Central Bank Policy Impacts on the Distribution of Future Interest Rates

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Reference notes for talks at the International Monetary Fund in Washington, D.C., and at central banks and universities in Southeast Asia.
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I. Overview of the Presentation

1. Before and after analysis shows that the U.S. Federal Reserve and the European Central Bank's policies often have significant impacts on the distributions for future interest rates. Fed, ECB and Bank of England research papers show central bank awareness.

2. Interest Rate Caps and Floors have been used for the last 30 years to hedge interest rate risks of financial institutions. They are portfolios of interest rate put and call options. We show how to use their prices to estimate the market's implied "insurance prices" for what LIBOR will be 3 to 5 years in the future.

3. Empirically, interest rate insurance prices 2003-2016 have shifted from bell-shaped curves to positively skewed ones. Some key market prices show "bipolar" views on future rates that reflect either (1) normalization or (2) fears of recession or deflation.

Breeden-Litzenberger Method (1978, 2013) used by Central Banks to find price distributions from option prices.

Probability distributions of future asset prices implied by option prices

By Bhupinder Bahra of the Bank's Monetary Instruments and Markets Division.

Introduction

Many monetary authorities routinely use the forward-looking information that is embedded in financial asset prices to help in formulating and implementing monetary policy. For example, they typically look at changes in the forward rate curve implied by government bond prices to assess changes in market perceptions of future short-term interest rates.(1) But, although implied forward rates are informative about the market's mean expectation for future interest rates, they tell us nothing about the range of expected outcomes around such estimates. For this, we can turn to options markets.

exercising it only if the price of the underlying asset lay above the strike price at that time.

Consider a set of European options on the same underlying asset, with the same time-to-maturity, but with different exercise prices. The prices of such options are related to the probabilities attached by the market to the possible values of the underlying security on the maturity date of the options. Intuitively, this can be seen by noting that the difference in the price of two options with adjacent exercise prices will reflect the value attached to the ability to exercise the options when the price of the underlying asset lies between the two exercise prices. This price difference in turn depends on the probability of the underlying asset price

1996 Bank of England Quarterly

The Breeden and Litzenberger approach

Breeden and Litzenberger (1978) derived a relationship linking the curvature of the call pricing function to the terminal RND function of the price of the underlying asset. In particular, they showed that the second partial derivative of the call pricing function with respect to the exercise price is directly proportional to the terminal RND function. Details about the derivation of the Breeden and Litzenberger result are given in Bahra (1996). The rest of this article focuses on how this result can be applied in order to estimate market RND functions for short-term interest rates in the future and how such RND functions can be used for policy analysis.

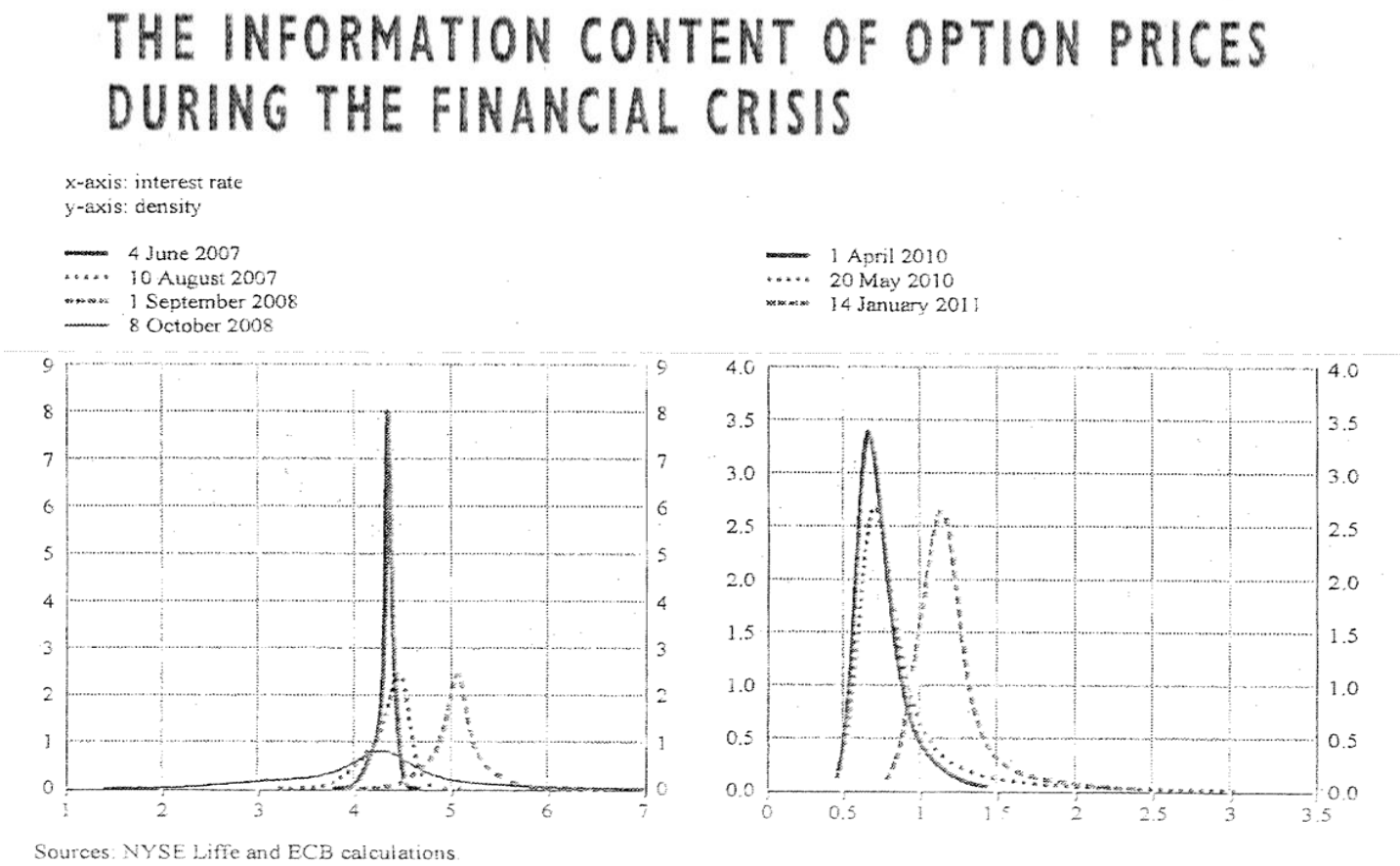
FEDERAL RESERVE BANK OF MINNEAPOLIS

BANKING AND POLICY STUDIES

Methodology for Estimating Risk Neutral Probability Density Functions

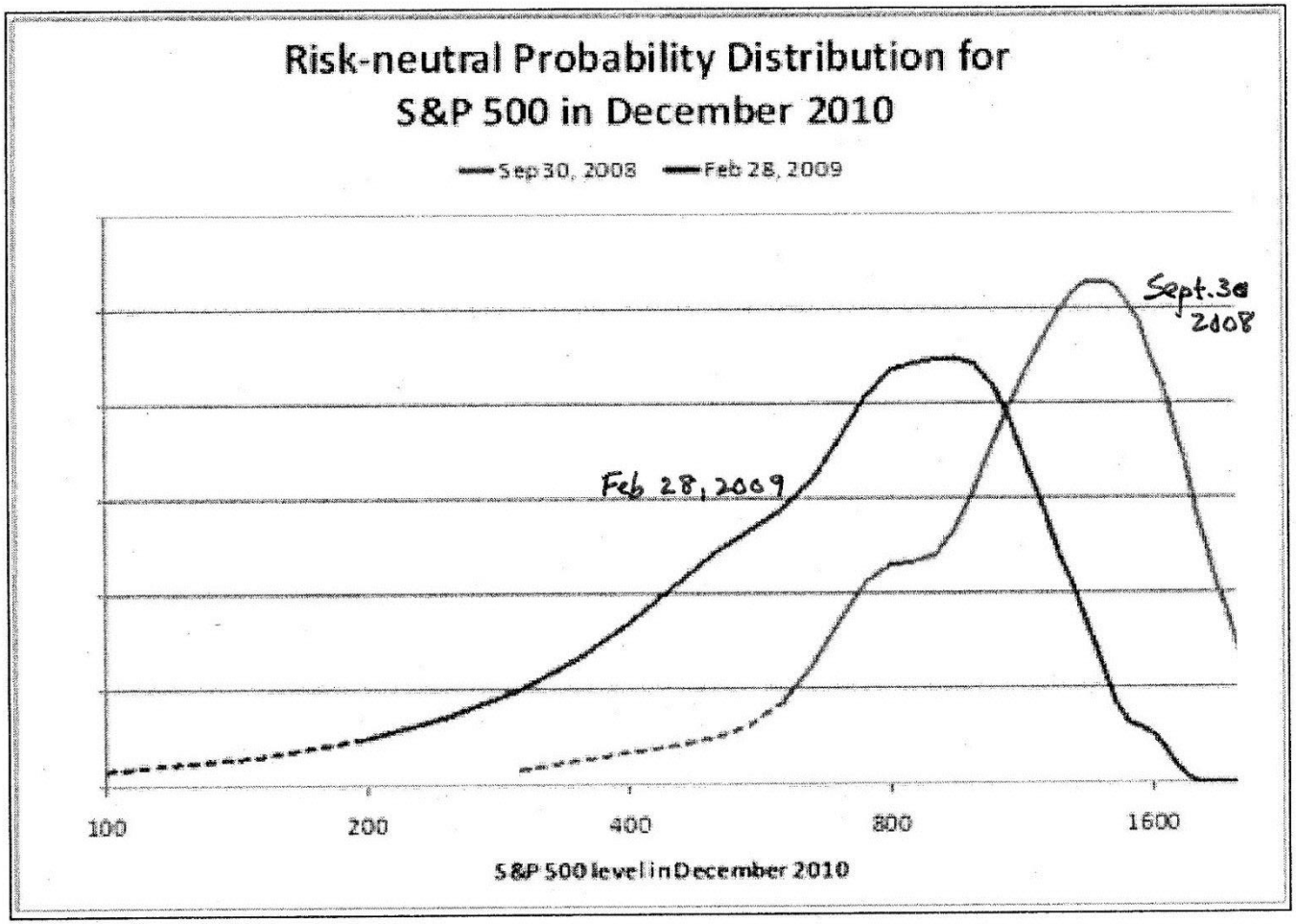
We estimate risk neutral probability density functions (RNPDs) for a variety of different asset classes using a variation of the technique developed by Shimko (1993). This procedure involves fitting a curve to the implied volatilities of a series of options and expressing the volatility as a function of the strike price. The implied volatilities are then translated into continuous call option prices, and the risk neutral distribution of the underlying asset is obtained through the Breeden-Litzenberger (1978) method.

European Central Bank's Monthly Bulletin, February 2011, uses the Breeden-Litzenberger 1978 method to estimate interest rate distributions for what Euribor will be in 3 Months:



Freakonomics article: "Quantifying the Nightmare Scenarios"

Eric Zitzewitz (Dartmouth) uses Breeden-Litzenberger 1978 technique to estimate pricing of disaster risks in ***Freakonomics*** Blog by Justin Wolfers, March 2, 2009



9/30/2008:
S&P500= 1166
VIX = 39.4%

2/28/2009
S&P500 = 735
VIX = 46.4%

Disadvantages of Many Prior Approaches

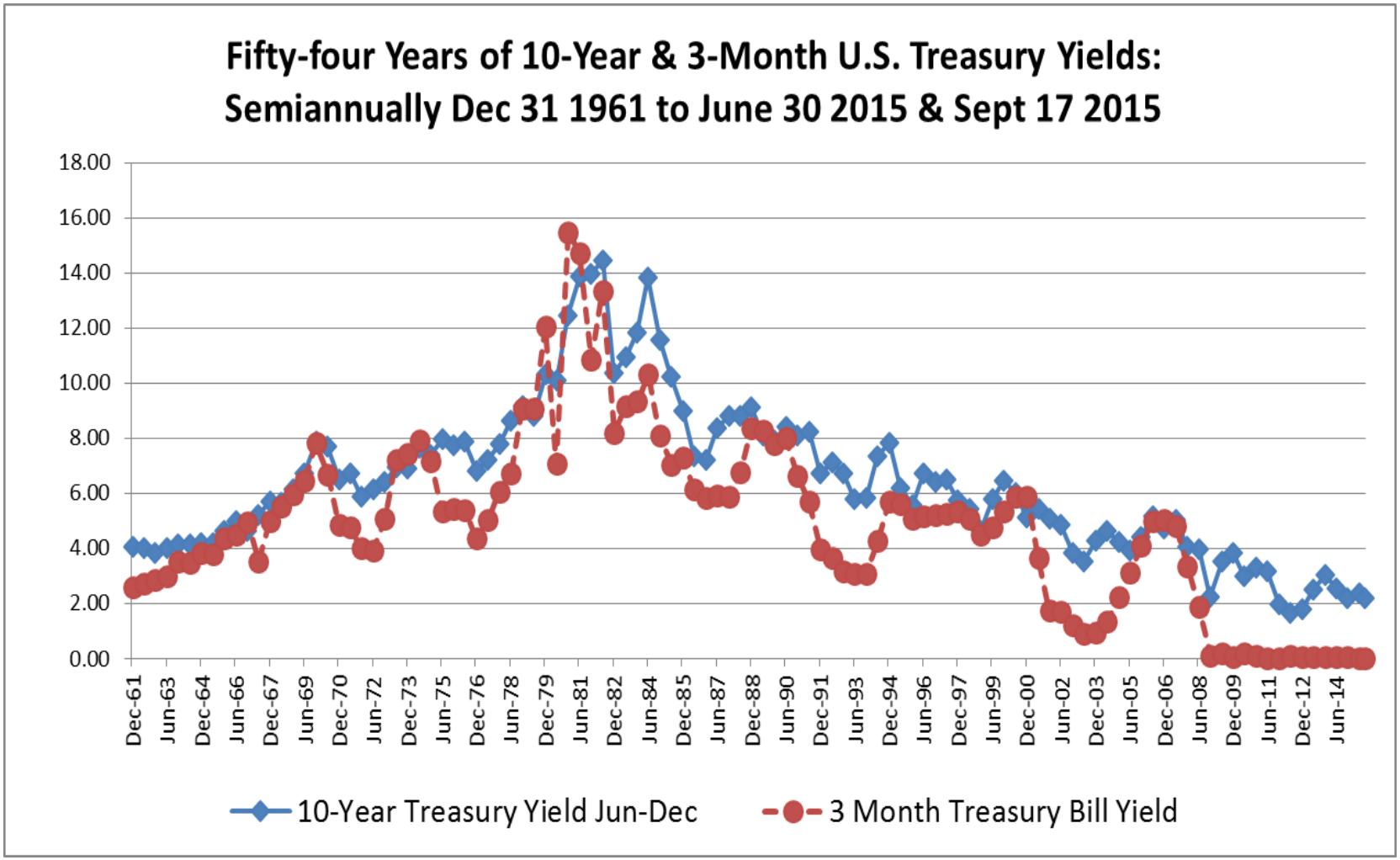
- 1. ***Short-term option prices used***.

Most options mature in 3 months to 18 months, as many markets only have active markets for those maturities. Often there are not options actively traded for a large number of standardized strike prices. We use interest rate caps and floors that have longer term maturities from 2 to 10 years.

- 2. ***Parametric vs. nonparametric approach.***

Applications often parameterize option prices with 3 or 4 parameters (mean, variance, skewness, kurtosis) and estimate implied volatility surfaces and entire risk-neutral densities. It is well-known among practitioners that these methods can be off significantly in estimating tail risks.

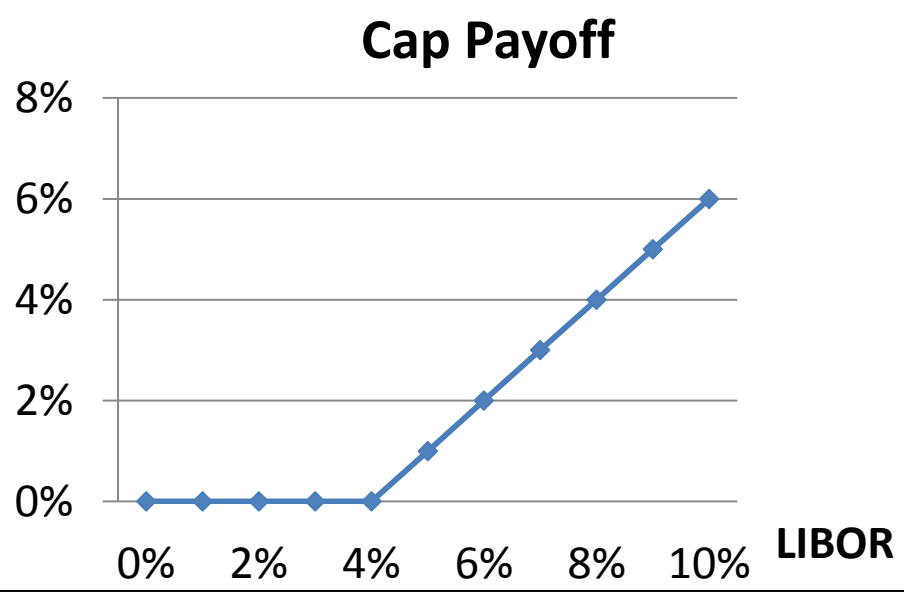
Interest Rate Caps and Floors are long-term interest rate protection agreements for hedging. Traded since interest rate surge in 1981-82.

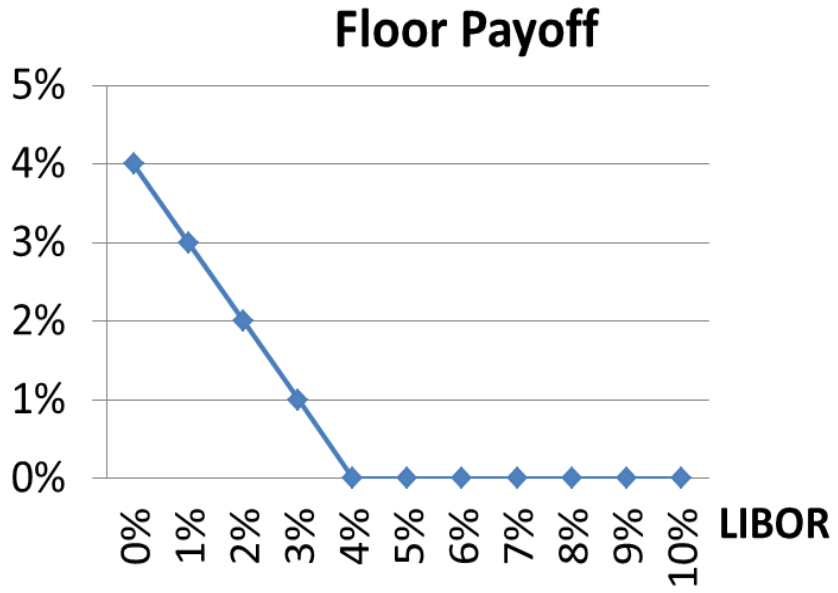


Payoffs on Interest Rate Caps and Floors

Purchaser of a 5-year **interest rate cap** on 3-month LIBOR, with a "strike rate" (exercise price) of 4% receives quarterly for 5 years the difference between then-current LIBOR and 4%. (0 if <4%).

Caps hedge against higher interest rates. As rates increase, the cap's cash flows increase and pay increased funding costs. Caps win when rates increase, like portfolios of put options on bond prices.





US Dollar: 3-Month LIBOR

Interest Rate Cap Prices (% Notional value)

	3-mnth	5-Yr	10-yr		2% Cap				3% Cap				4% Cap			
	LIBOR	Swap	Treas		2-Yr	3-yr	4-Yr	5-yr	2-Yr	3-yr	4-Yr	5-yr	2-Yr	3-yr	4-Yr	5-yr
12/31/2003	1.15	3.65	4.25		1.13	3.15	5.65	8.66	0.58	1.86	3.73	6.04	0.29	1.11	2.41	4.14
12/31/2004	2.56	4.03	4.22		2.70	4.60	6.79	9.11	1.07	2.23	3.75	5.34	0.34	0.98	1.97	2.98
12/30/2005	4.54	4.86	4.39		5.29	7.25	10.11	12.48	3.38	4.95	6.61	8.27	1.57	2.44	3.46	4.54
12/29/2006	5.36	5.10	4.70		5.89	8.44	10.94	13.33	3.98	5.62	7.40	9.04	2.14	3.04	4.12	5.14
6/29/2007	5.36	5.53	5.02		6.23	9.20	12.11	14.95	4.32	6.41	8.53	10.62	2.44	3.72	5.08	6.47
12/31/2007	4.70	4.20	4.02		3.50	5.45	7.56	9.89	1.83	3.02	4.39	6.05	0.64	1.26	2.04	3.09
6/30/2008	2.78	4.25	3.97		3.03	5.47	7.91	10.37	1.48	3.03	4.84	6.59	0.66	1.55	2.77	3.89
12/31/2008	1.43	2.10	2.21		0.39	1.18	2.11	3.32	0.13	0.43	0.98	1.77	0.04	0.17	0.42	0.88
3/31/2009	1.19	2.22	2.66		0.26	0.96	2.03	3.33	0.08	0.42	0.98	1.74	0.03	0.18	0.47	0.88
6/30/2009	0.60	2.96	3.53		0.74	2.25	4.41	6.71	0.40	1.33	2.89	4.52	0.23	0.78	1.89	3.01
12/31/2009	0.25	3.02	3.84		0.79	2.35	4.51	7.01	0.46	1.46	3.02	4.78	0.28	0.92	2.03	3.27
6/30/2010	0.53	2.05	2.93		0.21	0.92	2.06	3.67	0.08	0.48	1.14	2.21	0.03	0.26	0.63	1.31
12/31/2010	0.30	2.20	3.29		0.26	1.04	2.40	4.43	0.16	0.62	1.43	2.85	0.10	0.38	0.86	1.82
6/30/2011	0.25	2.04	3.16		0.15	0.79	2.07	3.98	0.08	0.44	1.20	2.49	0.04	0.25	0.70	1.53
9/30/2011	0.37	1.25	1.92		0.08	0.32	0.89	1.88	0.04	0.17	0.50	1.16	0.02	0.10	0.30	0.74
12/31/2011	0.58	1.25	1.88		0.09	0.34	0.88	1.76	0.04	0.20	0.54	1.15	0.02	0.13	0.36	0.78
6/30/2012	0.46	0.97	1.64		0.02	0.15	0.44	1.06	0.01	0.09	0.25	0.66	0.00	0.06	0.16	0.45
12/31/2012	0.31	0.83	1.78		0.01	0.10	0.38	0.92	0.01	0.05	0.22	0.57	0.00	0.03	0.14	0.38
6/28/2013	0.28	1.59	2.52		0.07	0.44	1.36	2.77	0.04	0.25	0.83	1.75	0.02	0.16	0.53	1.14
12/31/2013	0.27	1.78	3.04		0.07	0.52	1.69	3.53	0.04	0.29	1.02	2.27	0.02	0.18	0.64	1.48
6/30/2014	0.24	1.70	2.53		0.07	0.56	1.51	2.83	0.03	0.27	0.80	1.62	0.01	0.14	0.44	0.94
12/31/2014	0.30	1.79	2.17		0.17	0.79	1.67	2.75	0.05	0.34	0.81	1.45	0.02	0.16	0.41	0.79
6/30/2015	0.33	1.78	2.35		0.12	0.63	1.49	2.64	0.03	0.24	0.68	1.34	0.01	0.10	0.32	0.69
9/18/2015	0.33	1.60	2.13		0.08	0.44	1.09	2.02	0.02	0.16	0.49	1.01	0.01	0.07	0.24	0.54

Interest Rate Floor Prices (% Notional value)

US Dollar: 3-Month LIBOR

	3-mnth	5-Yr	10-yr	1%				2%				3%			
	LIBOR	Swap	Treas	2-Yr	3-yr	4-Yr	5-yr	2-Yr	3-yr	4-Yr	5-yr	2-Yr	3-yr	4-Yr	5-yr
12/31/2003	1.15	3.65	4.25	0.07	0.11	0.13	0.15	0.92	1.08	1.19	1.28	2.36	2.81	3.11	3.37
12/31/2004	2.56	4.03	4.22	0.00	0.01	0.03	0.04	0.02	0.08	0.19	0.24	0.35	0.59	0.91	1.08
12/30/2005	4.54	4.86	4.39	0.00	0.00	0.01	0.02	0.00	0.02	0.05	0.11	0.02	0.09	0.22	0.38
12/29/2006	5.36	5.10	4.70	0.00	0.00	0.01	0.02	0.00	0.01	0.04	0.07	0.01	0.07	0.14	0.25
6/29/2007	5.36	5.53	5.02	0.00	0.00	0.01	0.02	0.00	0.01	0.03	0.06	0.01	0.03	0.09	0.15
12/31/2007	4.70	4.20	4.02	0.02	0.05	0.07	0.10	0.09	0.20	0.25	0.37	0.36	0.66	0.82	1.09
6/30/2008	2.78	4.25	3.97	0.02	0.06	0.10	0.15	0.11	0.21	0.31	0.44	0.51	0.74	0.99	1.24
12/31/2008	1.43	2.10	2.21	0.33	0.57	0.73	0.95	1.50	2.14	2.51	3.03	3.23	4.50	5.29	6.25
3/31/2009	1.19	2.22	2.66	0.24	0.35	0.43	0.51	1.53	1.98	2.27	2.54	3.35	4.40	5.14	5.79
6/30/2009	0.60	2.96	3.53	0.43	0.46	0.53	0.58	1.74	1.94	2.19	2.35	3.41	3.98	4.56	4.94
12/31/2009	0.25	3.02	3.84	0.66	0.70	0.75	0.78	2.03	2.27	2.49	2.63	3.70	4.35	4.91	5.28
6/30/2010	0.53	2.05	2.93	0.66	0.87	1.00	1.11	2.31	2.98	3.38	3.67	4.19	5.53	6.42	7.05
12/31/2010	0.30	2.20	3.29	0.97	1.10	1.22	1.32	2.74	3.32	3.65	3.92	4.65	5.90	6.64	7.21
6/30/2011	0.25	2.04	3.16	1.03	1.20	1.30	1.39	2.83	3.50	3.82	4.07	4.77	6.15	6.92	7.46
9/30/2011	0.37	1.25	1.92	1.09	1.54	1.83	2.10	2.98	4.20	5.05	5.73	4.96	7.06	8.66	9.95
12/31/2011	0.58	1.25	1.88	0.83	1.28	1.64	1.98	2.68	3.93	4.92	5.77	4.66	6.81	8.58	10.09
6/30/2012	0.46	0.97	1.64	1.01	1.51	1.89	2.20	2.97	4.34	5.44	6.33	4.98	7.30	9.26	10.91
12/31/2012	0.31	0.83	1.78	1.31	1.85	2.25	2.54	3.29	4.73	5.90	6.80	5.30	7.71	9.76	11.45
6/28/2013	0.28	1.59	2.52	1.19	1.49	1.68	1.82	3.10	4.08	4.68	5.10	5.09	6.91	8.14	9.02
12/31/2013	0.27	1.78	3.04	1.23	1.50	1.65	1.76	3.15	4.00	4.47	4.78	5.14	6.79	7.78	8.45
6/30/2014	0.24	1.70	2.53	1.10	1.29	1.44	1.57	2.96	3.63	4.10	4.48	4.94	6.35	7.36	8.18
12/31/2014	0.30	1.79	2.17	0.81	1.00	1.20	1.43	2.44	2.99	3.53	4.06	4.34	5.53	6.64	7.67
6/30/2015	0.33	1.78	2.35	0.74	0.93	1.11	1.29	2.38	2.95	3.42	3.87	4.31	5.55	6.57	7.47
9/18/2015	0.33	1.60	2.13	0.80	1.07	1.34	1.60	2.52	3.27	3.93	4.53	4.48	5.99	7.29	8.45

II. How to Find Interest Rate Insurance Prices From Option Prices:

*See Ross (1976), Quarterly Journal of Economics article "Options and Efficiency" and Breeden and Litzenberger (1978) Journal of Business article, "Prices of State-Contingent Claims Implicit in Option Prices." B-L's MIT working paper in 2013 on "Central Bank Policy Impacts on the Distribution of Future Interest Rates" gives the method for calculations in this talk. .

Stripping A Zero Coupon Riskless Bond to Create Lottery Tickets (Insurance Payments) on 3-Month LIBOR

- Let us assume that you want to create securities that would allow portfolio managers to hedge interest rate risks for 3-month LIBOR in 1 year. To keep things simple, you assume that LIBOR might be

  {1%, 2%, 3%, 4%, 5%, 6%, 7%, or 8%} then.

- A financial institution buys $100 million face value of 1-year U.S. Treasury bills (zero coupon) and puts them into a Trust, paying $97 million for the position, reflecting an assumed discount rate today of approximately 3% on T-bills.

Stripping A Zero Coupon Riskless Bond to Create Lottery Tickets (Insurance Payments) on 3-Month LIBOR

The institution then "strips" the Tbill payoffs into 8 lottery tickets A1 to A8:

Lottery ticket A1 pays $1.00 if and only if LIBOR <1.5% in 1 year, zero otherwise.
Lottery ticket A2 pays $1.00 if and only if LIBOR is between 1.5% and 2.49%
Lottery ticket A3 pays $1.00 if and only if LIBOR is between 2.5% and 3.49%
Lottery ticket A4 pays $1.00 if and only if LIBOR is between 3.5% and 4.49%
Lottery ticket A5 pays $1.00 if and only if LIBOR is between 4.5% and 5.49%
Lottery ticket A6 pays $1.00 if and only if LIBOR is between 5.5% and 6.49%
Lottery ticket A7 pays $1.00 if and only if LIBOR is between 6.5% and 7.49%
Lottery ticket A8 pays $1.00 if and only if LIBOR is 7.5% or more in 1 year.

The institution could sell 100 million of each lottery ticket and pay off as promised.

Table of Payoffs on the Lottery Tickets for Different LIBOR Rates

3-MoLIBOR Rate (the "State")	True Probability of the State	Lottery Tickets for 3-Month LIBOR Rate in 1 Year: Ticket ID = A1	A2	A3	A4	A5	A6	A7	A8	A1-A8 Portfolio
1.0%	0.10	1	0	0	0	0	0	0	0	1
2.0%	0.12	0	1	0	0	0	0	0	0	1
3.0%	0.15	0	0	1	0	0	0	0	0	1
4.0%	0.20	0	0	0	1	0	0	0	0	1
5.0%	0.15	0	0	0	0	1	0	0	0	1
6.0%	0.12	0	0	0	0	0	1	0	0	1
7.0%	0.10	0	0	0	0	0	0	1	0	1
8.0%	0.06	0	0	0	0	0	0	0	1	1
Total =	1.00									

Note 1: Different "states" mean different economic scenarios for LIBOR in 1 year.

Note 2: The lottery tickets {A1,…,A8} are also called "**Arrow Securities**," named after their originator in 1964, Nobel Laureate Kenneth Arrow (Harvard/Stanford).

Note 3: Arrow securities are also like pure insurance for a certain rate/state, for if you had a loss of $100,000 on your portfolio if LIBOR goes to 4%, you could buy 100,000 of security A4 and it would insure that loss.

Lottery Tickets/Insurance Payments As Building Blocks for Caps and Floors and Calls and Puts

1. Note that If you bought 1 of each of the 8 lottery tickets, it would be like buying a ticket to win on each horse in a horse race, which is certain to get the winning horse.

2. Your portfolio payoff would be $1.00 for sure, just like a riskless Treasury bill. Thus, to avoid arbitrage, the price of the whole portfolio would have to be $0.97, as that's what the 1-year Tbill sells for per $1.00 par. Each individual lottery ticket would sell for less than that.

Lottery Tickets/Arrow Securities As Building Blocks for Caps and Floors and Calls and Puts

Note that these securities are great building blocks for derivative securities, like caps and floors or call and put options (See subsequent table for an illustration):

Buying a portfolio of 1 shares of the A4 securities, 2 shares of A5 securities, 3 shares of A6, 4 shares of A7 and 5 shares of A8 creates an interest rate cap's payoffs with a strike rate of 3%.

Buying a portfolio of 3 shares of the A1 security, 2 shares of A2 and 1 share of the A3 security creates an interest rate floor's payoffs with a strike rate of 4%. To avoid arbitrage or dominance, the cost of the floor must equal the cost of this portfolio.

True Probabilities vs. Insurance Prices or "Risk Neutral Probabilities"

Insurance prices or "risk neutral probabilities" differ from true, objective probabilities, because investors price assets higher for those that pay off most when times are bad (negative beta). Thus, their insurance prices (risk neutral probabilities) exceed their true probabilities.

States that correspond to good economies will have lower insurance prices, and their insurance prices will underestimate the true probabilities.

Illustration of State Prices vs. Probabilities and Cap and Floor Prices

				RND*			
3-MoLIBOR Rate (the "State")	Projected Real GDP Growth	True Probability of State	State Price	Normalized State Price	RiskNeut Probab/ TrueProb	3% Cap Payoffs	4% Floor Payoffs
1.0%	-2.3%	0.10	0.126	0.13	1.30	0	3
2.0%	-0.9%	0.12	0.140	0.15	1.25	0	2
3.0%	0.6%	0.15	0.165	0.17	1.13	0	1
4.0%	2.0%	0.20	0.194	0.20	1.00	1	0
5.0%	3.4%	0.15	0.126	0.13	0.87	2	0
6.0%	4.8%	0.12	0.097	0.10	0.83	3	0
7.0%	6.2%	0.10	0.078	0.08	0.80	4	0
8.0%	7.6%	0.06	0.039	0.04	0.67	5	0
Total =	2.3%	1.00	0.97	1.00	Value =	$1.24	$0.82

*RND = Risk Neutral Density

Breeden-Litzenberger Method
Constructs Pure Bet Insurance Prices from Call Option Prices

Breeden-Litzenberger 1978, *Journal of Business*

Underlying Asset Price

Spreads Butterfly Spread

				Call Option Portfolios		
	Payoffs on Call Options			Port. A	Port. B	Port.C=A-B
P	C(X=2)	C(X=3)	C(X=4)	C(2)-C(3)	C(3)-C(4)	C(2)-2C(3)+C(4)
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	1	0	0	1	0	1
4	2	1	0	1	1	0
5	3	2	1	1	1	0
6	4	3	2	1	1	0
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
N	N-2	N-3	N-4	1	1	0

"Butterfly Spreads" of Options Give Pure Insurance Prices.

Breeden and Litzenberger (1978) derived Arrow's state prices for different levels of the stock market (relative to today's level) and different maturities, using the Black-Scholes option pricing formula, as given on the following slide.

Breeden and Litzenberger found prices of $1 payoffs for various stock market Increases and declines for periods up to 20 years out. (1978, J. Business)

B-L derived that the price of $1.00 received if underlying price ends between Y_1 and Y_2 and the Black-Scholes formula holds is: $\Delta(Y_1, Y_2, T) = B(T)\{N[d_2(X = Y_1)] - N[d_2(X = Y_2)]\}.$

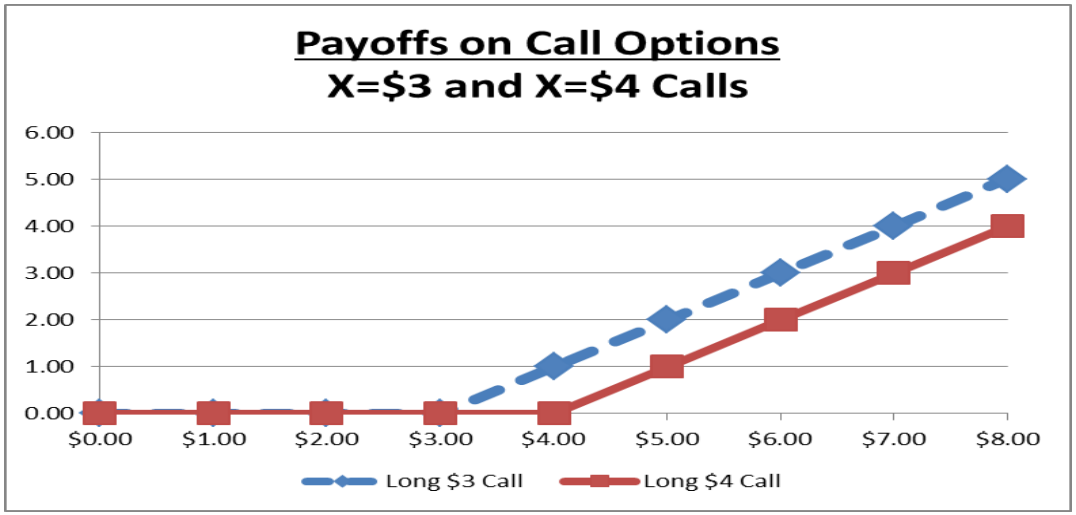
TABLE 3 Delta-Security Prices*

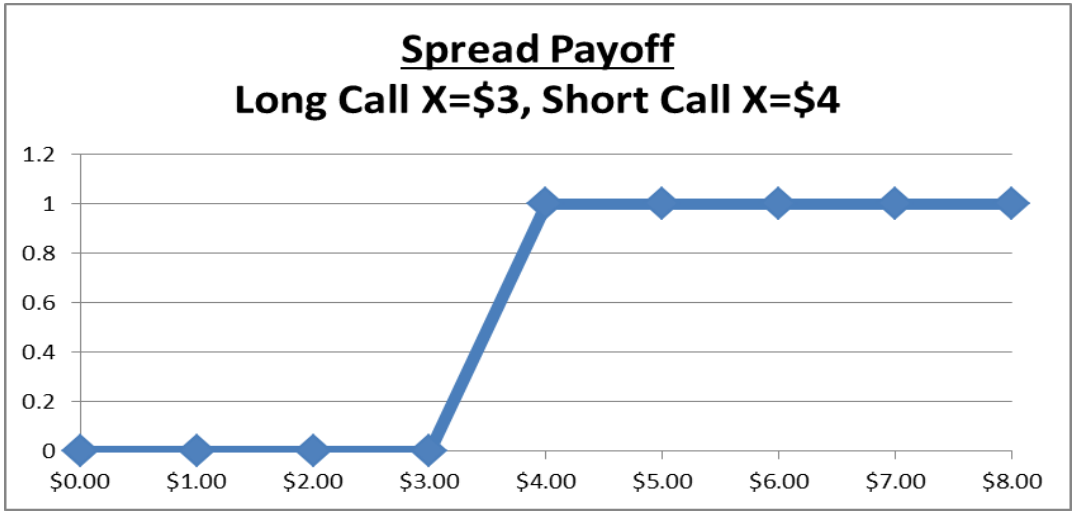
$\frac{Y_1}{M_0} - \frac{Y_2}{M_0}$	Time to Maturity									
	3 Mos.	6 Mos.	9 Mos.	1 Yr.	2 Yrs.	3 Yrs.	4 Yrs.	5 Yrs.	10 Yrs.	20 Yrs.
0–.1										.2¢
.1–.2									.3¢	.9
.2–.3							.1¢	.3¢	1.2	1.6
.3–.4					.1¢	.3¢	.7	1.2	2.5	1.9
.4–.5					.6	1.5	2.4	3.0	3.5	2.0
.5–.6			.2¢	.5¢	2.5	4.0	4.7	4.9	4.0	1.9
.6–.7		.6¢	1.7	3.0	6.1	6.8	6.7	6.4	4.2	1.8
.7–.8	1.2¢	5.0	7.6	9.0	9.9	9.0	8.0	7.1	4.2	1.7
.8–.9	13.1	16.6	16.5	15.7	12.4	10.1	8.5	7.3	4.0	1.6
.9–1.0	34.8	26.4	21.9	18.9	12.9	10.0	8.2	6.9	3.6	1.4
1.0–1.1	32.5	24.3	20.0	17.3	11.7	9.1	7.4	6.2	3.3	1.3
1.1–1.2	13.5	14.7	13.8	12.8	9.6	7.7	6.4	5.5	3.0	1.2
1.2–1.3	2.9	6.5	7.8	8.1	7.3	6.2	5.4	4.7	2.6	1.0
1.3–1.4	.4	2.2	3.7	4.6	5.3	4.9	4.4	3.9	2.3	.9
1.4–1.5		.6	1.5	2.3	3.7	3.7	3.5	3.2	2.0	.9
1.5–1.6				1.1	2.5	2.8	2.8	2.6	1.8	.8
1.6–1.7				.5	1.6	2.1	2.2	2.1	1.5	.7
1.7–1.8				.2	1.0	1.5	1.7	1.7	1.3	.6
1.8–1.9				.1	.6	1.1	1.3	1.4	1.2	.6
1.9–2.0					.4	.8	1.0	1.1	1.0	.5
2.0–2.1					.2	.5	.8	.9	.9	.5
2.1–2.2					.1	.4	.6	.7	.8	.4
2.2–2.3					.1	.3	.4	.6	.7	.4
2.3–2.4					.1	.2	.3	.5	.6	.4
2.4–2.5						.1	.3	.4	.5	.3
2.5–2.6						.1	.2	.3	.5	.3
2.6–2.7						.1	.1	.2	.4	.3
2.7–2.8							.1	.2	.4	.3
2.8–2.9							.1	.1	.3	.2
2.9–3.0							.1	.1	.3	.2
3.0–3.1							.1	.1	.2	.2
3.1–3.2								.1	.2	.2
3.2–3.3								.1	.2	.2
3.3–3.4									.2	.2
3.4–3.5									.1	.2
3.5–3.6									.1	.1
3.6–3.7									.1	.1
3.7–3.8									.1	.1
3.8–3.9									.1	.1
3.9–4.0									.1	.1
4.0–4.1									.1	.1

* Assumptions for all maturities are: $r = .06$, $\delta = .04$, $\sigma = .20$.

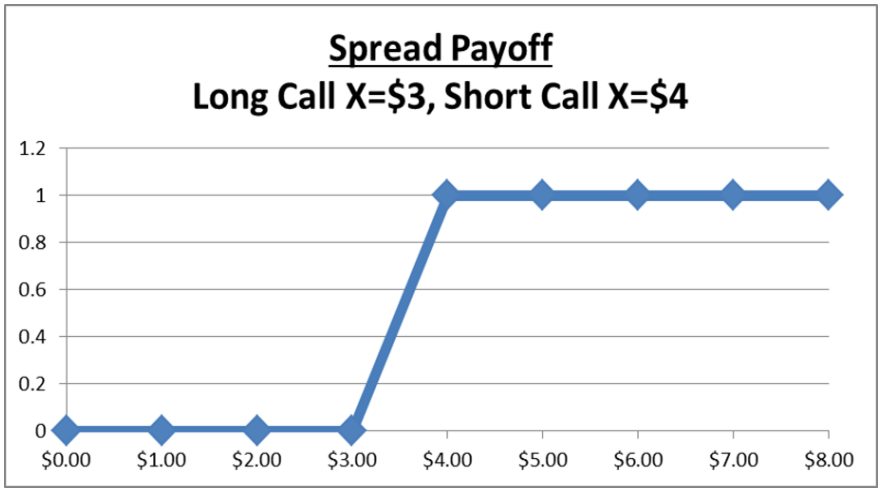
Journal of Business *State-contingent Claims in Option Prices*

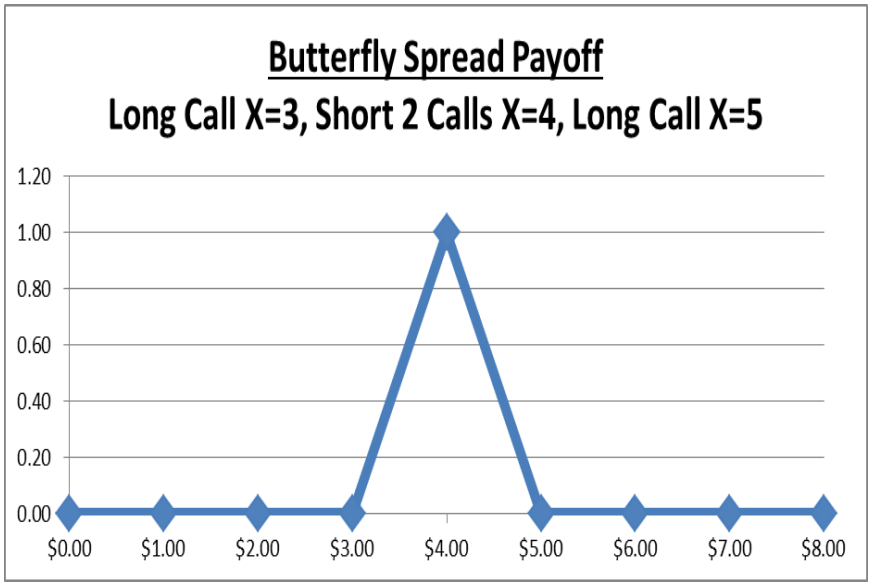
Option payoffs with continuous movements in the underlying asset prices…

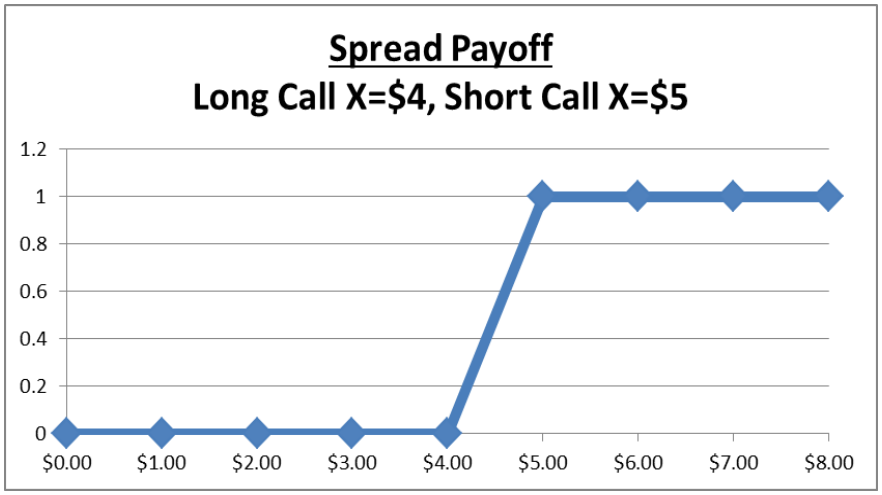




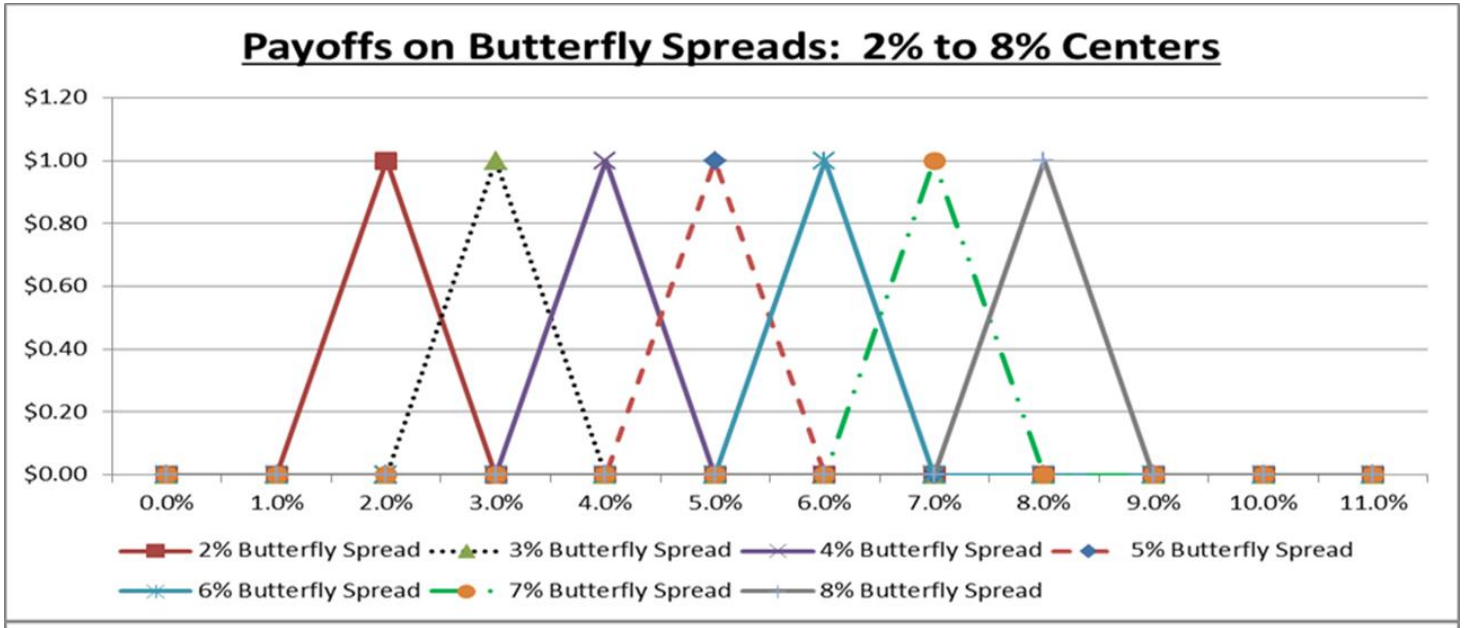
Butterfly spread of options is a spread of spreads: Payoffs are a pure bet on a specific range, zero elsewhere

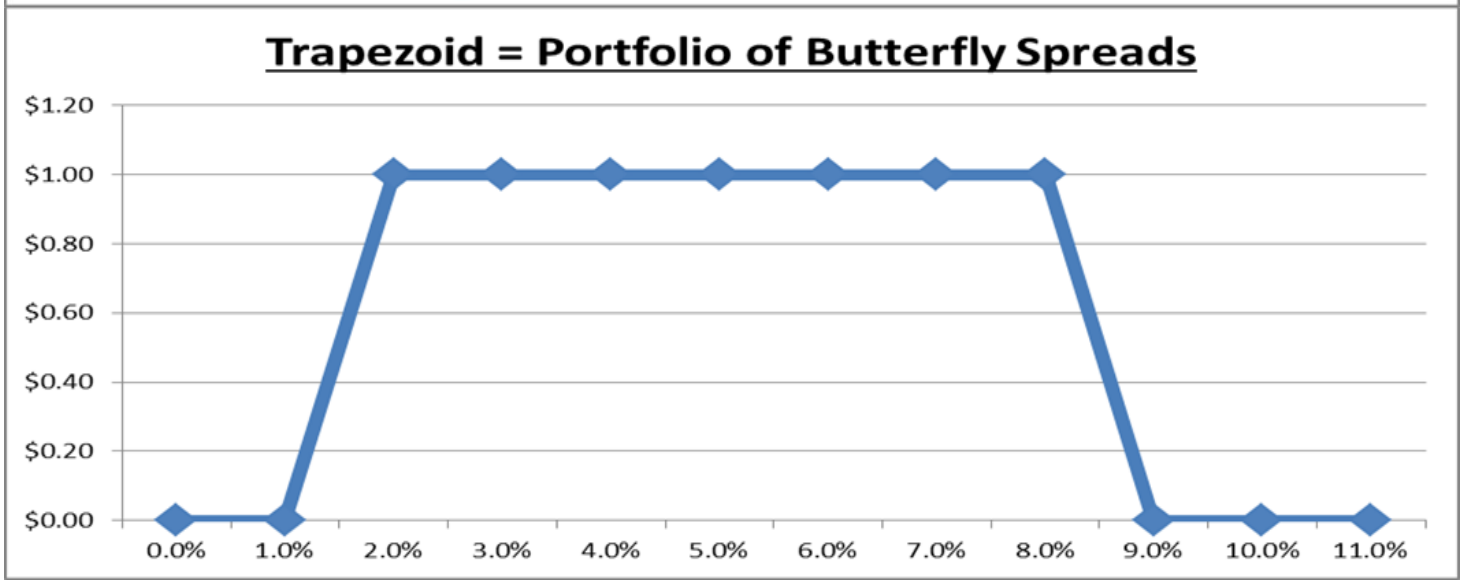


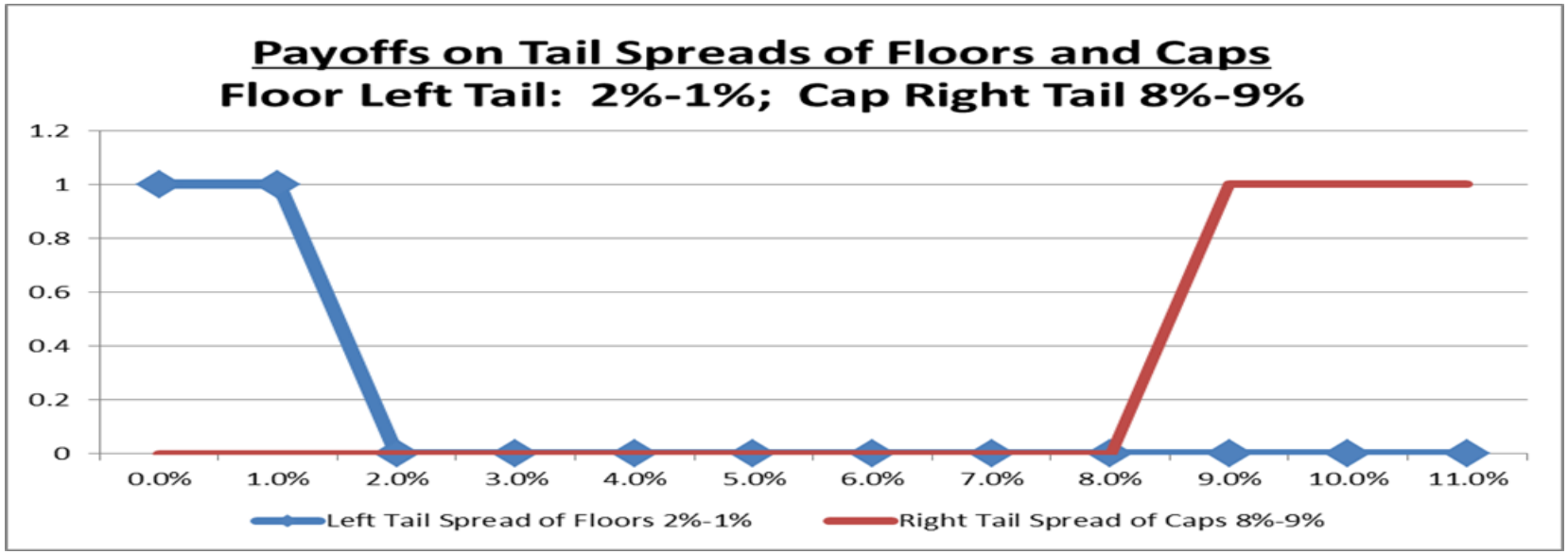


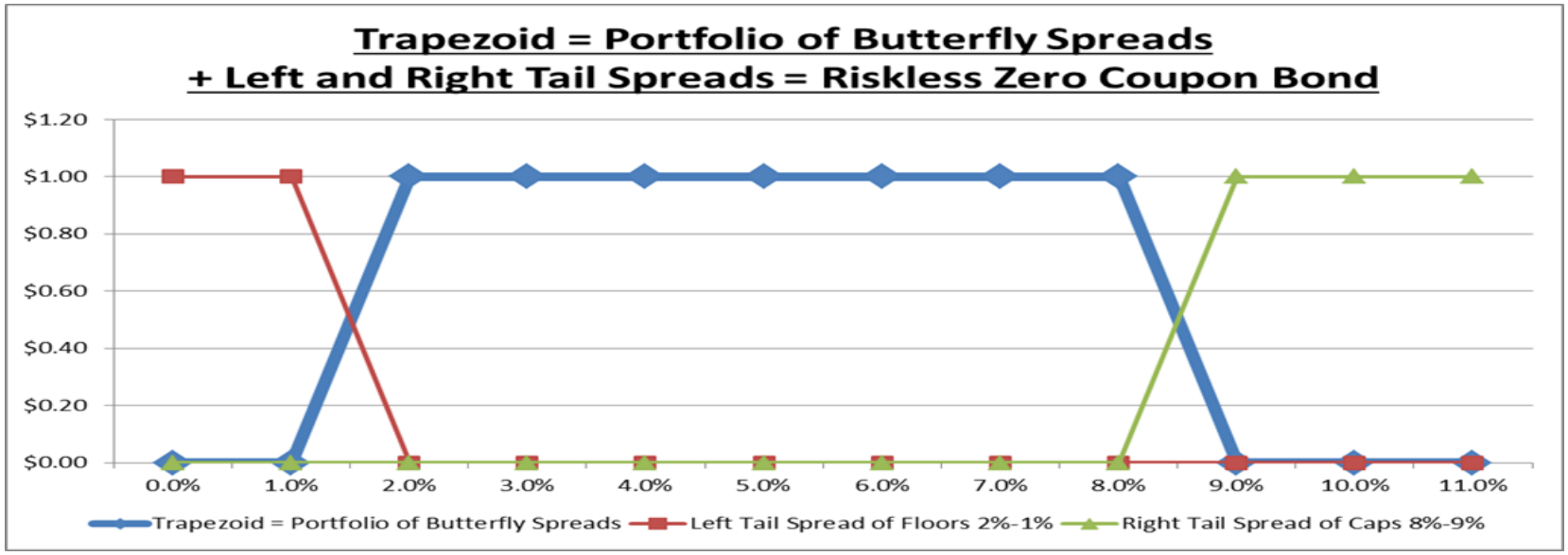


Butterfly spread with strikes of $3, $4, $5 pays off only if Underlying asset falls in that range.


Payoffs on Butterfly Spreads: 2% to 8% Centers
$1.20
$1.00
$0.80
$0.60
$0.40
$0.20
$0.00
0.0%
1.0%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9.0%
10.0%
11.0%
2% Butterfly Spread
3% Butterfly Spread
4% Butterfly Spread
5% Butterfly Spread
6% Butterfly Spread
7% Butterfly Spread
8% Butterfly Spread


Trapezoid = Portfolio of Butterfly Spreads
$1.20
$1.00
$0.80
$0.60
$0.40
$0.20
$0.00
0.0%
1.0%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9.0%
10.0%
11.0%


Payoffs on Tail Spreads of Floors and Caps
Floor Left Tail: 2%-1%; Cap Right Tail 8%-9%
1.2
1
0.8
0.6
0.4
0.2
0
0.0%
1.0%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9.0%
10.0%
11.0%
Left Tail Spread of Floors 2%-1%
Right Tail Spread of Caps 8%-9%


Trapezoid = Portfolio of Butterfly Spreads
+ Left and Right Tail Spreads = Riskless Zero Coupon Bond
$1.20
$1.00
$0.80
$0.60
$0.40
$0.20
$0.00
0.0%
1.0%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9.0%
10.0%
11.0%
Trapezoid = Portfolio of Butterfly Spreads
Left Tail Spread of Floors 2%-1%
Right Tail Spread of Caps 8%-9%

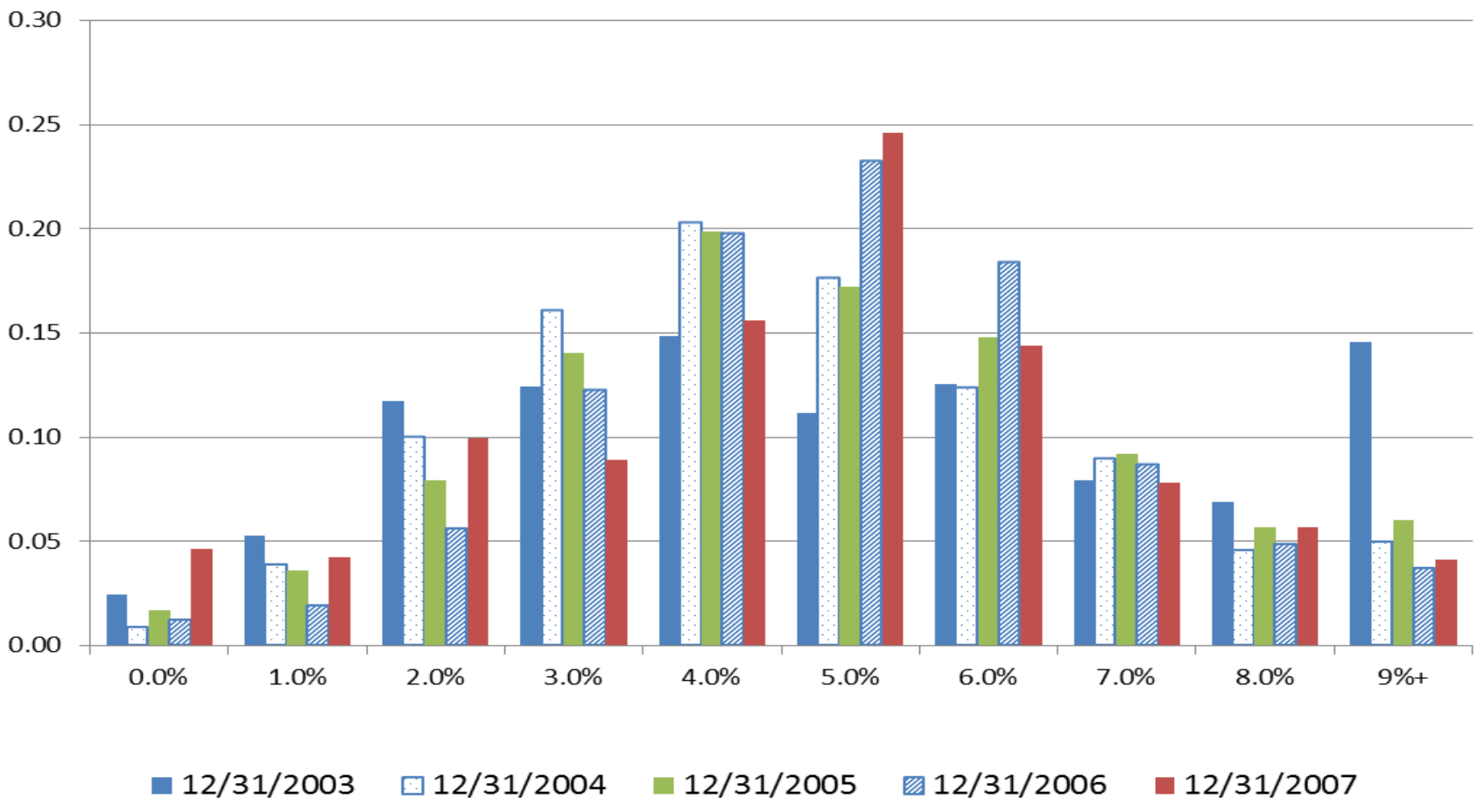
Butterfly Spread and Tail Spread Costs and Risk Neutral Probabilites (Insurance Prices)

	Spread Cost	"Risk-Neutral Probability"
"0%" = Left tail spread: Long 1%, Short 0% floorlet	$0.290	0.297
1% Butterfly spread (Long 0%, Short 2 1%, Long 2%)	$0.320	0.328
2% Butterfly spread (Long 1%, Short 2 2%, Long 3%)	$0.180	0.184
3% Butterfly spread	$0.080	0.082
4% Butterfly spread	$0.037	0.038
5% Butterfly spread	$0.028	0.028
6% Butterfly spread	$0.014	0.014
7% Butterfly spread	$0.007	0.007
8% Butterfly spread	$0.007	0.007
9%+ = Right tail spread: Long 8%, Short 9% caplet	$0.015	0.015
Totals	$0.977	1.000

III. Estimates of Interest Rate Insurance Prices Implicit in Prices of Interest Rate Caps and Floors 2003-2007.

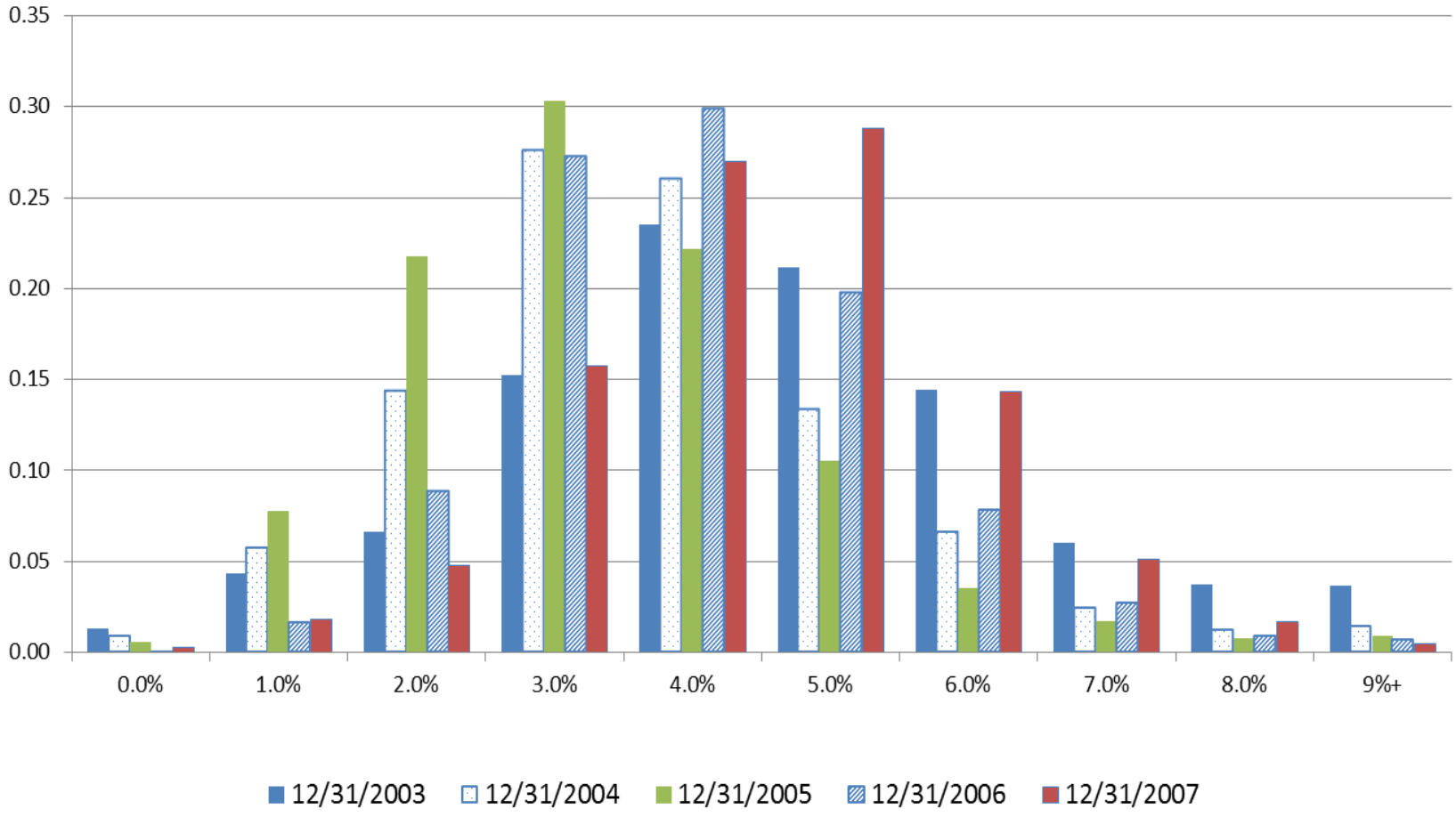
USA Insurance Prices for 3-Month LIBOR in 5 Years, as of December 31, 2003, 2004, 2005, 2006, 2007:

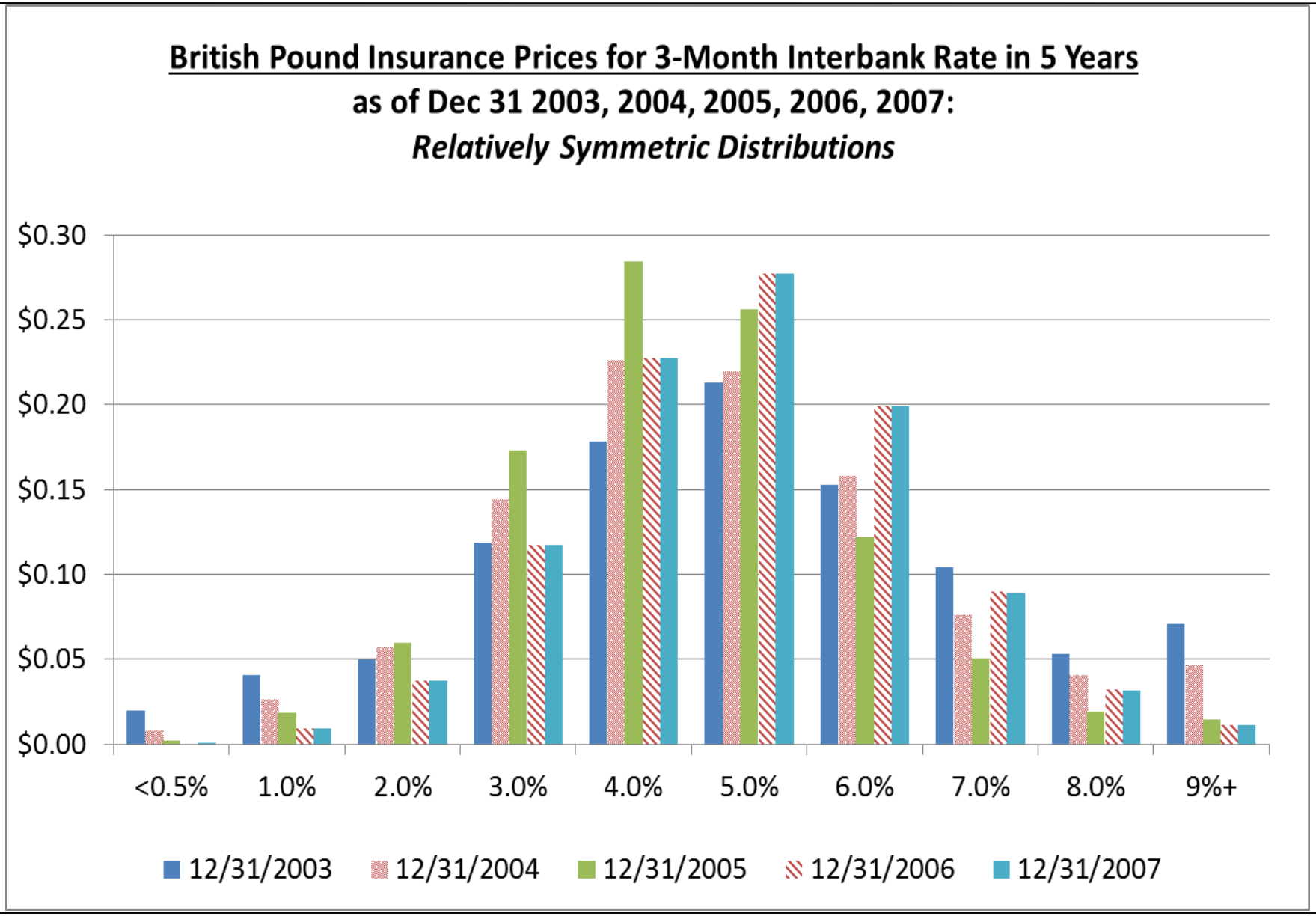
Relatively Symmetric Distributions



Euro Area Insurance Prices for 6-Month Euribor in 5 Years, as of December 31, 2003, 2004, 2005, 2006, 2007

Relatively Symmetric Distributions



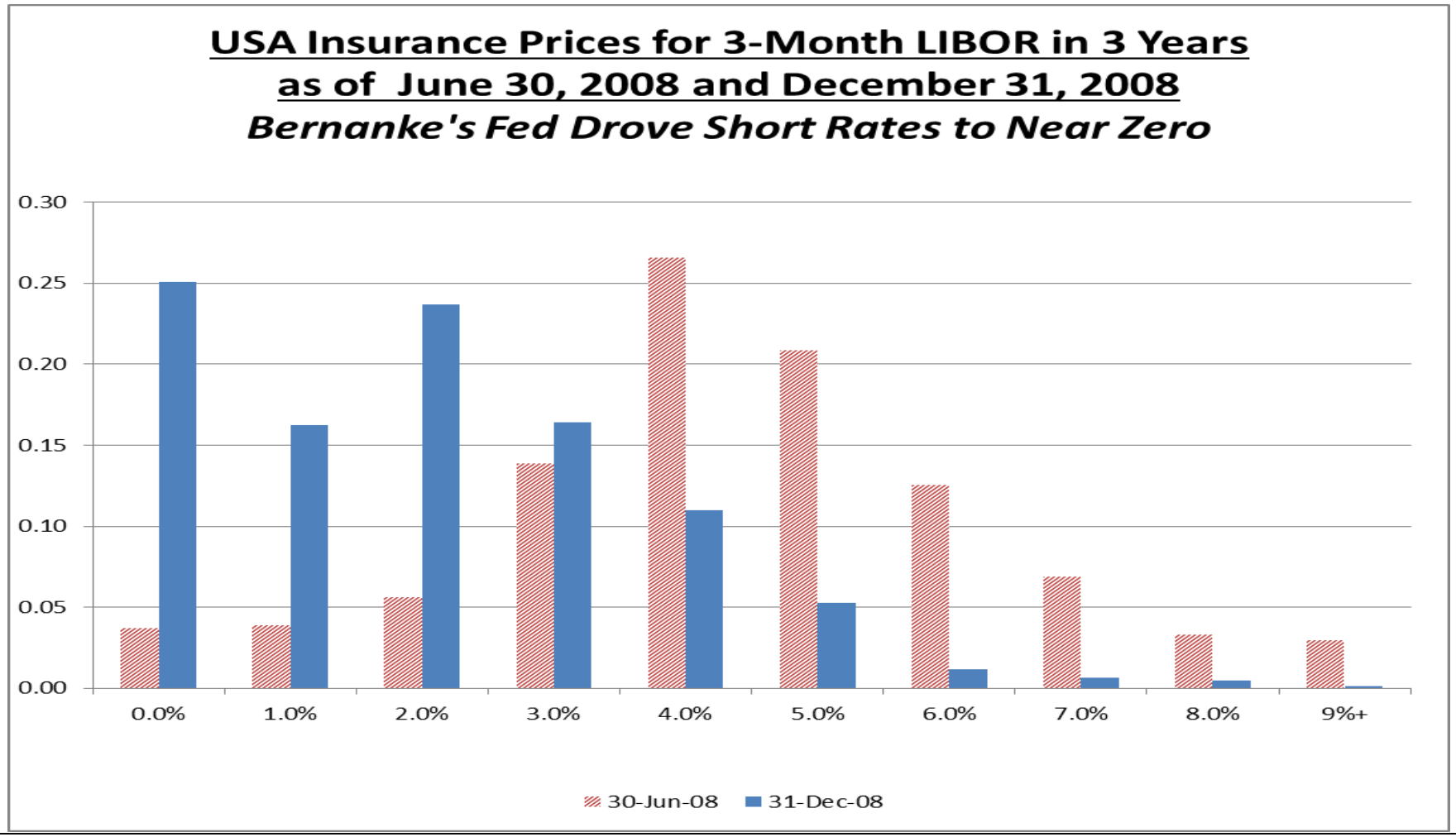

British Pound Insurance Prices for 3-Month Interbank Rate in 5 Years
as of Dec 31 2003, 2004, 2005, 2006, 2007:
Relatively Symmetric Distributions
$0.30
$0.25
$0.20
$0.15
$0.10
$0.05
$0.00
<0.5%
1.0%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9%+
12/31/2003
12/31/2004
12/31/2005
12/31/2006
12/31/2007

IV. Impact of U.S. Federal Reserve Policy Announcements on Interest Rate Insurance Prices for 3-Month LIBOR: 2008-2016

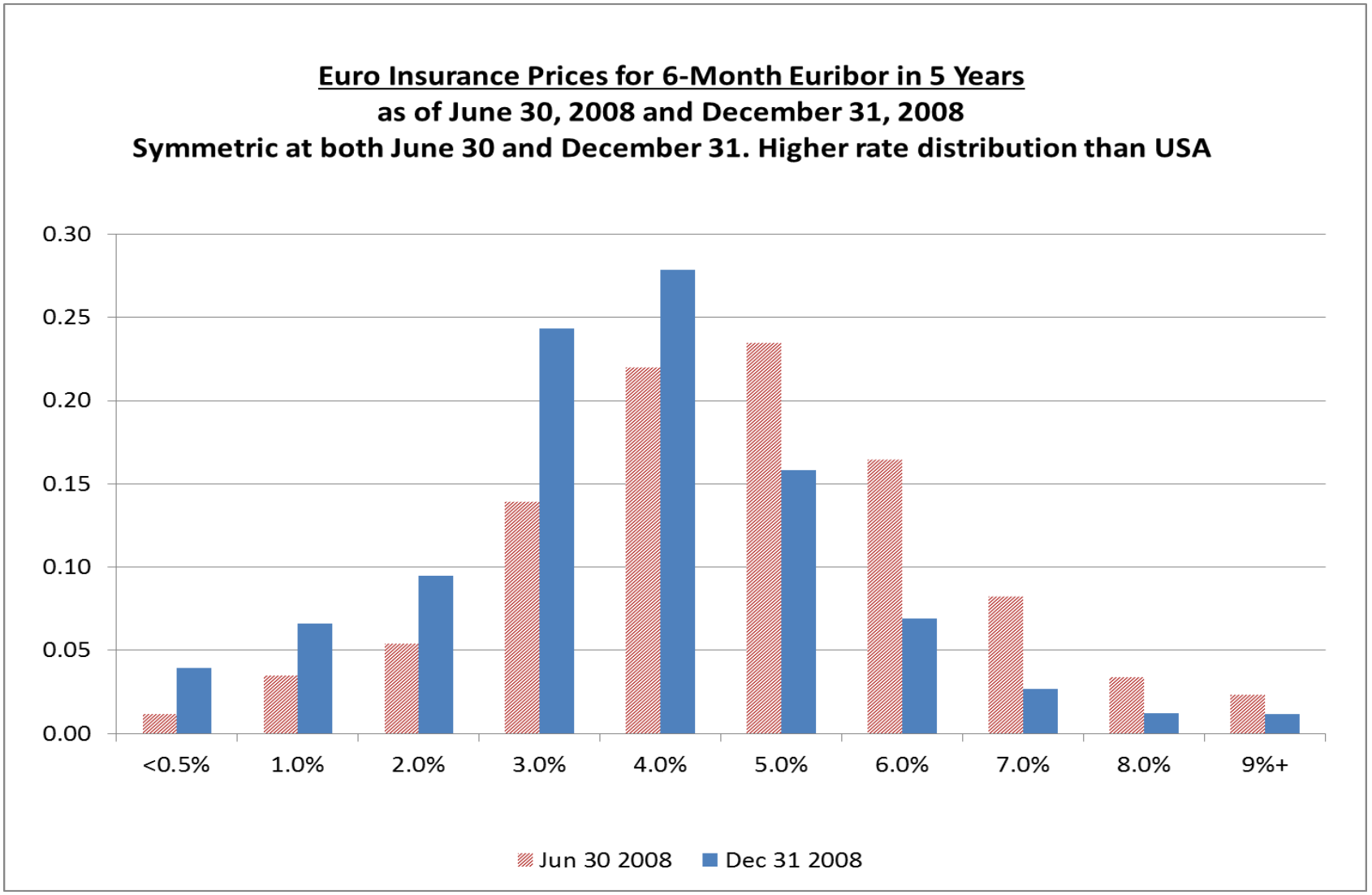
Major Federal Reserve Announcements 2008-2016

- **December 2008**. Cut rates to record lows in financial panic.
- **March 2009**: Will keep rates close to zero for "extended period." Stock market bottoms March 9th. Unemployment rate increases to peak of 10.0% in October 2009.
- **August 2011**: Budget impasse. Fed "will keep rates extremely low "at least until 2013."
- **September 2012**: Low "at least until 2015"
- **December 2012**: Will tie low rates to range in Unemployment (>6.5%), Inflation(<2%).
- **May/June 2013**: **May 22**: Given economic strength, Fed is seriously considering "tapering" asset purchases (QE3). **June 19:** Housing market is strong and supportive; tapering QE3 likely in 2nd half 2013.
- **Sept 18, 2013**: Fed announces "No tapering yet" and surprises markets.
- **Dec 18, 2013.** Bernanke Fed announces beginning of tapering, $10 billion/month.
- **March 19, 2014**. Yellen Fed indicates short rates may rise in 6 months after end of tapering, perhaps by mid-2015, earlier than markets expected.
- **April 30, 2014**. Job growth strong. Unemployment rate drops sharply: 6.7% to 6.3%.
- **October, 2014**. Unemployment at 5.9%. Yellen Fed ending asset purchases (QE).
- **March, 2015.** Unemployment at 5.5%, rapid job growth. Fed drops "patience" talk. "Dots" show that Fed members expect a slower ramping up of rates after liftoff.
- **December, 2015.** Fed "lifts off" and raises its policy rate 0.25%, first since the Great Recession.

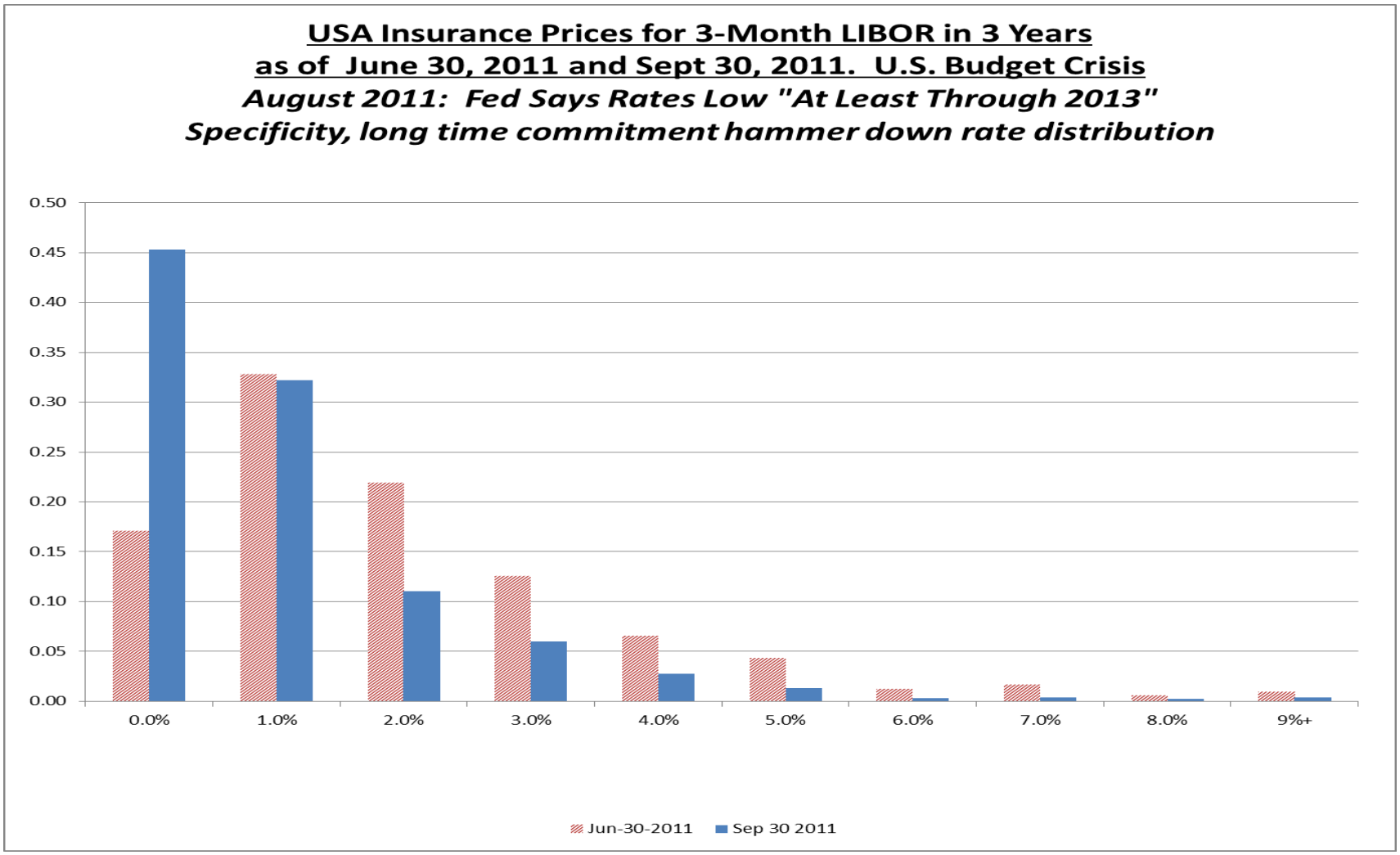
2008: U.S. Rate Distribution Transformed from Symmetric to Positive Skewness (Concentrated near zero, but long right tail)



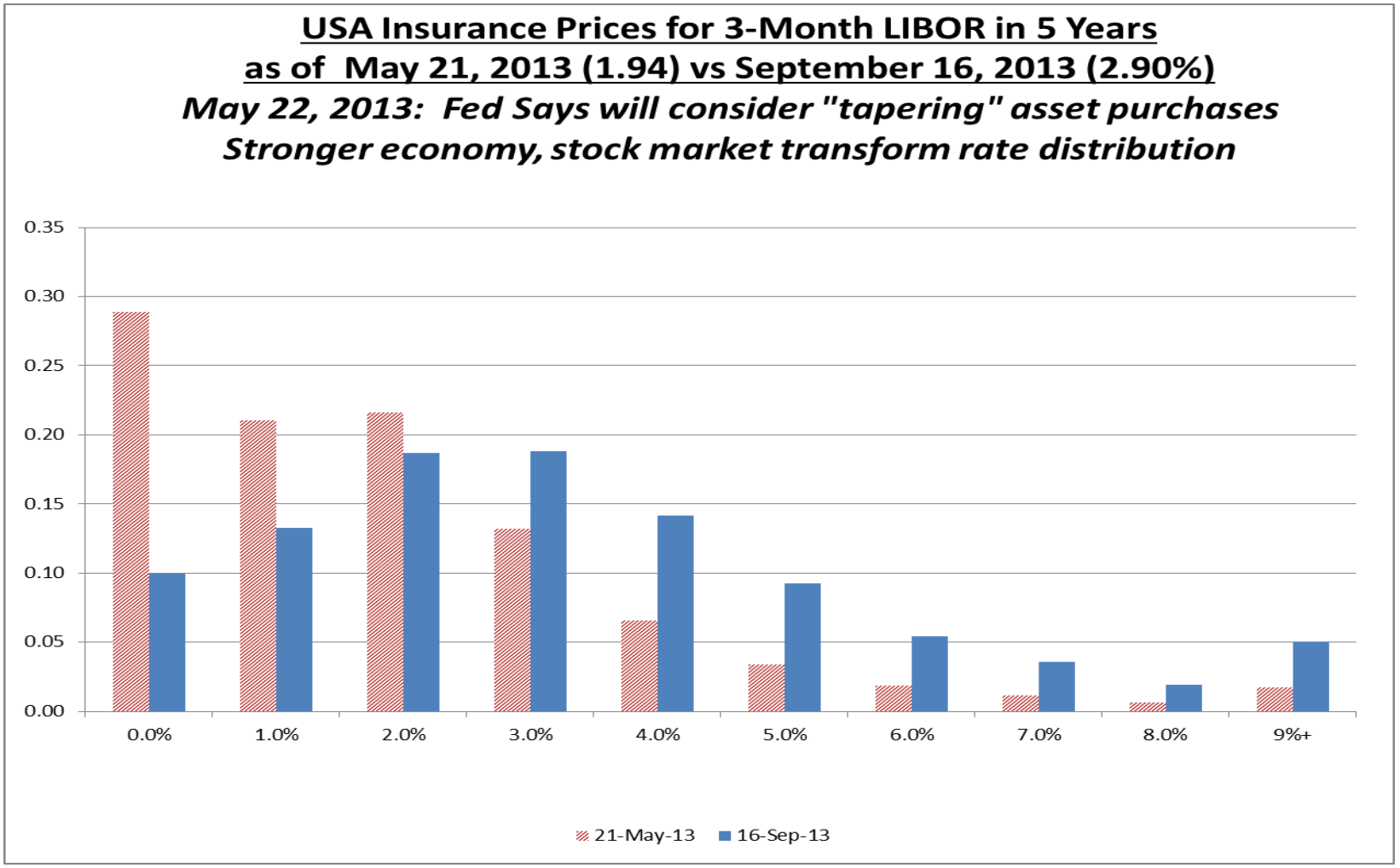
Dec 2008: Euro Area Rate Distribution Unaffected by USA problems



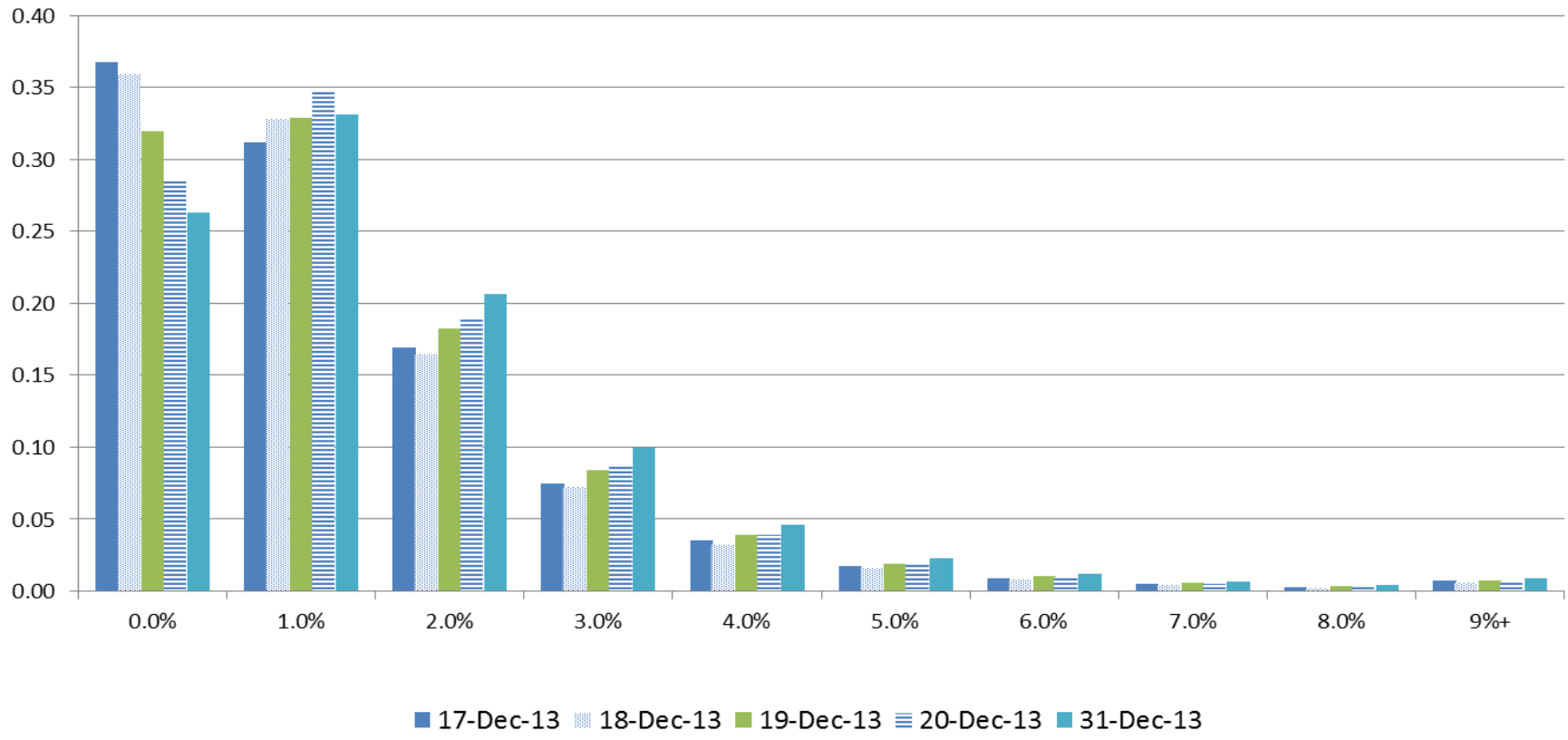
Panic during budget impasse causes Fed to commit low rates 2.5 years.



Summer 2013 Tapering Announcements: Stronger economy shifts distribution towards symmetry



USA Insurance Prices for 3-Month LIBOR in 3 Years
Daily: December 17 (2.85% 10 Yr) to Dec 31, 2013 (3.04%)
Dec 18: Bernanke Fed announces start of tapering asset purchases
Rate distribution shifts to higher rates

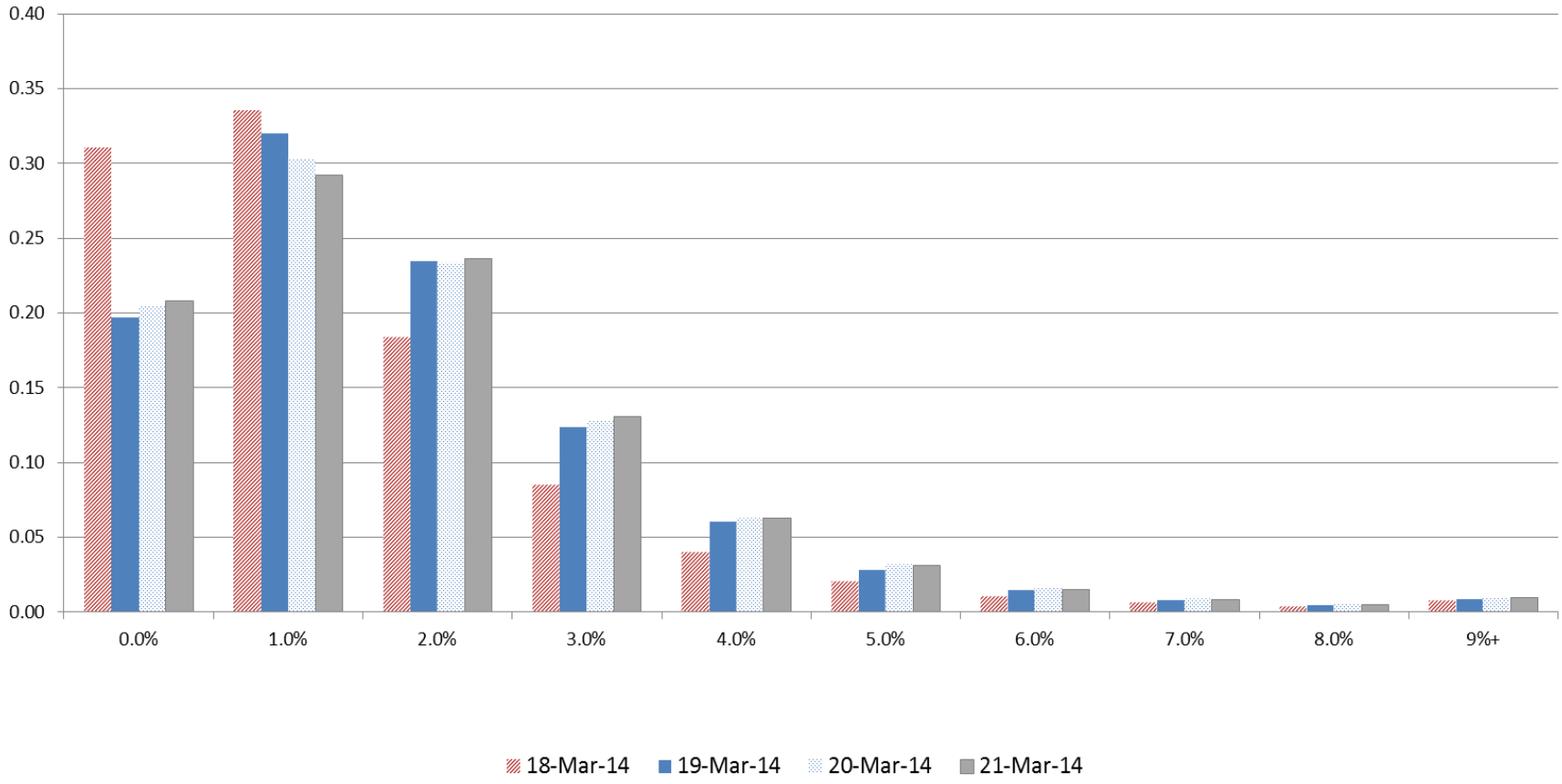


USA Insurance Prices for 3-Month LIBOR in 3 Years

Daily: March 18 2014 (2.68% 10 Yr) to March 21 2014 (2.75%)

March 19: Fed Chair Yellen says rates could increase in mid-2015 (6 mos after taper).

Rate distribution shifts higher for shorter term.



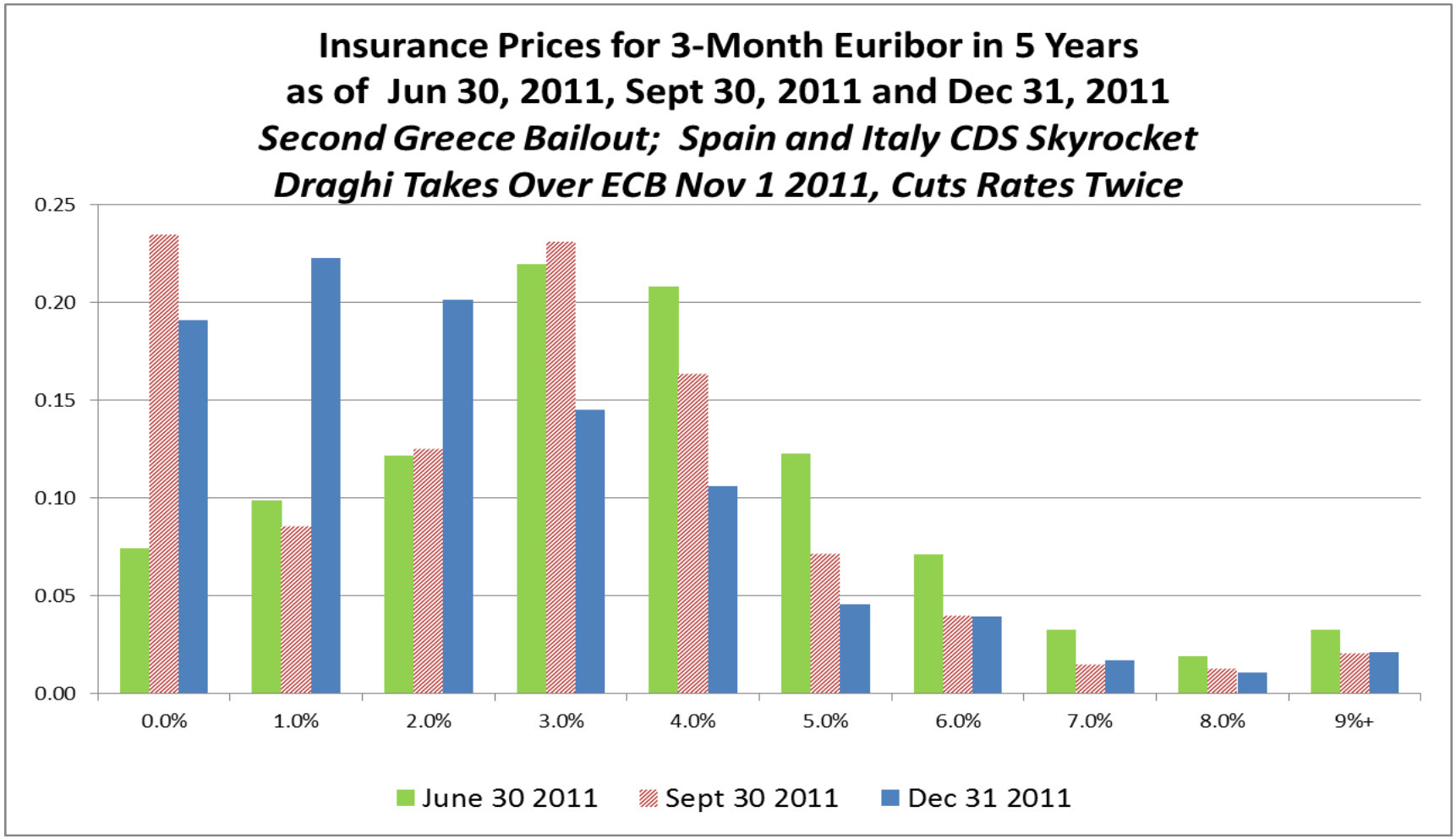
V. Interest Rate Insurance Prices for Euribor During the Sovereign Debt Crisis 2010-2015

Key Events in the European Sovereign Debt Crisis
European Central Bank 2010-2015

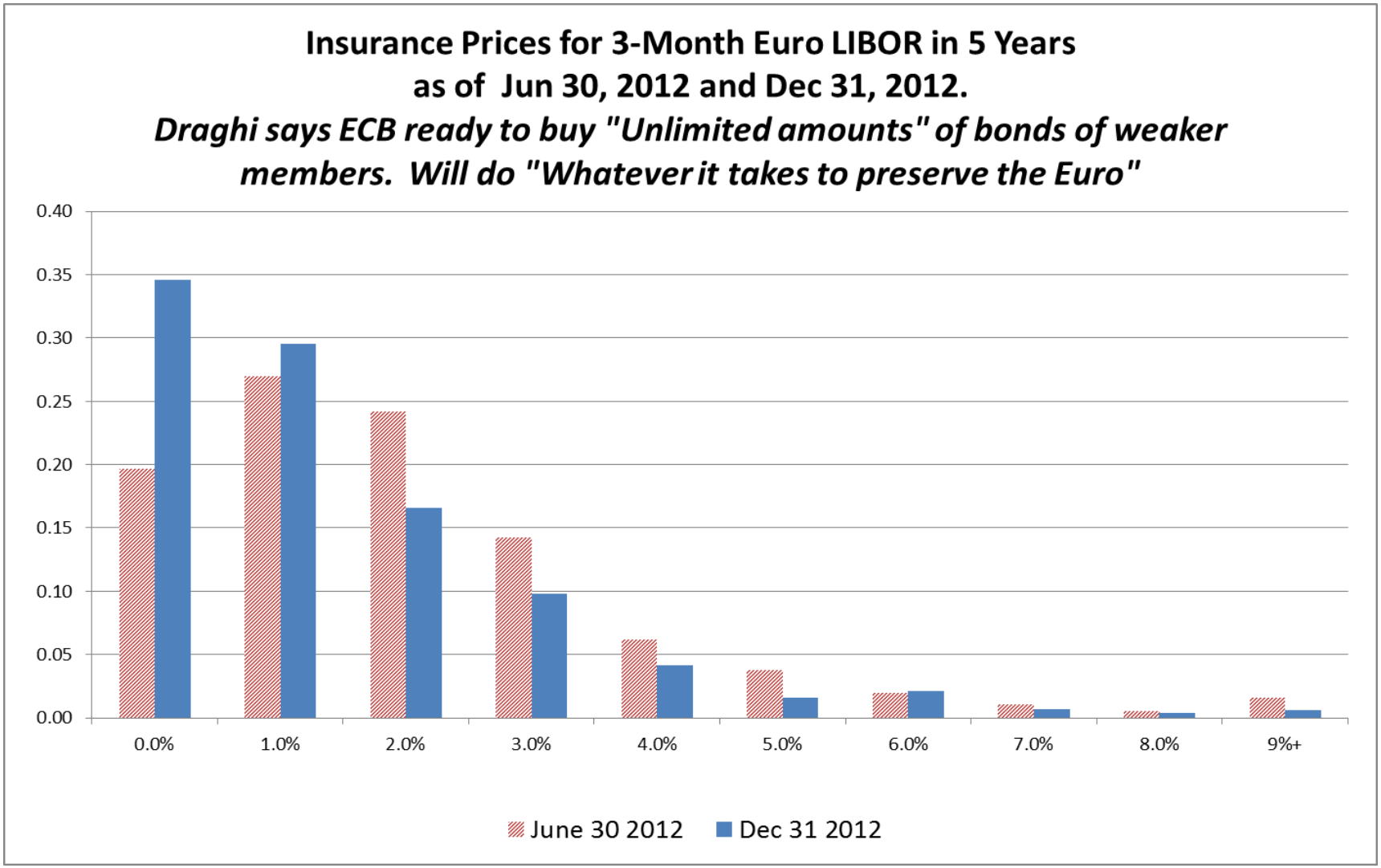
Sources: BBC,Reuters

- January-May 2010. Greek deficit revised upward from 3.7% to 12.7%. "Severe irregularities" in accounting. EU agrees to $30 billion, then $110 billion bailout of Greece. Ireland bailed out in November 2010.
- July-August 2011: Talk of Greek exit from Euro. Second bailout agreed. EC President Barroso: sovereign debt crisis spreading. Spain, Italy yields surge.
- November 1, 2011: Mario Draghi takes over European Central Bank from Jean-Claude Trichet. Draghi cuts rates twice quickly.
- September, 2012: ECB ready to buy "unlimited amounts" of bonds of weaker member countries. Draghi says ECB will do "whatever it takes to preserve the Euro." "…and believe me, it will be enough."
- May/June 2013: U.S.Fed considers "tapering" asset purchases, as economy strengthens. Long term interest rates move up sharply.
- June-October, 2014: European economies weak, inflation expectations lower. Draghi cuts rates twice to 0.05%. Announces QE, buying ABS, possibly even from Italy and Spain, up to 1 trillion Euro.
- January-March 2015: Draghi of ECB announces on January 22nd "Quantitative Easing" by massive asset purchases. Began QE March 9, 2015.

2011 Sovereign Debt Crisis: Draghi ECB cuts rates sharply. Massive shift in Euribor interest rate distribution to positive skewness like U.S.

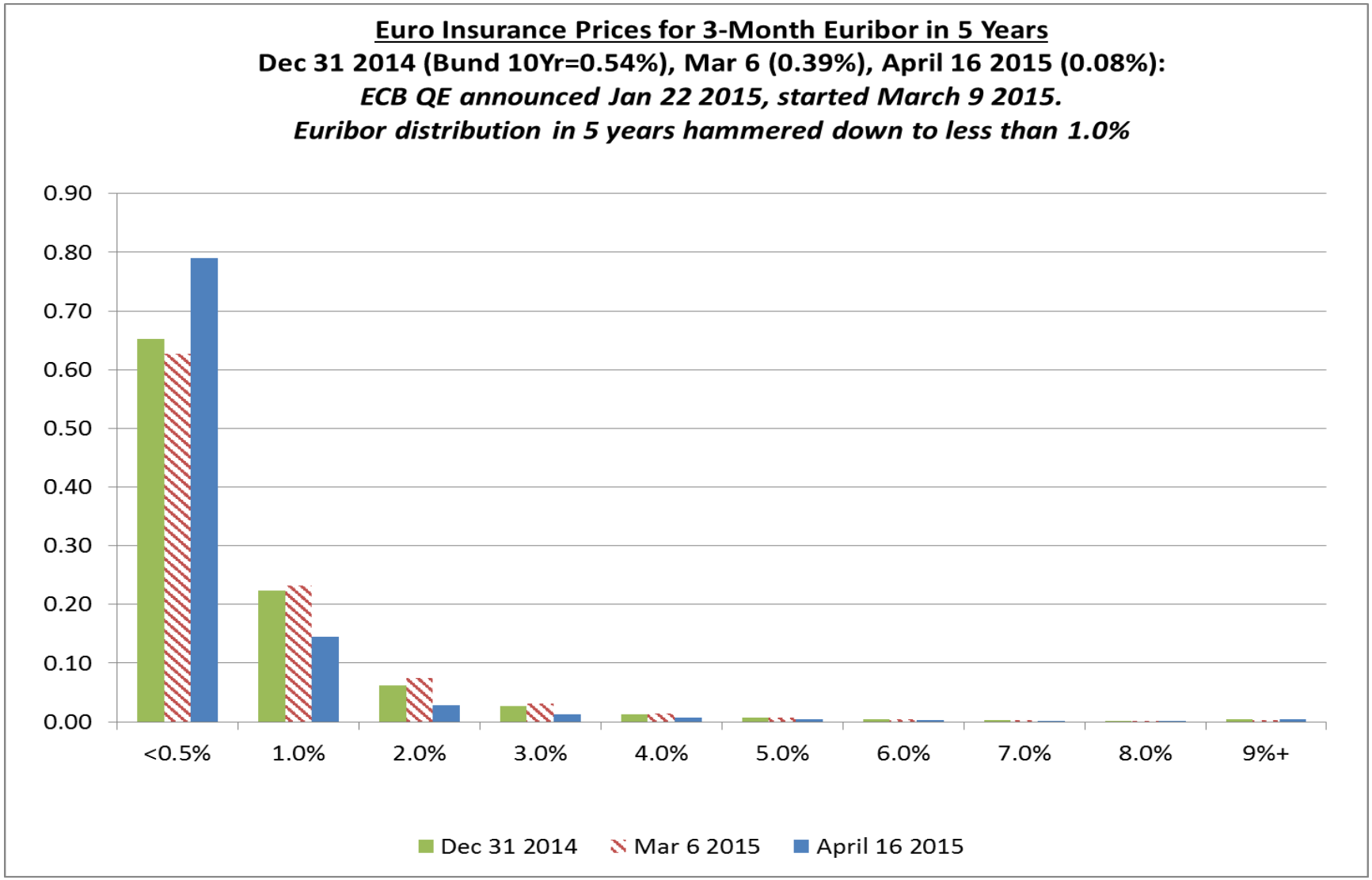


Draghi Rescues the Euro in 2012 with "Whatever it takes…"



VI. 2015: Draghi's European Central Bank Massive "Quantitative Easing" Program:

ECB announced QE January 22, 2015,
implemented it starting March 9 2015,
buying massive amounts of Eurozone bonds.
Long rates drop sharply in Eurozone, UK, USA.

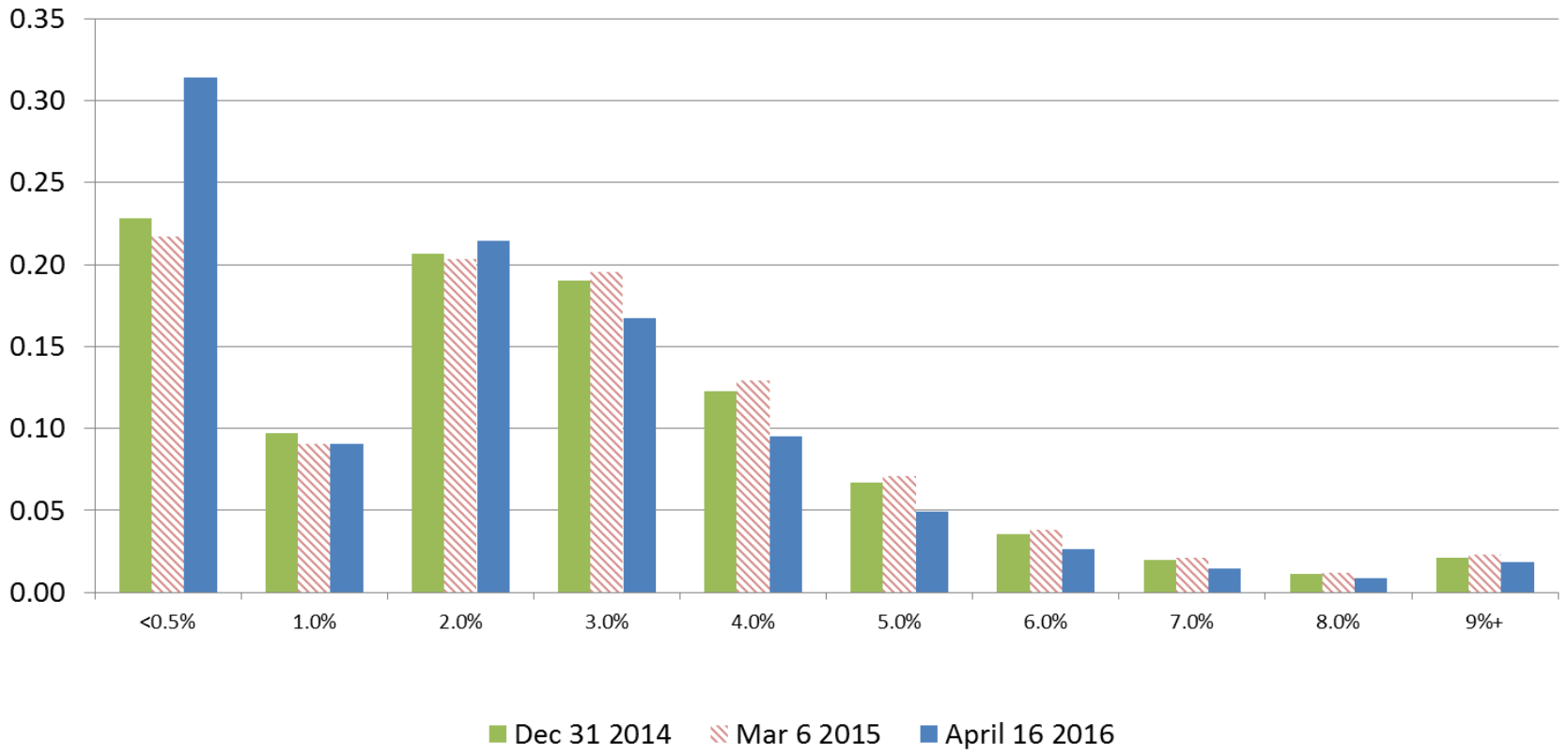

Euro Insurance Prices for 3-Month Euribor in 5 Years
Dec 31 2014 (Bund 10Yr=0.54%), Mar 6 (0.39%), April 16 2015 (0.08%):
ECB QE announced Jan 22 2015, started March 9 2015.
Euribor distribution in 5 years hammered down to less than 1.0%
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
<0.5%
1.0%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9%+
Dec 31 2014
Mar 6 2015
April 16 2015

USA Insurance Prices for 3-Month LIBOR in 5 Years

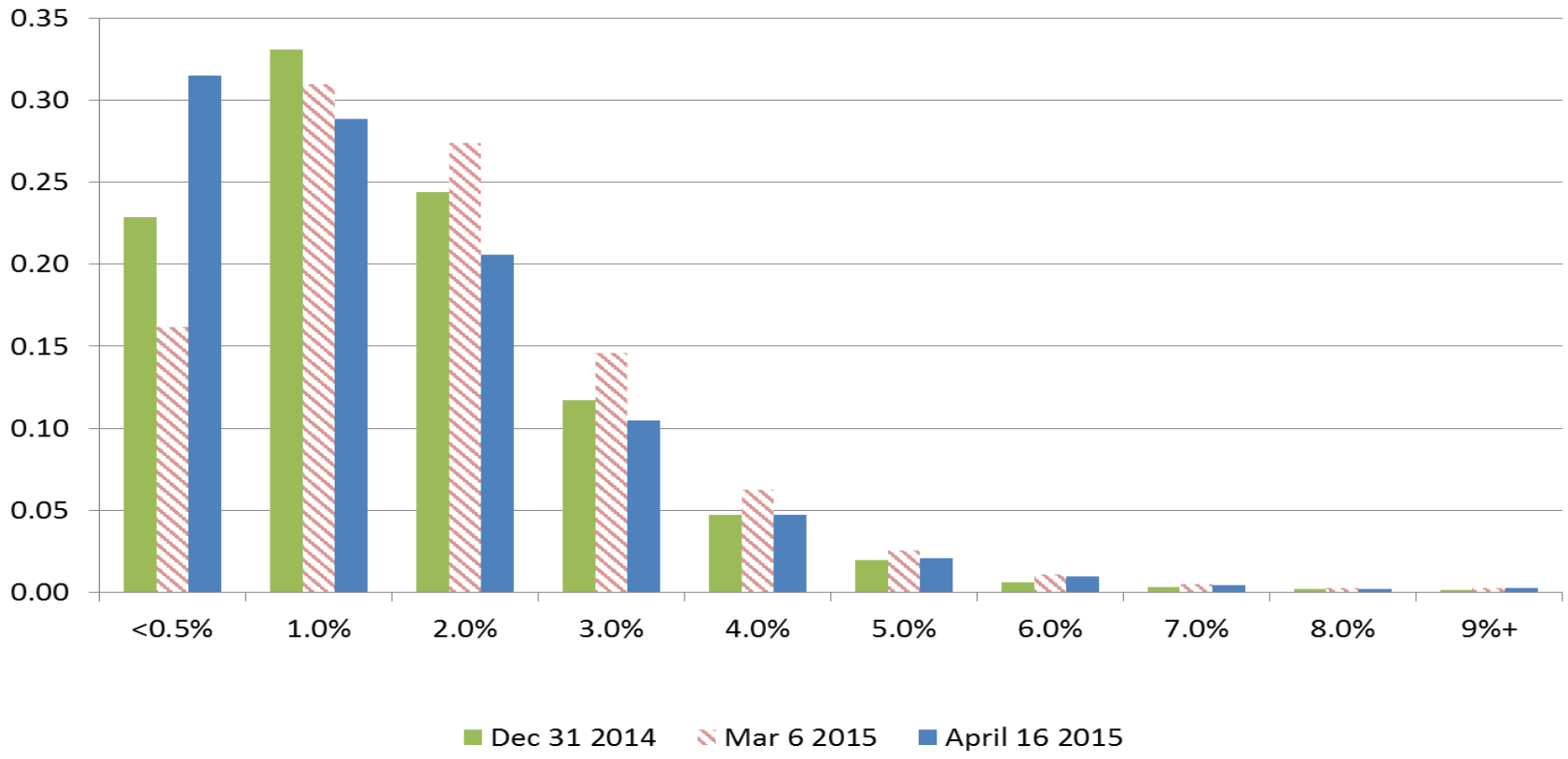
Dec 31 2014 (10 Yr=2.17%), March 6 (2.24%), April 16 2015 (1.88%):

Rates drop with ECB QE March 9th. Dots show flatter ramp after liftoff.

Bimodal dist'n for LIBOR 5 yrs. Economy weak or strong then?



British Pound Insurance Prices for 3-Month Interbank Rate in 3 Years
Dec 31 2014 (10 Yr Gilt=1.76%), March 6 (1.95%), April 16 2015 (1.61%):
ECB QE announced Jan 22, implemented Mar 9 2015. Rates drop.
Mode shifts down <0.5% for 3 years out.



VII. What are markets saying now in the USA, the Eurozone and the UK?

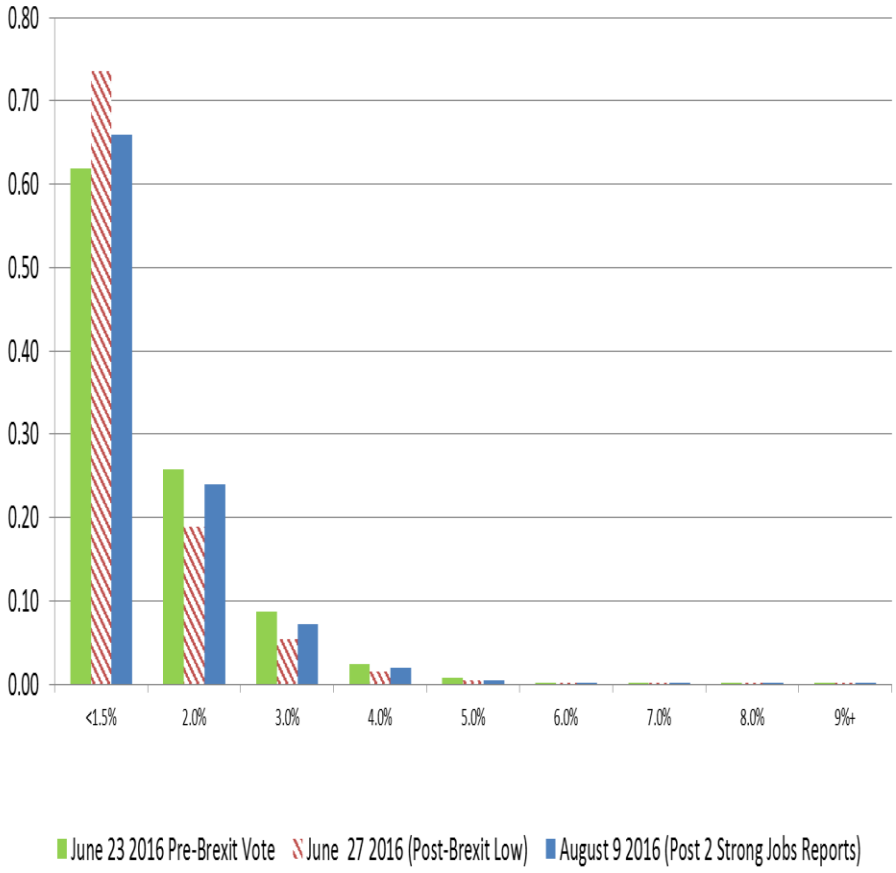
June 23, 2016's surprise vote for Brexit roiled markets and caused rates to drop sharply. Strong actions and statements by the Bank of England's Governor, Mark Carney, continued the shift towards low rates in the UK distribution. Strong jobs reports in the USA on July 8 and on August 5, 2016, caused some bounceback in rate distributions, especially in the USA.

USA Insurance Prices for 3-Month LIBOR in 3 Years

Jun 23 2016 (1.74% 10 Yr), Jun 27 (1.46%), August 9 2016 (1.55%)

June 23 - August 9 2016. USA rate dist'n drops sharply after Brexit vote.

Strong USA jobs reports July 8, Aug 5 aid bounceback in stocks,

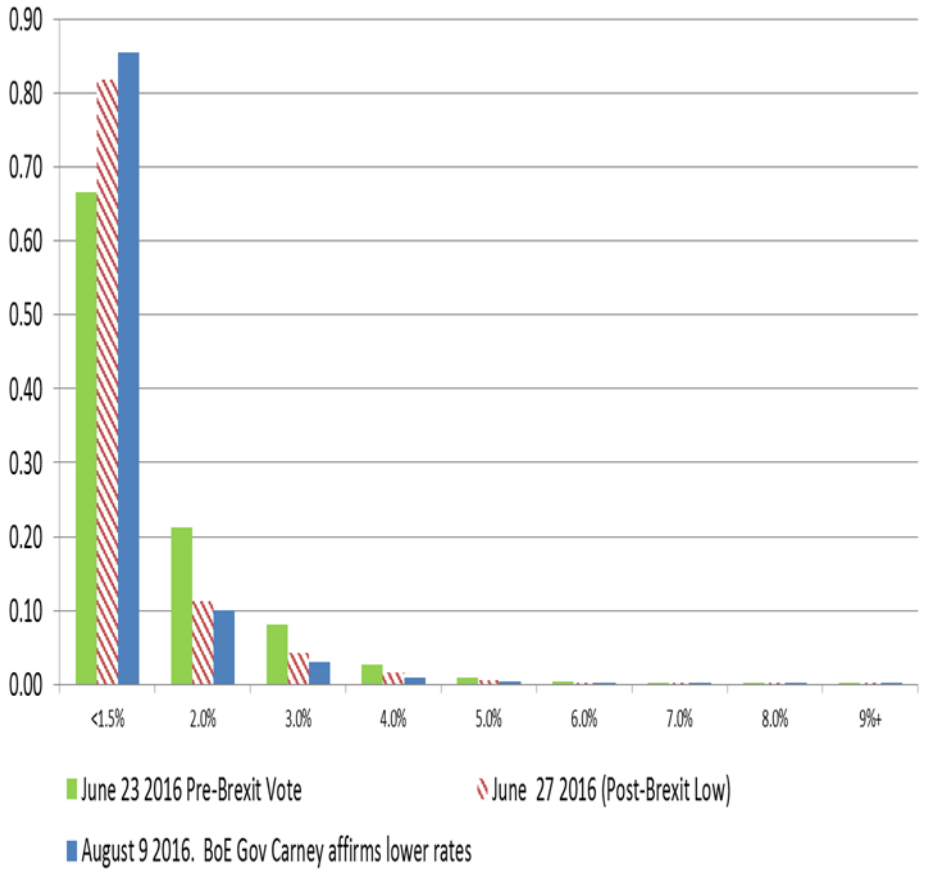


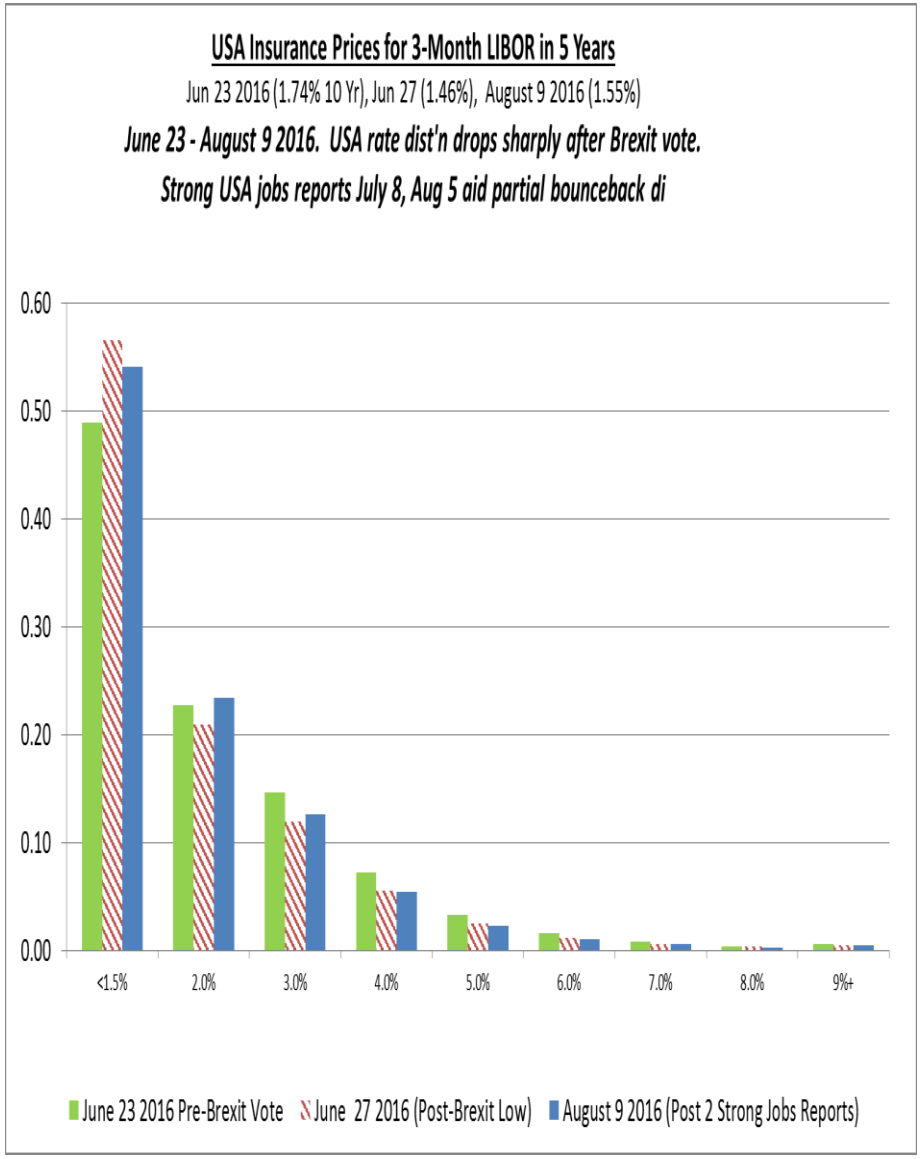
UK Insurance Prices for 3-Month Interbank Rate in 3 Years

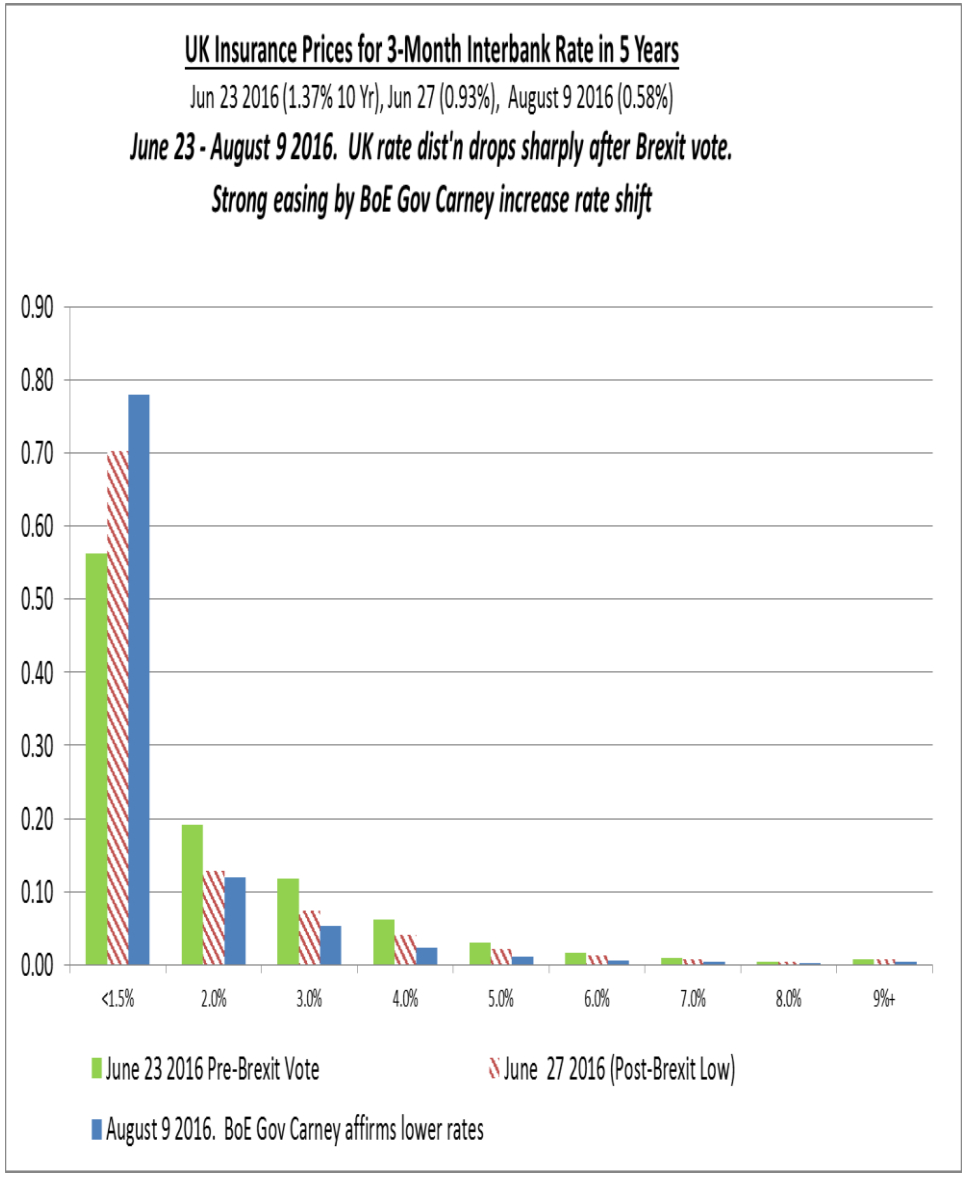
Jun 23 2016 (1.37% 10 Yr Gilt), Jun 27 (0.93%), August 9 2016 (0.58%)

June 23 - August 9 2016. UK rate dist'n drops sharply after Brexit vote.

Strong easing by BoE Gov Carney increases shift.




USA Insurance Prices for 3-Month LIBOR in 5 Years
Jun 23 2016 (1.74% 10 Yr), Jun 27 (1.46%), August 9 2016 (1.55%)
June 23 - August 9 2016. USA rate dist'n drops sharply after Brexit vote.
Strong USA jobs reports July 8, Aug 5 aid partial bounceback di
0.60
0.50
0.40
0.30
0.20
0.10
0.00
<1.5%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9%+
June 23 2016 Pre-Brexit Vote
June 27 2016 (Post-Brexit Low)
August 9 2016 (Post 2 Strong Jobs Reports)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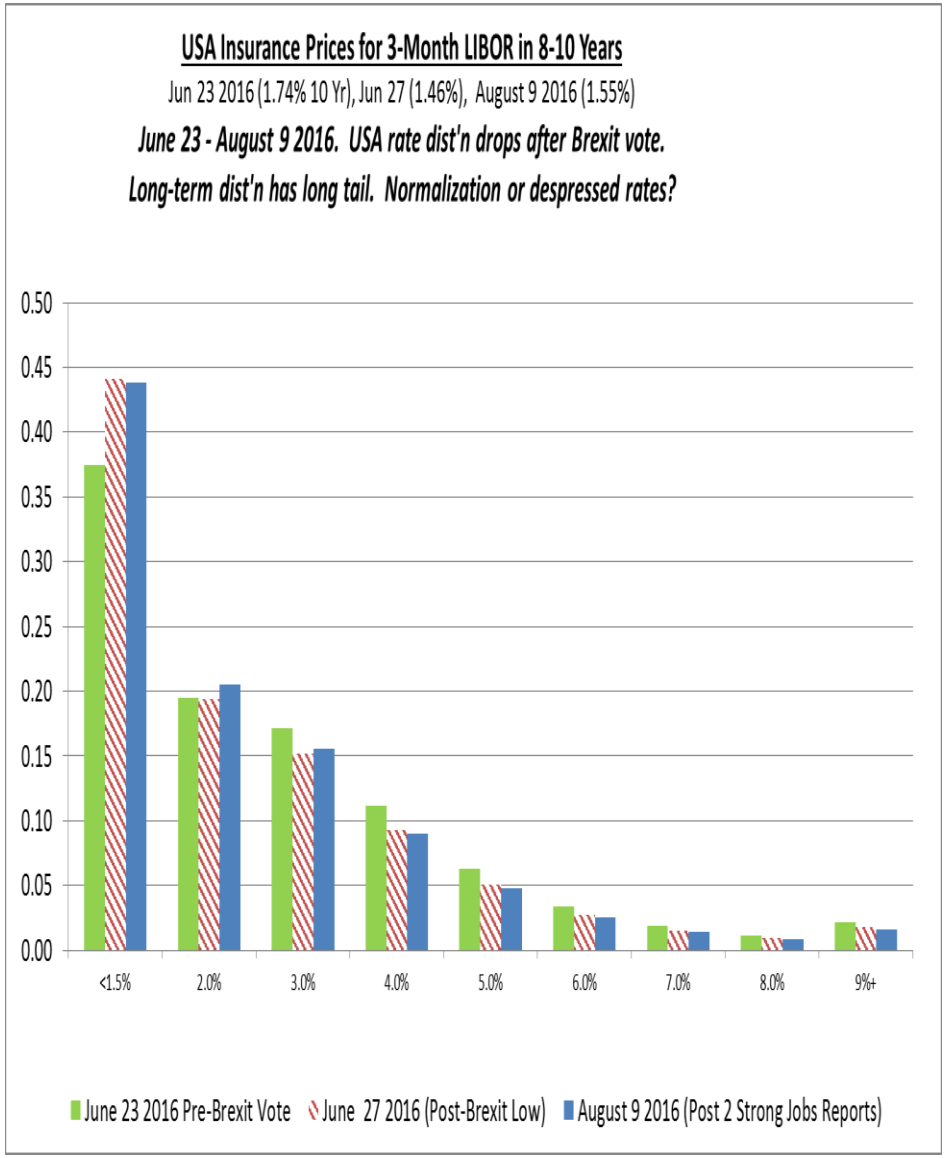
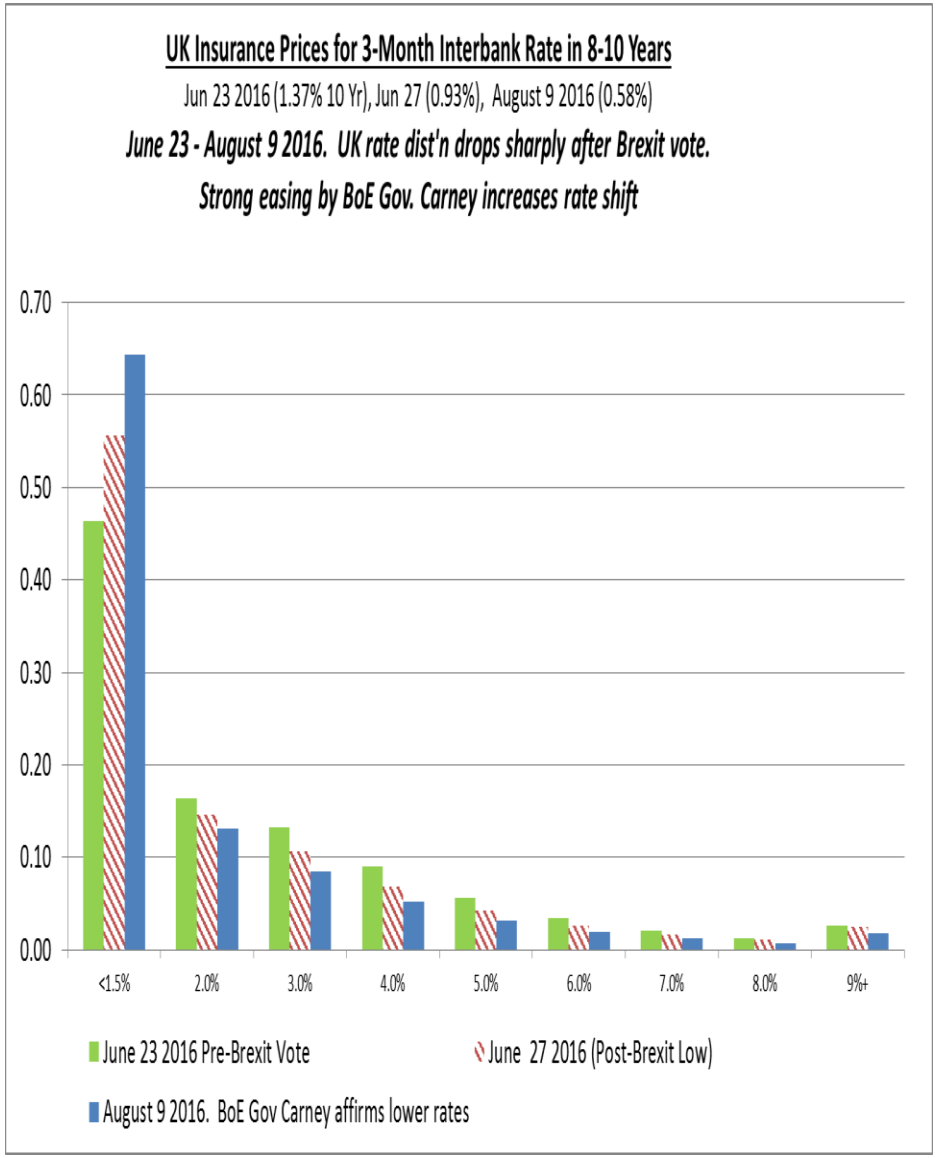
Jun 23 2016 (1.37% 10 Yr), Jun 27 (0.93%), August 9 2016 (0.58%)
June 23 - August 9 2016. UK rate dist'n drops sharply after Brexit vote.
Strong easing by BoE Gov Carney increase rate shift
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
<1.5%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9%+
June 23 2016 Pre-Brexit Vote
June 27 2016 (Post-Brexit Low)
August 9 2016. BoE Gov Carney affirms lower rates


USA Insurance Prices for 3-Month LIBOR in 8-10 Years
Jun 23 2016 (1.74% 10 Yr), Jun 27 (1.46%), August 9 2016 (1.55%)
June 23 - August 9 2016. USA rate dist'n drops after Brexit vote.
Long-term dist'n has long tail. Normalization or despressed rates?
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
<1.5%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9%+
June 23 2016 Pre-Brexit Vote
June 27 2016 (Post-Brexit Low)
August 9 2016 (Post 2 Strong Jobs Reports)


UK Insurance Prices for 3-Month Interbank Rate in 8-10 Years
Jun 23 2016 (1.37% 10 Yr), Jun 27 (0.93%), August 9 2016 (0.58%)
June 23 - August 9 2016. UK rate dist'n drops sharply after Brexit vote.
Strong easing by BoE Gov. Carney increases rate shift
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
<1.5%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9%+
June 23 2016 Pre-Brexit Vote
June 27 2016 (Post-Brexit Low)
August 9 2016. BoE Gov Carney affirms lower rates

VIII. Conclusions

1. Interest rate cap and floor prices show the market's implied pricing of insurance payoffs for interest rates from 1% to 9%+. A virtue of caps and floors is forecasts covering much longer periods (3, 5, 10 years), rather than 3-18 months from options.

2. The "Great Recession" in the USA and the "Sovereign Debt Crisis" in Europe show dramatic moves in distributions for rates after Central Bank interventions. Symmetric distributions shifted to highly positively skewed ones. Our technique is non-parametric, not relying on lognormality.

3. USA "lifted off" from zero rates in December 2015, given the relatively strong USA economy. Despite liftoff, US stock market is at all-time highs. Strong job market brought the unemployment rate down to 4.9%. However, markets are worried about economy in 5 years, as the "bipolar" rate distribution shows.

4. January-March 2015 Weakness in Europe caused European Central Bank President Mario Draghi to announce a massive "Quantitative Easing" program of asset purchases. Rates dropped sharply after that implementation, with German Bund 10-year yield down from 0.38% to 0.09% and to negative levels in 2016.

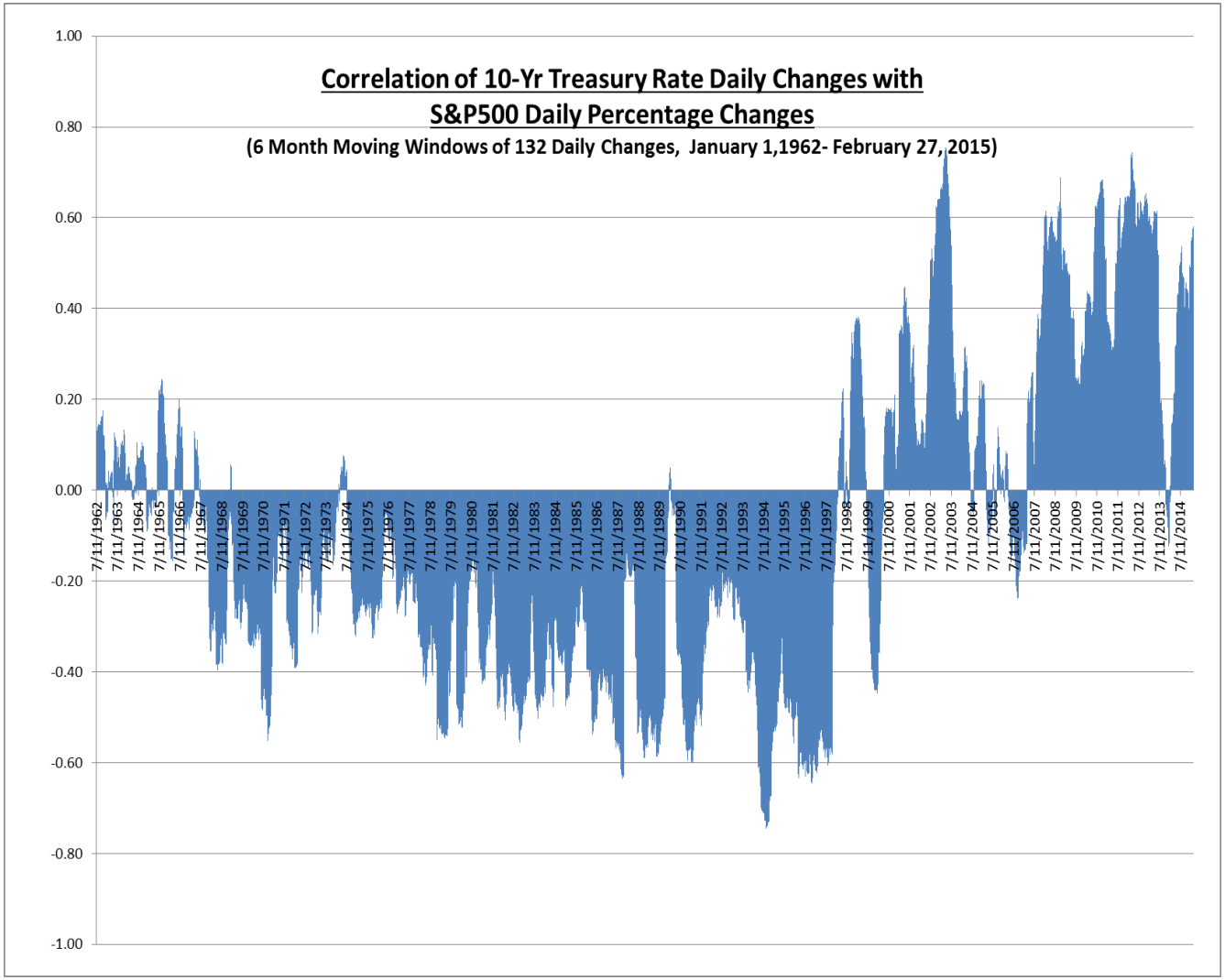
5. The U.K. vote for "Brexit" on June 23, 2016, roiled markets for a bit, with interest rates and the British Pound falling sharply. Stock prices dropped sharply, especially in the Eurozone, but have bounced back in most countries, partially due to strong jobs reports in the U.S. for June-July 2016. The Bank of England reduced its policy rate 25 basis points and prepared strong stimulus measures to combat weakness expected with Brexit and the uncertainties of negotiations with the EU. The insurance price distributions reflect lower rates anticipated in the UK for years.

Appendix

True Probabilities vs. Insurance Prices

Shifting Correlations of Interest Rates and the Economy

When rates are high, is marginal utility high or low? Depends on the time period.

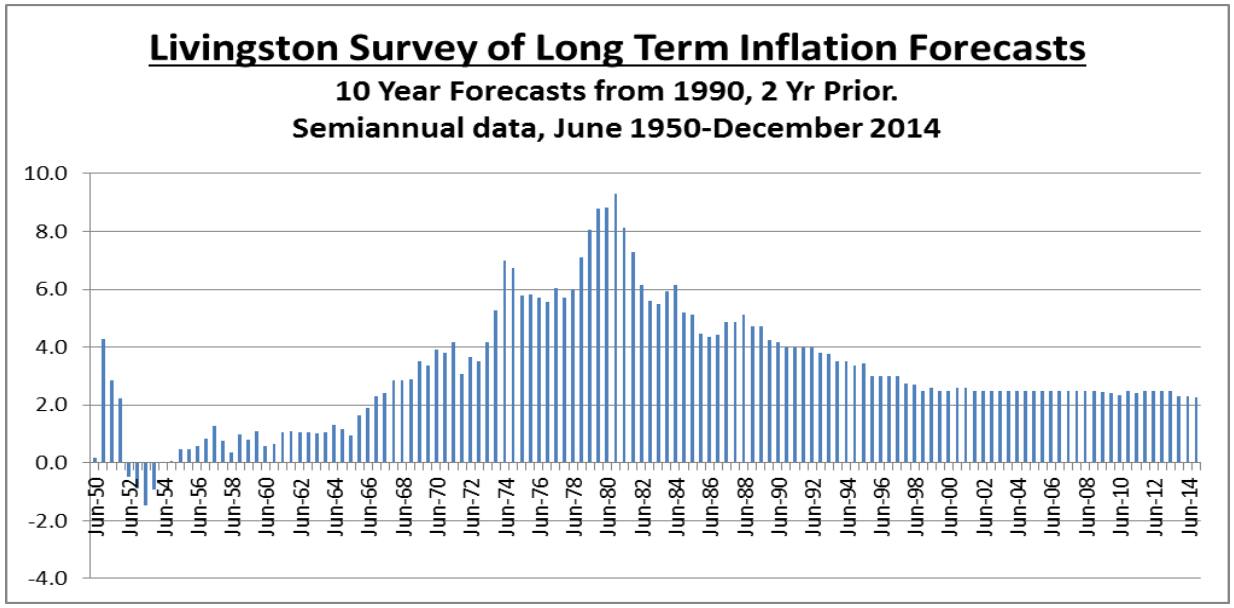


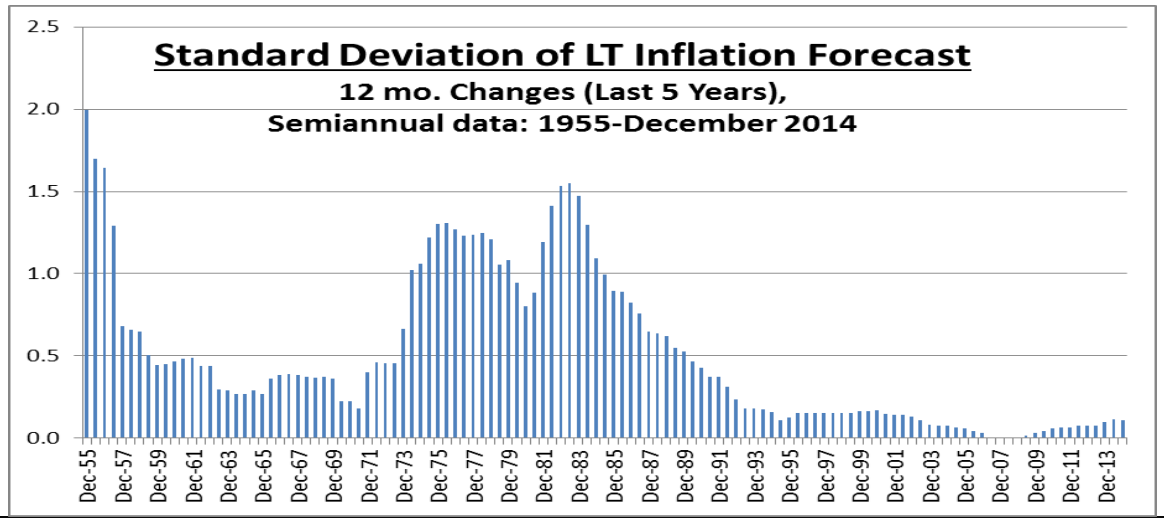
This graph shows the dramatic switch from negative to positive in 1999/2000 in the correlation between changes in the 10-year interest rate and moves in the S&P 500.

This switch in correlation reflects a shift from supply-oriented inflation concerns in the 1970s and 1980s to inflation concerns dominated more by demand issues.

The beta of long-term bond returns versus stock returns and the economy thus shifted from positive to negative. The fair risk premium on long-term bonds should have shifted from positive to negative, as long-term bonds became excellent hedges for risks of a bad economy.

What caused correlations of interest rates and stocks to change sign in 2000-2015?

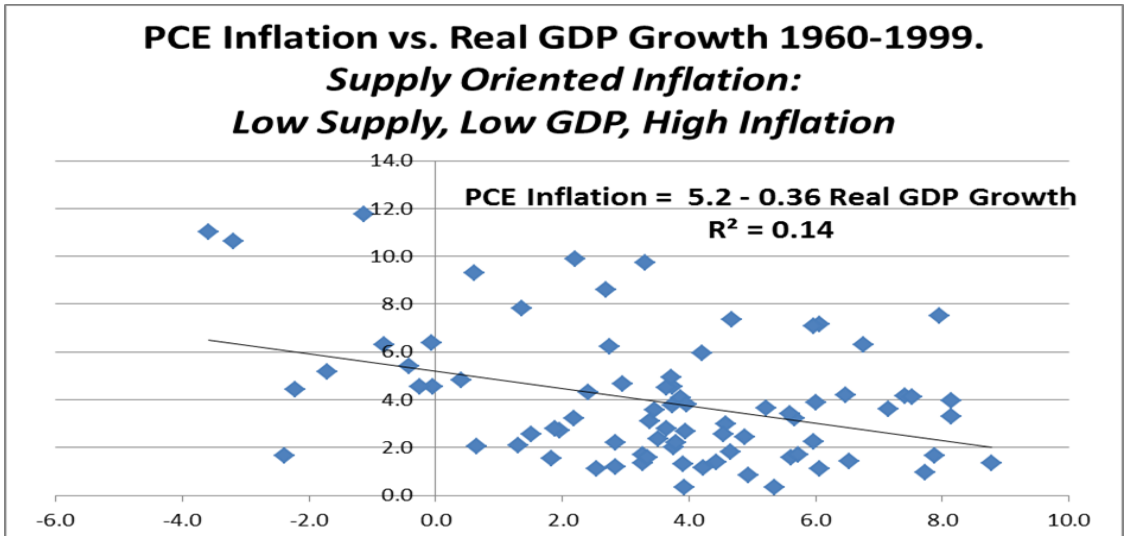




The Livingston/Philly Fed semiannual survey of inflation expectations shows the dramatically higher inflation rate in the 1970s and early 1980s, with notable surges in 1974/75 due to oil price and grain price shocks, as well as in 1981/82 when the second major round of OPEC oil price shocks occurred.

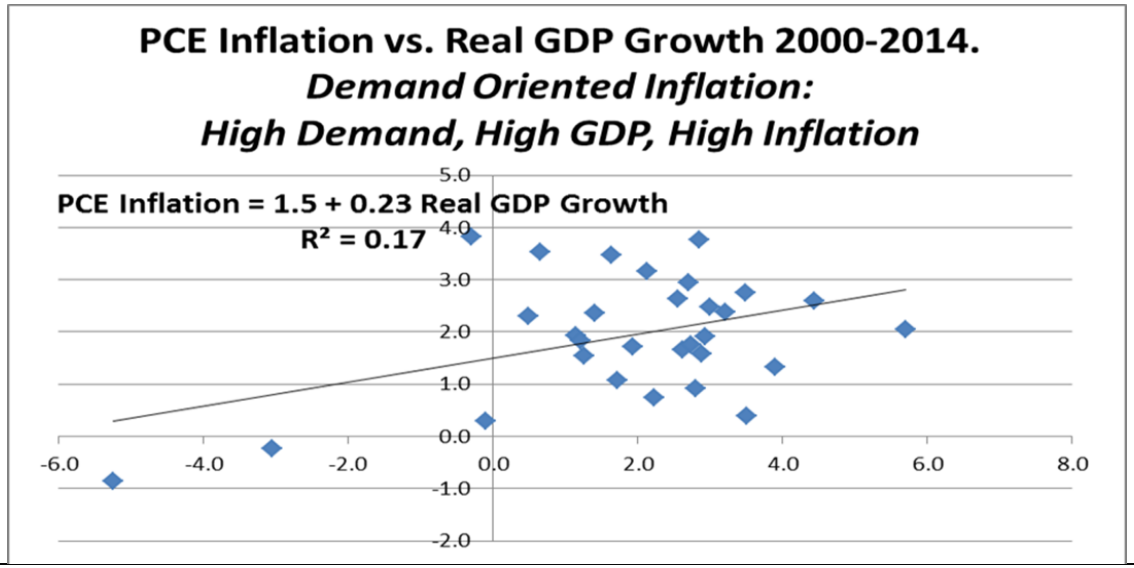
After the Volcker Fed in 1979-1981 let interest rates increase dramatically while focusing on controlling money supply growth, inflation was sharply reduced. Inflation expectations peaked in 1981 at 9% and dropped to less than 5% by the end of the 1980s. With continued monetary discipline, the 10-year inflation forecast dropped through the 1990s until it hit 2.5% in 1999/2000. The inflation rate forecast has remained anchored between 2.0% and 2.5% from 2000 to 2014, with very low volatility of the inflation forecast.

What caused correlations of interest rates and stocks to change sign in 2000-2015?



In the 40-year period from 1960 to 1999, higher real GDP growth occurred in conjunction with lower inflation, and recessions generally happened with high inflation, led by the big 1974/75 and 1981/82 recessions.

Constricted oil and grain supplies caused high inflation at times of these significant recessions. High inflation led to high interest rates, so the USA had high rates in recessions. Bond returns had positive stock correlations with supply risks: returns were negative when the economy was down, positive when the economy was strong.



In the 15-year period from 2000 to 2014, higher real GDP growth occurred in conjunction with higher inflation, and recessions generally happened with low inflation, led by the Great Recession of 2008/09.

Weak demand in the Great Recession led to very low interest rates. Supply uncertainties were dominated by demand uncertainties in this period.

Bond returns had negative stock correlations when demand risks dominated, as their returns were very positive (due to the very low rates) when the economy was down sharply in 2008/9.

Illustration of True Probabilities Related to Risk Neutral Probabilities

True probability = K*Risk Neutral x exp(Gamma*(gts - mu))

Assumes: CRRA-Lognormal real growth model

Real Growth on Nominal Rate: 1998 to 2011 Data					Real Growth on Nominal Rate: 1977 to 1997 Data				
Intercept	-3.71	(t= -2.2)			Intercept	4.11	(t= 3.2)		
Slope	1.42	(t= 3.8)			Slope	-0.12	(t= -0.8)		
MuCgrow	3				MuCgrow	3			
		Relative Risk Aversion (Gamma)					Relative Risk Aversion (Gamma)		
Nominal Rate	Real Growth	2	4	8	Nominal Rate	Real Growth	2	4	8
		Ratio of True Probability to Risk Neutral*					Ratio of True Probability to Risk Neutral*		
1	-2.29	0.90	0.81	0.65	1	3.99	1.02	1.04	1.08
2	-0.87	0.93	0.86	0.73	2	3.87	1.02	1.04	1.07
3	0.55	0.95	0.91	0.82	3	3.75	1.02	1.03	1.06
4	1.97	0.98	0.96	0.92	4	3.63	1.01	1.03	1.05
5	3.39	1.01	1.02	1.03	5	3.51	1.01	1.02	1.04
6	4.81	1.04	1.08	1.16	6	3.39	1.01	1.02	1.03
7	6.23	1.07	1.14	1.29	7	3.27	1.01	1.01	1.02
8	7.65	1.10	1.20	1.45	8	3.15	1.00	1.01	1.01
9	9.07	1.13	1.27	1.63	9	3.03	1.00	1.00	1.00
10	10.49	1.16	1.35	1.82	10	2.91	1.00	1.00	0.99
					11	2.79	1.00	0.99	0.98
					12	2.67	0.99	0.99	0.97
					13	2.55	0.99	0.98	0.96
					14	2.43	0.99	0.98	0.96
					15	2.31	0.99	0.97	0.95
*=Up to a scalar multiple					16	2.19	0.98	0.97	0.94

Douglas T. Breeden is the William W. Priest Professor of Finance and former Dean of Duke University's Fuqua School of Business. He also served on faculties at Chicago Booth, Stanford and North Carolina, where he was the Dalton McMichael Professor of Finance. He was the Fischer Black Visiting Professor of Financial Economics at MIT's Sloan School in 2011-2013, winning an "Outstanding Teacher" award.

Breeden published seminal research on insurance prices implicit in option prices, the Consumption CAPM, and hedging mortgage securities. His current research is "A Stocks, Bonds, Consumers Leading Indicator" and (with Robert Litzenberger) "Central Bank Policy Impacts on the Distribution of Future Interest Rates," which won a Roger Murray Prize from the Q-Group.

Breeden was Associate Editor of 5 top journals. He was also Founding Editor and Editor for 10 years of the Journal of Fixed Income. He was elected to the Board of Directors of the American Finance Association and in 2010 a lifetime Fellow. The International Association for Quantitative Finance named Breeden "Financial Engineer of the Year 2013" for being an "industry pioneer."

Breeden holds a Ph.D. in Finance from Stanford and an S.B. from M.I.T. He served on the MIT President's Council, the Sloan School Visiting Committee and the Stanford Business School Advisory Council. He was named an Honorary Professor of the Chinese Academy of Sciences and served on the Boards of Goethe Business School in Germany and the Financial Management Association. He is on the Board of Trustees of Commonfund. Breeden is a Senior Research Consultant for Amundi Smith Breeden, a money management firm that he co-founded.

September 2016