has higher volatility and correlation with the market risk premium than non-stockholder consumption growth.   As Table 17 shows, stockholders have a correlation of consumption growth with stock returns equal to 0.49, almost 5 times as large as non-stockholders, who have a correlation of 0.10. Volatility of stockholders' consumption growth is 0.032, almost 60% higher than for non-stockholders at 0.020. The combination of higher correlation and higher volatility gives stockholders a covariance of consumption with stock market returns that is 7 times higher. Mankiw and Zeldes then compute that the implied constant relative risk aversion for a "representative shareholder" that is consistent with these statistics drops to about a third the level implied by full participation, from 100 to 35.

Table 17

Calibrating the equity premium: Stockholders vs nonstockholders.

GC is the growth of consumption (based on the PSID) and $r^m - r^f$ is the difference between the return on the S&P 500 and the return on three-month Treasury bills. A is the coefficient of relative risk aversion implied by the corresponding estimates. Data are for 1970 to 1984.

	$\rho(GC, r^m - r^f)$	$\sigma(GC)$	$\sigma(r^m - r^f)$	$\text{cov}(GC, r^m - r^f)$	$E(r^m - r^f)$	Implied value of A
PSID all families	0.26	0.021	0.148	0.000796	0.080[b]	100.4
PSID[a] nonstockholders	0.10	0.020	0.148	0.000305	0.080[b]	261.9
PSID[a] stockholders	0.49	0.032	0.148	0.002270	0.080[b]	35.2

[a]Based on split 3 (a household is a stockholder if it holds at least $10,000 of stock and a nonstockholder otherwise).
[b]Uses value from 1948 to 1988.

Source: Mankiw and Zeldes (1991).

The estimate of 35 for the constant relative risk aversion of a representative shareholder is still implausibly large, and might be caused by the extremely small sample (1970-1984 annual data) of only 14 annual food consumption growth observations and the assumption that stockholding status remains unchanged for each household throughout the whole sample period, as well as the assumption of constant relative risk aversion. Therefore, the size of equity risk premium is reduced per their results, though it is still not fully resolved due to data limitations and/or misspecification of the form of the representative shareholder's utility function. Although the short sample period is a major limitation, the observed differences in estimtaes consumption correlation, volatility, covariance and the implied risk aversion parameters of these two groups shown are dramatic enough to stimulate future research.

Heaton and Lucas (1992, 1996) have two important articles on **incomplete markets** and the volatilities of individuals' consumption growths vis-a-vis aggregate consumption growth. In the 1992 paper, Heaton and Lucas cleverly employ a 3-period, 2-person model to demonstrate how market incompleteness, combined with market frictions (such as borrowing and short-sale constraints, as well as transactions costs), can explain the equity risk premium. The market is incomplete in having uninsurable labor income shocks, either transitory or permanent. By setting up 3 periods in the model, they are able to model trade, as well as to differentiate between transitory and permanent shocks to income. And with 2 heterogeneous agents in the model, they are able to model unemployment risk in a possible recession. They find that the assumed market structure has a large and systematic impact on predicted asset prices. They also note that whether consumers use asset markets to smooth consumption depends critically on the persistence of the idiosyncratic shocks. They point out that “The ability to self-insure diminishes as shocks become more persistent, because more persistent shocks have a larger impact on permanent income and hence on desired consumption.” This Heaton-Lucas (1992) model is very thought-provoking, but perhaps a problem is that there is so much model flexibility that it seems possible to explain almost any empirical data.

In their 1996 article, Heaton and Lucas decompose the two effects of transactions costs on the equity premium. The direct effect is because individuals equate marginal benefits, net of transactions costs. The indirect effect occurs because transactions costs result in individual consumption that more closely tracks individual income. In the simulations, they find that the direct effect dominates and can produce a sizable equity premium only if transactions costs are large or the quantity of tradable assets is limited. They cannot resolve whether or not there is a realistic assumption about transactions costs that can simultaneously explain the low volatility of short bond rates and the high volatility of stock returns. They do seem to recognize that taxation of capital gains based on realizations is a form of a very large transaction costs on assets with high appreciation that could lead to concentrated portfolios in their model.

Two insightful articles documenting inefficient allocations of consumption and analyzing their implications were published in the August 2002 issue of the *Journal of Political Economy* by Brav, Constantinides and Geczy (BCG), and by Vissing-Jorgensen (V-J), respectively. In both articles, the authors use the data set provided by the U.S. Bureau of Labor Statistics in its *Consumer Expenditure Survey*. BCG provide an excellent detailed explanation of this quarterly series of cross-sections of household level consumption data. Each quarter approximately 5,000 households are surveyed about their spending on a list of consumption goods and services that account for approximately 95% of all household expenditures. Households are chosen randomly according to stratification criteria determined by the U.S. Census and are surveyed for 5 quarters in a row, one training quarter and four regular ones. Every quarter 1/5 of the sample is replaced by a newly selected household, so 4/5 of the sample is the same from one quarter to the next. The data started in 1980 Q1; however, they omit the first two years since Attanasio and Weber (1995) explained that the 1980 and 1981 data are of questionable quality. The following table from BCG's article gives the summary statistics for per capita consumption of households that are sampled quarterly starting in January, February and March tranches, respectively:

Table 18

SUMMARY STATISTICS ON PER CAPITA QUARTERLY CONSUMPTION

	Number of Households			Household Consumption Level		Household Consumption Growth Rate	
	Minimum	Median	Maximum	Mean	Standard Deviation	Mean	Standard Deviation
				A. Total Assets ≥$0			
January	552	692	825	2,437	368	−.01	.06
February	569	682	761	2,466	370	−.01	.07
March	533	688	794	2,436	378	−.01	.06
				B. Total Assets ≥$2,000			
January	31	80	108	3,351	528	.00	.08
February	30	81	104	3,426	554	−.01	.09
March	39	81	113	3,469	606	.00	.09
				C. Total Assets ≥$10,000			
January	22	53	80	3,541	560	.00	.10
February	18	54	76	3,621	583	−.02	.10
March	23	56	83	3,665	631	−.01	.11
				D. Total Assets ≥$20,000			
January	14	40	69	3,657	605	.00	.10
February	13	40	63	3,764	606	−.02	.10
March	16	40	71	3,773	681	−.01	.12

NOTE—We present summary statistics on the quarterly per capita consumption of nondurables and services by households for the period 1982:1-1996:1. The household's consumption of nondurables and services is calculated by aggregating the household's quarterly consumption across the consumption categories that constitute the definition of nondurables and services. We employ aggregation weights that adhere to the NIPA definitions of consumption of nondurables and services. The household consumption data are filtered using the methods described in Sec. III *C* and are deflated to the 1996:1 level, using the CPI for consumption of nondurables and services. We obtain the CPI series from the BLS through Citibase. We report sample means and standard deviations for both the level of consumption and consumption growth for a variety of definitions of asset holders as well as summary statistics on the number of observations in the particular asset-holding layer. Asset holders are defined as the households in the database that report total assets, in 1996-adjusted dollars. We present summary statistics separately for each of the three tranches (interview groups) labeled January, February, and March.

Source: Brav, Constantinides, Geczy (2002).

The estimated standard deviation of real consumption growth for these subsets of households, which are given in the far right hand column of the above table is particularly informative. Quarterly growth rates have volatilities from 6% to 12%, with wealthier households having higher volatilities. If consumption growth was independent from quarter to quarter, these would annualize proportionally to the square root of time, so the annualized volatilities of real consumption growth would range from 12% to 24%, which is quite a lot of volatility and is considerable above the approximately 1.0% annualized volatility of aggregate per capita consumption in the postwar period in the U.S. and 3.6% from 1889-1978, as given in Mehra and Prescott's Table 2. This level of volatility in actual individual household level consumption and limited participation households is one potential rationale for the equity premium puzzle.

Brav, Constantinides and Geczy have a dual goal of investigating the pricing implications of the *incompleteness of markets* that insure against idiosyncratic income shocks and the *limited participation* of households in the capital markets. BCG define a "stochastic discount factor" (SDF) or "pricing kernel," m_t, as a function that has the property that for all assets j:

$$E[m_t R_{j,t} | F_{t-1}] = 1, \quad j = 1, \ldots, J. \tag{55}$$

And applying this to any two assets, such as the market portfolio M and riskfree rate F we have:

$$E[m_t (R_{M,t} - R_{F,t})] = 0. \tag{56}$$

They note that, quite generally, due to the Euler equations of 6' and 6'', every individual's marginal rate of substitution of consumption (u_{ts}'/u_0') across dates and states should be a valid SDF. And they point out that any weighted sum of the households' SDFs is also a valid SDF. This is true whether markets are complete or incomplete. Given this, they compute the "unexplained mean premium" statistic, u, from historic data for the equity risk premium as follows:

$$u = T^{-1} \sum_{t=1}^{T} m_t (R_{M,t} - R_{F,t}) \tag{57}$$

They also examine the "value premium" by using returns for high book/market and low book/market portfolios and computing a "conditional Euler equation, where the attribute of book/market is the conditioning variable." Their unexplained-premium statistic for this is:

$$u = T^{-1} \sum_{t=1}^{T} m_t (R_{H,t} - R_{L,t}). \tag{58}$$

Then, using an assumption of power utility/constant relative risk aversion, BCG estimate the individual household's marginal rate of substitution by raising the household's consumption growth to a power equal to the negative of the RRA coefficient. With a time discount factor of $\beta = e^{-\rho t}$ and α=RRA in BCG's notation, the SDF for a group of households is:

$$m_t = \beta \left(\frac{\sum_{i=1}^{I} c_{i,t}}{\sum_{i=1}^{I} c_{i,t-1}} \right)^{-\alpha}. \tag{59}$$

BCG filter the data so that a few extreme outliers do not dominate the results and then test the null hypothesis that the Euler equations hold and the mean values of the unexplained premium statistics are individually zero.

Some of Brav, Constantinides and Geczy's (2002) test results are as follows: using constant relative risk aversion values from 0 to 9, they find that the unexplained equity risk premium test statistic is positive for low values of CRRA and negative for high CRRA, crossing zero at CRRA between 3 and 4, a range that is viewed as plausible by many economists. When skewness in consumption growth is considered, the unexplained equity premium remains positive for all levels of CRRA from 0 to 9 and even increases as CRRA increases, but the amount is statistically insignificant for CRRA greater than 2. Without considering skewness in consumption growth, the unexplained premium of value stocks over growth stocks crosses zero when CRRA is between 3 and 4, and crosses with CRRA between 4 and 5 when skewness is considered. Thus, for very reasonable levels of a "representative household's" constant relative risk aversion, BCG appear to provide an explanation for a higher risk premium for the value stock portfolio compared to the growth stock portfolio based on limited participation of households in incomplete capital markets.

Vissing-Jorgensen (V-J, 2002) focused on estimating the "elasticity of intertemporal substitution" (EIS) with the CES data, using information on asset holdings to identify limited participation in stock and bond markets. As V-J states, "The elasticity of intertemporal substitution determines how much consumers change change their expected consumption growth rate in response to changes in the expected return to any such asset." Using the riskless rate, it would be the change in expected consumption growth for a 1% change in the interest rate. As the riskless interest rate increases, consumers reduce current consumption and have more to consume in the future, so the expected growth rate of consumption increases. The amount of that sensitivity is the EIS. A high EIS means that consumers are willing to vary their growth rate quite a lot in response to interest rate moves, giving a higher volatility of the lifetime consumption path. This would be consistent with a higher tolerance for lifetime consumption risk and a lower degree of risk aversion. Thus, as we saw in the term structure theory equation 30, in the CRRA-lognormal model, the EIS would equal the inverse of relative risk aversion, i.e., EIS = 1/RRA. However, as pointed out by Hall (1988) and others since then, EIS can certainly be quite different from relative risk tolerance (the inverse of RRA) when preferences have time complementarity. More precisely, risk aversion is perhaps best thought of with regard to intratemporal (timeless) gambles or choices among different risky assets at a point in time, whereas the EIS is more a descriptor of intertemporal responses to changes in the reward for deferring consumption.

Vissing-Jorgensen uses the micro data from the U.S. Consumer Expenditure Survey to argue that accounting for limited asset market participation is crucial for obtaining consistent estimates of the EIS. As she says, "The Euler equation should hold for a given household only if the household holds a nonzero position in the asset." If households do not hold an asset, there is little or no reason why they would vary their consumption in response to changes in its expected return. Including them in estimates of EIS could lead to substantially downward biased estimates. Using data for stockholders, V-J estimates EIS to be approximately 0.3 to 0.4, whereas for bondholders the EIS estimates are approximately 0.8 to 1.0. Inverting these numbers gives a CRRA estimate of 2.5 to 3.3 using the stockholder data, and gives an estimate of 1.0 to 1.25 using the bondholder data.

To estimate the EIS, Vissing-Jorgensen estimated the following equation:

$$\frac{1}{H_t^s}\sum_{h=1}^{H_t^s} \Delta \ln C_{t+1}^{h,s} = \sigma^s \ln (1 + R_{s,t}) + \delta_1^s D_1 + \delta_2^s D_2 + \cdots + \delta_{12}^s D_{12}$$

$$+ \alpha^s \frac{1}{H_t^s}\sum_{h=1}^{H_t^s} \Delta \ln (\text{family size})_{t+1}^{h,s} + u_{t+1}^s, \qquad (60)$$

where σ denotes the EIS, $D_1, \ldots, D_{12}$ are seasonal dummies, R_s denotes the real net stock return, and H_t^s denotes the number of stockholders in the cross section at date t.

Vissing-Jorgensen does the estimation using GMM with three different sets of instrumental variables: (1) has the log dividend/price ratio, (2) has that and lagged real stock return, and (3) has the dividend/price ratio, the bond horizon premium and the bond default premium. Her results are in Table 19. They show that the EIS for stockholders is approximately 0.30, which is much larger than the 0.06 for nonstockholders, a value that shows little sensitivity of nonstockholders to returns in the stock market. Thus, stockholders show much more willingness to respond to better returns than do nonstockholders. Further rows of the table show that this is even more true for the wealthier households, as the lowest 1/3 households by wealth have an estimated EIS of 0.05, the middle 1/3 have an EIS of 0.18 and the wealthiest 1/3 have an EIS of 0.49. Thus, Vissing-Jorgensen showed that limited asset market participation is very important for estimating the elasticity of intertemporal substitution, EIS. Differences across stockholders and nonstockholders, as well as between bondholders and nonbondholders, are large and statistically significant. This research should be quite useful to the Federal Reserve and other policy makers as they estimate the likely responses of consumers to changes in interest rates and risky investment returns.

Table 19

GMM ESTIMATION OF LOG-LINEARIZED EULER EQUATIONS: REAL TREASURY BILL RETURN AND REAL VALUE-WEIGHTED NYSE RETURN, SEPARATE ESTIMATIONS (CEX, 1982–96, Semiannual Data)

Instrument Set 1:	$\hat{\sigma}$	Wald Test Equals σ	$\hat{\sigma}$	Wald Test Equals σ
	A. Euler Equation for Stocks		. Euler Equation for Treasury Bil	
	1. All Household Sizes		1. All Household Sizes	
All	0.10 (0.07)		0.37 (0.23)	
Stockholders	**0.30** **(0.15)**		**0.93** **(0.37)**	
Nonstockholders	**0.06** **(0.08)**		**0.11** **(0.27)**	
Bottom layer	0.05 (0.19)		0.99 (0.66)	
Middle layer	0.18 (0.27)		0.29 (0.55)	
Top layer	0.49 (0.33)		1.65 (0.52)	
Nonstockholders vs. stockholders		3.26 (0.07)		4.03 (0.05)
Nonstockholders vs. top layer		2.15 (0.14)		8.06 0.01
	2. Single-Individual Households		2. Single-Individual Households	
All	0.20 (0.17)		0.68 (0.46)	
Stockholders	0.70 (0.5)		2.62 (0.89)	
Nonstockholders	0.08 (0.14)		0.05 (0.48)	
Nonstockholders vs. stockholders		1.73 (0.19)		8.37 (0.004)

NOTE.—Numbers in parentheses are standard errors for $\hat{\sigma}$ and *p*-values for the Wald test. For the overidentification test, the entries are *p*-values, and the test has two degrees of freedon for each of instrument sets 2 and 3. Twelve monthly dummies are included as explanatory variables and instruments. The estimations for all household sizes furthermore includ $\Delta \ln$ (family size) as an explanatory variable and instrument. In addition the instrument sets include the following variables. Instrument set 1: log dividend-price ratio. Instrument set 2: log dividend-price ratio, lagged log real value-weighted NYSE return, and lagged log real Treasury bill return. Instrument set 3: log dividend-price ratio, default premium, and bond horizor premium.

Source: Vissing-Jorgensen (2002).

Using updated household level CES data (with help from Dana Kiku, of the University of Illinois), we computed the volatility of real consumption growth for households of sizes 1, then for 2-3, and 4+ members, as well as by 3 levels of income – Upper 1/3, Middle 1/3 and Lower 1/3. Individual percentage consumption growth volatility in Table 20 below compares with volatility of just approximately 1.0% for aggregate per capita consumption growth of nondurables and services. The calculations that individuals actually experience real consumption volatility more than 7% should reduce constant relative risk aversion estimates by a factor of at least 7, which would bring many risk aversion estimates into the 1 to 10 range that many economists believe is reasonable.

Table 20

NDS Consumption Growth Volatility

		Income			
		Lower 1/3	Middle 1/3	Upper 1/3	Row Average
Family Size	1 person	11.7	12.4	12.5	9.4
	2-3 person	9.8	8.4	9.1	6.2
	≥4 person	13.5	10.3	7.6	6.7
	Column Average	8.6	8	7.3	6.4

Constantinides and Duffie (1996) developed an elegant theoretical model of the impact of substantial heterogeneity of individuals' incomes on asset pricing. They model consumers who have power utility functions (CRRA), but have uninsurable, persistent and heteroscedastic labor income shocks. This is quite a reasonable and important model, given the differences we have seen in the preceding tables for households' consumption volatility versus aggregate consumption volatility. Constantinides and Duffie observe (p. 223) "… the model predicts that a potential source of the equity premium is the covariance of the securities returns with the cross-sectional variance of individual consumers' consumption growth, a source that is (under typical conditions) irrelevant in an economy with full consumption insurance." With CRRA=α and their carefully built model of individual income shocks, the Euler equation is derived in terms of aggregate consumption, C_{t+1}, and a measure, y_{t+1}^2, which "…is interpreted as the variance of the cross-sectional distribution of log $[(C_{i,t+1}/C_{t+1})/(C_{i,t}/C_t)]$" where i=1 to N represents different individuals (p. 229):

$$E\left[R_{j,t+1}e^{-\rho}\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}\exp\left[\frac{\alpha(\alpha+1)}{2}y_{t+1}^2\right]\middle|\phi_t\right]=1 \qquad (61)$$

If consumers were identical in preferences and incomes, then y^2 would be zero and the Euler equation reduces to the familiar one for CRRA.

Constantinides and Duffie also prove the equilibrium solution of their heterogeneous consumer model is isomorphic to that of the representative consumer model, by assuming specific forms of the y_t function, such as the following:

$$y_{t+1}^2 = a + b \log\left(\frac{C_{t+1}}{C_t}\right) \tag{62}$$

Substituting this function into the general Euler equation gives a reduced form Euler equation corresponding to the classical one with representative consumers:

$$E\left[R_{j,t+1} e^{-\rho}\left(\frac{C_{t+1}}{C_t}\right)^{-\hat{\alpha}} \middle| \phi_t\right] = 1 \tag{63}$$

where the modified risk aversion coefficient $\hat{\alpha}$, deviates from the one without heterogeneity, α.

$$\hat{\alpha} = \alpha - \frac{\alpha(\alpha + 1)}{2} b \tag{64}$$

Though this specific function of y_t is rejected when they test the Euler equation with data, their discussion is of general interest in explaining the size of the equity risk premium. If the cross-sectional variation is countercyclical, b is negative and the modified risk aversion $\hat{\alpha}$ is higher than the classical risk aversion parameter α. Therefore, an econometrician unaware of the heterogeneity would mistake $\hat{\alpha}$ as α, and overestimate the risk aversion parameter under a homogenous model.

The Constantinides-Duffie derivation of the importance for asset pricing of the cross-sectional heterogeneity of individual consumers' actual incomes and consumption volatilities, rather than just the volatility of the aggregate, is a significant and economically intuitive result in asset pricing. It seems that almost nobody has consumption volatility as low as the aggregate measures, so is it not intuitive that market pricing would reflect covariances with individuals' actual marginal utilities, which reflect their individual consumption volatilities? This is an area worthy of additional research. We need to understand the consistency of this result with

Breeden's (1979) continuous-time CCAPM with heterogeneous consumers, wherein consumer spending can be aggregated in asset pricing results.

VIII. 1990s Models of Changing Conditional Risks: Risk Factors and Risk Premia

The perceived failure of consumption-based asset pricing models in explaining the equity premium and the cross-section of returns (e.g., see Mehra-Prescott (1985) and Breeden, Gibbons, Litzenberger (1989)), motivated over the subsequent decade both theoretical research on more general preferences with time complementarity and habits, and empirical research on multiple factors, conditional risks and changing risk premiums through time. As shown by Sundaresan (1989) and Epstein-Zin (1989), preferences with time complementarity justified multiple factors , (with market betas as well as consumption betas), and testing proceeded seeking multiple factors and multiple prices of risk, along the lines of Merton's Intertemporal CAPM and Ross's Arbitrage Pricing Theory (APT).

Several articles were produced that appeared to demonstrate predictability in mean returns, a result that researchers had doubted based on earlier research on market efficiency. However, researchers began to realize that if risks change through time and in different economic conditions (e.g., in risky recessions vs. stable growth periods), then it is economically sensible that mean returns should also vary with economic conditions to reward investors more when risk is higher. Keim and Stambaugh (1986) found that the credit yield spread of Baa rated bonds over Aaa rated bonds had some ability to predict future bond and stock returns. Fama and French (1988) and Campbell and Shiller (1988) found that trailing dividend yield, an easily measured variable, had ability to predict returns, especially over the longer term, as much as 7 years out. Kandel and Stambaugh (1989) used dividend yield, a credit risk yield spread and the slope of the term structure to model time-varying risk premiums. In a particularly insightful article, Ferson and Harvey (1991) built on this prior work to model both changing conditional betas and changing conditional risk premiums, finding that the changing risk premium for beta was a much larger explanatory variable in returns than are changing betas for 12 major industries, 10 deciles of size-ranked portfolios, and government and corporate bonds and Treasury bills. The right hand side of the following table from Ferson-Harvey shows these results:

Table 21

DECOMPOSING THE PREDICTED VARIATION OF MONTHLY PORTFOLIO RETURNS, 1964:5–1986:12 (272 Observations)

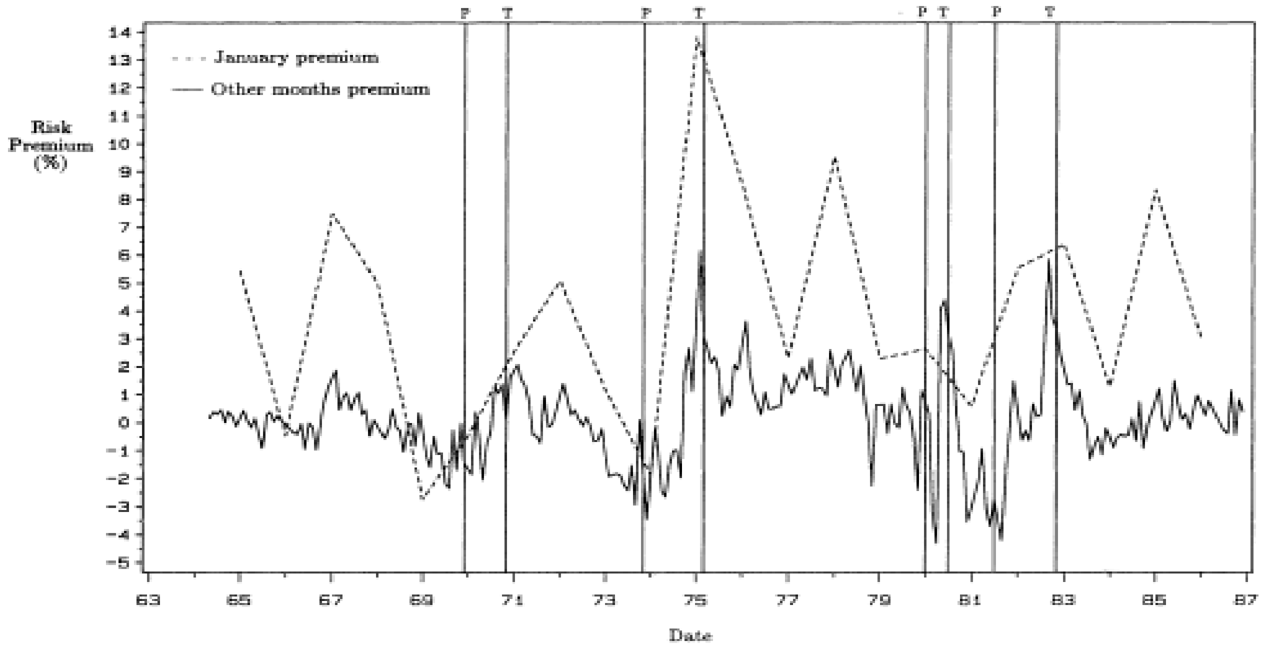
	Decomposition by Economic Risk Variables							Decomposition by Betas vs. Price of Beta		
Portfolio	XVW	PREM	ΔSLOPE	UI	CGNON	REALTB	Interaction Effects	Changing Beta	Changing Price of Beta	Interaction Effects
Decile:										
1	45.19	6.09	4.74	9.16	.31	23.22	11.29	1.41	67.80	30.79
2	51.42	3.90	3.00	5.59	.25	17.50	18.34	1.20	68.58	30.22
3	58.59	3.13	2.53	3.39	.15	14.38	17.83	1.31	75.32	23.37
4	64.45	1.45	2.01	2.57	.36	12.06	17.10	.85	78.71	20.44
5	69.68	1.81	1.43	1.99	.16	8.47	16.46	.46	80.03	19.51
6	80.05	.87	1.28	2.00	.19	8.36	7.25	.25	85.66	14.09
7	89.44	.38	.61	1.34	.12	3.62	4.49	.16	91.62	8.22
8	85.98	.38	.67	1.50	.14	5.32	6.01	.25	89.58	10.17
9	97.13	.20	.39	2.65	.14	1.78	−2.29	.04	90.87	9.09
10	105.45	.10	.26	.47	.06	1.36	−7.70	.04	111.44	−11.48
Industry:										
Petroleum	150.64	1.91	1.80	67.18	6.80	149.50	−277.83	.83	146.67	−47.49
Finance/real estate	112.22	1.32	.20	7.18	1.80	6.36	−29.08	.42	101.46	−1.88
Consumer durables	87.95	.89	.87	4.12	.73	15.90	−10.42	.04	93.87	6.09
Basic industries	89.07	.10	.22	2.01	1.91	4.09	2.60	.05	95.80	4.15
Food/tobacco	82.11	.96	.52	5.81	1.50	15.90	−6.80	.37	89.24	10.39
Construction	90.03	1.43	.66	4.80	.52	8.44	−5.88	.33	83.40	16.27
Capital goods	75.32	.61	1.11	15.26	3.20	13.07	−8.57	3.14	97.24	−.38
Transportation	75.47	.67	4.28	6.49	1.56	6.25	5.28	.66	84.03	15.31
Utilities	87.61	5.56	4.28	9.41	13.60	8.27	−28.73	1.73	87.66	10.61
Textiles/trade	65.80	.27	.37	7.73	6.37	35.00	−15.54	.15	80.40	19.45
Services	74.88	.44	.37	7.19	.32	20.71	−3.91	.75	68.66	30.59
Leisure	73.12	.10	1.31	17.07	1.13	43.64	−36.37	.80	84.01	15.19
Government bonds	7.23	132.27	3.24	1.96	6.56	9.23	−60.47	1.68	91.93	6.39
Corporate bonds	18.34	92.70	6.73	13.21	21.77	20.07	−72.82	3.92	59.83	36.25
6-month Treasury bill	.53	39.74	60.74	12.04	5.40	18.76	−37.21	1.19	46.16	52.65

NOTE—All rates of return are in excess of the 1-month Treasury bill rate. Decile 1 represents the excess returns on the decile of the smallest-valued firms on the NYSE. Decile 10 represents the excess returns on the largest decile of NYSE stocks. The figures are the percentages of the sample variances of predicted excess returns, using a multibeta asset pricing model, which are allocated to different sources of predictable variation.

Source: Ferson and Harvey (1991, Table 8)

Ferson and Harvey show that the estimated risk premium for equities varies with economic conditions, generally increasing in recessions (as risk and premiums per unit of risk increase) and decreasing during growth periods, when risk and premiums per unit of risk appear to subside. The following graph shows this result:

Figure 10



F_{IG}.1.—Fitted values from a regression of the price of market beta on the instrumental variables. The values for the price of market beta are the estimated coefficients from a cross-sectional regression each month of 25 portfolio returns on estimates of the market beta coefficients. The monthly estimates of the price of beta are regressed over time on the predetermined variables summarized in table 3. The regressions include dummy variables that allow each of the slope coefficients to differ in January from their values in the other months. The fitted values of the regression are shown in this graph. The dashed line represents the January observations; the solid line represents the other 11 months.

Source: Ferson and Harvey (1991), Figure 1.

In 1992 and 1993, Fama and French, in now classic articles, showed that average returns on stocks and bonds were related to five major explanatory factors (3 from stocks, 2 from bonds): (1) a general stock market factor risk estimated by market betas, (2) a factor related to differential performance of small stocks versus large stocks (SMB for small minus big), (3) a factor related to differential performance of high versus low book/market stocks, which picks up a premium of value versus growth stocks (HML for high minus low), (4) a term premium variable measured by the return on long-term government bonds, less that on 1-month Treasury bills, and (5) a default premium, measured by the return on corporate bonds, less that of government bonds. As the following regression from Fama and French (1993, eq. 1) shows, the market portfolio's excess return is a "hodgepodge" of the common risk factors, even showing statistically significant common variation with the two bond market related factors (t-statistics in parentheses, $R^2 = 0.38$):

$$RM - RF = \underset{(2.55)}{0.50} + \underset{(6.48)}{0.44SMB} - \underset{(-8.23)}{0.63HML} + \underset{(9.09)}{0.81TERM} + \underset{(4.62)}{0.79DEF} + e \quad (65)$$

This regression shows that the stock market portfolio tends to do better when small stocks outperform large stocks, when growth stocks beat value stocks, when long-term government bonds beat T-bills, and when credit risk is rewarded.

Fama and French's papers raised two new puzzles that stimulated a large literature in subsequent years. In their 1992 classic, they found that the "relation between market beta and average return is flat." Contrary to the prediction of the CAPM, they found no significant reward for taking equity beta risk in their 1963-1991 sample. The first puzzle they is that small stocks outperformed large stocks (SMB effect >0 on average), even after taking into account equity market beta differentials. The second puzzle is that value stocks outperformed growth stocks (HML effect >0 on average), also after taking into account equity market beta differentials.

Table 22

Average Returns (in % per year) for Portfolios Sorted on Size and Betas

	All Firms	Low Beta	High Beta
All Firms	15.0%	16.1%	13.7%
Small	18.2%	20.5%	17.0%
Large	10.7%	12.1%	6.7%

Table 23

Average Returns (in % per year) for Portfolios Sorted on Size and B/M Ratios

	All Firms	Low B/M	High B/M
All Firms	14.8%	7.7%	19.6%
Small	17.6%	8.4%	23.0%
Large	10.7%	11.2%	14.2%

Source: Fama and French (1992).

Fama and French's surprising results have been partially attributed to the relatively small sample (1963-1991) and it has been noted that their estimates have high standard errors. While the average returns versus betas relationship improves with longer data series and more asset classes, the size and book/market results have been demonstrated over longer time periods. As we shall see in key articles by Lettau and Ludvigson (2001b) and Jagannathan and Wang (2007) on consumption-based asset pricing, perhaps the size and value/growth effects can be explained by their differential conditional consumption risks. For example, value stocks are shown to have relatively higher consumption betas in recessions, when risks and risk premiums per unit of risk are high, and lower conditional consumption betas in growth periods, when risks and risk premiums per unit of risk are low. This effect makes the unconditional risk premium for value stocks larger, due to the positive correlation of their consumption betas with market risk premiums. Alternatively viewed, the growth stock portfolio has an unconditional convex relation to consumption growth indicated by a higher CCAPM beta conditional on expansion periods and a lower CCAPM beta conditional on recessionary periods. Conversely, the value stock portfolio has an unconditional concave relation to consumption indicated by a lower CCAPM beta conditional on expansion periods and higher CCAPM beta conditional on recessionary periods. Consistent with Jagannathan and Wangs findings, the analysis of Kraus and Litzenberger (1983) suggests that under decreasing absolute risk aversion individual would have a preference for positive skewness and the unconditional risk premium for the growth stock portfolio would cet. par. be less than that for the value stock portfolio.

Jagannathan and Wang (1996) significantly advanced the case for modeling conditional variation in betas and risk premiums. They modeled changes in betas as being related to the credit yield spread between low and high grade bonds, which is sensitive to perceived risks of default and is quite related to the state of the economy, as was shown earlier in Figure __ relating credit spreads to the unemployment rate. Additionally, they use a proxy for human capital to get a better estimate of returns on the true, but unobservable market portfolio. With the broader market portfolio, combined with changing conditional risks and conditional risk premiums, they are able to explain much of the size effect identified by Fama and French.

In the next section, we will examine habit formation models, which provided strong empirical support for consumption-based asset pricing. Subsequent sections will show how this evidence was then enhanced by works on conditional consumption betas (Lettau-Ludvigson (2001b) and Jagannathan-Wang (2007)), ultimate consumption betas (Parker-Julliard (2005)), and the long-run risk model of Bansal and Yaron (2004, and follow up articles).

IX. Habit Formation Models: Campbell-Cochrane (1999)

In a major "second generation" consumption-based asset pricing model, 20+ years after the original CCAPM, Campbell and Cochrane (1999) developed a landmark model of asset pricing using a utility function of a representative individual with an **external habit**. Under an external habit individual do not consider the impact of their current consumption decision on their habit in future periods, which simplifies the optimization problem. Campbell and Cochrane successfully employ the utility function to fit a countercyclical equity risk premium. Three features of their model are worthy of note: (1) a slow-moving external habit based on per capital consumption, (2) i.i.d. per capita consumption growth and (3) highly nonlinear utility and relative risk aversion that approaches infinity near the external habit level. By assuming a representative individual with an external habit, they sidestep the aggregation of heterogeneous individuals, limited participation issues, and the impact of current consumption decisions on the current or future habit. Their model is able to generate counter-cyclical fluctuations and long-term predictability of equity risk premium by having relative risk aversion become arbitrarily large as current consumption approaches the external habit.

Individuals are assumed to maximize:

$$\mathrm{E}\sum_{t=0}^{\infty}\delta^{t}\,\frac{(C_t-X_t)^{1-\gamma}-1}{1-\gamma} \tag{66}$$

Note that this preference function is similar to an extended power utility function with an intercept equal to minus the external habit. This utility function displays decreasing relative risk aversion, and relative risk aversion approaches infinity as the representative individual's consumption declines toward the external habit. Thus, the habit intuitively seems more like a subsistence level of consumption, rather than a habit motivated by consumption envy that the “keep up with the Joneses” motive seems to suggest. Since the individual views the habit as exogenous, the representative individual’s consumption decision does not consider the impact on the habit. Under the assumption of identical powers, γ, this preference function could be aggregated from individual extended power utility with diverse external habit levels. The modeling of the aggregated external habit as a lagged function of past per capita consumption is intuitive. The preference function is not defined for a negative habit level, which requires consumption to be strictly above the habit. The above preference function could be used with a stochastic process on per capita consumption that was consistent with this constraint. For example, if C_t followed a shifted lognormal process with a shift parameter of X_t, realizations of excess consumption would be positive without making X_t a function of C_t, which would not be intuitive for a subsistence level of consumption. However, Campbell and Cochrane assume that per capita consumption is lognormally distributed and make the external habit an implicit function of current consumption, such that the external habit’s downward moves assure that excess consumption is positive for all realizations of per capita consumption.

Campbell and Cochrane use a variable called the “surplus consumption ratio” as the difference between per capita consumption in the economy and the representative individual’s external habit level, X_t, expressed as a fraction of per capita consumption:

$$S_t^a = \frac{C_t^a - X_t}{C_t^a} \tag{67}$$

The log surplus consumption function s_t^a is modelled as an AR(1) process with a speed depending on parameter ϕ and a monotonically decreasing sensitivity function $\lambda(s_t^a)$, where lower case letters are logs of the upper case variables:

$$s_{t+1}^a = (1 - \phi)\bar{s} + \phi s_t^a + \lambda(s_t^a)(c_{t+1}^a - c_t^a - g), \tag{68}$$

Substituting the surplus consumption ratio into the AR(1) process demonstrates that the external habit, X_t, adjusts to C_t, as well as to the history of average per capita consumption. The external habit adjusts slowly and geometrically to past consumption with coefficient ϕ. The log transformation constrains the surplus consumption to be non-negative. They impose several restrictions on the parameters to produce a constant risk-free rate and a predetermined habit level around the steady state, so as to make sure the excess co-moves with consumption, but is always positive. Under their specification as C_t approaches zero, changes in X_t offset the impact of changes in C_t on the excess consumption ratio. The justification for this specification for a learned habit, which intuitively should be slowly moving in response to past levels of consumption of others, is not provided. The implications of their parameter specifications are shown in the graphs below.

Figure 11

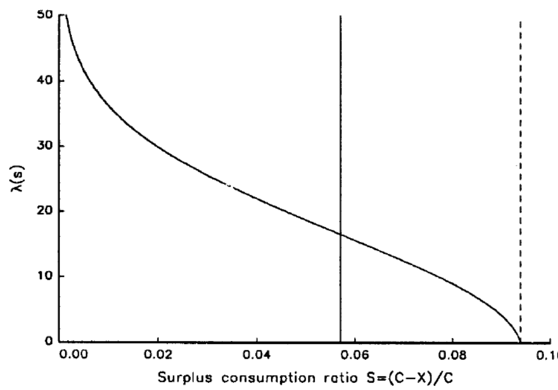
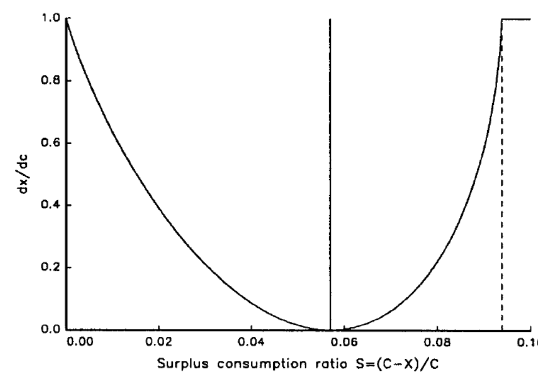


Figure 12



They then price bonds and stocks using classic Euler equations and choose the free parameters in the model to fit the moments of post-war data. Empirical calibration shows their model can fulfill its goals and generate a non-linear countercyclical risk premium and cyclical equity volatility. When surplus consumption drops to near zero during recessions, both the equity risk premium and volatility of stock returns increase at an increasing pace, as shown in the following figures from their article:

Figure 13

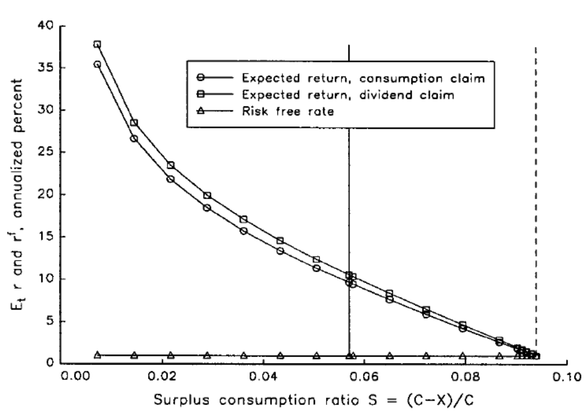
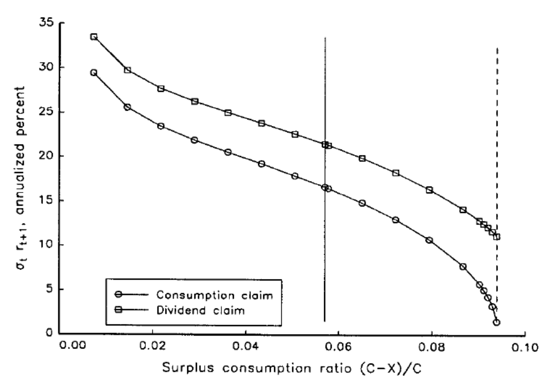


Figure 14



Also, as Table 24 from Campbell and Cochrane shows, there are enough parameters that their simulated data can fit the four moments of the post-war data quite well, fitting the equity risk premium and its Sharpe ratio almost perfectly, as well as consumption growth's mean and volatility and equity market volatility:

Table 24

MEANS AND STANDARD DEVIATIONS OF SIMULATED AND HISTORICAL DATA

Statistic	Consumption Claim	Dividend Claim	Postwar Sample	Long Sample
$E(\Delta c)$	1.89*		1.89	1.72
$\sigma(\Delta c)$	1.22*		1.22	3.32
$E(r^f)$	.094*		.094	2.92
$E(r - r^f)/\sigma(r - r^f)$	.43*	.33	.43	.22
$E(R - R^f)/\sigma(R - R^f)$	.50		.50	
$E(r - r^f)$	6.64	6.52	6.69	3.90
$\sigma(r - r^f)$	15.2	20.0	15.7	18.0
$\exp[E(p - d)]$	18.3	18.7	24.7	21.1
$\sigma(p - d)$	.27	.29	.26	.27

NOTE.—The model is simulated at a monthly frequency; statistics are calculated from artificial time-averaged data at an annual frequency. All returns are annual percentages.
* Statistics that model parameters were chosen to replicate.

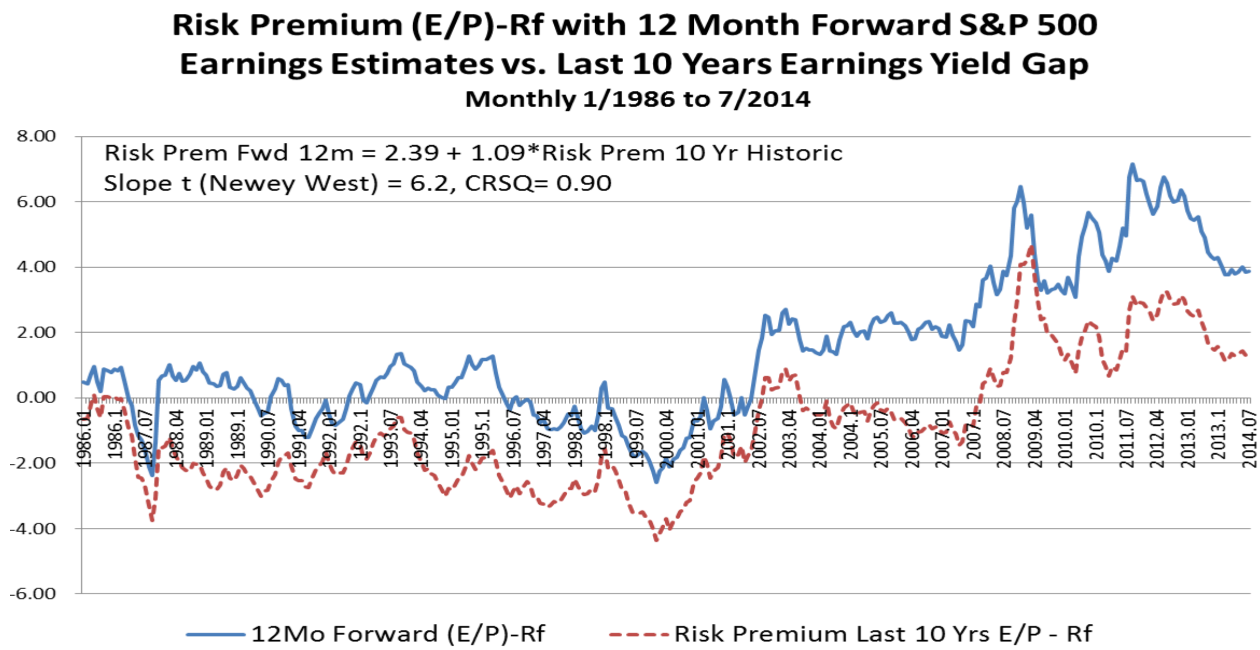
Source: Campbell and Cochrane (1999)

As one of the early papers modeling the effects of habit formation utility functions on asset pricing, Campbell and Cochrane (1999) plays an important role in modeling the time-varying and countercyclical risk premium by a having a slowly adjusting habit level and highly nonlinear utility responses. Effectively, by making relative risk aversion become very large as excess consumption approaches zero, large variations in risk premiums can be explained. C-C observe (p. 244): "Risk aversion is about 80 at the steady state … rises to values in the hundreds for low surplus consumption ratios and is still as high as 60 at the maximum surplus consumption ratio." Their empirical results do not seem to depend upon time complementarity per se. In retrospect, this helps us see that the whole literature on excess volatility seems to have implicitly focused on constant relative risk aversion. Of course, a potential drawback is that having many parameters and imposing some delicate restrictions gives considerable flexibility to overfit the data. Out of sample testing using the in sample parameter would be informative.

Campbell and Cochrane (1999) have a very important and plausible prediction that relative risk aversion and risk premiums increase quite nonlinearly as surplus consumption goes towards zero, as in major recessions. To amplify on this important aspect of changes in risk premiums, we have examined data for real consumption growth and the level and changes in the unemployment rate over long time horizons. We started with Shiller's (2014) long-term database for stock prices and consumption growth, and added to this long-term data on the unemployment rate from NBER and the St. Louis Federal Reserve historical database, FRED. The NBER has a monthly series of unemployment rates that goes back to April, 1929, and we used monthly data on employment changes from the 1932 Supplement to the Survey of Current Business to estimate monthly unemployment rates from January of 1923 to April 1929. Given this, our first data point for the 12-month change in the unemployment rate is for January 1924, so we have just over 90 years of monthly data to July 2014.

We invert Shiller's long-term estimates of his "Cyclically Adjusted Price Earnings" Ratio (CAPE) to get an earnings yield number (biased low), from which we subtract the long-term (10-year) U.S. Treasury interest rate to get a long data series of estimated "risk premiums" for U.S. equities, $E/P - R_f$. These are surely biased low, as earnings grow over time, so forward forecasts will be higher than these backward looking earnings numbers. To test the stochastic properties of this backward looking earnings yield with a forward one, we obtained monthly observations of the 12-month *forward* S&P 500 earnings estimates from 1986 to August 2014 from Ed Yardeni's website and computed a forward looking earnings yield based on that series. The following graph shows that the Shiller-type backward looking, long-term earnings yield gap is highly correlated ($\rho = 0.95$) from 1986 to 2014 with the forward looking earnings yields: As expected earnings yields based on next 12 months earnings estimates are persistently higher than the 10-year historic earnings divided by current price. The bias averages about 3% over the long term. Furthermore, corporate investment at rates of return in excess of capital cost would result in the forward looking earnings yield being a downward biased estimate of the expected rate of return. However, this bias should be greater in prosperous period with higher returns on real investment than in recessionary period with lower returns on real investment.

Figure 15



Given this high correlation of 10-year historical and 1-year forward "yield gaps", we feel it is not unreasonable to look at a Shiller-type earnings yield gap time series as an estimate of what the stochastic properties of the time series of investors' forward earnings yield spreads to riskfree rates were in past years. Given the cyclical bias of forward earnings yields as estimates of expected returns, higher earnings yield spreads in recessionary periods is consistent with higher risk premiums in recessionary periods.

Our next figure shows the relationship of the 12-month changes in the estimated equity risk premium to 12-month changes in the unemployment rate. As the graphs shows, the series are highly correlated. When the economy falls into recession and the unemployment rate jumps, the estimated risk premium also tends to jump. We find a very strong and nonlinear relationship of the estimated risk premium to the unemployment rate and to real consumption growth. This helps us to understand why the Campbell-Cochrane (1999) model is very helpful in modeling movements in the real economy. The correlation of the moves in estimated risk premium and changes in the unemployment rate is 0.40 over the entire 1924-2014 sample, and is 0.53 if the World War II years of 1939-1947 are excluded, as unemployment fell sharply in WWII, while risk and risk premia increased, an abnormal economy.

Figure 16

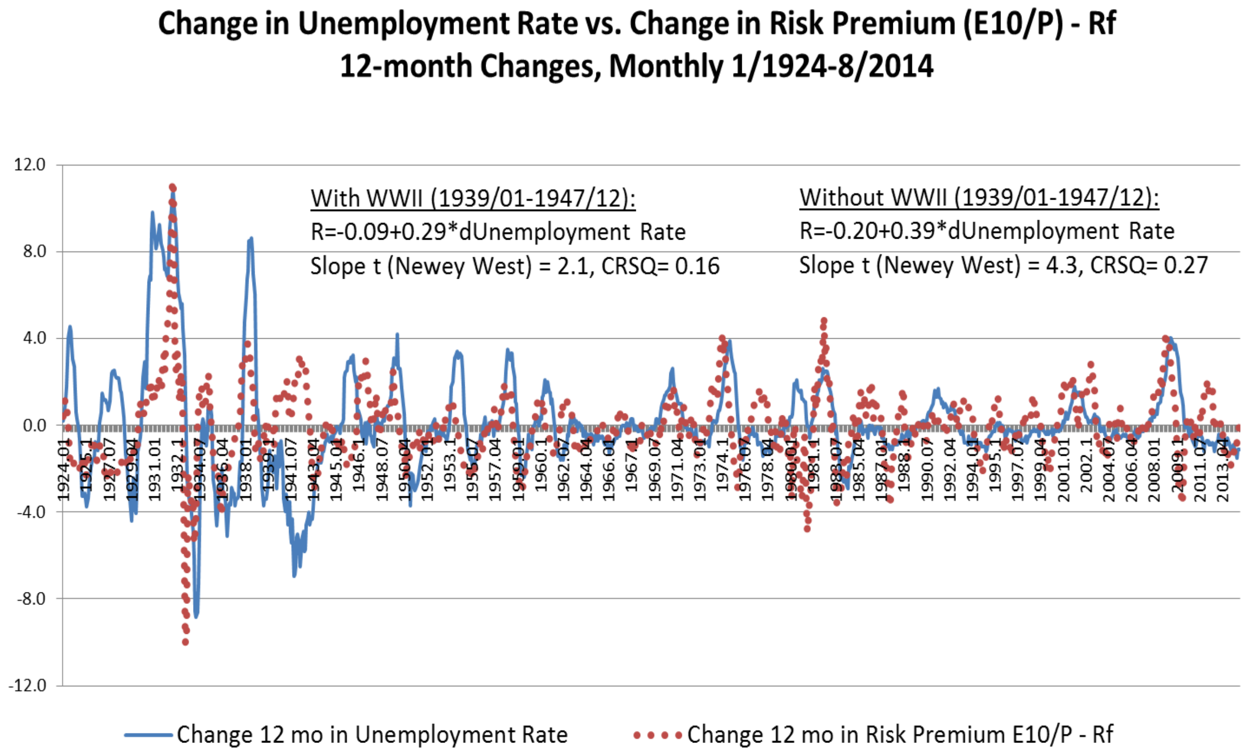
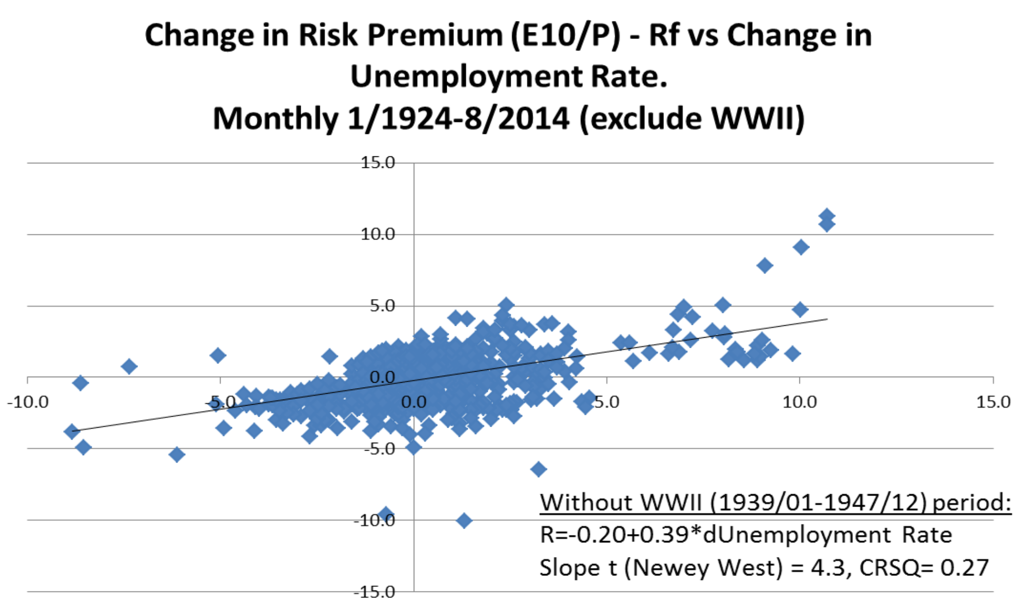


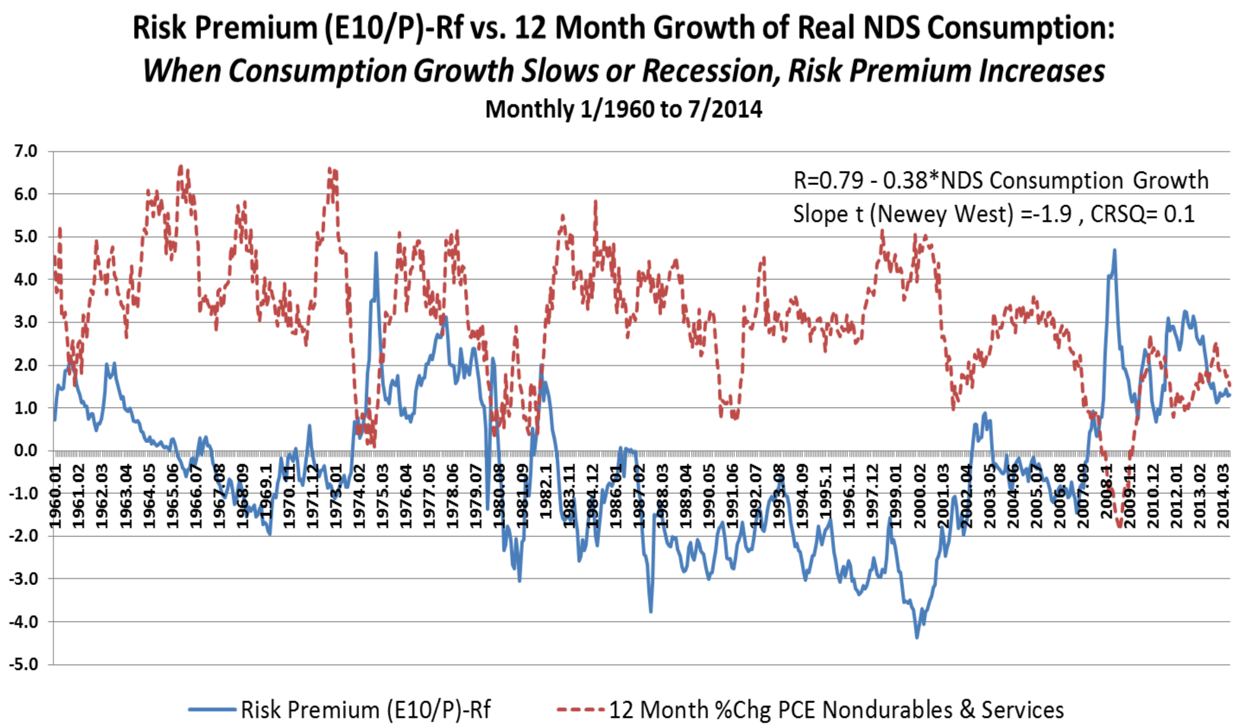
Figure 17, shows that changes in the unemployment rate explain changes ($R^2 = 0.27$) in the estimated risk premium, a statistically significant relation (with a Newey-West t-statistic of 4.3.)

Figure 17



The next figure shows that the picture is much the same if one uses real growth of consumption of nondurables and services in modeling changes in the equity risk premium. When real consumption growth is high and the economy is good, Campbell-Cochrane's surplus consumption increases and risk aversion likely drops, along with risk premiums. Note that as monthly consumption data begins in January 1959, our first data point of 12-month consumption growth is for January 1960.

Figure 18



The next two graphs show that the picture is very similar if we look at forecasted forward earnings yields, less the 10-year Treasury yield and compare that yield gap's moves to moves in real consumption growth and to changes in the unemployment rate. Once again, the relationships are all strong and in the right direction. Thus, we believe that, as predicted by Campbell and Cochrane model, it is quite plausible that relative risk aversion and risk premiums are indeed significantly countercyclical.

Figure 19

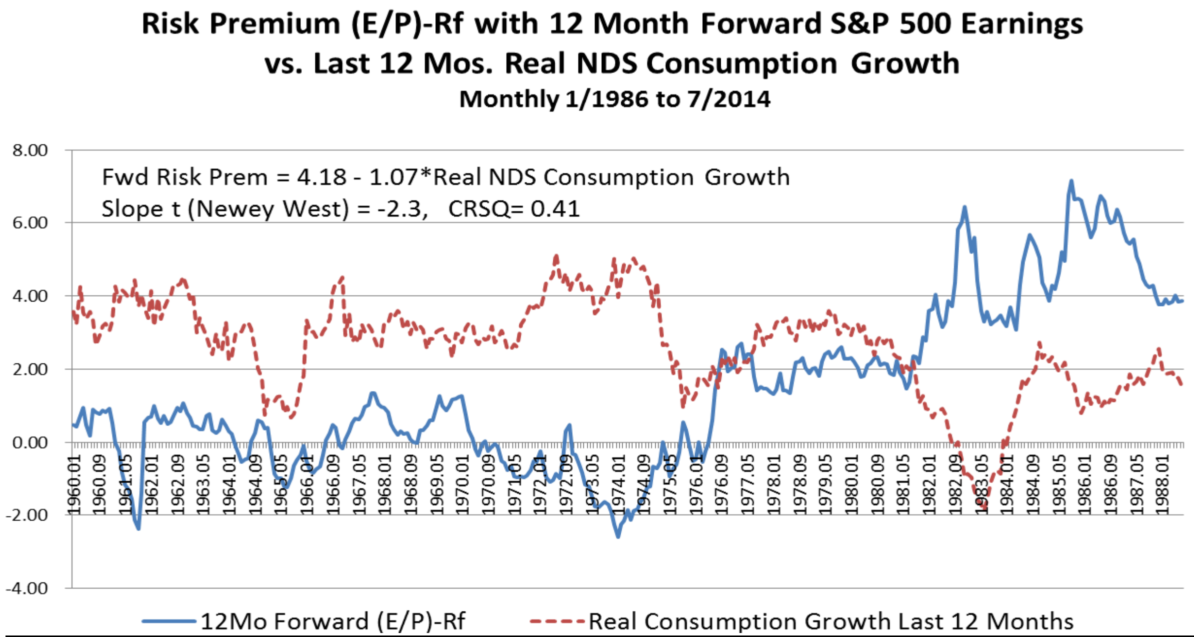
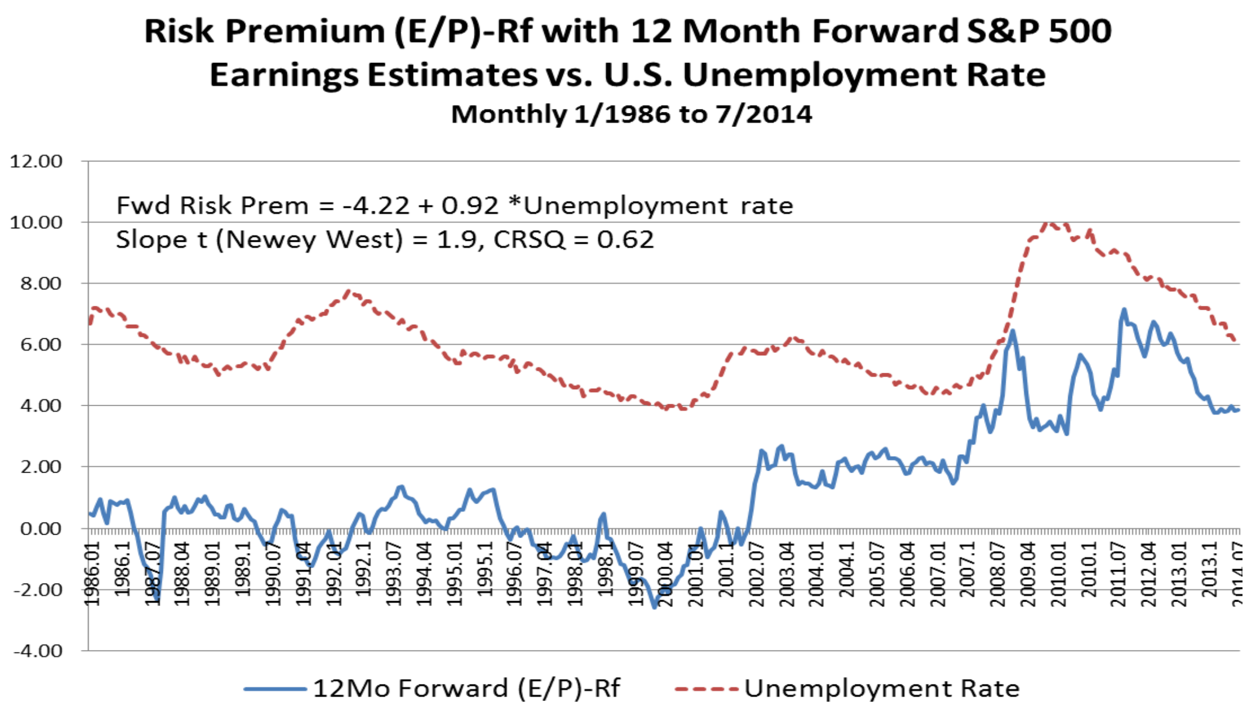


Figure 20



In conclusion on this segment, we believe that the external habit formation model and the forecasted large cyclical swings in relative risk aversion and risk premiums have much to offer to finance researchers and practitioners that is quite important economically and statistically.

X. Resurrecting the CCAPM: Conditional Consumption Risks, Ultimate Risk: Lettau-Ludvigson (2001), Parker-Julliard (2005) and Jagannathan-Wang (2007)

In a pair of innovative, impactful articles, Lettau and Ludvigson (2001a,b) built an econometric model where consumption, wealth and labor income are cointegrated, and consumption's deviations from the shared trend summarize agents' expectations of future returns on the market portfolio. This ties in very nicely with the continuous-time intertemporal portfolio theory of Merton (1971, 1973) and Breeden (1979, 1984). It also is consistent with prior analyses of consumption responses to shocks to permanent and transitory income. In Breeden's (1984) theoretical analysis (Section II, eq. 15 in this article), optimal consumption's sensitivities to the state variables that describe the investment and income opportunity set are proportional and opposite to the compensating variations in wealth for changes in those state variables. If the opportunity set improves, compensating variations in wealth are negative and we expect most individuals to respond positively with increased consumer spending. In contrast, if the opportunity set is believed to deteriorate, individuals optimally reduce consumption spending today to smooth their forward looking lifetime consumption paths.

Going backwards from consumption's moves, a high level of consumption relative to wealth and income indicates a good investment and income opportunity set, whereas a low consumption/wealth ratio is indicative of views of a poor investment and income opportunity set (perhaps a poor job market?). Lettau and Ludvigson (2001a) estimate the cointegrating relationship among consumption, wealth and income, and use positive deviations of consumption from the shared trend as a predictor of better than normal investment returns and negative deviations as predictors of poor future returns.[6] Using a dynamic least squares technique that accounts for both leading and lagging relationships among the cointegrated variables, they generated the following point estimates for the parameters of shared consumption, labor income and wealth (with small letters indicating natural logarithms of real nondurables and services

[6] Do note that it is entirely possible both in theory and in practice that movements in volatility could be offsetting to the impact of movements in mean returns, and cause the relation to not be as sought by Lettau and Ludvigson. If, for example, volatility dropped sharply at the same time that mean returns dropped modestly, the Sharpe ratio (slope of the capital market line) could actually improve, indicating a better investment opportunity set and causing optimal consumption to increase. Similarly, the mean return on the market could increase modestly at a time that volatility increased sharply, and optimal consumption would decline. These results were actually found by Breeden (1989) in an unpublished working paper presented at the French Finance Association, wherein high consumption growth was shown to be more a reflection of low future investment risk than it was a reflection of high future investment returns.

consumption, asset wealth and labor income per capita, respectively), using data from 1952Q4 to 1998Q3:

$$\begin{aligned} c_{n.t} &= 0.61 + 0.31 a_t + 0.59 y_t \\ &\quad\ (7.96) \quad (11.70) \quad (23.92) \end{aligned} \tag{69}$$

The residual term, which they denote as "*cay*," measures the difference between log consumption and its conditional expectation based of household net worth and labor income. From Lettau and Ludvigson's websites, given another 15 years of data, some substantial data revisions by the government in 2003, and some changes to their structural model, their current estimated log consumption trend deviation, using data from 1952Q1 to 2013Q3 is of the form:

$$cay_t = c_t - 0.87 - 0.12\, a_t - 0.78\, y_t \tag{70}$$

The change in coefficients from 1998 to 2013 indicates that, in describing consumption moves, labor income has become relatively more important than wealth, which is measured as "Household Net Worth" reported quarterly by the Federal Reserve, and includes real estate and bond values, as well as stocks.

Lettau and Ludvigson's (2001a) results during the time period studied (1952-1998) are quite strong, finding that "… a one-standard-deviation increase in cay leads to a 220 basis points rise in the expected real return (*in the next quarter*) on the S&P Index and about the same rise in the excess return, roughly a nine percent increase at an annual rate." The Newey West t-statistic, corrected for generalized autocorrelation, is above 3.0, which is statistically significant. Longer horizon forecasts are also impressive. As the following excerpt from their Table VI shows, the *cay* variable has significant explanatory power at all intervals from 1 quarter to 6 years, with robust t-statistics of 3.0 or more:

Table 25

Long-horizon Regressions

The table reports results from long-horizon regressions of excess returns on lagged variables. H denotes the return horizon in quarters. The dependent variable in Panel A is H-period consumption growth $\Delta c_{t+1} + \ldots + \Delta c_{t+H}$. In Panel B, the dependent variable is the sum of H log excess returns on the S&P Composite Index, $r_{t+1} - r_{f,t+1} + \ldots + r_{t+H} - r_{f,t+H}$. The regressors are one-period lagged values of the deviations from trend $\widehat{cay}_t = c_t - \hat{\beta}_a a_t - \hat{\beta}_y y_t$, the log dividend yield $d_t - p_t$, the dividend earnings ratio $d_t - e_t$, the detrended short-term interest rate $RREL_t$, and combinations thereof. For each regression, the table reports OLS estimates of the regressors, Newey–West corrected t-statistics in parentheses, and adjusted R^2 statistics in square brackets. Significant coefficients at the five percent level are highlighted in bold. The sample period is fourth quarter of 1952 to third quarter 1998.

		Forecast Horizon H							
Row	Regressors	1	2	3	4	8	12	16	24
		Panel A: Consumption Growth							
1	$\widehat{cay}_t$	0.11	0.62	1.23	1.98	2.29	0.33	−1.17	0.21
		(0.33)	(0.87)	(1.09)	(1.33)	(1.13)	(0.14)	(−0.41)	(0.05)
		[0.00]	[0.01]	[0.02]	[0.03]	[0.02]	[0.01]	[0.00]	[0.01]
		Panel B: Excess Stock Returns							
2	$\widehat{cay}_t$	**2.16**	**3.80**	**5.43**	**6.72**	**8.35**	**8.57**	**7.86**	**12.44**
		(3.44)	(3.34)	(3.37)	(3.70)	(3.73)	(3.24)	(2.99)	(3.41)
		[0.09]	[0.12]	[0.16]	[0.18]	[0.16]	[0.15]	[0.11]	[0.16]
3	$d_t - p_t$	0.03	0.06	0.10	0.13	0.24	0.27	0.30	**0.76**
		(1.40)	(1.23)	(1.16)	(1.22)	(1.18)	(1.27)	(1.36)	(3.12)
		[0.01]	[0.02]	[0.03]	[0.04]	[0.06]	[0.07]	[0.07]	[0.25]

Source: Lettau and Ludvigson (2001a), Table VI.

During this period, dividend yield (in log terms, log D minus log P) was less statistically significant as a forecaster of future real stock returns than in prior studies, but still had strong significance forecasting 6 years out returns. LL's cay variable had even stronger explanatory power for future stock returns for both shorter time horizons (1-4 years) and the longer horizon, 6 years. Panel A shows that consumption deviations were not successful in forecasting future real consumption growth, which is consistent with Hall's (1978) prior results. Updating the statistics with data from 1998 to 2013 from the website data of Lettau and Ludvigson, we find that a 1 standard deviation move in their cay variable is associated with a move that is about 65 bp less per quarter than in the original study, perhaps 155 bp/quarter, giving a still-large increment of returns of about 6%-6.5% annualized, rather than the original finding of 9.0%.

In the companion article by Lettau and Ludvigson (2001b) entitled "Resurrecting the (C)CAPM: A Cross-Sectional Test When Risk Premia Are Time-Varying," they use their new consumption trend deviation, cay_t, as a "scaling" variable for measuring conditional expected returns of assets.   Lettau and Ludvigson first illustrate the poor results of using **unconditional** beta estimates in cross-sectional fits of mean returns with market-based CAPM betas (figure a) and with Consumption CAPM betas (fig. c) for the 25 Fama-French portfolios sorted by size and book/market. They show the much better fit of mean returns from the Fama-French (1992) 3-factor statistical model (fig b), for which the underlying risk factors are unknown. Finally, using their *cay* variable for conditioning, they find that their **conditional** version of the **Consumption CAPM** fits nearly as well as the Fama-French, 3 factor statistical model (fig. d):

Figure 21

Conditional Consumption CAPM Fits Nearly As Well As Fama-French 3-Factor Model

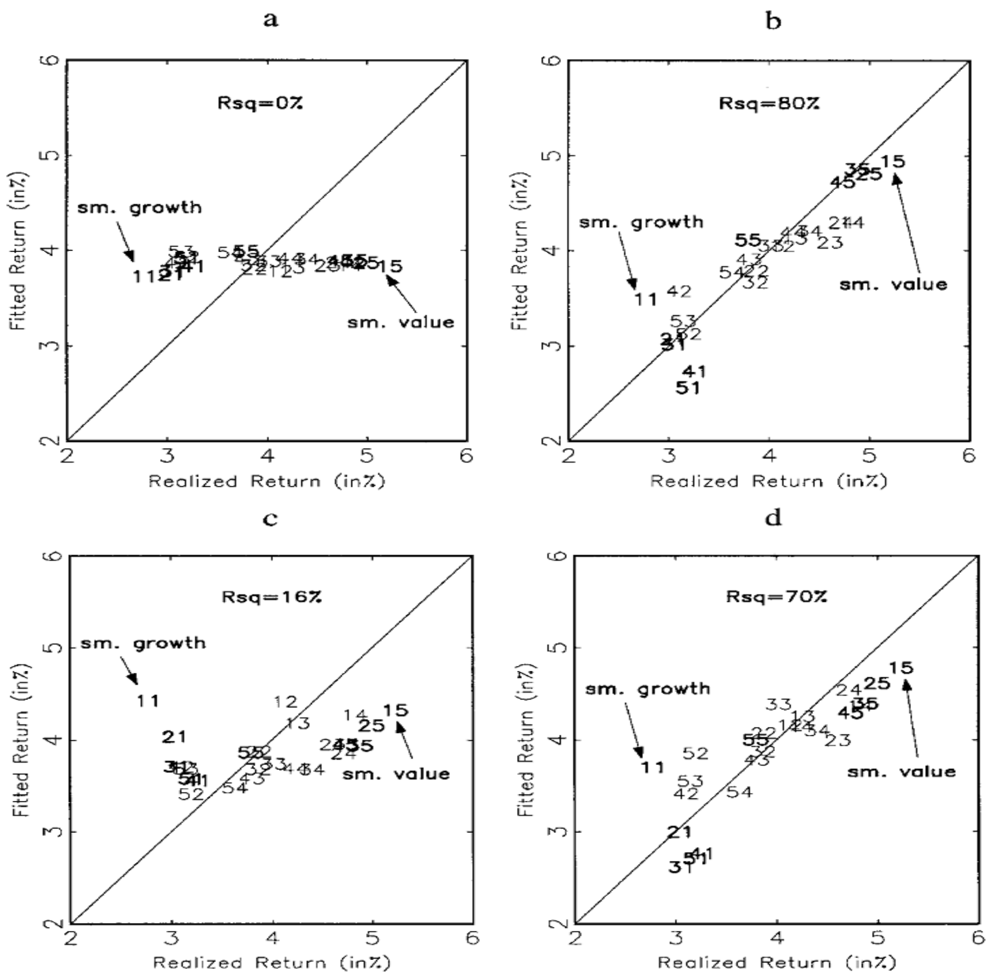


FIG.1.—Realized vs. fitted returns: 25 Fama-French portfolios: *a*, CAPM; *b*, Fama-French; *c*, consumption CAPM; *d*, consumption CAPM scaled. The figure shows the pricing errors for each of the 25 Fama-French portfolios for the four models. Each two-digit number represents one portfolio. The first digit refers to book-to-market quintiles (1 indicating the portfolio with the lowest book-to-market ratio, 5 with the highest). The pricing errors are generated using the Fama-MacBeth regressions. The scaling variable is $\widehat{cay}$.

Source: Lettau and Ludvigson (2001b), Figure 1.

Lettau and Ludvigson (2001b, p. 1241) note that: "Intuitively, conditioning improves the fit of the CCAPM because some stocks are more highly correlated with consumption growth in bad times, when risk or risk aversion is high, than they are in good times, when risk or risk aversion is low. This conditionality on risk premia is missed by unconditional models because they assume that those risk premia are constant over time." This logic is consistent with Campbell and Cochrane (2000), who argued that conditional models will perform far better than unconditional models based on the presence of an external habit.

To see the changes in conditional consumption betas between "good states" and "bad states," Lettau and Ludvigson denote the good states as those where cay was 1 standard deviation above the unconditional mean, and bad states as those where cay was 1 standard deviation below that mean. The estimated conditional consumption betas for the 25 Fama-French size and book/market sorted portfolios are in the following table.

Table 26

Conditional Betas in the Consumption CAPM

Portfolio		All States	Good States*	Bad States**	Change in Beta Bad-Good
S1B1	Growth	6.4	7.3	5.7	-1.6
S1B2		6.3	5.8	6.6	0.8
S1B3	***Small***	5.1	4.4	5.6	1.1
S1B4		5.5	4.4	6.2	1.8
S1B5	Value	5.8	3.4	7.4	4.0
S2B1	Growth	4.4	7.7	2.1	-5.6
S2B2		3.6	3.6	3.6	0.0
S2B3		3.9	4.1	3.8	-0.4
S2B4		3.5	2.1	4.5	2.4
S2B5	Value	4.8	3.1	6.0	3.0
S3B1	Growth	2.7	7.4	-0.5	-7.8
S3B2		2.8	3.5	2.3	-1.2
S3B3		2.9	2.0	3.5	1.6
S3B4		2.6	2.6	2.6	0.0
S3B5	Value	3.7	2.7	4.4	1.7
S4B1	Growth	2.0	6.2	-0.9	-7.1
S4B2		2.6	4.8	1.2	-3.6
S4B3		1.9	3.0	1.2	-1.8
S4B4		2.5	2.4	2.6	0.2
S4B5	Value	3.8	3.1	4.3	1.2
S5B1	Growth	1.6	6.1	-1.4	-7.5
S5B2		1.2	2.0	0.6	-1.5
S5B3	***Large***	2.3	4.1	1.2	-2.9
S5B4		1.2	3.4	-0.3	-3.7
S5B5	Value	3.1	3.3	2.9	-0.5

*Good states are states with *cay* more than 1 standard deviation above the mear
**Bad states are states with *cay* more than 1 standard deviation above the mean
Source: Lettau and Ludvigson (2001b)

Note the differences in the systematic changes in consumption betas between good and bad states and how they are related to whether the stocks are growth stocks (B1/B2) or value stocks (B4/B5). Consumption betas of value stocks increase in bad times, which is when risks are highest, so their equilibrium returns on average need to be higher, as the data has shown they are. Thus, Lettau and Ludvigson are able to explain the "value effect" with the conditional changes in consumption betas. In contrast, betas for growth stocks seem to fall in bad times, giving them lower consumption risk then. As previously discussed this pattern of conditional consumption beta indicated that the growth stock portfolio has an unconditional convex relation to consumption growth and conversely, the value stock portfolio has an unconditional concave relation to consumption growth. Under decreasing absolute risk aversion there is a preference for positive skewness and the unconditional risk premium for the growth stock portfolio would, *ceteris paribus,* be less than that for the value stock portfolio.

The Campbell and Cochrane (1999) model of external habit formation (and conditionally changing and nonlinear risks and relative risk aversion), and the Lettau and Ludvigson (2001a, b) articles were important in reestablishing the Consumption CAPM and consumption-based asset pricing as a leading model of asset pricing. Both demonstrated that we need more advanced econometric techniques to properly model changing consumption risks and changing risk aversion and risk premiums through time and economic scenarios.

In an important article, Parker and Julliard (2005) develop a model of "ultimate consumption risk," which captures the longer run relationship of consumption with asset returns. They argue that "… most importantly, the ultimate risk may be a better measure of the true risk of an asset if consumption is slow to adjust to returns. If consumption responds with a lag to changes in wealth, then the contemporaneous covariance of consumption and wealth understates or mismeasures the true risk of a portfolio. Ultimate consumption risk, on the other hand, can provide the correct measure of risk under several extant explanations of slow consumption adjustment, such as some models of (a) measurement error in consumption; (b) costs of adjusting consumption; (c) nonseparability of marginal utility of consumption from factors such as labor supply or housing stock, which themselves are constrained to adjust slowly; or (d) constraints on information flow or calculation so that household behavior is 'near-rational.'"

Their main finding is that ultimate consumption risk can largely explain the cross-sectional pattern of expected portfolio returns. While the covariance of each portfolio and *contemporaneous* consumption growth explains little of the variation in expected returns across portfolios, at a horizon of three years the ultimate risk to consumption explains 44 to 73 percent of the variation in expected returns across portfolios, depending on the specification. The performance of ultimate consumption risk rivals that of the Fama and French (1992) three-factor model and the Lettau and Ludvigson (2001b) three-factor model, two important linear factor models that have been used to price the expected returns in the Fama-French portfolios.

Parker and Julliard propose ultimate consumption risk to explain the cross-sectional heterogeneity of the 25 Fama-French portfolios. By ultimate consumption, they mean that increasing the consumption growth horizon to longer terms can increase cross-sectional explanatory power of the canonical CCAPM model, and the optimal point is achieved at 3 years with a 44-73% R-square, depending on the specifications. Parker and Julliard estimates of the relationship of the cross-section of returns to consumption risk measured over different horizons are in the following table:

Table 27

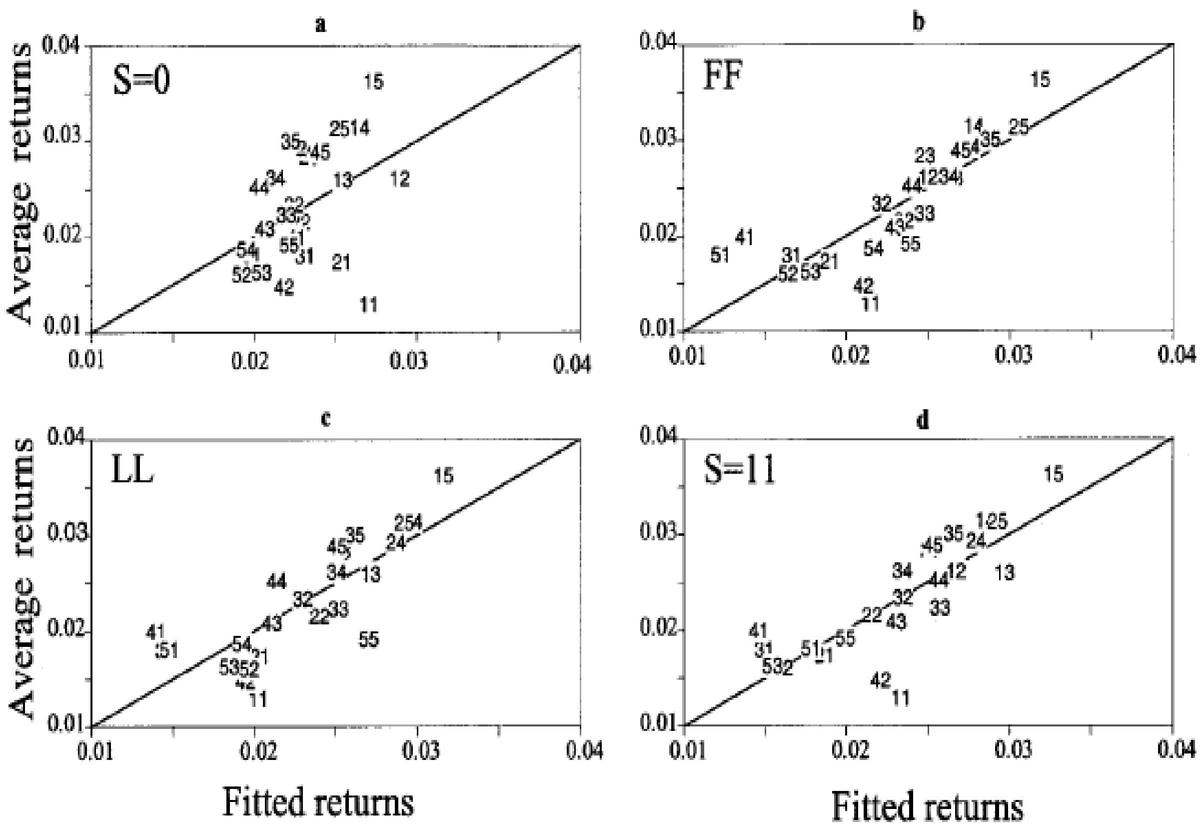
GMM with prespecified weighting matrix									
		Risk Aversion					Risk Aversion		
Horizon S	R^2 (%)	α	ϒ	Dist	Horizon S	R^2	α	ϒ	Dist
(Quarters)	*(1)*	*(2)*	*(3)*	*(4)*	*(Quarters)*	*(1)*	*(2)*	*(3)*	*(4)*
0*	4	0.029	19.9	0.37	7	10	0.019	11	0.36
		(0.006)	(33.3)	[0.000]			(0.008)	(14.3)	[0.000]
0	3	0.023	19	0.37	8	20	0.018	15.1	0.34
		(0.005)	(41.8)	[0.000]			(0.006)	(13.8)	[0.000]
1	2	0.023	10.7	0.37	9	30	0.018	17.9	0.31
		(0.007)	(27.5)	[0.000]			(0.005)	(12.5)	[0.000]
2	5	0.02	14.6	0.37	10	33	0.017	18.6	0.31
		(0.009)	(24.8)	[0.000]			(0.005)	(13.7)	[0.000]
3	10	0.018	17.9	0.36	11	44	0.015	25.4	0.28
		(0.009)	(23.5)	[0.000]			(0.006)	(16.4)	[0.000]
4	4	0.021	9.1	0.37	12	32	0.016	25	0.31
		(0.008)	(17.2)	[0.000]			(0.005)	(16.5)	[0.000]
5	7	0.019	11.7	0.36	13	35	0.012	38.5	0.3
		(0.008)	(16.3)	[0.000]			(0.006)	(14.0)	[0.000]
6	9	0.018	12.6	0.36	14	30	0.014	34.6	0.31
		(0.008)	(15.3)	[0.000]			(0.005)	(24.6)	[0.000]
					15	24	0.016	39.4	0.33
							(0.008)	(24.4)	[0.000]

Source: Parker and Julliard (2005), Table 1. Standard errors in parentheses, and Hansen-Jagannathan distance p-values are in brackets.

where γ is the estimate of relative risk aversion and α is a constant term. From the table, it is seen that R^2 grows from near zero for contemporaneous consumption risk to 44 percent when consumption risk is measured by cumulative real nondurables consumption growth over the next 11 quarters, almost 3 years. Note that while ultimate consumption risk helps explain the cross-section of returns, the relative risk aversion estimate is quite high at approximately 25.

Parker and Julliard compare the empirical performance of four models: (1) CCAPM with only contemporaneous consumption risk, (2) the Fama-French 3-factor model, (3) Lettau and Ludvigson's conditional test of the CCAPM using their consumption/wealth ratio, cay, as a conditioning variable, and (4) ultimate consumption risk over 11 quarters. The results are quite apparent in the graphs below:

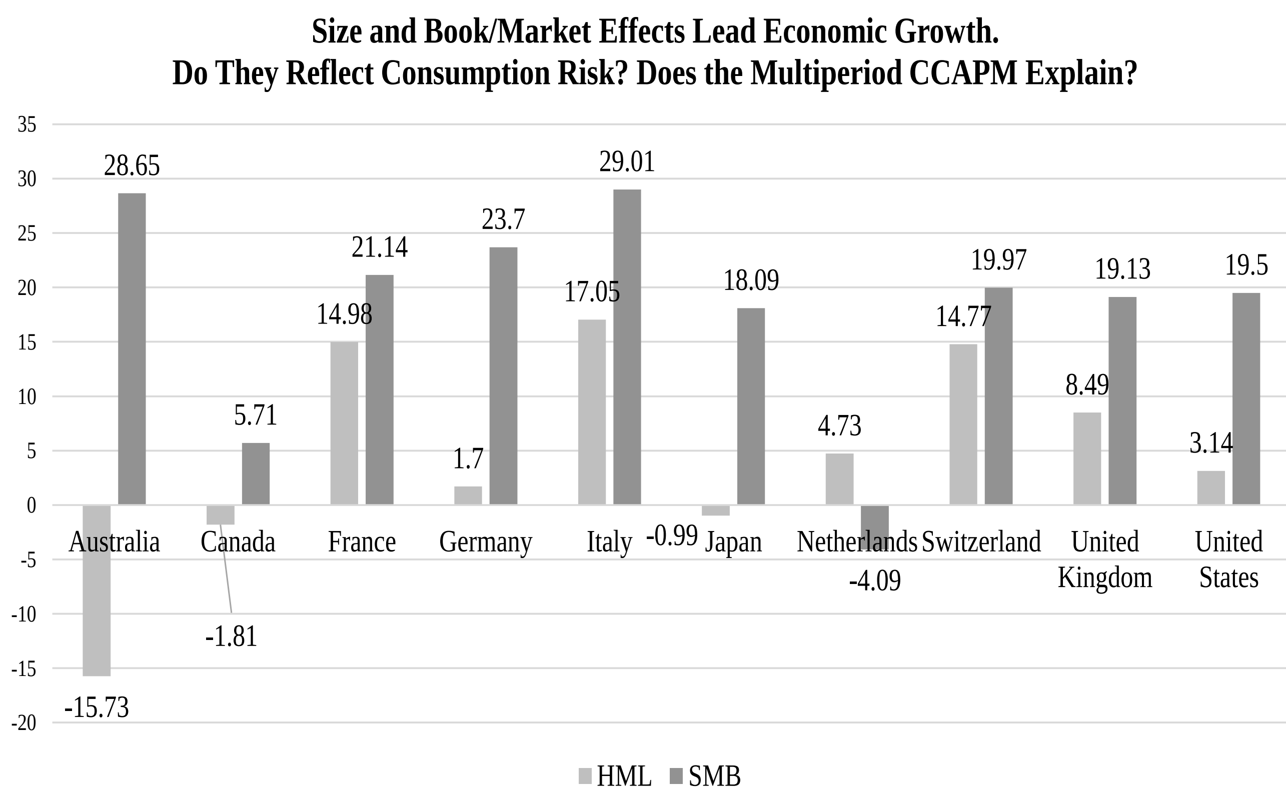
Figure 22



Source: Parker and Julliard (2005).

What drives the excellent results for Parker-Julliard's ultimate consumption risk measure is the fact that the excess returns of the Fama-French portfolios predict future consumption growth. As they say, "… both the excess return of small firms less large firms and the excess return of high-value stocks less low-value stocks predict consumption growth. The joint predicting power of SMB and HML peaks at 3 years." However, using the GMM estimator, the ultimate consumption model with 26 moments and only 3 parameters is still rejected by the over-identification test, so further analysis is required. While their observations concerning longer term consumption effects are intuitive, the specification based on declining weights over time would be more intuitive than equal weighting over a fixed horizon. Figure 24, from Liew and Vassalou (2000), shows the difference in return to factor portfolios in the year prior to above average versus below average GDP growth. (See Bodie, Kane and Marcus's *Investments* (2010, Fig. 13.1)). Both small minus big and high minus low portfolios are typically higher prior to improving GDP growth:

Figure 23



Source: Liew and Vassalou (2000).

In 2007, Jagannathan and Wang (J-W) also provided strong empirical support for consumption based asset pricing. They propose a theory of consumers who use discretion in periodically reviewing consumption and investment decisions, as opposed to continuously optimizing their consumption and investment rules. They argue that consumers are more likely to review their decision making at tax year ends (in the 4th quarter) and during economic contractions. They demonstrate that this helps explain the unsatisfactory previous testing results of the CCAPM. They compute the excess returns and consumption betas for size-ranked and book/market ranked (5x5 = 25) portfolios to be as in Table 28:

Table 28

Annual Excess Returns and Consumption Betas

Panel A reports average annual excess returns on the 25 Fama–French portfolios from 1954 to 2003. Annual excess return is calculated from January to December in real terms. All returns are annual percentages. Panel B reports these portfolios' consumption betas estimated by the time-series regression:

$$R_{i,t} = \alpha_i + \beta_{i,c}\Delta c_t + \varepsilon_{i,t},$$

where $R_{i,t}$ is the excess return over the risk-free rate, and Δc_t is Q4-Q4 consumption growth calculated using fourth quarter consumption data. Panel C reports t-values associated with consumption betas.

	Low		Book-to-market		High
Panel A: Average Annual Excess Returns (%)					
Small	6.19	12.47	12.24	15.75	17.19
	5.99	9.76	12.62	13.65	15.07
Size	6.93	10.14	10.43	13.23	13.94
	7.65	7.91	11.18	12.00	12.35
Big	7.08	7.19	8.52	8.75	9.50
Panel B: Consumption Betas					
Small	3.46	5.51	4.26	4.75	5.94
	2.89	3.03	4.79	4.33	5.21
Size	2.88	4.10	4.35	4.79	5.71
	2.57	3.35	3.90	4.77	5.63
Big	3.39	2.34	2.83	4.07	4.41
Panel C: t-values					
Small	0.93	1.71	1.59	1.83	2.08
	0.98	1.27	2.02	1.83	2.10
Size	1.15	1.93	2.17	2.07	2.39
	1.14	1.75	1.90	2.26	2.39
Big	1.71	1.32	1.67	2.15	2.00

Source: Jagannathan and Wang (2007).

A simple scatter plot of returns versus consumption betas for the 25 Fama-French portfolios sorted on size and value shows a strong relationship:

Figure 24

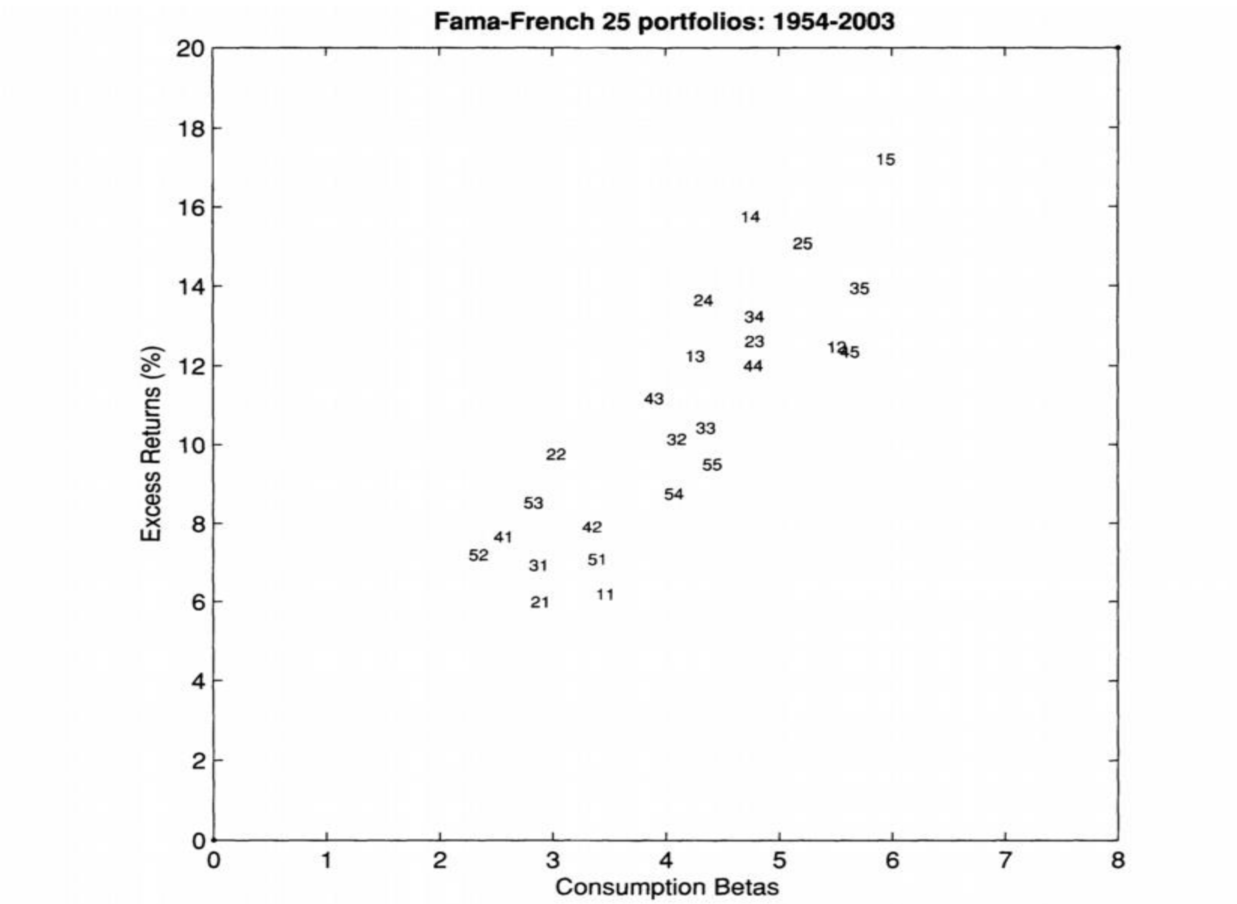


Figure 1. Annual excess returns and consumption betas. This figure plots the average annual excess returns on the 25 Fama–French portfolios and their consumption betas. Each two-digit number represents one portfolio. The first digit refers to the size quintile (1 smallest, 5 largest), and the second digit refers to the book-to-market quintile (1 lowest, 5 highest).

Source: Jagannathan and Wang (2007) , Figure 1.

Jagannathan and Wang show that the differencing interval and data series matters for relations of cross-sectional consumption betas to average returns. They find that 4-quarter percentage changes in real consumption growth (Q4-Q4) and returns are most significant, as their model of normal decision making and planning would suggest, as shown in Table 29:

Table 29

CCAPM with Different Frequency Data

We use different frequency returns and consumption data to test the CCAPM. Panel A describes how consumption growth is calculated. For example, with monthly consumption data, annual consumption growth is measured using December consumption of one year and December consumption of the following year. Panel B reports cross-sectional regression estimation results for the CCAPM:

$$E[R_{i,t}] = \lambda_0 + \lambda_1 \beta_{i,c}.$$

Test portfolio returns are annualized excess returns on the 25 Fama–French (1993) portfolios from 1960 to 2003. (Monthly consumption data are available from 1959.)

Panel A: Consumption Growth

	Monthly Consumption Data	Quarterly Consumption Data	Annual Consumption Data
Monthly growth	Month-Month		
Quarterly growth	Dec–Mar, Mar–Jun Jun–Sep, Sep–Dec	Quarter-Quarter	
Annual growth	Dec-Dec	Q4-Q4	Annual-Annual

Panel B: Cross-Sectional Regression Results

	Monthly Consumption Data			Quarterly Consumption Data			Annual Consumption Data		
	λ_0	λ_1	R^2	λ_0	λ_1	R^2	λ_0	λ_1	R^2
Monthly return	7.70	0.02	0.00						
t-value	2.61	0.17	−0.04						
Quarterly return	8.34	0.03	0.00	4.52	0.33	0.22			
t-value	2.80	0.15	−0.04	1.83	1.59	0.18			
Annual return	−1.83	2.01	0.41	−1.19	2.68	0.69	10.12	1.32	0.21
t-value	−0.51	2.33	0.38	−0.37	3.49	0.68	3.70	1.61	0.18

Source: Jagannathan and Wang (2007). Table VII.

J-W model conditional consumption betas by using NBER business cycle dates for a dummy variable for regressions. Table 30 shows that this is important in modeling risks, as consumption betas increase during contractions, relative to expansions:

Table 30

Consumption Beta in Contractions and Expansions

This table reports cross-sectional regression results of the CCAPM during different subperiods. First, we estimate the contraction consumption beta and the expansion consumption beta by the time-series regression

$$E_t[R_{i,t+4}] = \alpha_{i,\text{cont}} I_t + \alpha_{i,\text{exp}}(1 - I_t) + \beta_{i,\text{cont}} \Delta c_{t+4} I_t + \beta_{i,\text{exp}} \Delta c_{t+4}(1 - I_t),$$

where $I_t = 1$ if the economy is contracting according to the NBER Business Cycle Dating, otherwise $I_t = 0$; $\beta_{i,\text{cont}}$ is the contraction consumption beta and $\beta_{i,\text{exp}}$ is the expansion consumption beta. Then we run the cross-sectional regression

$$E[R_{i,t+4}] = \lambda_0 + \lambda' \beta_i.$$

$R_{i,t+4}$ are annual excess returns of the 25 Fama–French portfolios from quarter t to quarter $t + 4$ for all quarters from 1954 to 2003. The total number of observations is 200, including 43 quarters of contractions and 157 quarters of expansions. Within the 43 recession quarters, there are 11 Q1s, 9 Q2s, 11 Q3s, and 12 Q4s.

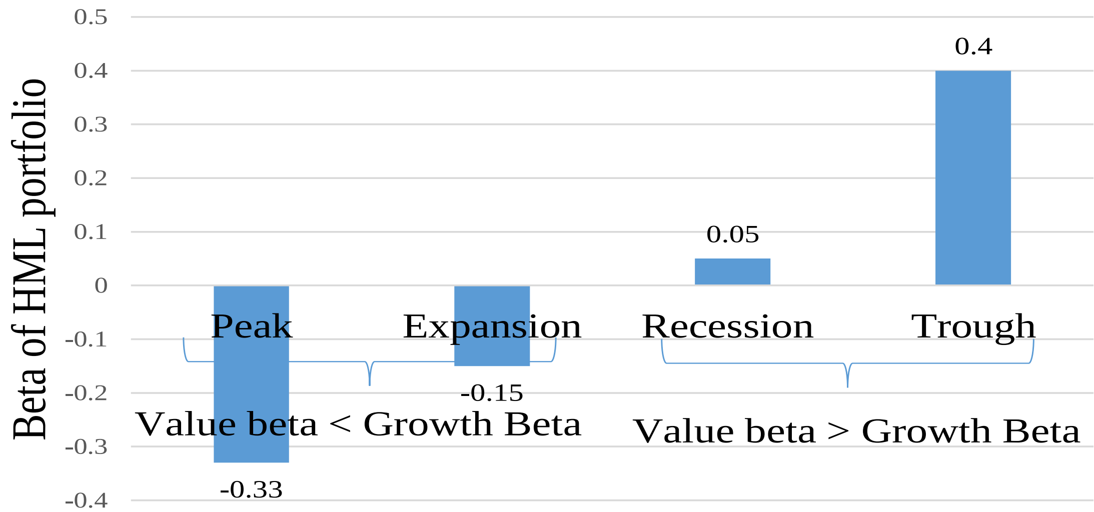
	Intercept	Contraction	Expansion	R^2(adj R^2)
Estimate	0.86	0.98	0.23	0.65
t-value	0.50	6.11	0.67	0.62
Estimate	0.84	1.06		0.65
t-value	0.50	7.51		0.62
Estimate	6.10		1.40	0.33
t-value	4.71		4.78	0.26

Source: Jagannathan and Wang (2007), Table XI.

Another way to see the changing conditional risks in comparisons of betas for value and growth stocks is from the Petkova and Zhang (2005) graph:

Figure 25

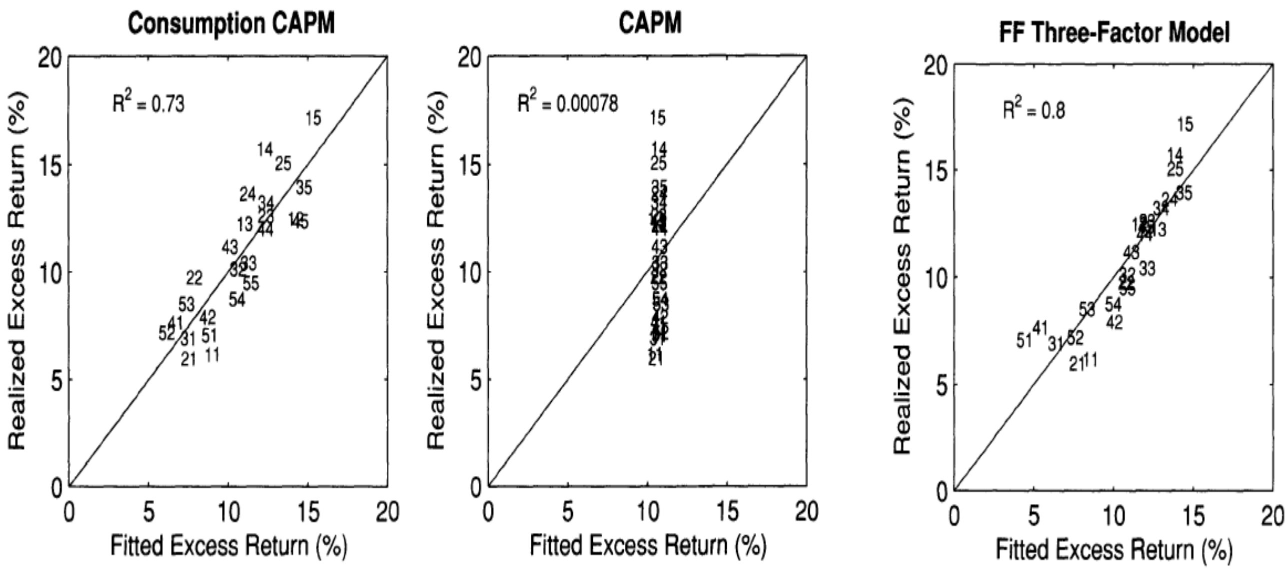
"Conditional Betas" Show Risks of Value Stock Increase in Recessions



Source: Petkova and Zhang (2005), printed in Bodie, Kane and Marcus (2010), Figure 13.2.

In their testing results, Jagannathan and Wang (2007) find that the CCAPM performs as well as, if not better than, the Fama-French three factor model in explaining cross-sectional return differences among the 25 F-F portfolios sorted by size and book/market, using the year-over-year fourth quarter consumption growth rate. In addition, the contraction beta is tested to be more significant than expansion beta. Both findings support their hypothesis. Results are in the following graphs:

Figure 26



Source: Jagannathan and Wang (2007).

The Jagannathan-Wang discretionary consumption decision model was motivated by several important prior empirical tests, including conditioning, long-run risk and ultimate consumption, as all of these models take into consideration the large errors in estimating short-term consumption data and the discretionary psychology involved in consumption decision making. As they point out, discretion is perhaps less consistent with Constantinides (1990) and Campbell-Cochrane (1999) habit formation models, as consumption is likely less dependent on the historical levels if more discretion is taken to review the decision. They reconcile the two effects and show that the habit formation effect is dominated by the classic consumption beta in asset pricing when the time between consumption decisions increases. However, this analysis is based on the assumption of simultaneous and infrequent decision making of all consumers, which is exogenous to the model and requires scrutiny. When a high implied risk aversion parameter is found, they attribute it to the lack of realism of the assumption that consumers simultaneously make choices only once a year. Overall, the Jagannathan-Wang paper added to the string of significant papers that found consumption-based asset pricing, based on alternative specifications of individuals' consumption and portfolio choices, is able to explain cross-sectional returns and the Fama-French size and book/market anomalies. However, the different specifications lead to different economic understandings of the causal behavior generating the observed returns.

XI. Long-Run Risks. Bansal-Yaron (2004)

In another major second generation consumption-based asset pricing model, 25+ years after the original Consumption CAPM derivations, Bansal and Yaron (BY, 2004) developed a model of "long run risks" in consumption growth. This has been an influential version of consumption-based asset pricing for the past decade, spawning a substantial amount of additional research. Their key innovations are: (1) to model expected consumption and dividend growth rates as containing a small, persistent long-run predictable component, and (2) to model changing volatility of consumption growth rates. They use Epstein and Zin's (1999) forward-looking preferences, which are recursive and exhibit time complementarity for consumption.

With regard to the modeling of a small, persistent long-run fluctuation in consumption growth rates, Bansal and Yaron note that Shephard and Harvey (1990) show that in finite samples, it is very difficult to distinguish between a purely i.i.d. process and one that incorporates a small persistent component. While it is hard to distinguish econometrically between the two alternative processes, the asset pricing implications across them are very different. Bansal and Yaron observe that: "If, indeed, news about consumption has a nontrivial impact on long-term expected growth rates or economic uncertainty, then asset prices will be fairly sensitive to small growth rate and consumption volatility news. … For these channels to have a significant quantitative impact on the risk premium and volatility of asset prices, the persistence in expected growth rate has to be quite large, close to 0.98." They show that their combination of assumptions for consumption and dividend growth rates, which incorporate the fluctuating persistent component, are consistent with the historic data and helps them explain several of the literature's puzzling aspects of asset price levels and fluctuations.

The Epstein-Zin preferences assumption drives a wedge between the elasticity of intertemporal substitution (EIS) and relative risk aversion (RRA), which allows separate modeling of each. Flexibility in fitting EIS to a high level (EIS=1.5) allows them to match the low level of real short-term interest rates. Flexibility in fitting RRA to a relatively high level (RRA=10) allows them to fit the relatively large risk premium on equities. Their modeling of changing conditional volatility of the growth rate of consumption across time allows them to model time-varying risk and risk premia. As shown by Bansal, Khatchatrian and Yaron (2002) and confirmed in this paper, there is a significant negative correlation between price-dividend ratios and consumption volatility. When consumption volatility is high, stock prices are low in relation to dividends. They note that "about half of the volatility of the price-dividend ratios in the model can be attributed to variation in expected growth rates, and the remaining can be attributed to variation in economic uncertainty."

Some of the specifics of Bansal and Yaron's LLR model are as follows. Defining consumption growth as g_t, dividend growth as $g_{d,t}$, and letting x_t be the small, predictable component of consumption growth, they specify that (eq. 4, p. 1485):

$$\begin{aligned} x_{t+1} &= \rho x_t + \varphi_e \sigma e_{t+1} \\ g_{t+1} &= \mu + x_t + \sigma \eta_{t+1} \\ g_{d,t+1} &= \mu_d + \phi x_t + \varphi_d \sigma u_{t+1} \\ & e_{t+1}, u_{t+1}, \eta_{t+1} \sim N.i.i.d.(0, 1), \end{aligned} \tag{71}$$

with the three shocks, e_{t+1}, u_{t+1}, and η_{t+1} being mutually independent.[5] Two additional parameters, $\phi > 1$ and $\varphi_d > 1$, allow us to calibrate the overall volatility of dividends (which in the data are significantly larger than that of consumption) and its correlation with consumption. The parameter ϕ, as in Abel (1999), can be interpreted as the leverage ratio on expected consumption growth.[6]

The following table shows the fit of Bansal and Yaron's model with the statistical properties of historic real consumption growth:

Table 31

Annualized Time-Averaged Growth Rates

The model parameters are based on the process given in equation (4). The parameters are $\mu = \mu_d = 0.0015$, $\rho = 0.979$, $\sigma = 0.0078$, $\phi = 3$, $\varphi_e = 0.044$, and $\varphi_d = 4.5$. The statistics for the data are based on annual observations from 1929 to 1998. Consumption is real nondurables and services (BEA); dividends are from the CRSP value-weighted return. The expression $AC(j)$ is the j^{th} autocorrelation, $VR(j)$ is the j^{th} variance ratio, and *corr* denotes the correlation. Standard errors are Newey and West (1987) corrected using 10 lags. The statistics for the model are based on 1,000 simulations each with 840 monthly observations that are time-aggregated to an annual frequency. The *mean* displays the mean across the simulations. The 95% and 5% columns display the estimated percentiles of the simulated distribution. The *p*-val column denotes the number of times in the simulation the parameter of interest was larger than the corresponding estimate in the data. The *Pop* column refers to population value.

	Data		Model				
Variable	Estimate	SE	Mean	95%	5%	*p*-Val	*Pop*
$\sigma(g)$	2.93	(0.69)	2.72	3.80	2.01	0.37	2.88
$AC(1)$	0.49	(0.14)	0.48	0.65	0.21	0.53	0.53
$AC(2)$	0.15	(0.22)	0.23	0.50	−0.17	0.70	0.27
$AC(5)$	−0.08	(0.10)	0.13	0.46	−0.13	0.93	0.09
$AC(10)$	0.05	(0.09)	0.01	0.32	−0.24	0.80	0.01
$VR(2)$	1.61	(0.34)	1.47	1.69	1.22	0.17	1.53
$VR(5)$	2.01	(1.23)	2.26	3.78	0.79	0.63	2.36
$VR(10)$	1.57	(2.07)	3.00	6.51	0.76	0.77	2.96
$\sigma(g_d)$	11.49	(1.98)	10.96	15.47	7.79	0.43	11.27
$AC(1)$	0.21	(0.13)	0.33	0.57	0.09	0.53	0.39
$corr(g, g_d)$	0.55	(0.34)	0.31	0.60	−0.03	0.07	0.35

Source: Bansal and Yaron (2004), Table I.

From the table, comparing the historic statistics with the means from simulations of the LLR model, we see that the fit of consumption volatility and autocorrelation is quite good, on average. The variance ratio statistics for both the data and the model are all above 1.0 for the data shown (up to 10 years), which is consistent with positive autocorrelation in consumption growth and persistent shocks, as the ratio would be 1.0 and variance would grow proportionally through time, absent those effects. However, note that the 10-year variance ratio in the historic data is less than the 5-year variance ratio (as in Cochrane (1988)), whereas the B-Y long run risk model has the variance ratio continuing to increase from 5 to 10 years out. Perhaps at some point, mean reversion of real consumption growth sets in (which seems quite plausible) and offsets the effects of the persistent shocks. This long-run risk model would likely miss that effect.

Bansal and Yaron's model for the time-varying variance of real consumption growth has both a purely random component and a mean reverting component, with υ_1 describing the speed of mean reversion:

$$\sigma_{t+1}^2 = \sigma^2 + \nu_1(\sigma_t^2 - \sigma^2) + \sigma_w w_{t+1} \tag{72}$$
$$e_{t+1}, u_{t+1}, \eta_{t+1}, w_{t+1} \sim N.i.i.d.(0, 1),$$

The risk premium comes as a compensation for three consumption risks: short-run, long-run and volatility risk. Time-variation in the risk premium is governed by the conditional variance of consumption growth (i.e., it is high when current volatility is high). With fluctuating economic uncertainty, as well as the persistent growth shocks, Table 32 shows that Bansal and Yaron's long run risk model can replicate historic data quite well for many key asset market returns when constant relative risk aversion is between 7.5 and 10:

Table 32

Asset Pricing Implications—Case II

The entries are model population values of asset prices. The model incorporates fluctuating economic uncertainty (i.e., Case II) using the process in equation (8). In addition to the parameter values given in Panel A of Table II ($\delta = 0.998, \mu = \mu_d = 0.0015, \rho = 0.979, \sigma = 0.0078, \phi = 3, \varphi_e = 0.044$, and $\varphi_d = 4.5$), the parameters of the stochastic volatility process are $\nu_1 = 0.987$ and $\sigma_w = 0.23 \times 10^{-5}$. The predictable variation of realized volatility is 5.5%. The expressions $E(R_m - R_f)$ and $E(R_f)$ are, respectively, the annualized equity premium and mean risk-free rate. The expressions $\sigma(R_m), \sigma(R_f)$, and $\sigma(p - d)$ are the annualized volatilities of the market return, risk-free rate, and the log price-dividend, respectively. The expressions $AC1$ and $AC2$ denote, respectively, the first and second autocorrelation. Standard errors are Newey and West (1987) corrected using 10 lags.

	Data		Model	
Variable	Estimate	SE	$\gamma = 7.5$	$\gamma = 10$
		Returns		
$E(r_m - r_f)$	6.33	(2.15)	4.01	6.84
$E(r_f)$	0.86	(0.42)	1.44	0.93
$\sigma(r_m)$	19.42	(3.07)	17.81	18.65
$\sigma(r_f)$	0.97	(0.28)	0.44	0.57
		Price Dividend		
$E(\exp(p - d))$	26.56	(2.53)	25.02	19.98
$\sigma(p - d)$	0.29	(0.04)	0.18	0.21
$AC1(p - d)$	0.81	(0.09)	0.80	0.82
$AC2(p - d)$	0.64	(0.15)	0.65	0.67

Source: Bansal and Yaron (2004), Table 4.

Another nice feature of Bansal and Yaron's long run risks model is that it is able to broadly mimic the predictability of returns, growth rates and price/dividend ratios, as shown in the next table:

Table 33

Predictability of Returns, Growth Rates, and Price–Dividend Ratios

This table provides evidence on predictability of future excess returns and growth rates by price–dividend ratios, and the predictability of price–dividend ratios by consumption volatility. The entries in Panel A correspond to regressing $r^e_{t+1} + r^e_{t+2} \cdots + \cdots r^e_{t+j} = \alpha(j) + B(j)\log(P_t/D_t) + v_{t+j}$, where r^e_{t+1} is the excess return, and j denotes the forecast horizon in years. The entries in Panel B correspond to regressing $g^a_{t+1} + g^a_{t+2} \cdots + \cdots g^a_{t+j} = \alpha(j) + B(j)\log(P_t/D_t) + v_{t+j}$, and g^a is annualized consumption growth. The entries in Panel C correspond to $\log(P_{t+j}/D_{t+j}) = \alpha(j) + B(j)|\epsilon_{g^a,t}| + v_{t+j}$, where $|\epsilon_{g^a,t}|$ is the volatility of consumption defined as the absolute value of the residual from regressing $g^a_t = \sum_{j=1}^{5} A_j g^a_{t-j} + \epsilon_{g^a,t}$. The model is based on the process in equation (8), with parameter configuration given in Table IV and $\gamma = 10$. The entries for the model are based on 1,000 simulations each with 840 monthly observations that are time-aggregated to an annual frequency. Standard errors are Newey and West (1987) corrected using 10 lags.

	Panel A: Excess Returns			Panel B: Growth Rates			Panel C: Volatility		
Variable	Data	SE	Model	Data	SE	Model	Data	SE	Model
$B(1)$	−0.08	(0.07)	−0.18	0.04	(0.03)	0.06	−8.78	(3.58)	−3.74
$B(3)$	−0.37	(0.16)	−0.47	0.03	(0.05)	0.12	−8.32	(2.81)	−2.54
$B(5)$	−0.66	(0.21)	−0.66	0.02	(0.04)	0.15	−8.65	(2.67)	−1.56
$R^2(1)$	0.02	(0.04)	0.05	0.13	(0.09)	0.10	0.12	(0.05)	0.14
$R^2(3)$	0.19	(0.13)	0.10	0.02	(0.05)	0.12	0.11	(0.04)	0.08
$R^2(5)$	0.37	(0.15)	0.16	0.01	(0.02)	0.11	0.12	(0.04)	0.05

Source: Bansal and Yaron (2005), Table VI.

Bansal and Yaron observe that "The model can justify the equity premium, the risk-free rate, the volatility of the market return and the price-dividend ratio. As in the data, dividend yields predict returns and the volatility of returns is time-varying." Three critical observations are: (1) the number of tuning parameters is so large that they have considerable flexibility to overfit the data, (2) their model requires constant relative risk aversion near 10 to duplicate the data, which is relatively high, and (3) EIS needs to be large (1.5) to replicate the negative correlation between consumption volatility and price-dividend ratio present in the data, which is well above the EIS indicated by Vissing-Jorgensen's (2002) research. Nonetheless, the fact that they are able to provide results that provide an economic rationale for the equity premium puzzle and duplicate a 6% risk premium, while having a low risk free rate and reasonable volatility, is quite impressive. To be able to get these results with simply a small persistent growth term and with time-varying volatility is surprising. Subsequent sections each have articles on the Bansal-Yaron (2004) long-run risk model, as it continues to be one of the most impactful articles in consumption-based asset pricing in the past decade.

XII. Cash Flow Betas: Consumption Risk vs. Market Risk

In their original derivation of the Consumption CAPM, Breeden and Litzenberger (BL, 1978) realized that measuring consumption betas from cash flows such as dividends or profits, rather than from stock market prices and returns, could be useful in practice, as they noted:

> "…the correct beta to be used in finding the risk-adjusted discount rate is the cash flow's volatility with respect to aggregate consumption, not with respect to the market portfolio. For capital budgeting, these betas may be easier to estimate than "market" betas, since the cash flows of many projects may be more closely related to GNP or aggregate consumption than to the level of the market portfolio." (BL 1978, p. 646)

Breeden (1989b) presented OLS regression results that confirmed this intuition by estimating (unconditional) cash flow betas, using National Income and Product Accounts (NIPA) annual data for real growth of profits, dividends and cash flows versus real market returns and real consumption growth, (separately for total and nondurables and services consumption):[7]

Table 34

Estimates of Cash Flow Risks: 1930–1988

Regression form: Y = a + b X, where X = {Market, PCE Tot, PCE NDS}
Time period: 1930–1988 (excl 1939–1948)

	X=Market			X=Total Consumption			X=NDS Consumption		
NIPA Variable	b	t(b)	RSQ	b	t(b)	RSQ	b	t(b)	RSQ
AT Profits	.059	5.34	.36	.357	6.26	.44	.400	5.50	.37
AT Prof IVA CCA	.047	4.63	.30	.354	8.09	.57	.406	7.21	.51
BT Prof IVA CCA	.062	5.48	.37	.441	9.02	.62	.494	7.50	.53
Net Dividends	1.79	4.67	.31	15.53	9.40	.65	18.88	9.33	.65
Net Cashflow	3.56	4.09	.25	15.79	2.77	.12	15.16	2.11	.07

Breeden (1989b), Table 3.

In Breeden (2005), unconditional consumption and market betas for various industries were presented, based upon 2-quarter changes in real profits over the 1948-2004 period:[8]

[7] Note that Durbin-Watson statistics were approximately 1.6 to 1.9 in these regressions, with Dividends having the lowest DW, given their stickiness. Note also that the Net Cashflow results were dominated by an outlier in 1933. Breeden (2005) presented similar results with data through 2003 in his handout as the Distinguished Speaker at the Western Finance Association meetings in Portland, Oregon in June, 2005.

[8] To deal with negative profits or near-zero profits in certain industries, actual calculations were of changes in real profits, divided by prior quarter employee compensation, then divided by overall average profits/compensation.

Table 35

Cash Flow Risks Ranked by Industry: Consumption Betas vs. Market Betas

Nonoverlapping 2-Quarter Real Percentage Changes in Real Profits, 1948-2004.
S&P500 betas are K. French Real Industry Returns vs. S&P500 Real Return

Industry	Profit Beta, Real 2Q, vs. %Chg Real NDS Consumption	Relative Consumption Beta (Industry divided by US Total)	Industry Stock Beta vs. Real Return on S&P 500
Motor Vehicles	15.1	3.51	1.08
Durables	10.4	2.42	1.16
Retail Trade	9.2	2.14	1.09
Construction	8.5	1.98	1.23
Wholesale Trade	3.6	0.84	1.10
Food & Beverage	2.9	0.67	0.84
Banks	1.7	0.40	1.03
Oil	1.6	0.37	0.76
Utilities	-1.0	-0.23	0.61
U.S. Total	4.3	1.00	1.00

Source: Breeden (2005).

The consumption betas from profits are quite different from betas estimated with stock market returns, though they are significantly correlated. Profits do reflect leverage and are highly volatile. For the U.S. Total, profits move by 4.3% when NDS real consumption growth moves by 1%, and motor vehicles, which are quite cyclic, have 1-year profits growth that move by 15.1% then (or 3.51 times as much as aggregate profits move). Motor vehicles, durables and wholesale trade all have stock market betas that are quite similar, between 1.08 and 1.16, but their relative profit betas are 3.51, 2.42 and 0.84, respectively. Utilities, oils and banks had low consumption betas of profits, but higher stock market betas. Banks' profit betas should increase in more recent samples that include the recent Great Recession. Breeden (1989b) also found that industry profits were often mean-reverting over 3-5 years, which makes sense, given competition and freedom of entry. Very high profits entice additional supplies and competition. Stock market betas and dividends should pick up these longer term effects.

Updated annual data for 1930-2012 for the relation of real profits changes to real S&P 500 returns and to the growth rate of real, nondurables and services consumption are in Figures 27 and 28. It can be seen that annual profit growth is more related to consumption changes than to stock returns.

Figure 27

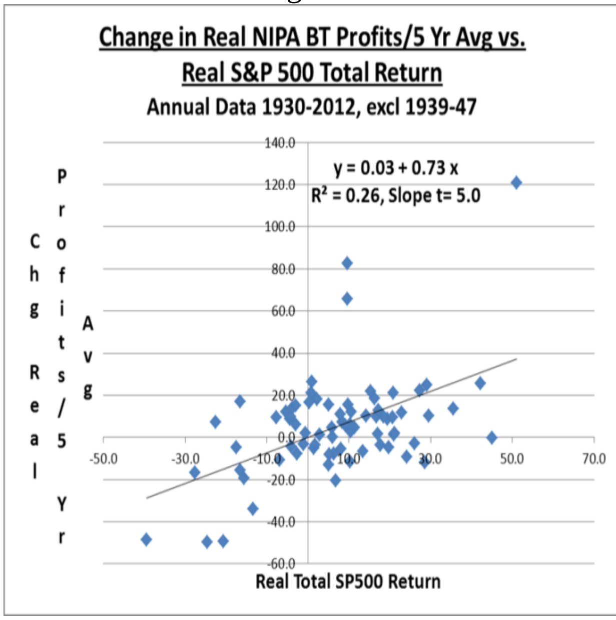
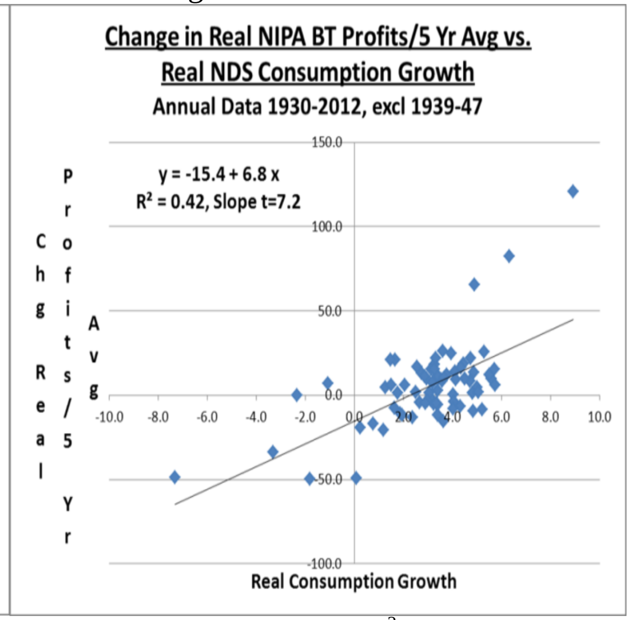


Figure 28



Note: When real total consumption growth is used, instead of NDS consumption, R^2=.51 and t-statistic for the slope is 8.6.

In a modern, econometrically stronger test of whether consumption risks of cash flows are helpful measures of risk, Bansal, Dittmar and Lundblad (BDL, 2005) examined dividends and share repurchases versus real NDS consumption growth for 30 portfolios: 10 decile portfolios ranked by size, book/market and momentum, respectively. They say (p. 1640): "We show that the cross-sectional dispersion in the measured cash flow beta explains approximately 62% of the cross-sectional variation in observed risk premia. Further, the estimated market price of consumption risk is sizable, statistically significant, and positive in all cases." They are able to duplicate much of the spread in mean returns of the extreme momentum portfolio (winner minus loser) and similarly defined size and value portfolios, using quarterly data from 1967 to 2001. Means and standard deviations of returns for the 30 portfolios are in Table 36. They display the well-known effects during this period that small stocks, high book/market, and positive momentum portfolios have the highest mean returns:

Table 36

Summary Statistics: Portfolio Returns for Size, Book/Market, Momentum Decile Portfolios

Quarterly returns: 1967-2001

Size	Mean	Std.Dev	Book/Market	Mean	Std.Dev	Momentum	Mean	Std.Dev
S1 (Small)	0.023	0.137	B1 Low	0.015	0.106	M1 (Losers)	-0.010	0.154
S2	0.023	0.127	B2	0.020	0.096	M2	0.007	0.119
S3	0.023	0.120	B3	0.021	0.092	M3	0.012	0.109
S4	0.023	0.117	B4	0.022	0.092	M4	0.020	0.094
S5	0.024	0.111	B5	0.020	0.080	M5	0.014	0.087
S6	0.021	0.105	B6	0.023	0.081	M6	0.016	0.088
S7	0.022	0.104	B7	0.024	0.084	M7	0.020	0.089
S8	0.022	0.100	B8	0.026	0.084	M8	0.024	0.083
S9	0.021	0.091	B9	0.027	0.089	M9	0.028	0.093
S10 (Large)	0.018	0.083	B10 (High)	0.033	0.103	M10 (Winners)	0.036	0.114

Source: Bansal, Dittmar, Lundblad (2005), Table I.

The estimated cash flow (dividend) growth rates for the 30 portfolios are interesting and in the next table:

Table 37

Summary Statistics: Portfolio Real Cash Flow (Dividend) Growth

Size	Mean	Std.Dev	Book/Market	Mean	Std.Dev	Momentum	Mean	Std.Dev
S1 (Small)	0.011	0.055	B1 Low	-0.001	0.040	M1 (Losers)	-0.039	0.228
S2	0.010	0.039	B2	0.002	0.051	M2	-0.019	0.130
S3	0.008	0.038	B3	0.003	0.072	M3	-0.009	0.112
S4	0.007	0.039	B4	0.005	0.070	M4	-0.002	0.080
S5	0.007	0.040	B5	0.003	0.047	M5	-0.003	0.090
S6	0.003	0.030	B6	0.006	0.032	M6	0.002	0.075
S7	0.005	0.037	B7	0.005	0.034	M7	0.004	0.104
S8	0.004	0.065	B8	0.009	0.040	M8	0.012	0.092
S9	0.002	0.042	B9	0.008	0.046	M9	0.021	0.122
S10 (Large)	0.000	0.018	B10 (High)	0.011	0.089	M10 (Winners)	0.028	0.178

Source: Bansal, Dittmar, Lundblad (2005), Table II.

Small firms having the highest cash flow growth is as expected. However, high book/market firms, “value stocks,” display higher growth rates than the “growth stocks,” which helps BDL duplicate the value effect. Key to BDL being able to duplicate momentum effects is the result that the high positive momentum stocks display faster cash flow growth, while the negative momentum stocks have negative growth, perhaps not surprising, but interesting.

To estimate consumption risks, BDL use two alternative methods. First, they estimate γ_i to be the "projection of portfolio-specific dividend growth on the moving average of consumption growth," using an 8-quarter moving average of prior consumption growth with a lag length of 4 quarters, which is akin to Parker and Julliard's (2005) 11-quarter consumption calculations for "ultimate" consumption betas. Secondly, they estimate the sensitivity of the innovation in dividend growth rates to the estimated innovation in consumption growth, which gives their β_{ig} estimates in Table 38:

Table 38

Portfolio Risk Measures

The table presents two alternative measures of the cash flow risk for 30 characteristic-sorted portfolios. Portfolios are formed on momentum (M), market capitalization (S), and book-to-market ratio (B). The variable $M1$ represents the lowest momentum (loser) decile, $S1$ the lowest size (small firms) decile, and $B1$ the lowest book-to-market decile. Data are converted to real using the PCE deflator. The data are sampled at the quarterly frequency, and cover the first quarter of 1967 through fourth quarter of 2001. The column labeled "γ_i" presents the projection coefficient from the following regression

$$g_{i,t} = \gamma_i \left(\frac{1}{K} \sum_{k=1}^{K} g_{c,t-k} \right) + u_{i,t},$$

where $g_{i,t}$ represents *demeaned* log real dividend growth rates on portfolio i and $g_{c,t}$ the *demeaned* log real growth rate in aggregate consumption. Standard errors for this regression are reported in the columns labeled "SE," and associated R^2 are presented in the adjacent column. We also present risk measures and standard errors obtained from regressing the cash flow innovation on the consumption innovation, as in equation (10). These measures are presented in the columns labeled "$\beta_{i,g}$," and standard errors are presented in the adjacent columns. Finally, we present the slope coefficients from regressing the innovation in dividend growth rates, $\upsilon_{i,t}$, from equation (9), on the innovation in consumption growth, $\eta_{c,t}$. These coefficients are presented in the column labeled "ϕ_i" with standard errors in the adjacent column. Standard errors are corrected for heteroskedasticity and autocorrelation using the procedure in Newey and West (1987).

	γ_i	SE	R^2	$\beta_{i,g}$	SE		γ_i	SE	R^2	$\beta_{i,g}$	SE		γ_i	SE	R^2	$\beta_{i,g}$	SE
S1	1.2	2.9	0.003	2.4	4.5	*B1*	3.0	2.9	0.039	5.9	5.1	*M1*	-8.9	7.5	0.010	-12.8	12.8
S2	2.8	2.5	0.034	6.1	4.7	*B2*	-3.4	2.3	0.029	-3.6	4.1	*M2*	-1.4	4.8	0.001	0.2	8.2
S3	0.8	1.9	0.003	1.4	3.3	*B3*	0.02	2.4	0.000	0.4	4.2	*M3*	-1.7	5.4	0.002	1.3	9.0
S4	0.8	1.4	0.003	1.0	2.1	*B4*	-0.3	2.8	0.000	-0.9	4.8	*M4*	-0.5	2.5	0.000	-0.1	4.1
S5	0.8	1.5	0.002	1.7	2.5	*B5*	0.5	1.8	0.001	0.5	3.3	*M5*	-0.7	3.8	0.000	2.0	5.6
S6	2.0	1.1	0.028	3.8	2.2	*B6*	1.7	1.5	0.018	3.5	2.9	*M6*	2.5	2.0	0.007	3.7	3.8
S7	1.4	1.6	0.009	2.5	2.8	*B7*	0.8	1.1	0.003	2.2	2.2	*M7*	4.2	4.1	0.010	5.0	7.0
S8	1.1	1.7	0.002	2.8	2.9	*B8*	4.4	1.7	0.076	7.0	3.1	*M8*	6.3	3.5	0.030	7.4	6.4
S9	1.1	1.0	0.004	2.2	1.7	*B9*	4.7	3.1	0.071	10.3	4.9	*M9*	7.6	5.8	0.025	11.6	10.0
S10	1.1	0.7	0.022	2.7	1.2	*B10*	8.4	4.1	0.057	16.7	7.7	*M10*	11.6	5.7	0.027	15.2	12.4

Source: Bansal, Dittmar, Lundblad (2005), Table III.

There is some instability in these cash flow consumption risk estimates, and there is no clear relation of beta estimates to size. However, for book/market portfolios, the value stocks do have noticeably higher cash flow consumption betas than do the growth stocks, and the high momentum stocks also have higher cash flow consumption betas than the negative momentum stocks. Note that the standard errors are high relative to risk estimates, and R^2 are very low. So it is surprising that the estimated cross-sectional relations for returns versus these cash flow consumption risks show such a strong relationship. The cross-sectional evidence (using GMM estimates) is in Table 40, with cash flows measured in 2 ways and risks measured in 2 ways:

Table 39

Cross-Sectional Evidence: Returns vs. Cash Flow Consumption Betas

	$\lambda 0$	Std Error	t-stat	λc	Std Error	t-stat	R^2
			Panel A:Dividends				
			Independent Variable Is γi				
Coeff.	1.754	(0.815)	2.15	0.177	(0.072)	2.46	0.663
			Independent Variable Is $\beta i,g$				
Coeff.	1.658	(0.837)	1.98	0.118	(0.027)	4.37	0.620
			Panel B:Dividends Plus Repurchases				
			Independent Variable Is γi				
Coeff.	1.741	(0.851)	2.05	0.166	(0.057)	2.91	0.607
			Independent Variable Is $\beta i,g$				
Coeff.	1.697	(0.859)	1.98	0.105	(0.030)	3.50	0.456

Source: Bansal, Dittmar, Lundblad (2005), Table IV. GMM estimates.

Bansal, Dittmar and Lundblad's table shows a significant cross-sectional relationship of risk premiums with cashflow consumption betas from dividends, which explains 62% to 66% of return premiums. With repurchases added, the relationship is a bit weaker, but still strong, with R^2 ranging from 46% to 61%. All in all, this is quite a good showing for cash flow consumption betas. Note that BDL confirmed the Lettau and Ludvigson result that unconditional consumption betas fit returns very poorly.

Hansen, Heaton and Li (HHL, 2008), in the colorful title "Consumption Strikes Back? Measuring Long Run Risk," carefully developed an interesting model that contained both short-term, transitory shocks and long-term, permanent shocks in an economy of consumers with Epstein Zin and Weil preferences. Impulse response functions for both shocks are in Figure 29:

Figure 29

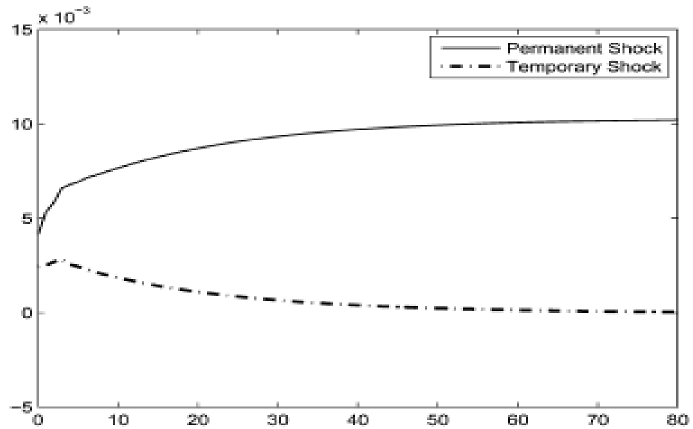


FIG. 3.—Impulse responses of consumption to permanent and temporary shocks implied by bivariate VARs where consumption and earnings are assumed to be cointegrated. Each shock is given a unit impulse. Responses are given at quarterly intervals.

Source: Hansen, Heaton and Li (2008), Figure 3.

HHL model corporate profits as being cointegrated with consumption and study the valuation of growth and value stocks as an application of their model. The following figure shows how differently cash flows of value and growth stocks move, relative to consumption:

Figure 30

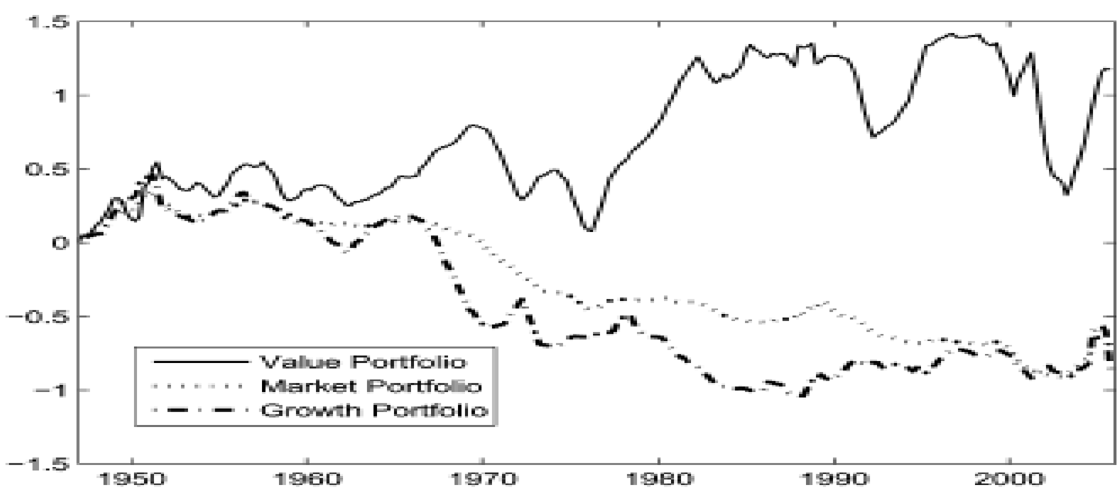


FIG. 1.—Cash flows relative to consumption for two portfolios. Quarterly logarithms of the ratios of portfolio cash flows to consumption are depicted.

Source: Hansen, Heaton and Li (2008), Figure 1.

While the return of value stocks over growth is well-known, the much greater long-run volatility of value stocks' performance in consumption units is quite noticeable. By contrast, movements in growth stocks and the market portfolio are tame. As Hansen, Heaton and Li observe (p. 261): "We find that the cash flows of value portfolios exhibit positive comovement in the long run with macroeconomic shocks, whereas the growth portfolios show little covariation with these shocks. Equilibrium pricing reflects this heterogeneity in risk exposure: risk-averse investors must be compensated to hold value portfolios." Using data for the Fama-French (1992) high book/market portfolio 5 versus that for the growth portfolio 1, they graph the impulse responses of cash flows of both to a permanent shock in consumption as in the next figure:

Figure 31

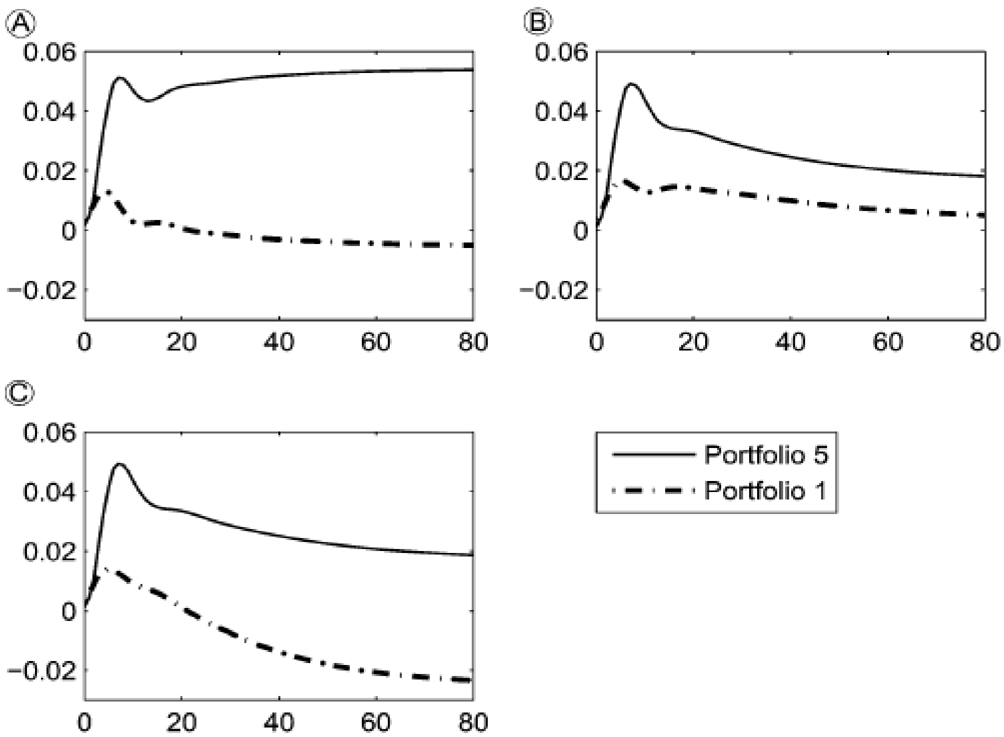


FIG. 8.—Impulse responses to a permanent shock to consumption of the cash flows to portfolios 1 and 5: *A*, from the first-difference specification used as our baseline model; *B*, from the level specification without time trends; *C*, from the level specification with time trends.

Source: Hansen, Heaton and Li (2008), Figure 8.

This shows that consumption shocks have transitory impacts on growth stock performance, but permanent impacts on value stocks, leading to the value premium.

In another significant article with conditional consumption betas, Bansal, Dittmar and Kiku (BDK, 2009) found that betas generated by an error correction, vector autoregressive (EC-VAR) model with cointegration restrictions between dividends and consumption can explain cross-sectional differences at many horizons, outperforming the model without the cointegration restriction. As they say (p. 1344), "…the deviation of the level of dividends from consumption (the error correction variable) is important for predicting dividend growth rates and returns at all horizons." And then they say (p.1344): "Imposing cointegration, we are able to predict on average 11.5% of the variation in one-year returns, compared to 7.5% when we do not impose cointegration. This difference is even starker at longer horizons: at the 10-year horizon, the EC-VAR specification results in an average 44.0% adjusted R^2, compared to 9.9% for the standard growth-rate VAR specification." This predictability evidence, they claim, has important implications for measuring return innovations and, consequently, conditional consumption betas. They argue that the error correction term of the cointegrated VAR contains important information and can predict future dividend growth by conditioning, which translates into longer-term return predictability. Their empirical results suggest that return predictability increases more than dividend predictability does when adding the EC term into the VAR model.

Please note that Bansal, Dittmar and Kiku (2009) assume that all individuals are identical and have a standard, time-additive power utility function with everyone having the same CRRA. This is a special case of the original derivations of the CCAPM in the late 1970s (See Section II), some of which were derived for heterogeneous individuals and more general time-additive utility and do not assume that anyone has CRRA utility. So all of the results and insights of those original models must apply to this one and the contributions here are empirical, not theoretical. Bansal, Dittmar and Kiku use annual data from 1929-2002, quite a different sample than for Bansal, Dittmar, Lundblad's 1963-2001 quarterly sample. Their data for dividends and returns are in the next table for dividend growth and returns for size and book/market decile portfolio: The BDK longer-term calculations use more stable annual data and appear to have more continuous estimated movements in returns and risk measures for size and book/market portfolios than in BDL (2005):

Table 40

Summary statistics

Portfolio	Cash flow growth Mean	Cash flow growth SD	Returns Mean	Returns SD
S1	0.0657	0.3969	0.1345	0.3388
S2	0.0829	0.4181	0.1235	0.2911
S3	0.0269	0.3518	0.1278	0.3163
S4	0.0238	0.2637	0.1256	0.2957
S5	0.0215	0.2463	0.1199	0.2797
S6	0.0168	0.2019	0.1106	0.2642
S7	0.0195	0.1684	0.1076	0.2517
S8	0.0120	0.1549	0.0994	0.2308
S9	0.0092	0.1303	0.0904	0.2181
S10	0.0034	0.1053	0.0758	0.1906
B1	−0.0026	0.1634	0.0701	0.2157
B2	0.0179	0.1598	0.0834	0.1902
B3	0.0041	0.1562	0.0727	0.1904
B4	0.0009	0.2194	0.0771	0.2224
B5	0.0160	0.1552	0.0963	0.2225
B6	0.0146	0.2461	0.0989	0.2305
B7	0.0165	0.2667	0.1069	0.2484
B8	0.0548	0.2194	0.1265	0.2187
B9	0.1009	0.3486	0.1360	0.2287
B10	0.0467	0.6799	0.1337	0.3315

Source: Bansal, Dittmar, Kiku (2009), Table 1. 1929-2002 annual data.

As BDK observe, "Cointegration implies that the dividend growth rates are predicted by the cointegrating residuals. That is, the current deviations of an asset's cash flows from their long-run relation with consumption should forecast the dynamics of dividend growth rates while dividends are moving back toward equilibrium." As dividends are a key element in investment returns, especially in the longer term, this may also translate into return predictability. Comparisons of the predictability of dividend growth and stock returns for the 20 portfolios with an error-correction vector autoregression (EC-VAR) with a standard VAR model are in Tables 41 and 42, respectively:

Table 41

Predictability evidence: dividend growth

	Horizon					
	1 year		5 years		10 years	
Portfolio	EC-VAR	VAR	EC-VAR	VAR	EC-VAR	VAR
S1	0.00	0.04	0.14	0.20	0.21	0.22
S2	0.11	0.00	0.15	0.01	0.14	0.01
S3	0.44	0.52	0.10	0.25	0.05	0.29
S4	0.11	0.10	0.09	0.19	0.07	0.21
S5	0.11	0.07	0.13	0.20	0.10	0.24
S6	0.15	0.16	0.12	0.21	0.10	0.22
S7	0.12	0.08	0.21	0.22	0.17	0.25
S8	0.12	0.06	0.22	0.22	0.17	0.28
S9	0.17	0.15	0.24	0.29	0.17	0.30
S10	0.19	0.03	0.43	0.15	0.35	0.19
B1	0.27	0.06	0.55	0.21	0.46	0.29
B2	0.25	0.08	0.52	0.31	0.37	0.37
B3	0.24	0.12	0.54	0.40	0.48	0.48
B4	0.39	0.25	0.48	0.51	0.42	0.39
B5	0.14	0.19	0.24	0.31	0.21	0.24
B6	0.23	0.25	0.21	0.37	0.11	0.24
B7	0.34	0.37	0.15	0.34	0.07	0.22
B8	0.17	0.14	0.16	0.26	0.09	0.22
B9	0.13	0.24	0.24	0.40	0.15	0.26
B10	0.07	0.10	0.17	0.22	0.19	0.27

This table presents the adjusted R^2 for dividend projections implied by the EC-VAR specification and the growth-rate VAR model that does not assume the long-run relation between assets' cash flows and consumption. The entries are reported for the 20 portfolios sorted by market capitalization (S1–S10) and book-to-market ratio (B1–B10). Data are sampled at the annual frequency, expressed in real terms, and cover the period 1929–2002.

Source: Bansal, Dittmar, Kiku (2009), Table 3.

Table 42

Predictability evidence: returns

	Horizon					
	1 year		5 years		10 years	
Portfolio	EC-VAR	VAR	EC-VAR	VAR	EC-VAR	VAR
S1	0.17	0.06	0.40	0.04	0.36	0.01
S2	0.13	0.11	0.53	0.35	0.59	0.44
S3	0.10	0.03	0.40	0.08	0.53	0.13
S4	0.15	0.10	0.44	0.18	0.50	0.20
S5	0.16	0.15	0.36	0.21	0.49	0.26
S6	0.11	0.07	0.37	0.17	0.47	0.18
S7	0.09	0.08	0.29	0.17	0.46	0.23
S8	0.08	0.05	0.32	0.14	0.53	0.21
S9	0.04	0.01	0.28	0.12	0.46	0.16
S10	0.03	0.02	0.23	0.15	0.43	0.24
B1	0.04	0.00	0.19	0.03	0.32	0.04
B2	0.07	0.00	0.23	0.04	0.36	0.06
B3	0.23	0.14	0.27	0.06	0.43	0.07
B4	0.06	−0.01	0.26	−0.02	0.44	−0.03
B5	0.04	0.02	0.24	0.07	0.37	0.06
B6	0.08	0.06	0.26	0.03	0.37	0.00
B7	0.15	0.06	0.44	0.11	0.42	0.05
B8	0.17	0.11	0.49	0.21	0.41	0.18
B9	0.20	0.06	0.42	0.03	0.35	−0.01
B10	0.20	0.09	0.48	0.19	0.51	0.17

This table presents the adjusted R^2 for return projections implied by the EC-VAR specification and the growth-rate VAR model that does not assume the long-run relation between assets' cash flows and consumption. The entries are reported for the 20 portfolios sorted by market capitalization (S1–S10) and book-to-market ratio (B1–B10). Data are sampled at the annual frequency, expressed in real terms, and cover the period 1929–2002.

Source: Bansal, Dittmar, Kiku (2009), Table 4.

Examining Tables 42 and 43, we see the following: the cointegrated model, EC-VAR, is better than the standard VAR in the short and medium term, which makes sense as the error correction model picks up transitory variation in dividend growth rates. With regard to return predictability, the cointegrated model, EC-VAR, does has much better predictive accuracy, causing BDK to observe that (p. 1358): "…the cointegrating residual, included in the error-correction specification, contains distinct information about future returns beyond that in the growth-rate-based model." As return innovations differ across the two models, consumption betas will also differ between the models. Table 44 gives the estimated consumption betas in the cointegrated model. Notice the large differences in EC-VAR versus the VAR estimates:

Table 43

Consumption betas by horizon

		Horizon					
		1 year		5 years		10 years	
Portfolio	Unconditional	EC-VAR	VAR	EC-VAR	VAR	EC-VAR	VAR
S1	0.71 (1.47)	4.12 (2.43)	1.77 (2.10)	4.51 (4.66)	−1.46 (3.94)	6.54 (4.26)	−1.55 (4.01)
S2	0.80 (1.38)	2.09 (1.12)	0.89 (1.09)	1.82 (2.65)	−1.24 (2.42)	4.21 (3.02)	0.13 (2.37)
S3	0.52 (1.36)	4.14 (1.29)	2.95 (1.98)	2.14 (2.38)	0.04 (2.44)	3.38 (2.29)	0.35 (2.86)
S4	0.77 (1.13)	3.52 (1.21)	2.66 (1.50)	2.09 (1.97)	0.79 (1.94)	3.56 (2.40)	1.59 (1.76)
S5	0.36 (1.21)	2.76 (0.99)	2.36 (1.22)	0.99 (1.72)	0.39 (1.52)	2.05 (2.12)	1.14 (1.37)
S6	0.64 (1.10)	3.07 (0.81)	2.58 (1.07)	1.42 (1.51)	0.71 (1.87)	2.50 (1.83)	1.41 (1.56)
S7	0.33 (1.10)	2.37 (0.86)	2.20 (0.82)	0.46 (1.37)	0.14 (1.28)	1.00 (1.67)	0.61 (1.18)
S8	−0.31 (1.14)	1.62 (0.68)	1.43 (0.76)	−0.12 (1.35)	−0.44 (1.62)	0.35 (1.68)	−0.13 (1.64)
S9	0.13 (1.09)	1.58 (0.77)	1.38 (0.80)	0.60 (1.41)	0.27 (1.62)	1.02 (1.59)	0.62 (1.49)
S10	0.69 (0.83)	1.54 (0.56)	1.64 (0.46)	0.31 (1.09)	0.51 (1.09)	0.34 (1.07)	0.67 (1.04)
B1	0.82 (0.97)	1.81 (0.54)	2.16 (0.33)	−0.58 (1.40)	0.31 (1.48)	−0.83 (1.32)	0.14 (1.63)
B2	−0.18 (0.84)	0.16 (0.56)	0.65 (0.39)	−1.69 (0.91)	−0.76 (0.98)	−2.05 (0.86)	−0.86 (1.06)
B3	−0.33 (0.84)	−0.09 (0.37)	0.32 (0.37)	−1.79 (0.77)	−1.34 (0.85)	−1.70 (0.84)	−1.33 (0.98)
B4	0.29 (1.10)	1.48 (1.29)	1.94 (1.50)	−0.67 (1.99)	−0.07 (2.10)	−0.59 (2.06)	−0.28 (2.22)
B5	0.27 (1.11)	1.94 (0.86)	1.67 (0.96)	1.18 (1.58)	0.81 (1.79)	1.60 (1.69)	1.10 (1.57)
B6	2.24 (1.01)	3.18 (1.49)	2.64 (1.97)	2.75 (2.42)	1.91 (1.96)	3.27 (2.37)	2.17 (1.77)
B7	0.21 (1.24)	2.74 (0.98)	1.46 (1.68)	2.67 (1.53)	1.01 (1.59)	4.22 (1.63)	1.70 (1.69)
B8	0.84 (1.23)	4.34 (1.83)	2.18 (2.14)	4.39 (2.72)	0.37 (2.22)	6.36 (2.69)	0.98 (2.34)
B9	−0.39 (1.54)	5.47 (2.31)	2.16 (2.91)	6.11 (3.75)	−1.55 (2.72)	8.32 (3.43)	−2.03 (2.79)
B10	0.14 (1.63)	3.89 (1.06)	2.87 (1.03)	2.14 (2.60)	0.54 (3.15)	4.33 (3.38)	1.49 (3.54)

This table presents consumption betas for investment horizons of 1, 5, and 10 years for each of the 20 portfolios sorted by market capitalization (S1–S10) and book-to-market ratio (B1–B10). In columns labeled "EC-VAR," betas are measured using the error correction specification for consumption and asset returns. Columns labeled "VAR" present betas measured using a growth-rate VAR omitting the error correction information. These consumption betas are estimated as in Equation (17), using the covariance matrices implied by the relevant time-series model. The column labeled "Unconditional" represents the standard consumption beta. Robust standard errors are reported in parentheses. The number of lags used in the Newey and West (1987) covariance estimator is 8.

Source: Bansal, Dittmar, Kiku (2009), Table 6.

A close examination of Tables 42 and 43 confirm that : "Neither the unconditional betas nor those based on the VAR reflect the cross-sectional differences in mean returns on size and book-to-market-sorted portfolios." Table 44 gives the one-step GMM estimates of the market prices of risk that are jointly estimated with the time-series parameters. The R^2 values for the error-correction model are very high, at 73% to 84%, in contrast to the standard VAR results. Also, the estimated prices of risk are all estimated as 2 to 3 times their standard errors.

Table 44

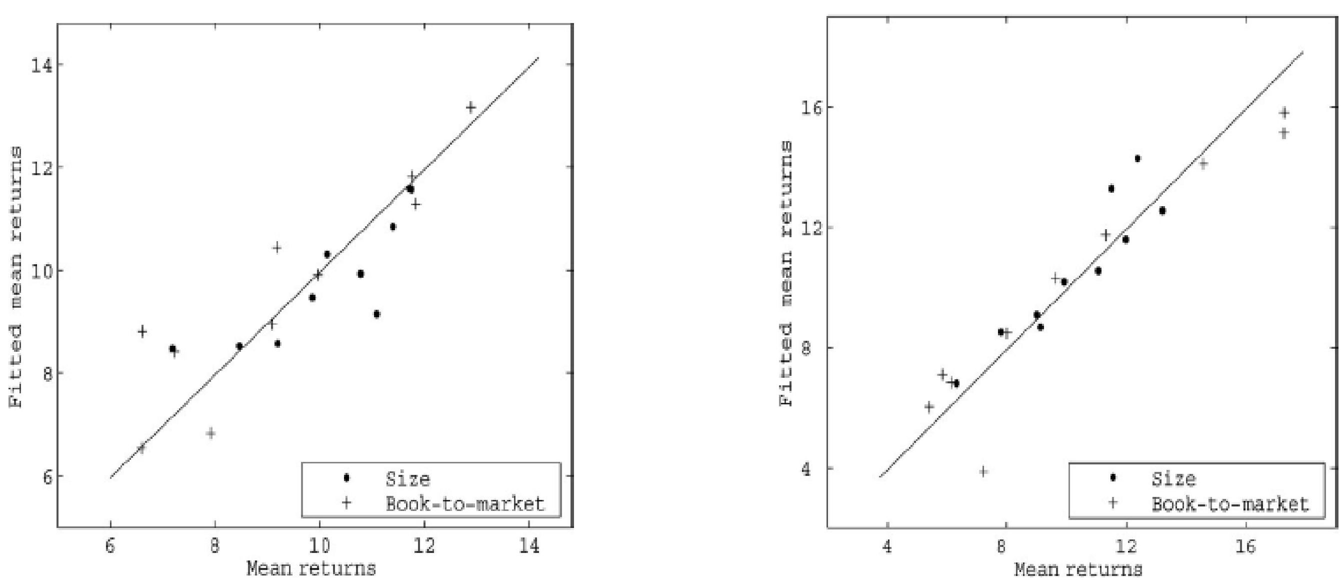
Cross-sectional regressions by horizon

		Horizon					
		1 year		5 years		10 years	
	Unconditional	EC-VAR	VAR	EC-VAR	VAR	EC-VAR	VAR
$\lambda_{1,s}$	0.51	1.19	1.28	0.73	−0.31	0.65	−0.07
SE	(2.24)	(0.41)	(1.57)	(0.32)	(0.40)	(0.24)	(0.44)
$\bar{R}^2$	−0.04	0.75	0.22	0.73	−0.03	0.84	−0.05

Source: Bansal, Dittmar, Kiku (2009), Table 7.

Graphs of the very strong 1-year and long-term fits of the EC-VAR model are in Figure 32:

Figure 32



(a) Short-run horizon ($s = 1$)

(b) Long-run horizon

Source: Bansal, Dittmar, Kiku (2009). Figure 2.

Strong results with consumption betas require modeling of conditional consumption betas. From the works of Lettau and Ludvigson (L-L, 2001b), Bansal, Dittmar, Lundblad (2005), Jagannathan and Wang (2007), and Bansal, Dittmar, Kiku (2009), we learned that it is very important to have a conditioning variable for consumption betas, as they change over time and economic states. L-L conditioned upon *cay,* their variable that represents consumption's deviations from a broad wealth variable (stocks, bonds and real estate) that also includes capitalized wage income. In contrast, J-W quite reasonably conditioned upon NBER-designated recession and expansion periods. Bansal, Dittmar and Kiku conditioned upon the deviations of dividends from their long-term trend with consumption.

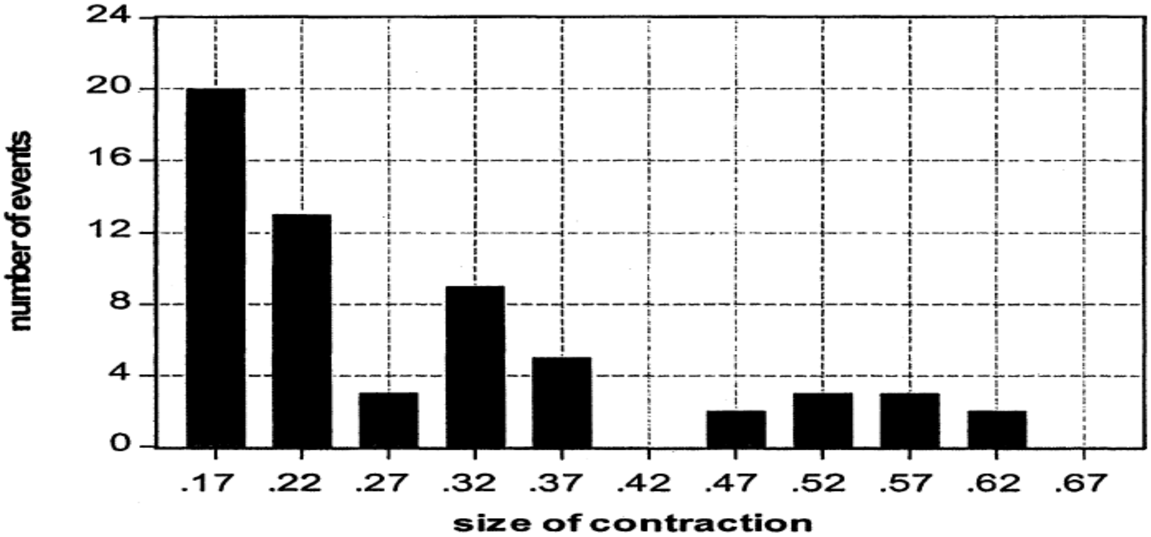
XIII. More Consumption-Based Pricing Research in the Past Decade

The literature on consumption-based asset pricing is vast and burgeoning, so it is impossible to properly present and review all of the excellent research in a single article. In this section, we examine additional articles that were published in the past decade, 2004-2014.

A: Risks of Rare Disasters

Almost two decades after Rietz's (1988) classic work on a disaster risk explanation of Mehra and Prescott's (1985) equity premium puzzle, Barro (2006) and Barro and Ursua (2008) and then Wachter (2013) gathered new data and built new models of the risks of rare disasters. As Barro (2006, p. 823) said "I think that Rietz's basic reasoning is correct, but the profession seems to think differently…" and "… the major reason for skepticism about Rietz's argument is the belief that it depends on counterfactually high probabilities and sizes of economic disasters." Barro, using data from Maddison (2003) (with some corrections), found 60 instances among 20 OECD countries of peak-to-trough declines in real per capita GDP of 15% or more in the twentieth century. The average decline was 29% of GDP. Barro computes the probability of such a -15% or more disaster to be 1.7% per year, on average, using data for the 20 OECD countries for which there was data. Barro finds the frequency distribution of contractions to be as in the following graph:

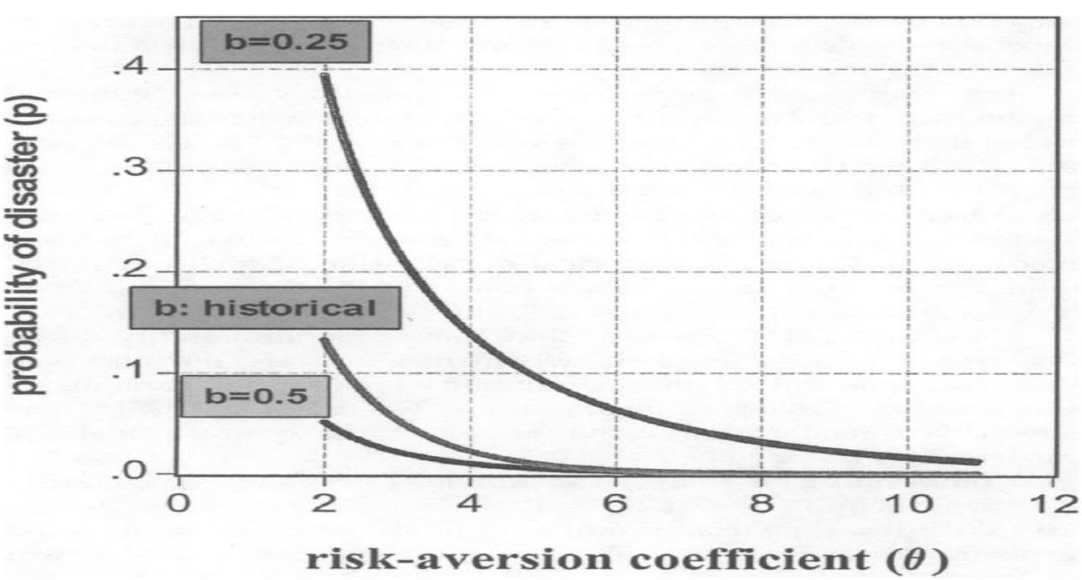
Figure 33



Source: Barro (2006), Figure I.

Barro also compute the combination of disaster probability and relative risk aversion that would explain the observed equity risk premium. The result is in the following graph, where *b* is the loss that occurs in a typical disaster, which is simulated to be a 25% to 50% decline in real GDP, peak to trough. Note that CRRA = 4.3, a not implausible level of risk aversion, combined with the historic probability of 1.7% per year, fits the historic equity risk premium.

Figure 34



Source: Barro (2006), Figure II.

More recently, Wachter (2013) pointed out that Rietz's and Barro's models would have stock market volatility equal to the volatility of dividends, which is so low as to not be realistic. Wachter shows that modeling the probability of a consumption disaster as time-varying can solve the problem. As she says (p. 987), "The possibility of this poor outcome substantially increases the equity premium, while time variation in the probability of this outcome drives high stock market volatility and excess return predictability." Wachter's specific model for the stochastic process for aggregate consumption is as follows:

$$dC_t = \mu C_{t^-}\, dt + \sigma C_{t^-}\, dB_t + (e^{Z_t} - 1)C_{t^-}\, dN_t, \qquad (73)$$

where B_t is a standard Brownian motion and N_t is a Poisson process time-varying intensity λ_t.[2] This intensity follows the process

$$d\lambda_t = \kappa(\bar{\lambda} - \lambda_t)\, dt + \sigma_\lambda \sqrt{\lambda_t}\, dB_{\lambda,t}, \qquad (74)$$

Thus, Wachter has a mixed jump-diffusion process. In normal times, when no disaster takes place, consumption follows a continuous diffusion process. Disasters are captured by the Poisson jumps downward in consumption, where: "Roughly speaking, λ_t can be thought of as the disaster probability over the course of the next year." (p. 991)

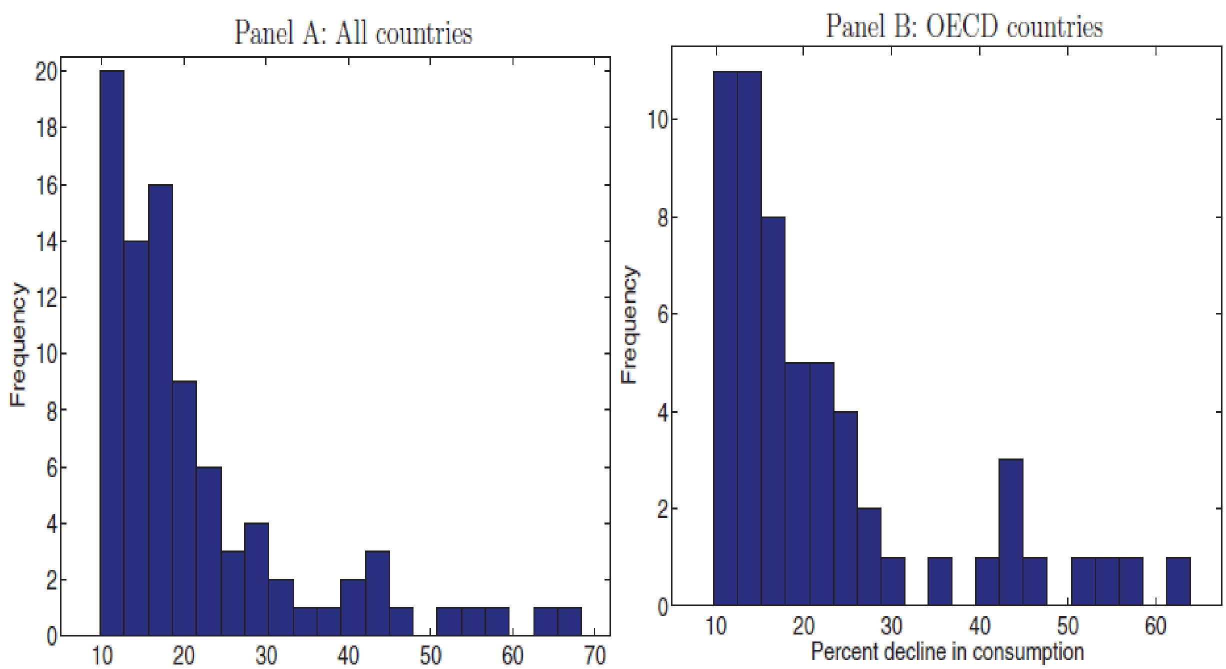
The equity premium in Wachter's model is different from previous models, reflecting the presence of disaster risk, and then also reflecting the time variation in disaster risk, as follows (her eq. 28):

$$r_t^e - r_t = \underbrace{\phi\gamma\sigma^2}_{\text{standard model}} \underbrace{- \lambda_t \frac{G'}{G} b\sigma_\lambda^2 + \underbrace{\lambda_t E_\nu[(e^{-\gamma Z} - 1)(1 - e^{\phi Z})]}_{\text{static disaster risk}}}_{\text{time-varying disaster risk}}. \qquad (75)$$

Note that the second term produces time variation in the equity premium that reflects changes in the disaster jump intensity, λ_t, while the third term gives the impact of a static amount of rare disaster risk.

For her calibration and simulation, Wachter uses data from Barro and Ursua (2008) on consumption declines, wherein a 10% or more decline is termed a disaster (rather than a 15% decline as in Barro (2006)). With this definition, the disaster probability, λ, equals 3.55%, using data from 22 countries from 1870 to 2006. A developed country subset (termed "OECD countries") has a slightly lower disaster probability of 2.86%. The frequency distributions of consumption declines for the two data sets are in the next figures.

Figure 35



Source: Wachter (2013), Figure 7.

Wachter assumes relative risk aversion of 3.0 and a rate of time preference equal to 1.2%, which allows her to match the average real return on the 3-month Treasury bill in postwar U.S. data. Her simulation results, using the data for all 22 countries are in the following table:

Table 45

Population Moments from Simulated Data and Sample Moments from the Historical Time Series

The model is simulated at a monthly frequency and simulated data are aggregated to an annual frequency. Data moments are calculated using overlapping annual observations constructed from quarterly U.S. data, from 1947 through the first quarter of 2010. With the exception of the Sharpe ratio, moments are in percentage terms. The second column reports population moments from simulated data. The third column reports moments from simulated data that are calculated over years in which a disaster did not occur. The last column reports annual sample moments. R^b denotes the gross return on the government bond, R^e the gross equity return, Δc growth in log consumption, and Δd growth in log dividends.

	Model		
	Population	Conditional	U.S. Data
$E[R^b]$	0.99	1.36	1.34
$\sigma(R^b)$	3.79	2.00	2.66
$E[R^e - R^b]$	7.61	8.85	7.06
$\sigma(R^e)$	19.89	17.66	17.72
Sharpe Ratio	0.39	0.49	0.40
$\sigma(\Delta c)$	6.36	1.99	1.34
$\sigma(\Delta d)$	16.53	5.16	6.59

Source: Wachter (2013), Table II.

Note that Wachter's "Conditional" results, when there was no disaster, match the U.S. postwar data quite well. However, the "Population" results, which include a normal fraction of disasters, have consumption volatility over 6%, in contrast to the 2% volatility with no disasters. Volatility of dividends depends also greatly upon whether or not there was a disaster in the sample period. In summary, Wachter's mixed jump-diffusion model does a good job at mimicking historic data when there are no disasters, and shows what data should look like over the very, very long term when we have a normal (small) proportion of disasters in the sample.

B. Predictability of Dividends and Returns

In 2005, Lettau and Ludvigson, modeled consumption, stock dividends and labor income (dividends from human capital) as having a three-way cointegrated relationship, much as in their 2001 article that developed *cay*, but with asset wealth being replaced by dividend growth via the Campbell-Mankiw (1989) derivation. In the post-World War II era, dividend yield has been a less useful predictor of stock returns. Lettau and Ludvigson argue that this is in part the result of the offsetting effects of dividend growth and equity risk premiums. In a recession, risks likely increase, dividend yields increase as stock prices fall more than dividends, and projected returns on stocks increase to compensate for higher risk. However, if dividends have fallen, expected dividend growth may increase, partially offsetting the effects of the higher risk premium. Lettau and Ludvigson find that this positive covariation of dividend growth with market risk premiums masks greater volatility in both in the post-War period. In the pre-war era, they find no evidence of this cointegrating relation and find little predictive power in the pre-War data.

Santos and Veronesi (2006) have a straightforward, but powerful model of time variation in the equity risk premium and changes in conditional risks of assets. They model consumer spending as being composed of two parts – the part funded by labor income and the part funded by financial assets, such as stock returns. Their hypothesis is that when labor income provides a larger fraction of financing for consumption (and stock returns provide a smaller portion), stock returns will be less correlated with optimal consumption at those times and will earn smaller risk premiums due to their smaller consumption betas then. In contrast, when stocks provide a large fraction of funds for consumption, covariance of consumption with stock returns increases and risk premiums on stocks should increase. Thus, as the ratio of labor income (compensation of employees data) to consumption (shown in Figure 36) increases, the equity risk premium should fall, a negative relationship.

Figure 36

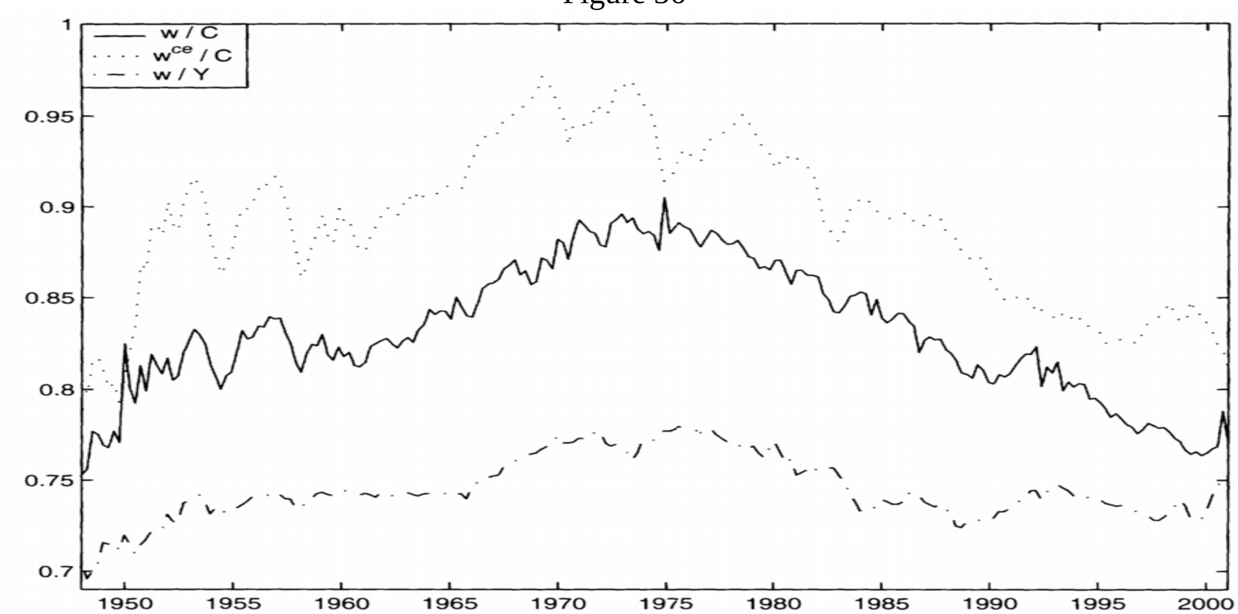


Figure 1
Shares of labor income to consumption
Current share of labor income to consumption, s_t^w, defined as (i) the ratio of labor income over consumption w/C (solid line), (ii) the ratio of compensation of employees over consumption w^{ce}/C (dotted line), or (iii) the ratio of labor income over disposable income w/Y (dash–dotted line). Data is quarterly and the sample period is 1948–2001.

Santos and Veronesi (2006), Figure 1.

In the graph, compensation of employees, the dotted line, is the broader measure of labor income, including bonuses, as well as wages and salaries. Note that in the sharp recessions in 1974/75 and 1981/82, the share of consumption paid for by labor income (which was quite high) dropped sharply, so that paid by financial assets provided a larger fraction and the conditional consumption beta of stocks and the time-varying risk premium should have increased. The next graph compares visually the ability of the labor income share to (negatively) predict subsequent 4-year returns on equities, compared with the ability of dividend yields to do so:

Figure 37

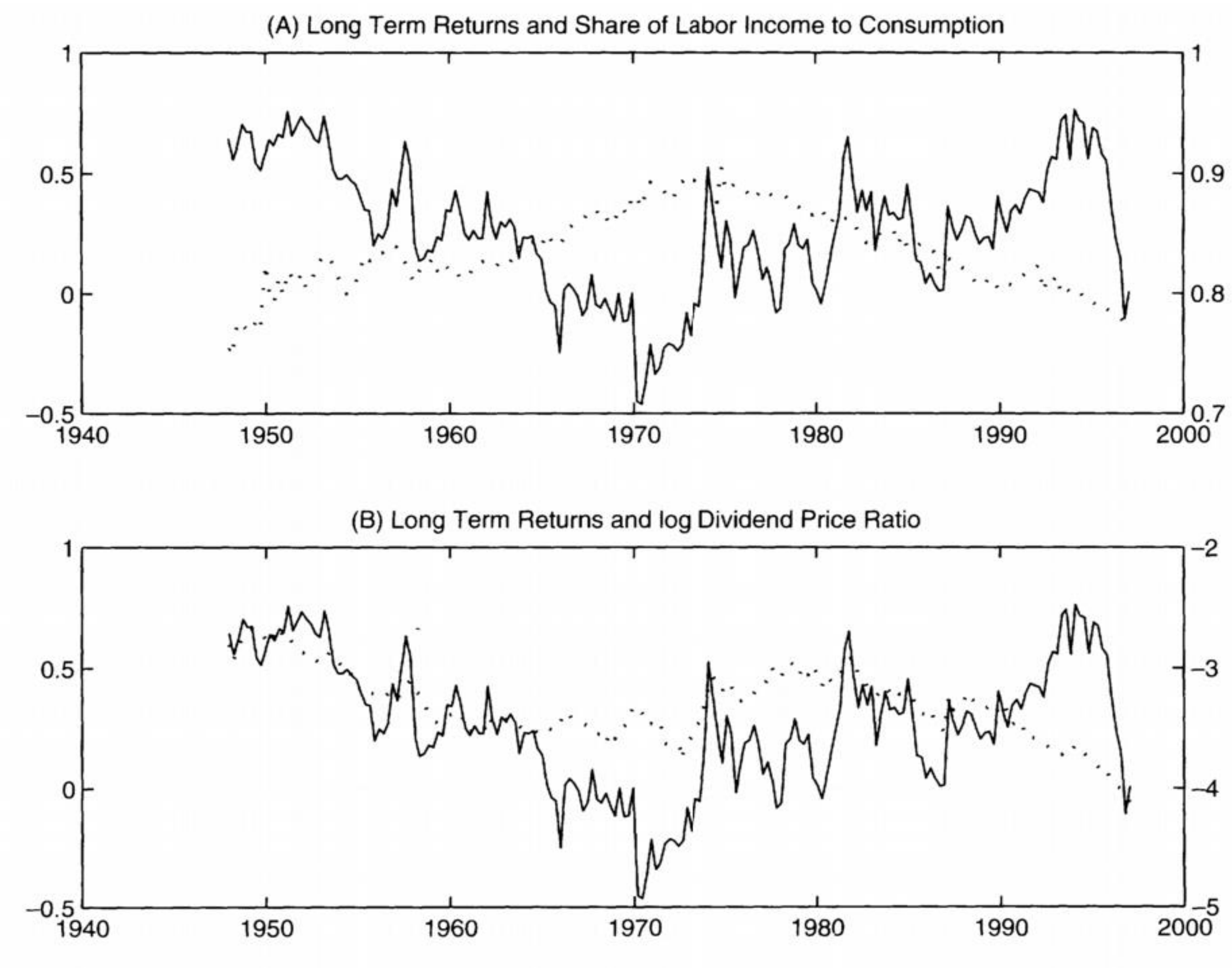


Figure 2
Long-term returns and predictive variables
Four-year cumulative market returns (solid line) lagged four years and the current share of labor income to consumption (dotted line, panel A), and the log dividend–price ratio (dotted line, panel B). Data is quarterly and the sample period is 1948–2001.

Source: Santos and Veronesi (2006), Figure 2.

Statistics for the ability of the labor income/consumption ratio and the dividend yield ratio to predict stock returns are in Table 46. The labor income/consumption ratio has greater ability to predict returns than does dividend yield during the post-War period. Long-term (4-year) predictability is quite high, with $R^2 = 0.42$ for the labor income/consumption ratio, versus 0.14 for dividend yield. Combining both predictors is better yet, with $R^2 = 0.57$ over the 1948-2001 sample period, all statistically significant.

Table 46

Forecasting regressions—$s_t^w = w_t/C_t$

	Panel A: Sample 1948–2001				Panel B: Sample 1948–1994			
Regression 1								
Horizon	4	8	12	16	4	8	12	16
s_t^w	−0.93	−2.48*	−4.01*	−5.25*	−1.39*	−3.04*	−4.41*	−5.54*
	(−1.44)	(−3.07)	(−4.24)	(−4.92)	(−2.30)	(−4.33)	(−4.39)	(−4.72)
	[−1.38]	[−1.93]	[−2.09]	[−2.14]	[−1.89]	[−2.11]	[−2.07]	[−1.99]
Adjusted R^2	0.04	0.16	0.32	0.42	0.07	0.21	0.35	0.44
Regression 2								
Horizon	4	8	12	16	4	8	12	16
$\log(D/P)$	0.13*	0.20	0.26	0.35	0.28*	0.48*	0.63*	0.78*
	(2.13)	(1.65)	(1.34)	(1.29)	(4.04)	(4.00)	(4.49)	(5.41)
	[2.24]	[1.75]	[1.49]	[1.60]	[3.66]	[2.95]	[2.50]	[2.43]
Adjusted R^2	0.09	0.10	0.11	0.14	0.19	0.32	0.43	0.54
Regression 3								
Horizon	4	8	12	16	4	8	12	16
s_t^w	−1.43*	−2.97*	−4.31*	−5.30*	−0.83	−2.00*	−3.03*	−3.72*
	(−2.49)	(−4.56)	(−5.56)	(−6.06)	(−1.51)	(−3.72)	(−4.92)	(−5.34)
	[−2.20]	[−2.32]	[−2.25]	[−2.17]	[−1.08]	[−1.36]	[−1.40]	[−1.27]
$\log(D/P)$	0.17*	0.26*	0.30*	0.35*	0.25*	0.40*	0.49*	0.60*
	(2.96)	(3.31)	(3.54)	(3.25)	(4.12)	(3.90)	(4.99)	(7.15)
	[2.97]	[2.40]	[1.85]	[1.75]	[3.12]	[2.37]	[1.88]	[1.75]
Adjusted R^2	0.17	0.33	0.47	0.57	0.21	0.40	0.57	0.71

The table summarizes the result of the predictive regression

$$r_{t,t+K} = \alpha + \beta(K)\mathbf{x}_t + \varepsilon_{t+K},$$

where $x_t = s_t^w, \log(D_t/P_t)$, or both, where K is the number of quarters ahead and $r_{t,t+K}$ is the cumulative log excess return over K quarters and $s_t^w = w_t/C_t$. Here, w_t is labor income as defined in Lettau and Ludvigson (2001a) and C_t is aggregate consumption of nondurable goods and services. Numbers in parenthesis show the Newey–West adjusted t-statistics, where the number of lags is double that of the forecasting horizon. The t-statistics computed with the Hodrick (1992) type 1B standard errors are reported in brackets. Data are quarterly and the sample is 1948–2001 (panel A) and 1948–1994 (panel B). *significance at the 5% level using the Newey–West adjusted t-statistic.

Source: Santos and Veronesi (2006), Table 2.

Using the labor income/consumption ratio as a conditioning variable, much as Lettau and Ludvigson (2001b) did with *cay*, Santos-Veronesi also get very positive results for the conditional CAPM, in cross-sectional fits of returns of the 25 Fama-French portfolios sorted by size and book/market.

C. Durable Goods Consumption, Systematic Risk and Asset Pricing

Yogo (2006) did very interesting research on durables consumption flows, finding asset pricing, risk measurement and risk premium results that were quite strong. This is especially interesting, as so many researchers simply use the nondurables and services part of consumption and exclude durables, due to the fact that only a portion of the durable is consumed annually (6%, as estimated by the U.S. Bureau of Economic Analysis). Yogo found that the ratio of durables consumption flows to nondurables and services is highly procyclic, with durables moving more sharply up and down with the economy than NDS consumption. This seems plausible introspectively, in that when individuals have moved up significantly in wealth or income, often they buy nice durable goods and luxury items, such as cars, jewelry, or vacation homes and control (reduce) marginal utility optimally in that way. Of course, in recessions, durables purchases are sharply curtailed and households live off their old durables stocks. Thus, durables spending could be quite an excellent signal of changes in marginal utility, which is indeed what Yogo found. A graph of the ratio of Durables stocks to nondurables consumption is in Figure 33. It shows the very procyclic nature described, as well as the increasing share of real durables consumption in total consumption, in part responding to the relative price decline of durables.

The correlations of the three Fama-French factors with nondurables and durables consumption with quarterly data from 1951 to 2001 are in the following table, along with mean, volatility and autocorrelation statistics. Note that durables consumption growth is highly autocorrelated, much more so than is nondurables (0.88 vs. 0.28). Correlations of durables consumption growth with the stock market and SMB and HML factors are all low, and the correlation of these quarterly changes with nondurables spending is only 0.19.

Figure 38

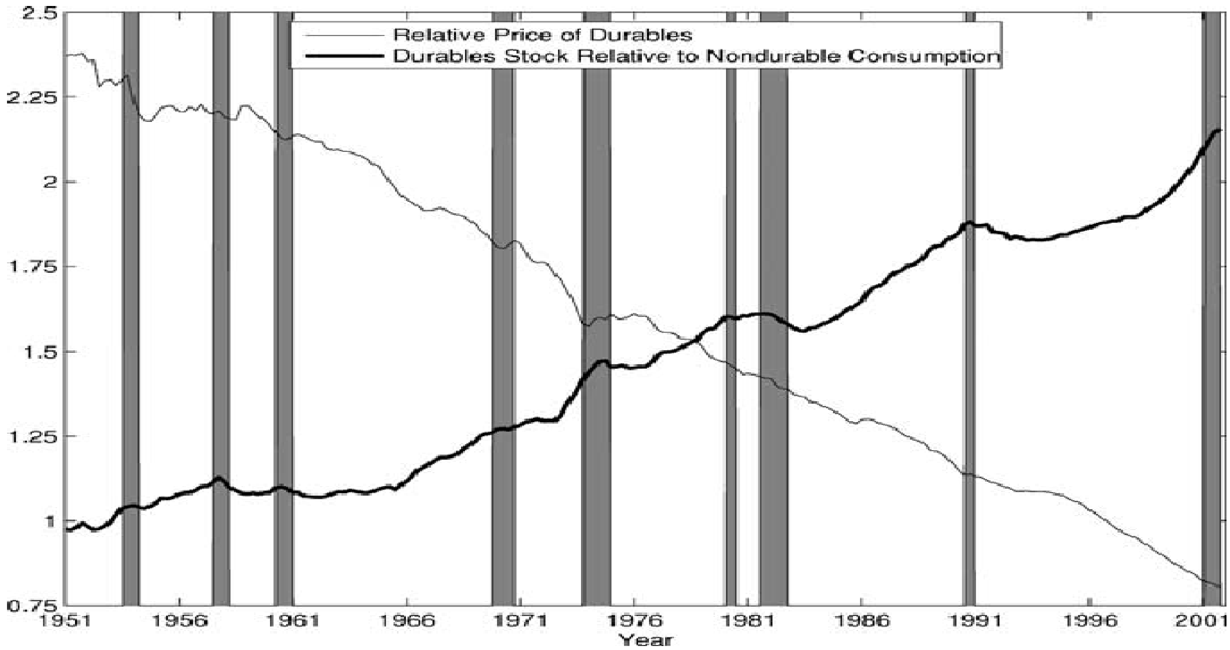


Figure 1. Price and Stock of Durables Relative to Nondurables. The figure is a time-series plot of (1) the price of durables as a ratio of the price of nondurables and (2) the real stock of durables as a ratio of real nondurable consumption. The sample period is 1951:1–2001:4; the shaded regions are NBER recessions.

Source: Yogo (2006), Figure 1.

Table 47

Descriptive Statistics

The table reports the mean, standard deviation, and first-order autocorrelation of excess market return, SMB return, HML return, and nondurable and durable consumption growth. It also reports the correlations among these variables.

Variable	Mean (%)	*SD* (%)	Autocorrelation	Correlation			
				Market	SMB	HML	Nondurables
Market	1.880	8.186	0.048				
SMB	0.508	5.580	−0.034	0.423			
HML	1.089	5.543	0.154	−0.386	−0.143		
Nondurables	0.513	0.542	0.282	0.281	0.130	0.004	
Durables	0.915	0.535	0.875	−0.110	−0.038	0.036	0.192

Source: Yogo (2006), Table I.

Yogo provides first-stage GMM estimates of consumption betas of the 25 (5x5) Fama-French portfolios sorted by size and book/market, as in Table 47. Note the much sharper correspondence of stocks' average excess returns between stocks' betas with durables consumption than with their betas with nondurables.

Table 48

Average Returns and Consumption Betas for the Fama–French Portfolios

Panel A reports average excess returns (per quarter) on the 25 Fama–French portfolios sorted by size and book-to-market equity. Panels B and C report nondurable and durable consumption betas, implied by the first-stage GMM estimate of the durable consumption model, respectively. The last row reports the difference between small and big stocks, and the last column reports the difference between high and low book-to-market stocks.

	Book-to-Market Equity					
Size	Low	2	3	4	High	High–Low
Panel A: Average Excess Return (%)						
Small	1.121	2.448	2.531	3.160	3.464	2.343
2	1.458	2.225	2.716	2.929	3.150	1.692
3	1.707	2.345	2.313	2.756	2.937	1.230
4	1.896	1.797	2.417	2.568	2.725	0.829
Big	1.686	1.652	2.015	1.987	2.140	0.454
Small−Big	−0.565	0.796	0.516	1.173	1.324	
Panel B: Nondurable Consumption Beta						
Small	6.512	6.126	5.814	5.438	6.216	−0.296
2	6.071	5.119	5.241	5.436	5.899	−0.172
3	5.457	5.142	5.057	5.159	5.926	0.469
4	4.923	4.302	4.465	5.225	5.061	0.137
Big	4.759	3.547	2.974	4.242	3.967	−0.792
Small−Big	1.754	2.578	2.841	1.196	2.249	
Panel C: Durable Consumption Beta						
Small	0.317	1.209	1.638	2.271	2.502	2.185
2	0.120	1.089	1.838	1.834	1.967	1.847
3	0.517	1.193	1.434	1.857	1.979	1.461
4	0.904	0.676	1.347	1.798	1.838	0.934
Big	0.956	0.750	1.268	1.396	1.325	0.368
Small−Big	−0.640	0.459	0.370	0.875	1.177	

Source: Yogo (2006), Table IV.

Graphically, the relation of cross-sectional returns vs. risks measured by market betas, the Fama-French 3-factor model, nondurables consumption betas and durables are in Figure 39:

Figure 39

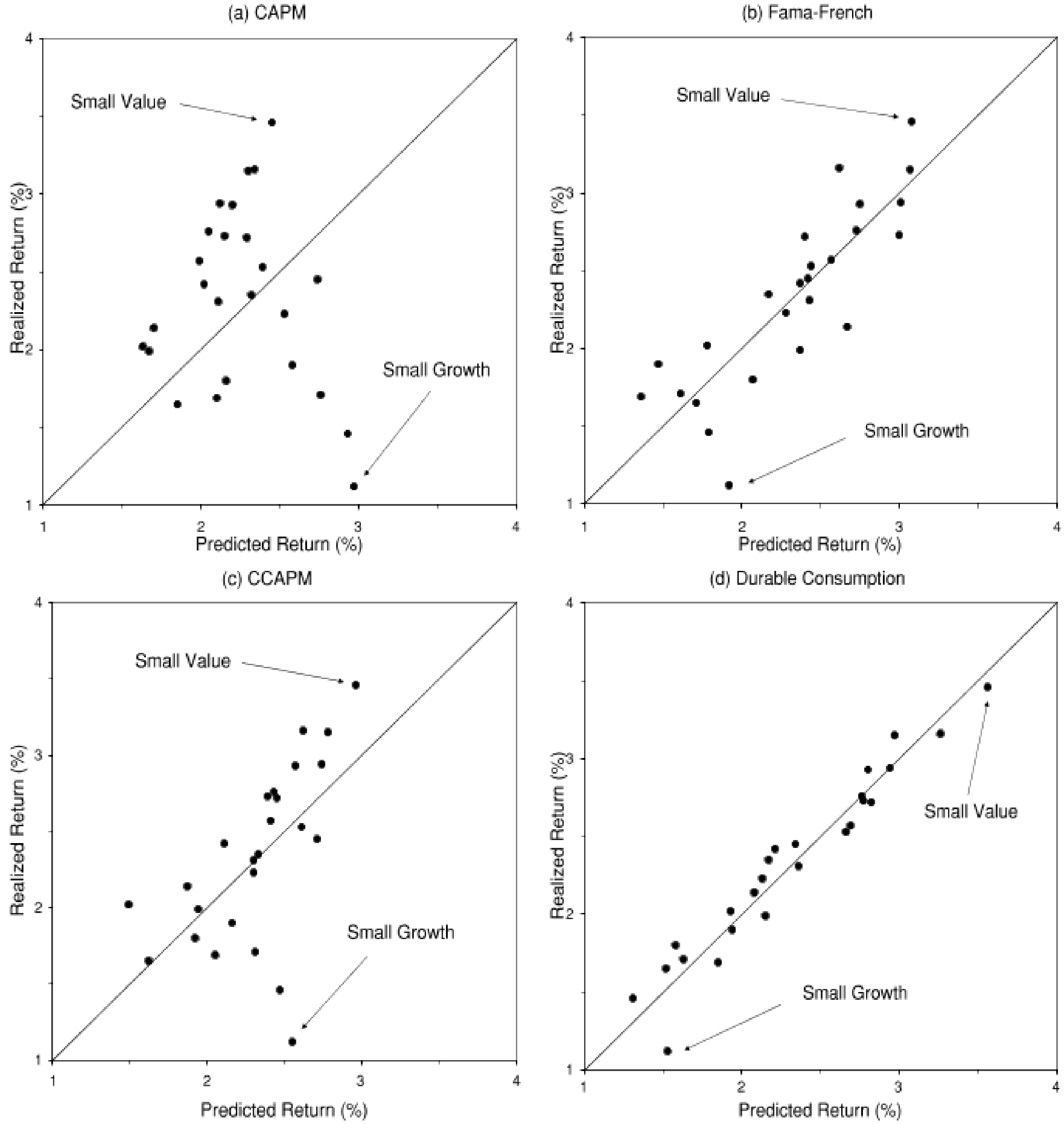


Figure 4. Realized versus Predicted Returns for the Fama–French Portfolios. The figure plots realized versus predicted excess returns (per quarter) for the 25 Fama–French portfolios sorted by size and book-to-market equity. The estimated models are (a) the CAPM, (b) the Fama–French three-factor model, (c) the CCAPM, and (d) the durable consumption model.

Source: Yogo (2006), Figure 4.

Interestingly, Yogo also has the consumption betas for various industries relative to nondurables and durables, estimated by first stage GMM:

Table 49

Average Returns and Consumption Betas for Portfolios Sorted by Book-to-Market Equity within Industry

Panel A reports average excess returns (per quarter) on 24 portfolios sorted by book-to-market equity (B/M) within industry. Panels B and C report nondurable and durable consumption betas, implied by the first-stage GMM estimate of the durable consumption model. See notes to Table VI for details on portfolio formation.

	Panel A: Average Return (%)			Panel B: Nondurable Beta			Panel C: Durable Beta		
Industry	Low B/M	Med B/M	High B/M	Low B/M	Med B/M	High B/M	Low B/M	Med B/M	High B/M
Manufacturing									
Nondurables	1.904	2.270	2.819	4.185	4.336	5.068	1.201	1.268	1.554
Durables	1.727	2.397	3.744	6.056	5.882	8.383	−0.050	0.770	2.796
Other	1.516	1.894	2.664	5.485	3.714	4.843	0.574	1.167	2.242
Retail									
Nondurables	1.961	2.627	2.522	6.035	4.465	4.918	0.397	0.894	1.035
Durables	2.259	2.052	3.480	5.732	6.309	5.552	0.375	−0.055	0.899
Services	1.670	1.298	2.182	4.381	3.406	5.815	−0.207	0.110	1.511
Finance	1.537	2.584	3.104	5.041	4.980	3.813	0.285	1.341	1.645
Natural resource	0.277	1.627	2.928	2.031	3.543	4.304	−0.334	1.255	2.782

Source: Yogo (2006), Table VII.

Note that value stocks (high B/M) have much higher durables betas than do growth stocks, whereas the nondurables betas are not much different. This helps explain the ability of the durables betas to do so well in explaining the cross-section of average returns.

Finally, Yogo also is able to demonstrate that durables can also model time variation in the estimated equity premium, as durables move sharply down in recessions at a time when risk and risk aversion cause the equity premium to surge, shown in Figure 40:

Figure 40

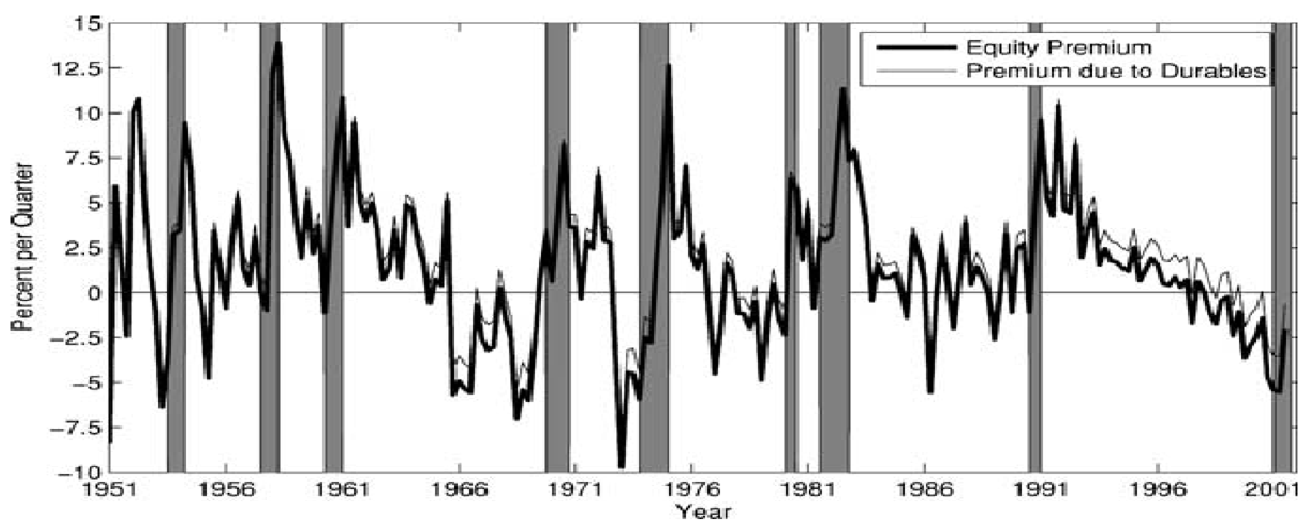


Figure 5. Time Variation in the Equity Premium. The figure is a time-series plot of expected excess returns on the market portfolio. The sample period is 1951:1–2001:3; the shaded regions are NBER recessions.

Source: Yogo (2006), Figure 5.

A weakness in Yogo's model is that the implied level of relative risk aversion is extremely high at CRRA=174. This high estimate is due to the significant risk premiums, despite the very low volatility of durables consumption stocks and durables consumption flows.

Gomes, Kogan and Yogo (GKY, 2009) follow up on Yogo's (2006) analysis of the consumption of durable goods. An innovation for this article is their construction of better industry classifications than the U.S. government provides, using the BEA's input/output tables for cash flows in the economy. GKY (2009, p. 943) document four new facts in the cross-section of cash flows and stock returns: (1) cash flows of durable goods producers are more volatile and are more correlated with aggregate consumption than are other industries, (2) returns on the durable goods portfolio are higher and more volatile on average, (3) cash flows of durable-good producers are conditionally more volatile when the durable expenditure/stock ratio is low, which generally coincides with recessions, and (4) the returns on the durable goods portfolio are more predictable. Supporting point (2), over the 1927-2007 period, a portfolio that is long durable goods and short the services portfolio earned an average annual return over 4.0%. Supporting (4), a portfolio that is long durables and short the market portfolio has countercyclical expected returns, reliably predicted by the durables expenditure/stock ratio.

A key mechanism in the Gomes, Kogan, Yogo model is that (p. 944): "…a proportional change in the service flow (or the stock) of durable goods requires a much larger proportional change in the expenditure on durable goods." They argue that: "the difference in the conditional cash flow risk between durable-goods producers and nondurable-good producers is relatively high when the existing stock of durables is high relative to current demand. This mechanism leads to a testable implication that the durable expenditure-stock ratio predicts cross-sectional differences in the conditional moments of cash flows and stock returns…" The next graph shows the cyclical movements of the durable expenditure-stock ratio. Note the sharp drops in the Great Depression, as well as in sharp recessions in 1974/75 and 1981/82.

Figure 41

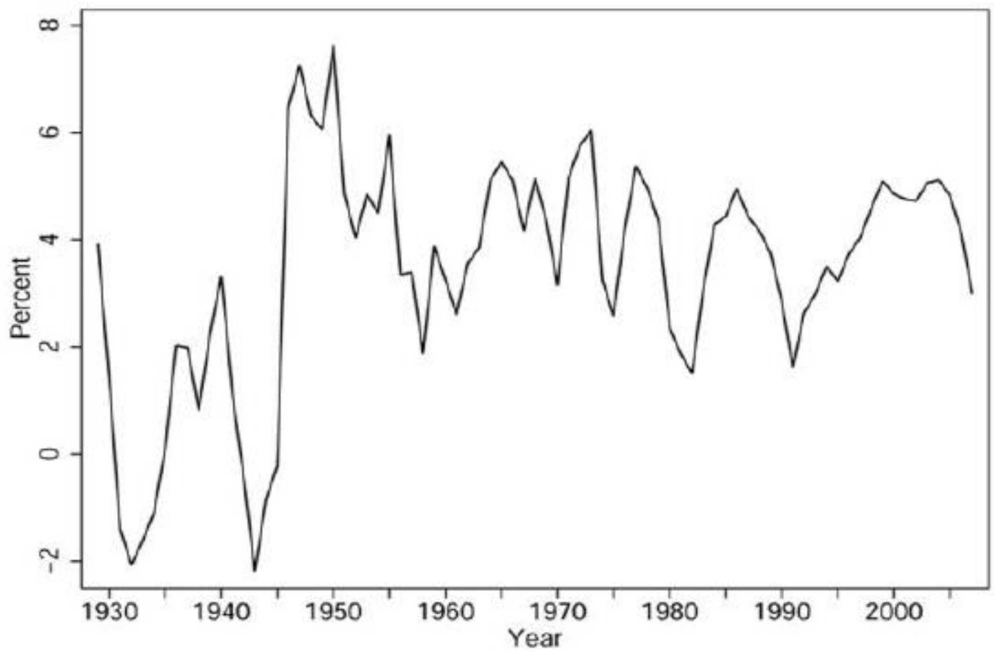


FIG. 2.—Ratio of net durable expenditure to the stock of durables. The stock of durables is the sum of the stock of consumer durable goods and the stock of private residential fixed assets. The sample period is 1929–2007.

Source: Gomes, Kogan, Yogo (2009), Figure 2.

GKY use the durables expenditure-stock ratio as a predictor variable for excess returns and Table 49 shows that it has performance that is comparable to that of aggregate dividend yield, especially in the postwar period. Consistent with the conditionally higher expected returns in recessions, additional results (not shown) confirm that cash flow risk and 5-year dividend growth risk are higher when the durables expenditure-stock ratio is low, as in recessions. Thus, in summary, the Gomes, Kogan, Yogo article demonstrates the greater cyclicality of durables and the potential use of the durables expenditure-stock ratio as a conditioning variable for modeling risk changes in the economy.

Table 50

PREDICTABILITY OF EXCESS RETURNS ON THE INDUSTRY PORTFOLIOS

Lagged Forecasting Variable	Services		Nondurables		Durables		Durables−Market	
				A. 1927–2007 Sample Period				
Durable expenditure–stock ratio	−1.71	−1.13	−1.20	−.59	−3.23	−3.72*	−1.48	−1.70*
	(1.04)	(1.07)	(1.14)	(.92)	(1.97)	(1.93)	(.98)	(.98)
Dividend yield		3.26***		5.46***		3.84**		1.62*
		(1.20)		(1.32)		(1.65)		(.87)
R^2 (%)	3.84	11.30	1.90	16.32	5.82	12.31	6.00	11.99
				B. 1951–2007 Sample Period				
Durable expenditure–stock ratio	−3.52**	−2.87	−.15	.30	−5.38**	−6.52***	−3.41***	−3.88***
	(1.72)	(1.73)	(1.94)	(1.71)	(2.67)	(2.43)	(1.24)	(1.17)
Dividend yield		1.93		4.53***		3.52**		1.44
		(1.22)		(1.17)		(1.62)		(.88)
R^2 (%)	6.75	10.63	.01	13.99	8.35	15.09	15.06	20.09

NOTE.—The table reports predictive regressions for annual excess returns, over the 3-month T-bill, on the industry portfolios. The lagged forecasting variables are the durable expenditure–stock ratio and each portfolio's own dividend yield. Heteroskedasticity-consistent standard errors are reported in parentheses.

* Significant at the 10 percent level.

** Significant at the 5 percent level.

*** Significant at the 1 percent level.

Source: Gomes, Kogan, Yogo (2009), Table 5.

D. Real Estate.

Piazzezi, Schneider and Tuzel (2007) examine housing and consumption and the "composition risk" between housing and nonhousing consumption and its impact on asset pricing. They argue several points for the importance of this split, both theoretically and empirically. Perhaps the most persuasive result that they find is that individuals really dislike reducing housing consumption (habit formation), so they only do that if circumstances are very bad. So, when housing expenditures decline sharply in relation to other expenditures (the "housing share" declines), the economic state is very poor, almost in an "extreme risk" situation. They observe that (p. 532): "In our model, investors' concern with composition risk implies that recessions are perceived as particularly severe when the share of housing consumption is low." Housing is a necessity and people don't reduce it unless circumstances are very bad and marginal utility is very high. They further state (p. 540) that: "stocks have …especially low payoffs in severe recessions, when housing consumption is relatively low (and α is high). This generates higher equity premia than under the standard model."

Movements in the expenditure share on housing services, α, are in Figure 43, comparing the expenditure share (α) on nonhousing services in the years from 1929 to 2001 with dividend yield:

Figure 42

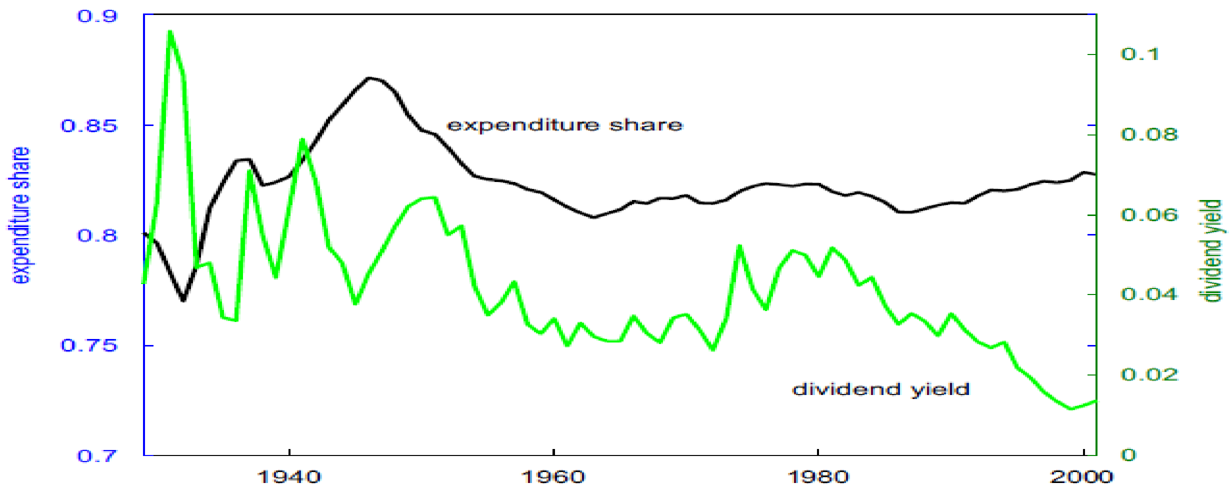


Fig. 1. Expenditure share α_t and dividend yield v_t^s, annual data 1929–2001.

Source: Piazzezi, Schneider and Tuzel (2007), Figure 1.

During World War II, resources were devoted to the war effort, and housing investment declined relative to defense needs and other consumer spending. Post-World War II, there was a large catch-up in housing expenditures as soldiers returned home, and spending on housing services was very high (nonhousing low) and then reverted more to its long-term fraction of consumption. After those moves, spending on housing as a fraction of total consumption has not varied as much (1960-2007).

Piazzezi, Schneider and Tuzel make another significant point that: (p. 548) "… times characterized by relatively little housing correspond to times when the volatility of shocks is higher. In other words, times with little housing are times of high uncertainty." And further, they observe that: (p. 560) "Interestingly, the model implies that a macroeconomic variable, the (nonhousing) expenditure share, α_t, should be a good forecasting variable. Intuitively, the model implies that α_t is high in severe recessions, when expected excess returns are high."

Table 51

Predicting excess stock returns with the expenditure share

Horizon	Hi perceived risk model		Hi risk aversion model		Long sample			Post-war sample		
(yr)	Slope	R^2	Slope	R^2	Slope	t-stat	R^2	Slope	t-stat	R^2
Panel A. Regressions on expenditure share										
1	2.00	0.05	2.40	0.06	1.36	1.47	0.02	1.42	1.68	0.03
2	3.80	0.09	4.55	0.10	3.30	2.03	0.07	3.68	2.24	0.08
3	5.42	0.13	6.50	0.14	5.01	2.40	0.14	6.25	3.21	0.20
4	6.88	0.16	8.25	0.18	6.58	2.84	0.18	8.63	3.95	0.28
5	8.19	0.19	9.83	0.21	8.44	3.65	0.22	10.73	4.92	0.30

Horizon	Long sample					Post-war sample				
	$\ln 1/v_t^s$		$\ln \alpha_t$			$\ln 1/v_t^s$		$\ln \alpha_t$		
(yr)	Slope	t-stat	Slope	t-stat	R^2	Slope	t-stat	Slope	t-stat	R^2
Panel B. Regression on expenditure share and dividend yield										
1	0.10	2.04	0.43	0.44	0.07	0.10	1.84	0.50	0.13	0.08
2	0.17	1.60	1.75	1.11	0.14	0.16	1.39	2.14	0.75	0.14
3	0.15	1.01	3.65	1.77	0.18	0.10	0.66	5.30	2.11	0.21
4	0.16	0.86	5.08	2.29	0.20	0.06	0.30	8.09	3.04	0.28
5	0.28	1.49	5.87	2.64	0.26	0.15	0.81	9.24	3.43	0.31

Note: Panel A reports regression results of log excess stock returns $\sum_{j=1}^{n} r_{t+j}^s - r_{t+j}^f$ on a constant and the log expenditure share $\ln \alpha_t$ for $n = 1, \ldots, 5$ years. The "Model" columns contain the average slope and R^2 over 50,000 simulated samples with 65 observations. The model with *hi perceived risk* is the bold-face parameterization in Table 3 with $\varepsilon = 1.05$, $\beta = 0.99$, and $1/\sigma = 5$. The model with *hi risk aversion* is the bold-face parameterization in Table 3 with $\varepsilon = 1.25$, $\beta = 1.24$, and $1/\sigma = 16$. The "Long sample" columns run regressions with 1936–2001 historical data, and the "Post-war sample" columns use 1947–2001 data. t-statistics are based on Newey–West standard errors to correct for overlapping observations. Panel B reports regression results of $\sum_{j=1}^{n} r_{t+j}^s - r_{t+j}^f$ on a constant, $\ln \alpha_t$, and the log dividend yield $\ln 1/v_t^s$.

Source: Piazzezi, Schneider, Tuzel (2007), Table 5.

E. Limited Participation

Malloy, Moskowitz and Vissing-Jorgensen (2009), following on Vissing-Jorgensen's (2002) significant work on limited participation with Consumer Expenditure Survey data (CEX) in Section VII, examine the consumption of stockholders versus nonstockholders, as well as consumption by the wealthiest 1/3 of stockholders, in the long-run risk context of Bansal and Yaron (2004) and Hansen, Heaton and Li (2008). Using the CEX data, they estimate the sensitivities of stockholder, top stockholder and nonstockholders to fluctuations in aggregate consumption over horizons from 1 quarter to 6 years, as shown in the next table:

Table 52

Sensitivity of Stockholder, Top Stockholder, and Nonstockholder Consumption Growth to Aggregate Consumption Growth Across Horizons

The sensitivity of stockholder, top stockholder, and nonstockholder consumption growth to aggregate consumption growth from NIPA is reported over horizons of $S = 1, 2, 4, 8, 12, 16, 20$, and 24 quarters. The sensitivity of each group's consumption growth to aggregate consumption growth is computed as the regression coefficient from regressing a group's discounted consumption growth over horizon S on aggregate discounted consumption growth over the same horizon. Standard errors (in parentheses) on the regression sensitivity measure are computed using a Newey–West estimator that allows for autocorrelation of up to $S \times 3 - 1$ month lags. Group consumption growth rates are calculated using data from the Consumer Expenditure Survey over the period March 1982 to November 2004.

$S =$	1	2	4	8	12	16	20	24
Stockholder	0.68	0.93	1.21	1.57	2.12	2.68	2.68	2.42
(s.e.)	(0.35)	(0.37)	(0.32)	(0.36)	(0.39)	(0.49)	(0.49)	(0.41)
Top stockholder	0.70	1.01	1.56	2.14	2.88	3.94	3.91	3.48
(s.e.)	(0.90)	(0.77)	(0.62)	(0.49)	(0.53)	(0.67)	(0.73)	(0.63)
Nonstockholder	0.51	0.41	0.59	0.84	0.96	1.01	0.95	0.79
(s.e.)	(0.23)	(0.27)	(0.26)	(0.27)	(0.29)	(0.26)	(0.24)	(0.27)

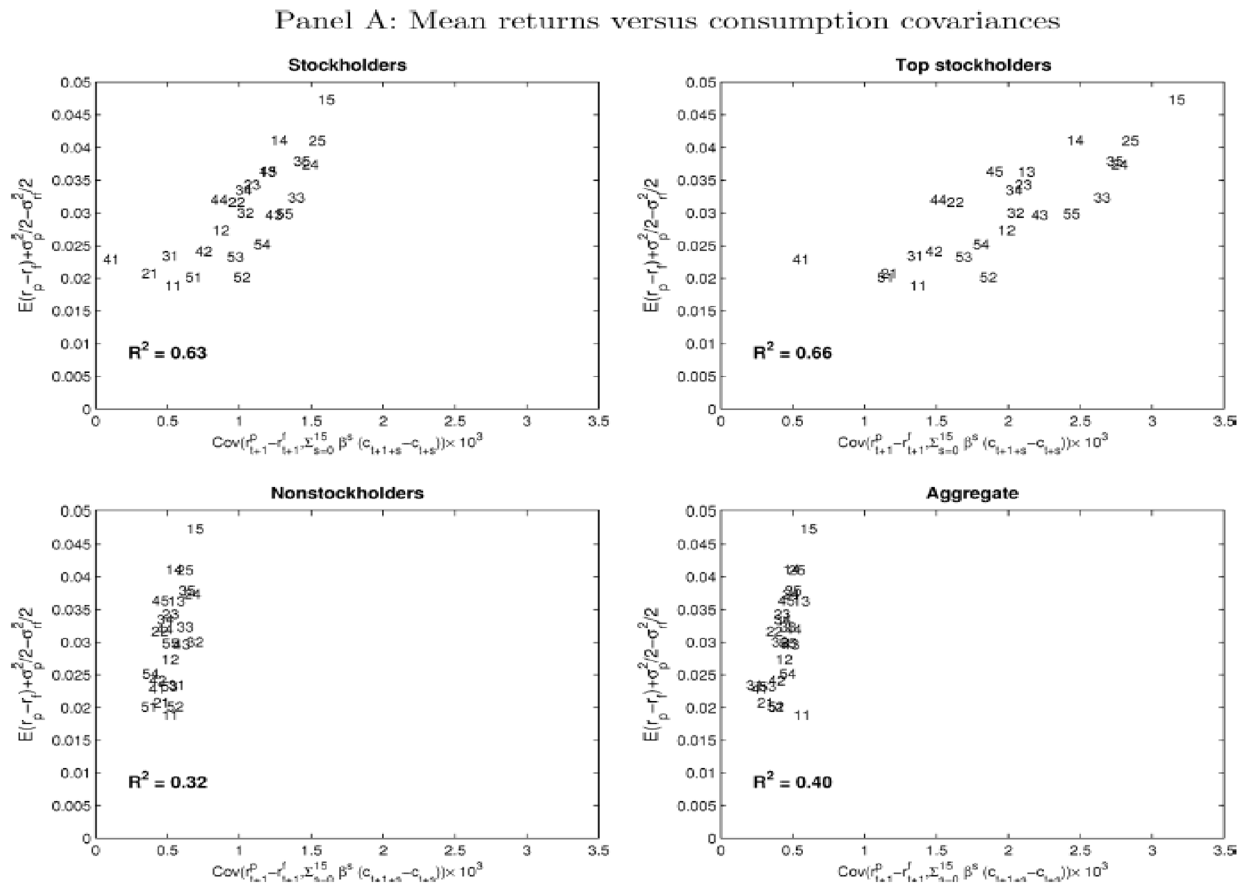
Source: Malloy, Moskowitz and Vissing-Jorgensen (2009), Table 1.

They find that, for example, at a horizon of 4 years, nonstockholders' real consumption growth moves approximately 1-for-1 with aggregate real growth, whereas "All Stockholders" move by 2.7 times as much and the wealthiest 1/3, "Top Stockholders," move by 3.9 times as much as the aggregate. There is no reason for the Euler equations for stocks to hold for nonstockholders, whereas they should for stockholders, the results of stockholders, and especially for the ones with the most invested. Given the much higher sensitivities to aggregate consumption (picking up only systematic consumption fluctuations), MMV-J estimates of relative risk aversion are much lower "at around 10" (p. 2427) and nearing a range that many economists find more plausible.

MMV-J estimated consumption growth covariances of the 25 Fama-French portfolios with the different groups's consumption rates and found substantial differences, depending upon whether Top Stockholders, Stockholders, or Nonstockholders' consumption growth were used, as shown in Table 50. Note the dispersion of consumption covariances (betas) between value and growth portfolios for stockholders versus for nonstockholders. Using stockholder consumption shows value stocks to have much higher consumption risk than for nonstockholders.

This makes the cross-sectional relation of average stock returns (using 1926-2004 data) to consumption covariances (using the CEX data period 1982-2004) have a much better fit for stockholders, (especially Top Stockholders), than for Nonstockholders, as shown in Figure 43.

Figure 43



Source: Malloy, Moskowitz and Vissing-Jorgensen (2009), Figure 1.

Table 53

Dispersion in Consumption Growth Covariances across the 25 Fama–French Portfolios

Panel A reports the first-stage covariance estimates and GMM t-statistics allowing for autocorrelation up to 47 monthly lags of each of the 25 Fama–French portfolios with stockholder discounted consumption growth over a 16-quarter horizon. An F-test on the joint equality of the covariances is reported (with the p-value in parentheses). Panels B, C, and D report the covariance estimates for top stockholder, nonstockholder, and aggregate consumption growth, respectively.

	Growth 1	2	3	4	Value 5	Avg.	Growth 1	2	3	4	Value 5
Panel A: Stockholder Consumption Growth											
	Consumption growth covariance $\times 10^{-4}$						GMM t-statistics				
1 (small)	4.80	8.14	11.37	12.17	15.45	10.38	0.90	1.56	1.98	2.48	2.37
2	3.18	9.19	10.27	14.34	14.83	10.36	0.90	2.20	2.23	3.49	2.94
3	4.61	9.82	13.37	9.68	13.75	10.25	1.12	2.11	3.34	2.47	3.21
4	0.51	6.90	11.79	7.98	11.40	7.72	0.62	2.50	3.08	2.76	3.32
5 (large)	6.22	9.59	9.13	10.98	12.54	9.69	1.58	3.12	3.67	3.49	4.52
Avg.	3.86	8.73	11.19	11.03	13.59		F-stat = 3.42 (p-value = 0.050)				
Panel B: Top Stockholder Consumption Growth											
	Consumption growth covariance $\times 10^{-4}$						GMM t-statistics				
1 (small)	13.14	19.26	20.66	24.06	31.12	21.65	1.03	1.87	2.08	2.73	3.16
2	11.12	15.70	20.44	27.16	27.88	20.46	1.10	2.13	2.82	4.16	4.13
3	12.93	19.90	25.94	19.79	26.81	21.08	1.39	2.37	3.94	3.40	3.69
4	4.99	14.23	21.57	14.53	18.45	14.75	0.74	2.44	3.24	2.70	3.47
5 (large)	10.86	18.01	16.29	17.50	23.80	17.29	1.44	2.50	3.09	3.24	4.99
Avg.	10.61	17.42	20.98	20.61	25.61		F-stat = 3.54 (p-value = 0.046)				
Panel C: Nonstockholder Consumption Growth											
	Consumption growth covariance $\times 10^{-4}$						GMM t-statistics				
1 (small)	4.61	4.58	5.02	4.86	6.33	5.08	1.72	2.64	2.89	3.09	3.30
2	3.96	3.85	4.60	6.14	5.63	4.84	1.59	2.30	3.24	5.18	3.28
3	5.06	6.32	5.63	4.25	5.76	5.40	1.87	3.32	4.02	3.70	4.47
4	3.70	3.71	5.39	4.20	3.91	4.18	1.58	2.44	3.59	3.38	2.52
5 (large)	3.13	4.93	4.53	3.22	4.62	4.08	1.53	2.60	2.79	3.31	4.89
Avg.	4.09	4.68	5.04	4.53	5.25		F-stat = 0.23 (p-value = 0.794)				
Panel D: Aggregate Consumption Growth, NIPA Sample Period											
	Consumption growth covariance $\times 10^{-4}$						GMM t-statistics				
1 (small)	5.06	3.81	4.99	4.32	5.50	4.74	1.77	1.51	2.25	2.16	2.74
2	2.51	3.13	3.66	4.26	4.69	3.65	1.05	1.58	2.16	2.47	2.89
3	1.76	3.50	4.07	3.69	4.44	3.49	0.99	2.00	2.59	2.56	2.93
4	2.15	3.29	4.27	4.45	3.96	3.62	1.24	2.03	3.44	3.51	3.15
5 (large)	3.29	3.21	2.66	4.05	4.09	3.46	2.10	2.30	2.87	3.56	3.83
Avg.	2.95	3.39	3.93	4.15	4.54		F-stat = 1.50 (p-value = 0.245)				

Source: Malloy, Moskowitz and Vissing-Jorgensen (2009), Table III

MMV-J extend their computations by using the “consumption mimicking portfolio” (CMP) technique of Breeden, Gibbons and Litzenberger (1989), as in Section V. Their maximum correlation portfolio is from regressions of 16 quarters of consumption growth on portfolios of stocks representing small growth, large growth, small value and large value stocks. With this CMP, they extend their data to the entire CRSP period from 1926-2004. With this long data set, they are able to compute time-varying factor loadings and to compute conditional consumption risks by having cross products with *cay*, the deviation of consumption from wealth and income of Lettau and Ludvigson (2001a). They also show that cay and the consumption share of stockholders are quite strongly negatively correlated (-0.44), as shown in Figure 44:

Figure 44

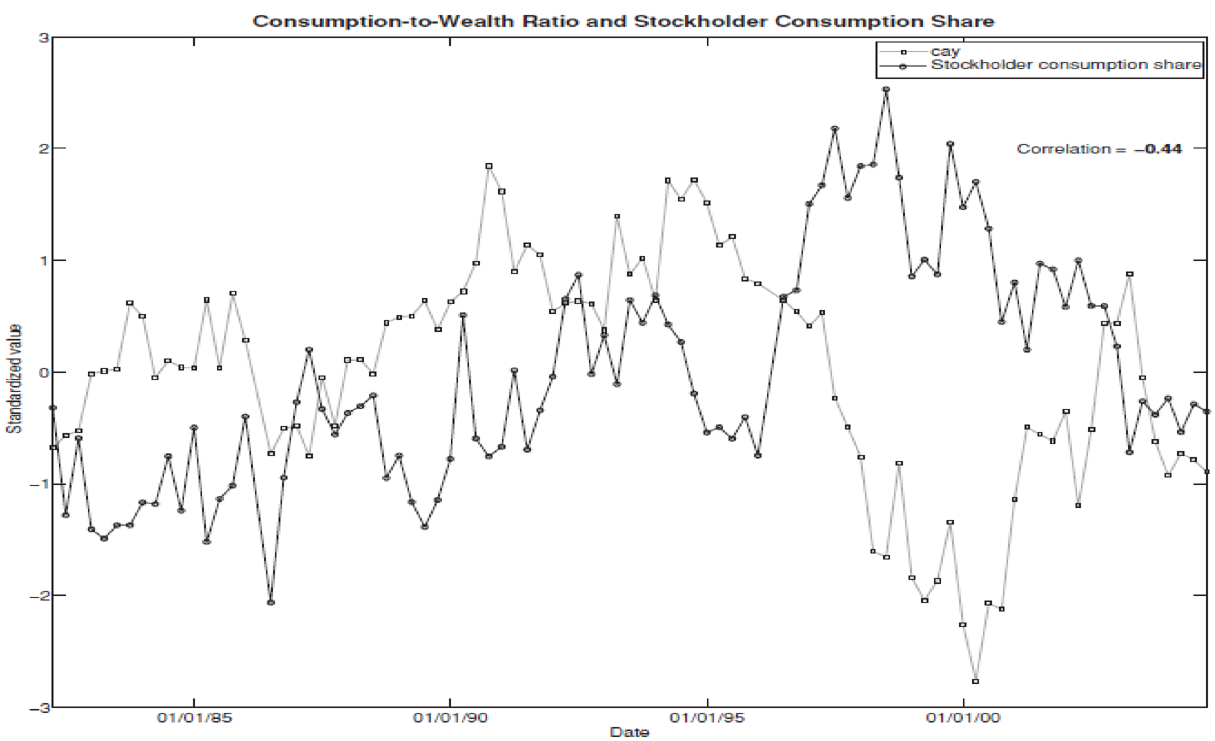


Figure 2. Plot of the consumption-to-wealth ratio *cay* and the stockholder consumption share. The figure plots the consumption-to-wealth ratio of Lettau and Ludvigson (2001a) along with the ratio of the quarterly consumption of stockholders (using our baseline stockholder definition) to total quarterly CEX consumption. For readability, each time series is standardized by subtracting its mean and dividing by its standard deviation calculated over the CEX sample period.

Source: Malloy, Moskowitz and Vissing-Jorgensen (2009), Figure 2.

MMV-J find that (p. 2459): “For each group, risk aversion estimates are similar in magnitude” to their prior estimates and indicates “that accounting for time variation in factor loadings from 1951 to 2004 does not seem to alter our estimates.”

F. Foreign Exchange

Foreign exchange rates are quite important in the global economy. Their movements make goods, services and labor cheaper or more expensive country by country (or for the Eurozone) and, thereby, direct the flow of international investment and employment around the world. When a country's economy is strong, its unemployment rate drops, there is less slack in the economy and its foreign exchange rate strengthens, which directs global investment to other, less strong areas in the world. So there is and should be a positive correlation of foreign exchange rates in a country with the country's economic strength relative to other countries.

Lustig and Verdelhan (2007) examine data for 8 different currency portfolios, selected by their interest rate spreads to the U.S. dollar, with portfolio 1 in low interest rate currencies, and portfolio 8 in high rate currencies. Table 54 gives the rate spread to the US dollar, the average rate of depreciation and average inflation rate in the countries over the 1953-2002 full sample, as well as the post-Bretton Woods period of 1971-2002.

Table 54

TABLE 2—EXCHANGE RATES AND INTEREST RATES

Portfolio	1	2	3	4	5	6	7	8
				1953–2002				
$E_T(\Delta R^j)$	−2.46	−1.20	−0.77	0.14	1.12	2.52	4.69	16.36
$E_T(-\Delta e^j)$	0.34	0.26	0.41	0.29	1.69	3.08	2.18	15.72
$E_T(\Delta p^j)$	4.12	4.66	4.19	5.14	5.63	6.19	7.67	15.20
				1971–2002				
$E_T(\Delta R^j)$	−2.94	−1.43	−0.44	0.74	2.31	4.00	6.84	22.96
$E_T(-\Delta e^j)$	0.74	−0.83	0.47	0.33	2.96	4.17	3.65	23.74
$E_T(\Delta p^j)$	4.72	5.53	4.93	6.05	6.95	7.72	10.23	20.92

Notes: This table reports the time-series average of the average interest rate differential ΔR_t^j (in percentage points), the average rate of depreciation Δe_{t+1}^j (in percentage points), and the average inflation rate Δp^j (in percentage points) for each of the portfolios. Portfolio 1 contains currencies with the lowest interest rates. Portfolio 8 contains currencies with the highest interest rates. This table reports annual interest rates, exchange rate changes, and inflation rates for annually rebalanced portfolios.

Source: Lustig and Verdelhan (2007), Table 2.

Excess returns to the 8 portfolios show that the high rate currencies provided positive excess returns, while the low rate currencies provided negative excess returns over both periods:

Table 55

TABLE 1—US INVESTOR'S EXCESS RETURNS

Portfolio	1	2	3	4	5	6	7	8
				1953–2002				
mean	−2.34	−0.87	−0.75	0.33	−0.15	−0.21	2.99	2.03
SR	−0.36	−0.13	−0.11	0.04	−0.02	−0.03	0.37	0.16
				1971–2002				
mean	−2.99	−0.01	−0.83	1.14	−0.69	−0.00	3.94	1.48
SR	−0.38	−0.00	−0.10	0.11	−0.07	−0.00	0.39	0.10

Notes: This table reports the mean of the real excess returns (in percentage points) and the Sharpe ratio for a US investor. The portfolios are constructed by sorting currencies into eight groups at time t based on the nominal interest rate differential at the end of period $t-1$. Portfolio 1 contains currencies with the lowest interest rates. Portfolio 8 contains currencies with the highest interest rates. The table reports annual returns for annually rebalanced portfolios.

Source: Lustig and Verdelhan (2007), Table 1.

If one considers investing in a currency, or playing a "carry trade" that is long a high interest currency and short a low interest currency, one likely has a position that has nontrival relative "consumption risk," in that returns from the trade will depend upon the relative performances of the economies, which most likely are reflected in their relative growths in real consumption. Lustig and Verdelhan's estimates of "unconditional" consumption betas show that the higher interest rate currencies on average have higher consumption betas, whether betas are measured relative to nondurables or relative to durables (as suggested by Yogo (2006)):

Table 56

TABLE 6—ESTIMATION OF FACTOR BETAS FOR EIGHT CURRENCY PORTFOLIOS SORTED ON INTEREST RATES

Portfolios	1	2	3	4	5	6	7	8
				Panel A: 1953–2002				
Nondurables	0.105	0.762	0.263	0.182	0.634	0.260	1.100	0.085
Durables	0.240	0.489	0.636	0.892	0.550	0.695	1.298*	0.675
Market	−0.066*	−0.027	−0.012	−0.119*	−0.000	−0.012	−0.056	0.028
				Panel B: 1971–2002				
Nondurables	0.005	0.896	0.359	0.665	0.698	0.319	1.546	−0.461
Durables	0.537	0.786	1.288*	2.032*	1.225*	1.359	2.183*	0.845
Market	−0.106*	−0.099*	−0.026	−0.171*	−0.017	−0.007	−0.083	0.052

Notes: Each column of this table reports OLS estimates of β^j in the following time-series regression of excess returns on the factor for each portfolio j: $R^{j,e}_{t+1} = \beta^j_0 + \beta^j_1 f_t + \varepsilon^j_{t+1}$. The estimates are based on annual data. Panel A reports results for 1953–2002 and Panel B reports results for 1971–2002. We use eight annually rebalanced currency portfolios sorted on interest rates as test assets. * indicates significance at 5-percent level. We use Newey-West heteroskedasticity-consistent standard errors with an optimal number of lags to estimate the spectral density matrix following Donald W. K. Andrews (1991).

Source: Lustig and Verdelhan (2007), Table 6.

Lustig and Verdelhan also do conditional estimates of betas, using the interest rate differential as the sole conditioning variable. Thus, when risks get larger and a portfolio's yield spreads widen, estimated consumption betas increase, and when spreads shrink, risks decrease. The authors observe (p. 102) that: "For every 4-percentage point reduction in the interest rate gap, the nondurable consumption betas decrease by about 100 basis points."

Table 57 gives the results for the estimated prices of consumption and market risks from Fama-MacBeth regressions of cross-sectional returns on conditional betas for four consumption models: (1) what they describe as the original CCAPM (using nondurables), (2) the Durables Consumption CAPM of Yogo (2006) and (3) and (4) Epstein-Zin preference versions of both, as well as for the Sharpe-Lintner CAPM and the Fama-French 3-factor and bond factor models:

Table 57

TABLE 11—ESTIMATION OF LINEAR FACTOR MODELS WITH EIGHT CURRENCY PORTFOLIOS SORTED ON INTEREST RATES

Panel A. Consumption models

	CCAPM	DCAPM	EZ-CCAPM	EZ-DCAPM
Nondurables	1.705	1.617	2.496	2.422
	[1.087]	[1.095]	[0.914]	[0.914]
Durables		2.556		2.916
		[0.959]		[0.905]
Market			15.260	8.481
			[7.804]	[7.259]
MAE	2.647	1.661	2.283	1.283
R^2	0.259	0.535	0.361	0.641
p − *value*	[0.312]	[0.535]	[0.222]	[0.479]

Panel B. Factor models

	CAPM	FF-equity	FF-bonds
Market	1.943	5.174	
	[8.443]	[8.684]	
SMB		9.530	
		[5.188]	
HML		−6.525	
		[5.965]	
Slope			3.967
			[9.628]
Default			0.661
			[2.393]
Stats			
MAE	3.549	2.905	3.457
R^2	0.006	0.186	0.032
p − *value*	[0.001]	[0.001]	[0.001]

Notes: This table reports the Fama-MacBeth estimates of the factor prices (in percentage points) using eight annually rebalanced currency portfolios as test assets. The sample is 1971–2002 (annual data). The standard errors are reported between brackets. The factors are demeaned. The last three rows report the mean absolute pricing error (in percentage points), the R^2, and the p-value for a χ^2 test.

Source: Lustig and Verdelhan (2007), Table 11.

All four consumption-based models pass the chi-squared test and are not rejected, but "only the models with durable consumption growth as a factor explain a large fraction of the cross-sectional variation in returns." Their benchmark model, using Durables Consumption measures explains 54% to 64% of the variation, depending whether CRRA or the E-Z preferences are used. Furthermore, "In this subsample, the CAPM explains none of the variation, and the Fama-French factor models explain less than 18 percent…" Thus, as Lustig and Verdelhan state (p. 89): "Because high interest rate currencies depreciate on average when domestic consumption growth is low and low interest rate currencies appreciate under the same conditions, low interest rate currencies provide domestic investors with a hedge against domestic aggregate consumption growth risk." For providing that consumption hedge, lower returns are earned on average on the low interest rate currencies, even negative excess returns. It would be interesting for researchers to also study the nonlinear risks, as it is plausible that there are substantial nonlinear risks in currency movements.

XIV. Conclusion

In this article, we first reviewed the late 1970s major theoretical derivations that led to consumption-based asset pricing models, one of the leading theories of asset pricing in the past four decades. Next, we examined empirical testing and evolution of both theory and empirical testing and of applications of consumption-based asset pricing, We began with individuals who have heterogeneous time-additive utility functions and derived that their optimal responses to market price signals should lead individuals to coordinate their consumption plans so that they all have high consumption when aggregate real consumption increases and have low consumption when aggregate real consumption decreases, leading us to the major aggregation theorem. Even without effectively complete markets with a full set of hedges available, it was shown that each individual would optimally choose a dynamic portfolio and optimal consumption plan to achieve the maximum correlation possible of the individual's consumption with aggregate, per capita consumption. In that economy in the continuous-time model, the Consumption CAPM was derived. Consumption betas were found to capture all of the risks and risk premiums in Merton's continuous-time Intertemporal CAPM, even with heterogeneous individuals and incomplete markets. No assumption of identical investors or the existence of a representative individual was required.

The 1980s empirical tests showed that there is a risk premium for consumption risk, much as there was for market risk, but that the CCAPM was rejected, much like the original market-oriented CAPM was. Assets' conditional risk premiums were not found to be higher in proportion to their conditional connsumption betas, as predicted by the CCAPM and the market-oriented CAPM. An "equity premium puzzle" was identified in the 1980s, which identified a mismatch between a relatively high risk premium on equities and very low volatility of real aggregate consumption and relatively low correlation of equity returns with consumption fluctuations. Researchers also found that models often had real riskless interest rates that were higher than historical data, which gave rise to what some identified as a "riskless rate puzzle."

Research proceeded along several dimensions, first by recognizing measurement of consumption as an integral of daily/instantaneous consumption, versus stock returns as "point to point returns." Next, researchers noted the considerable noise in consumption measurements versus consumers' fundamental economic desires, reflecting often large but often transitory impacts of weather, tax changes, labor strikes and such on income and consumption. Monthly data for real consumption growth showed negative autocorrelation (presumably due to weather, which can cause large monthly swings that dominate regression results), while quarterly, semiannual and annual autocorrelations were positive, reflecting business cycle impacts. Tests that relied upon monthly consumption growth estimates were found to be misleading relative to subsequent results. Subsequent researchers found it more sensible to use 6-month or 2-quarter returns, or 4-quarter returns, or even "ultimate consumption betas" that measured covariances over almost 3 years (11 quarters).

Another dimension for research that was established in the late 1980s and early 1990s was on models that exhibited time complementarity in utility for consumption, as in "habit formation" models (which are backward looking, in that individuals' utilities for current consumption depended upon their habits formed from prior consumption levels). Another popular model is of forward looking, recursive models of utility maximization (think of an anticipated future inheritance, for example). The asset pricing implications of these more general models allowed more flexibility in replicating financial and economic data and historic means, volatilities and correlations. Perhaps not surprisingly, as they have more degrees of freedom, these models were able to fit the data considerably better than prior models.

In the decade or so from 1991-2002, researchers used survey data compiled by the U.S. Government on actual consumer spending by individual households to see how individual consumption volatility differed from aggregate consumption volatility. Differences were presumed to be due to "incomplete markets" for insuring major risks (such as for labor income), as well as "limited participation," in that some consumers simply did not invest in the stock market. If they didn't invest, their first-order conditions for an optimum likely were not met. Thus, examining the consumption of stockholders versus that of nonstockholders yielded considerable insights. First, it was found that volatility for a typical household was 5-10 times as large as for aggregate real consumption, which helped to resolve the risk premium puzzle caused in part by the low volatility of aggregate consumption. Secondly, it was found that both risk aversion and elasticity of intertemporal substitution estimates (which closely related to riskless interest rates) are much different between stockholders and nonstockholders and between bondholders and nonbondholders.

These findings on incomplete markets and limited participation helped researchers to resolve both the risk premium puzzle and the riskless rate puzzle. However, in the early 1990s, two new puzzles were found in cross-sectional stock returns over long periods of time – the "size effect" (small stocks earned higher returns than large stocks, beta adjusted) and the "value premium" (where high book/market stocks, value stocks, earned more than growth stocks, beta adjusted). Subsequent researchers tested their models on 25 (5x5) portfolios stratified by size and value to see if they could explain these important results.

The 1990s and 2000s were periods of great progress in understanding and modeling changing risks and changing risk premiums ("conditional risks") in asset returns. It was shown that in recessions, consumer spending was reduced towards "habit" levels and consumers became very risk averse, (as they did not want to consume below their habit levels) just at a time when risk was often very high. Thus, risk premiums skyrocketed in big recessions, as both risk aversion became very high and risk was very high, and risk premiums were the product of the two. Studying this, researchers found the clue to the value/growth puzzle, as value stocks were found to have higher consumption risks when risk premiums were high, whereas growth stocks had higher risks when risk was low, so the unconditional returns on value stocks should be higher than for growth stocks, *ceteris paribus*. Thus, value and growth stocks had different concavity or convexity, which relates very much to equilibrium returns on these investments. With this new understanding and risk estimates, several researchers were able to explain the value premium and the size premium.

Along with continuing development of the implications of habit formation on utility and asset pricing, a new model of "long run risks" in consumption was developed in 2004. The model is one where small movements in the changes in real consumption growth are so persistent that they have really large implications for consumer spending long term and for asset prices now, which anticipate long term effects. This model of long run risks has spawned a great amount of research and has had a lot of empirical success.

At the present time, the "long-run risk model" and the "habit formation model" are the two main empirical models in the consumption-based asset pricing space. Surely, each model has its strengths, its weaknesses and its truths. Most of the theoretical insights still fit relatively comfortably within the most general original models of asset pricing of the late 1970s. In this article, with few exceptions, we have presented the outstanding research in this area in the past four decades much as the authors did, often using their tables, graphs and descriptions for authenticity and correctness. As well there should be, there is substantial academic debate about the merits and challengers of the various models. We have not taken the time and space to review the various claims and counterclaims by the authors of those competing models, but instead refer the readers to the following articles by the authors: Campbell-Cochrane (2000), Beeler-Campbell (2012) and Bansal, Kiku, Yaron (2012). In addition, two articles, Lewellen-Nagel (2006) and Nagel-Singleton (2010), raised important concerns and objections to significant amounts of the important research reviewed here and should also be examined carefully, so that the reader more fully understands the larger issues of econometric testing of consumption-based asset pricing models.

Consumption-based asset pricing has yielded many insights in the past four decades, both theoretical and empirical. The financial economics literature has produced a vast body of competing economic rationales for what earlier authors characterized as puzzles and paradoxes. To borrow an econometric term: we have over-identified these puzzles and paradoxes. Further research should attempt to integrate models and sort through them for the most important results and develop tests based on different predictions of the models.

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