Midwest Finance Association: Keynote Address Reference Notes

Stock and Bond Insurance Prices Implicit in Option Prices

Central Bank Policy Impacts on Interest Rate Insurance And Risk Aversion Predictions of Stock Returns

*Douglas T. Breeden**

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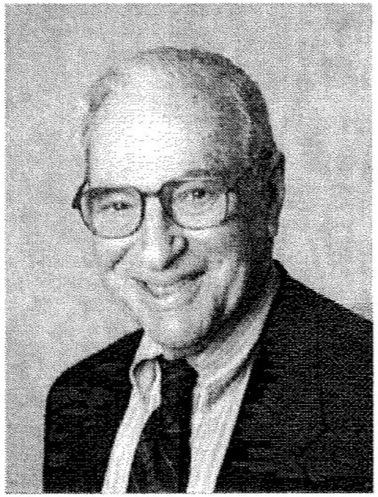
Much of this work is joint with Robert Litzenberger. We thank Robert Merton, Myron Scholes, Stephen Ross, Robert Litterman, and Bjorn Flesaker for helpful comments, and Tingyan Jia, and Research Assistants Jingyi Wu, Tuo Yang, Ling Chen, Gloria Zeng, Song Xiao and their predecessors.

Developers of the Arrow-Debreu Time-State Preference Model of Economies Under Uncertainty

,Nobel Laureates from Harvard/Stanford (Arrow) and Berkeley (Debreu)

- Fundamental theoretical contributions to analysis of equilibrium under uncertainty by Kenneth Arrow and Gerard Debreu , e.g., “Existence of a competitive equilibrium for a competitive economy,” 1954.



Kenneth Arrow (1921-2016)
Harvard/Stanford

Gerard Debreu (1921-2000)
U.C. Berkeley

I. Prices of State Contingent Claims Implicit in Option Prices

Stephen Ross (Yale,1976, Quarterly Journal of Economics)

Breeden-Litzenberger (Stanford, 1978, Journal of Business)

Breeden-Litzenberger Method
Constructs Pure Bet Insurance Prices from Call Option Prices

Breeden-Litzenberger 1978, *Journal of Business*

Underlying Asset Price | Spreads | Butterfly Spread

				Call Option Portfolios		
	Payoffs on Call Options			Port. A	Port. B	Port.C=A-B
P	C(X=2)	C(X=3)	C(X=4)	C(2)-C(3)	C(3)-C(4)	C(2)-2C(3)+C(4)
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	1	0	0	1	0	1
4	2	1	0	1	1	0
5	3	2	1	1	1	0
6	4	3	2	1	1	0
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
N	N-2	N-3	N-4	1	1	0

"Butterfly Spreads" of Options Give Pure Insurance Prices.

Breeden and Litzenberger (1978) derived Arrow's state prices for different levels of the stock market (relative to today's level) and different maturities, using the Black-Scholes option pricing formula, as given on the following slide.

More generally, B-L showed that 2nd derivatives of option pricing functions w.r.t. the exercise price provides the state price density.

The State Price Density can be used to price general derivative claims.

With Continuous Underlying Asset Price, but Discrete Exercise prices:

Butterfly spread: $\frac{[c(x-\Delta)-c(x)]-[c(x)-c(x+\Delta)]}{\Delta}=\frac{[c(x-\Delta)-2c(x)+c(x+\Delta)]}{\Delta}$

Values of derivative assets:

$$PV(f(\tilde{P}))=\int_{\tilde{P}} c_{xx}(x=\tilde{P})\cdot f(\tilde{P})d\tilde{P},$$

where $c(x,P)$ = price of European call option with exercise price x,

and c_{xx} is its second partial derivative with respect to x.

A similar formula holds with regard to European put formula, e.g.:

$$PV(f(\tilde{P}))=\int_{\tilde{P}} g_{xx}(x=P)\cdot f(P)dP.$$

These are pure arbitrage relations. Preferences and probabilities are reflected in c_{xx} and g_{xx}, but are not otherwise needed. Don't need homogeneous probability beliefs

With Black-Scholes Model Assumptions, BL Derived State Prices:

Douglas T. Breeden
University of Chicago

Robert H. Litzenberger
Stanford University

Prices of State-contingent Claims Implicit in Option Prices*

$$\Delta(Y_1,\ Y_2,\ T) = B(T)\{N[d_2(X = Y_1)] - N[d_2(X = Y_2)]\}$$

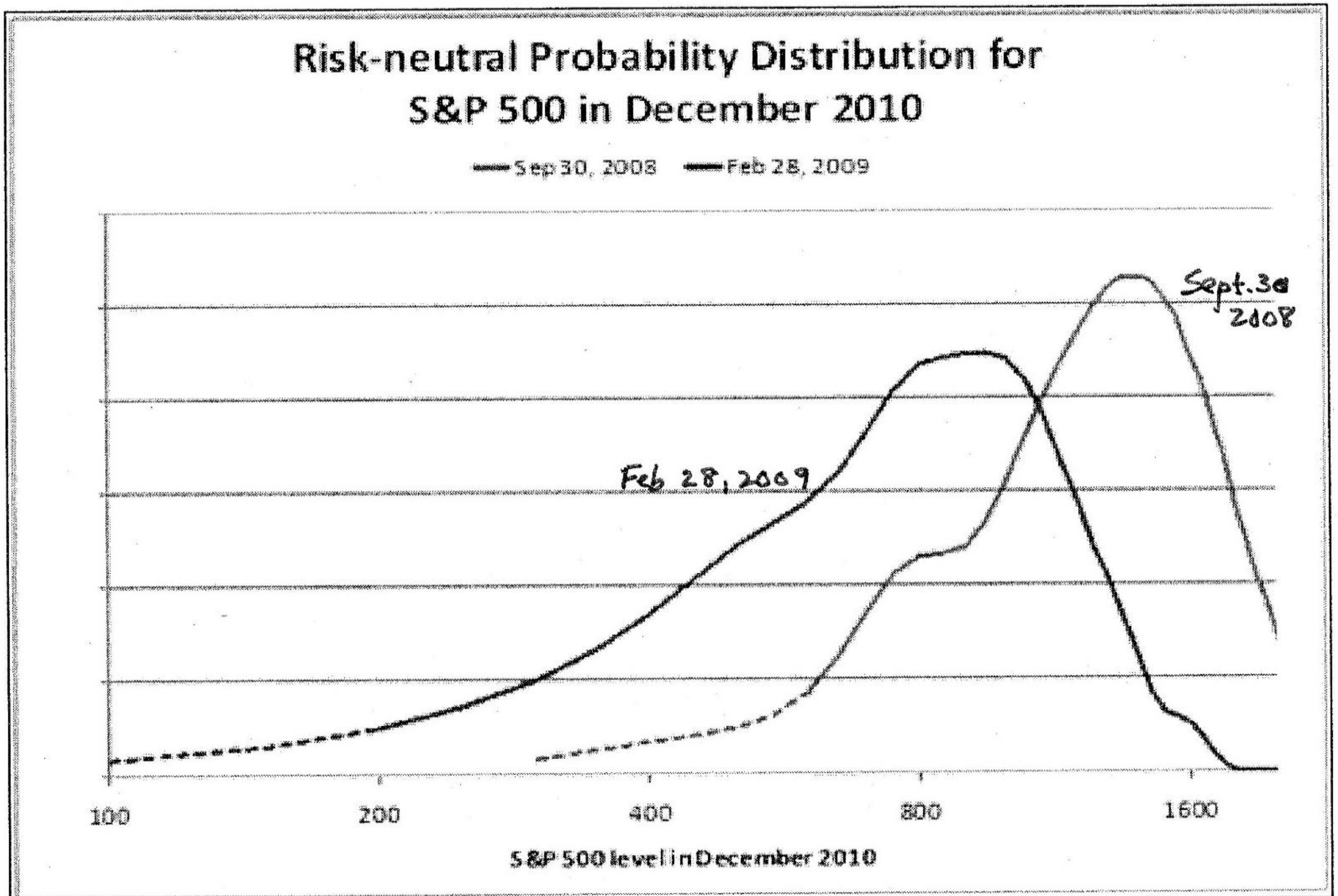
$\frac{Y_1}{M_0} - \frac{Y_2}{M_0}$	Time to Maturity									
	3 Mos.	6 Mos.	9 Mos.	1 Yr.	2 Yrs.	3 Yrs.	4 Yrs.	5 Yrs.	10 Yrs.	20 Yrs.
0–.1										.2¢
.1–.2									.3¢	.9
.2–.3							.1¢	.3¢	1.2	1.6
.3–.4					.1¢	.3¢	.7	1.2	2.5	1.9
.4–.5					.6	1.5	2.4	3.0	3.5	2.0
.5–.6			.2¢	.5¢	2.5	4.0	4.7	4.9	4.0	1.9
.6–.7		.6¢	1.7	3.0	6.1	6.8	6.7	6.4	4.2	1.8
.7–.8	1.2¢	5.0	7.6	9.0	9.9	9.0	8.0	7.1	4.2	1.7
.8–.9	13.1	16.6	16.5	15.7	12.4	10.1	8.5	7.3	4.0	1.6
.9–1.0	34.8	26.4	21.9	18.9	12.9	10.0	8.2	6.9	3.6	1.4
1.0–1.1	32.5	24.3	20.0	17.3	11.7	9.1	7.4	6.2	3.3	1.3
1.1–1.2	13.5	14.7	13.8	12.8	9.6	7.7	6.4	5.5	3.0	1.2
1.2–1.3	2.9	6.5	7.8	8.1	7.3	6.2	5.4	4.7	2.6	1.0
1.3–1.4	.4	2.2	3.7	4.6	5.3	4.9	4.4	3.9	2.3	.9
1.4–1.5		.6	1.5	2.3	3.7	3.7	3.5	3.2	2.0	.9
1.5–1.6				1.1	2.5	2.8	2.8	2.6	1.8	.8
1.6–1.7				.5	1.6	2.1	2.2	2.1	1.5	.7
1.7–1.8				.2	1.0	1.5	1.7	1.7	1.3	.6
1.8–1.9				.1	.6	1.1	1.3	1.4	1.2	.6
1.9–2.0					.4	.8	1.0	1.1	1.0	.5
2.0–2.1					.2	.5	.8	.9	.9	.5
2.1–2.2					.1	.4	.6	.7	.8	.4
2.2–2.3					.1	.3	.4	.6	.7	.4
2.3–2.4					.1	.2	.3	.5	.6	.4
2.4–2.5						.1	.3	.4	.5	.3
2.5–2.6						.1	.2	.3	.5	.3
2.6–2.7						.1	.1	.2	.4	.3
2.7–2.8							.1	.2	.4	.3
2.8–2.9							.1	.1	.3	.2
2.9–3.0							.1	.1	.3	.2
3.0–3.1							.1	.1	.2	.2
3.1–3.2								.1	.2	.2
3.2–3.3								.1	.2	.2
3.3–3.4									.2	.2
3.4–3.5									.1	.2
3.5–3.6									.1	.1
3.6–3.7									.1	.1
3.7–3.8									.1	.1
3.8–3.9									.1	.1
3.9–4.0									.1	.1
4.0–4.1									.1	.1

* Assumptions for all maturities are: $r = .06$, $\delta = .04$, $\sigma = .20$.

Freakonomics article: "Quantifying the Nightmare Scenarios" for the Stock Market

Eric Zitzewitz (Dartmouth) Uses Breeden-Litzenberger 1978 Technique to Estimate the market's state price density at the depth of the Great Recession.

In ***Freakonomics*** Blog by Justin Wolfers, March 2, 2009



9/30/2008:
S&P500= 1166
VIX = 39.4%

2/28/2009
S&P500 = 735
VIX = 46.4%

II. A Method for Estimating Central Bank Policy Impacts on the Distribution of Insurance Prices for Future Interest Rates

*Douglas T. Breeden and Robert H. Litzenberger**

Reference notes for central bank talks
in America, Europe, Asia, Latin America and the Middle East 2013-2020.

Sources for graphical data: OECD data,
IHS Global Insight, and Bloomberg

While we were "sleeping"…Breeden-Litzenberger Method (1978)
<u>used by Central Banks to find price distributions from option prices.</u>

Probability distributions of future asset prices implied by option prices

By Bhupinder Bahra of the Bank's Monetary Instruments and Markets Division.

Introduction

Many monetary authorities routinely use the forward-looking information that is embedded in financial asset prices to help in formulating and implementing monetary policy. For example, they typically look at changes in the forward rate curve implied by government bond prices to assess changes in market perceptions of future short-term interest rates.(1) But, although implied forward rates are informative about the market's mean expectation for future interest rates, they tell us nothing about the range of expected outcomes around such estimates. For this, we can turn to options markets.

exercising it only if the price of the underlying asset lay above the strike price at that time.

Consider a set of European options on the same underlying asset, with the same time-to-maturity, but with different exercise prices. The prices of such options are related to the probabilities attached by the market to the possible values of the underlying security on the maturity date of the options. Intuitively, this can be seen by noting that the difference in the price of two options with adjacent exercise prices will reflect the value attached to the ability to exercise the options when the price of the underlying asset lies between the two exercise prices. This price difference in turn depends on the probability of the underlying asset price

1996 Bank of England Quarterly

The Breeden and Litzenberger approach

Breeden and Litzenberger (1978) derived a relationship linking the curvature of the call pricing function to the terminal RND function of the price of the underlying asset. In particular, they showed that the second partial derivative of the call pricing function with respect to the exercise price is directly proportional to the terminal RND function. Details about the derivation of the Breeden and Litzenberger result are given in Bahra (1996). The rest of this article focuses on how this result can be applied in order to estimate market RND functions for short-term interest rates in the future and how such RND functions can be used for policy analysis.

FEDERAL RESERVE BANK OF MINNEAPOLIS

BANKING AND POLICY STUDIES

Methodology for Estimating Risk Neutral Probability Density Functions

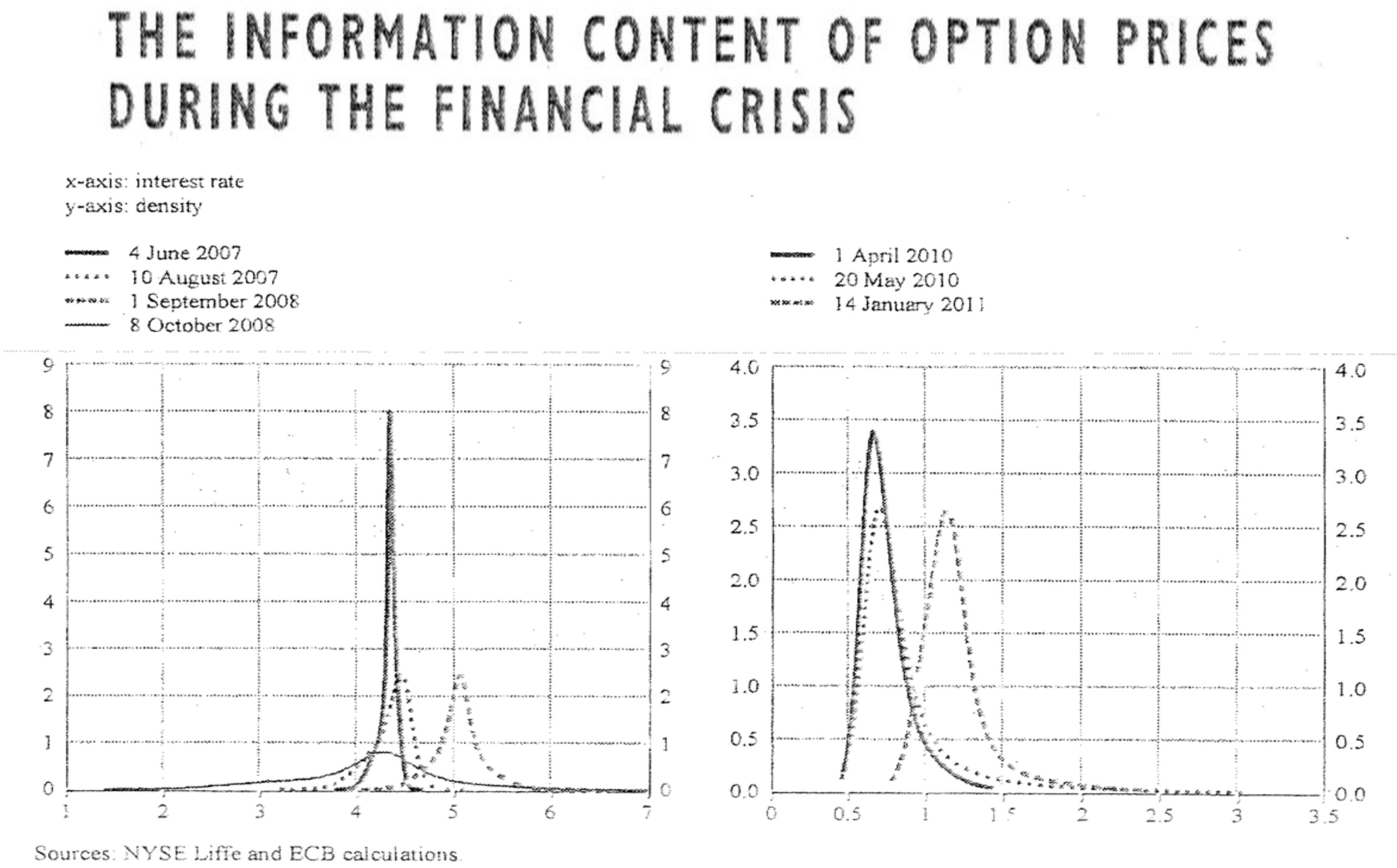
We estimate risk neutral probability density functions (RNPDs) for a variety of different asset classes using a variation of the technique developed by Shimko (1993). This procedure involves fitting a curve to the implied volatilities of a series of options and expressing the volatility as a function of the strike price. The implied volatilities are then translated into continuous call option prices, and the risk neutral distribution of the underlying asset is obtained through the Breeden-Litzenberger (1978) method.

Related Research by Central Banks

Central banks have estimated "option implied (risk-neutral) probability distributions" using the Breeden-Litzenberger (1978) technique. Central bank applications are discussed in articles of Bahra (1996, 1997), Clews, Panigirtzoglous and Proudman (2000), and Smith (2012) of the Bank of England, Maltz (1995,1997) of the Federal Reserve Board of New York; and Durham (2007), Kim (2008) and Kitsul and Wright (2013) of the Federal Reserve Board in Washington.

Kocherlakota's (2013) research group at the Federal Reserve Bank of Minneapolis uses Shimko's (1993) statistical method applying the Breeden-Litzenberger formula to regularly estimate and publish risk neutral density functions and tail risks (e.g., risk neutral probabilities of moves of +/- 20% or more) **for many assets, such as stocks, crude oil, wheat, real estate, and foreign exchange**.

European Central Bank's Monthly Bulletin, February 2011, uses the Breeden-Litzenberger 1978 method to estimate *interest rate distributions for what Euribor will be in 3 Months:*



Key Disadvantages of Many Approaches. Our Approach.

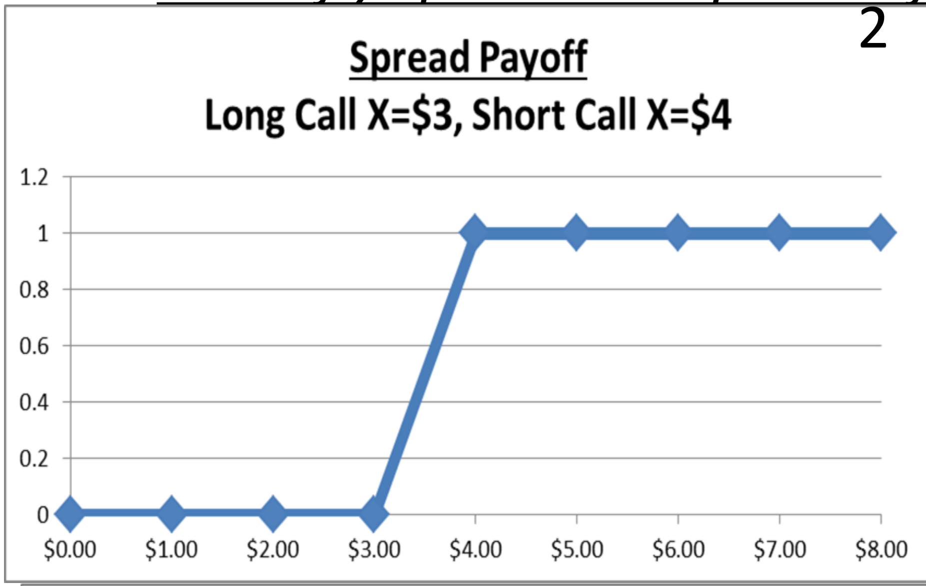
■ 1. ***Short-term option prices used***.

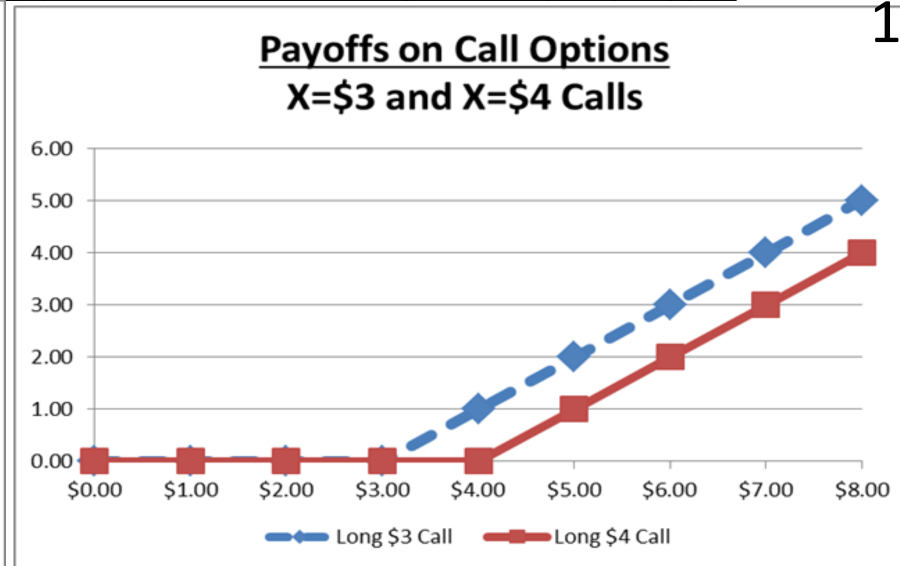
Most options mature in 3 months to 18 months, as many markets only have active markets for those maturities. Often there are not options actively traded for a large number of standardized strike prices. We use interest rate caps and floors that have longer term maturities from 2 to 10 years.

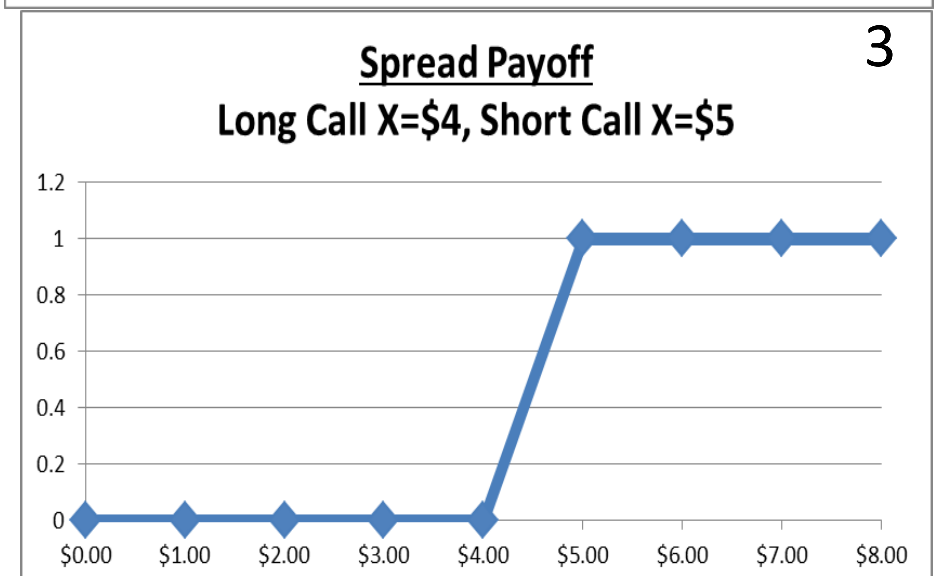
■ 2. ***Parametric vs. nonparametric approach.***

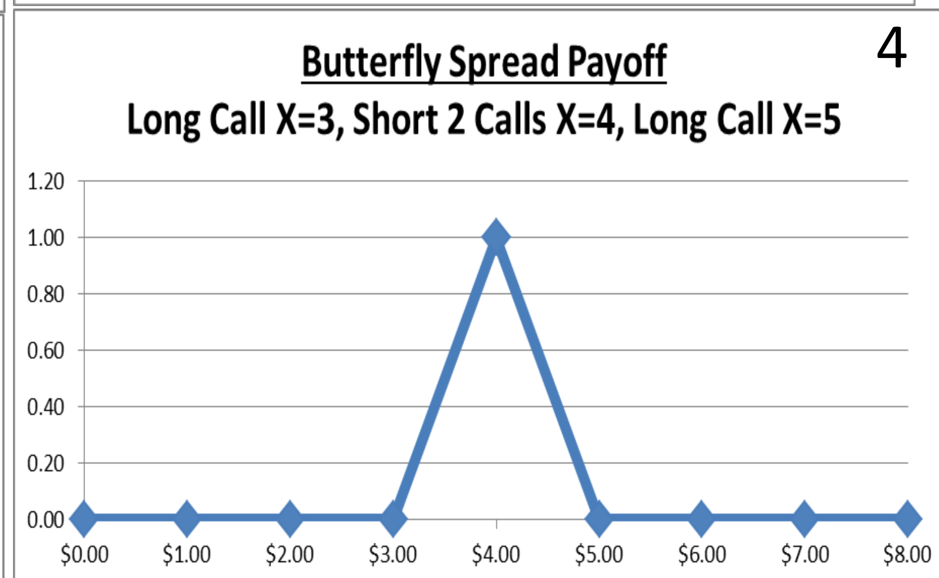
Applications often parameterize option prices with 3 or 4 parameters (mean, variance, skewness, kurtosis) and estimate implied volatility surfaces and entire risk-neutral densities. It is well-known among practitioners that these methods can be off significantly in estimating tail risks. For interest rate options, we use Bloomberg's volatility cube estimates of cap and floor prices, which are smoothly fitted from daily option market prices and give sensible insurance price distributions. In our approach for S&P 500 options, we use (nonparametric) traded option prices from Bloomberg, which give implied volatility smiles, smirks and skews that may be of any shape.

Butterfly spread is a spread of spreads. Triangular payoffs



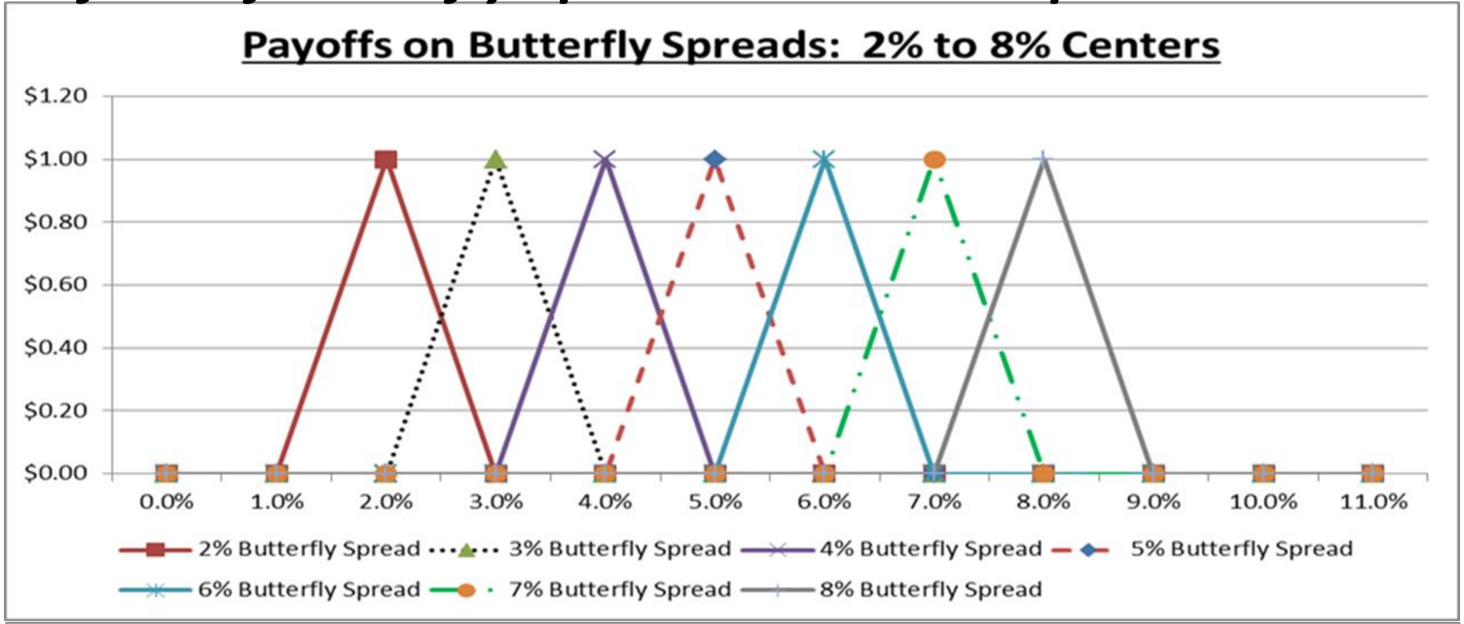


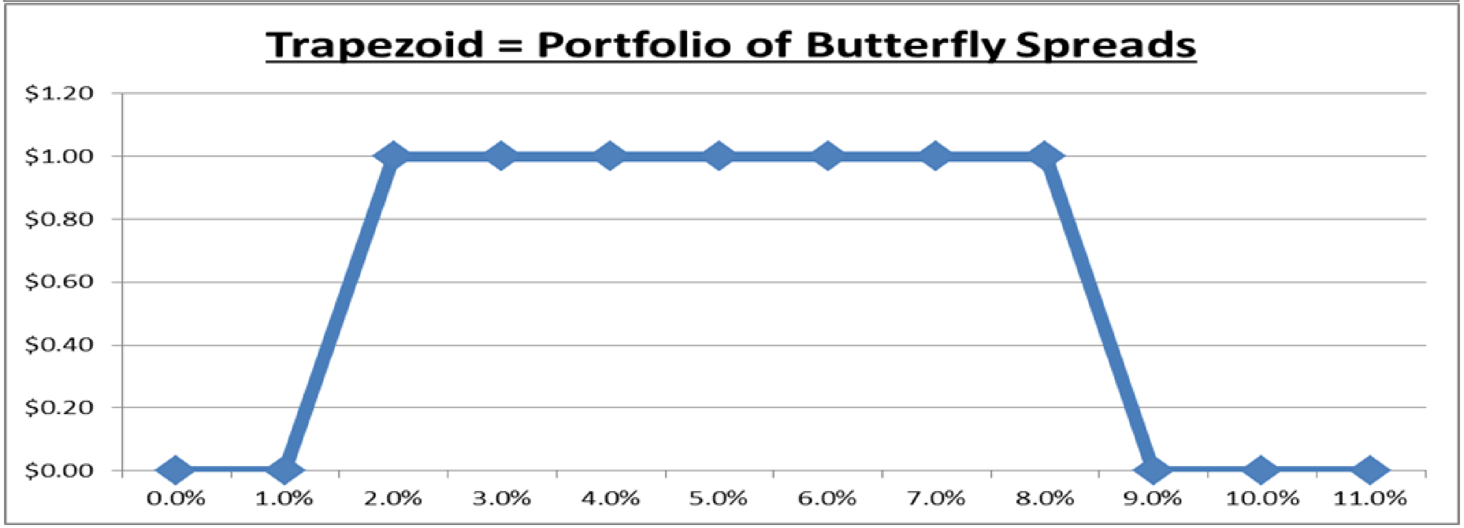




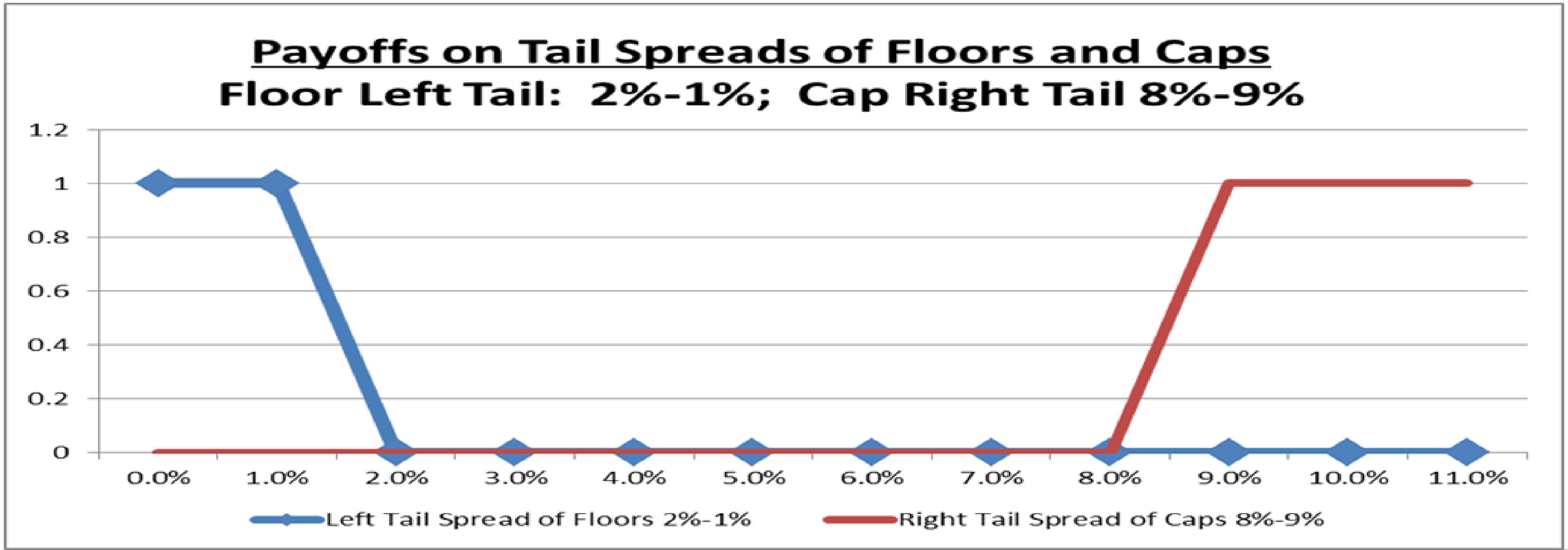
Example with Options on Interest Rates (Caps and Floors)

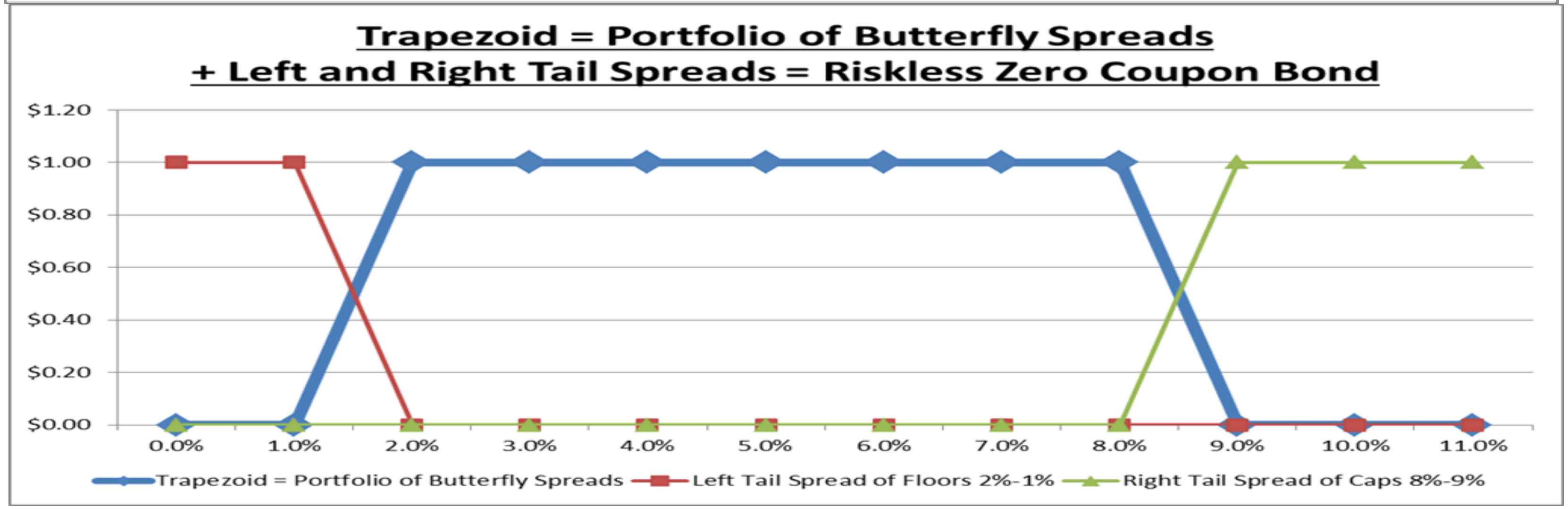
Porfolio of Butterfly Spreads Gives A Trapezoidal Distribution.





Left and Right Tail Spreads, Plus Portfolio of Butterflies Trapezoid = Riskless Bond





Butterfly Spreads and Tail Spread Costs Divided by Riskless Bond Price Gives the State Price Density

a.k.a. "Risk Neutral Probabilites" or "Insurance Prices"

	Spread Cost	"Risk-Neutral Probability"
"0%" = Left tail spread: Long 1%, Short 0% floorlet	$0.290	0.297
1% Butterfly spread (Long 0%, Short 2 1%, Long 2%)	$0.320	0.328
2% Butterfly spread (Long 1%, Short 2 2%, Long 3%)	$0.180	0.184
3% Butterfly spread	$0.080	0.082
4% Butterfly spread	$0.037	0.038
5% Butterfly spread	$0.028	0.028
6% Butterfly spread	$0.014	0.014
7% Butterfly spread	$0.007	0.007
8% Butterfly spread	$0.007	0.007
9%+ = Right tail spread: Long 8%, Short 9% caplet	$0.015	0.015
Totals	$0.977	1.000

Notes for Nerds: Theorem: If Risk Neutral Density is Linear in the Rate Range, then Digital Option (Arrow) Value Equals Butterfly Cost

Proposition: The relationship between butterfly spread values and digital option values:

If the risk-neutral density (RND) is a linear function of the interest rate within the range of the butterfly strikes, then the value of a digital option that pays off $1.00 over the middle half of the range is equal to the value of the butterfly.

Proof: Let x be the interest rate, such that $x = c$ at the lower strike of the butterfly, $x = c+1$ at the mid-point strike of the butterfly, and $x = c+2$ at the high strike of the butterfly.

Assume that between c and c+2 the risk-neutral density = RND $= a + b(x-c)$

The forward value of a digital option that pays off $1.00 between $x = c+0.5$ and $x = c+1.5$ is:

$$\int_{c+0.5}^{c+1.5} [a + b(x-c)] \cdot 1dx = a + b$$

The forward value of a butterfly is $\int_{c}^{c+1} \{[a + b(x-c)](x-c)\}dx + \int_{c+1}^{c+2} \{[a + b(x-c)](c+2-x)]\}dx$

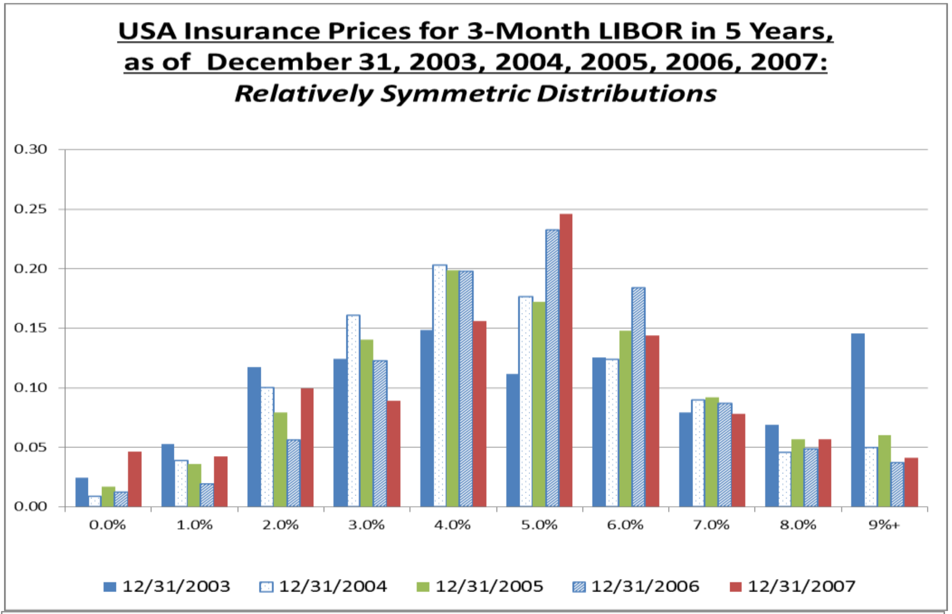
$$= \frac{1}{3}bx^3 + \frac{1}{2}(a-2bc)x^2 + (bc^2 - ac)x\Big|_{c}^{c+1} - \frac{1}{3}bx^3 + (bc + b - \frac{1}{2}a)x^2 + (2a - 2bc - bc^2 + ac)x\Big|_{c+1}^{c+2} = a$$

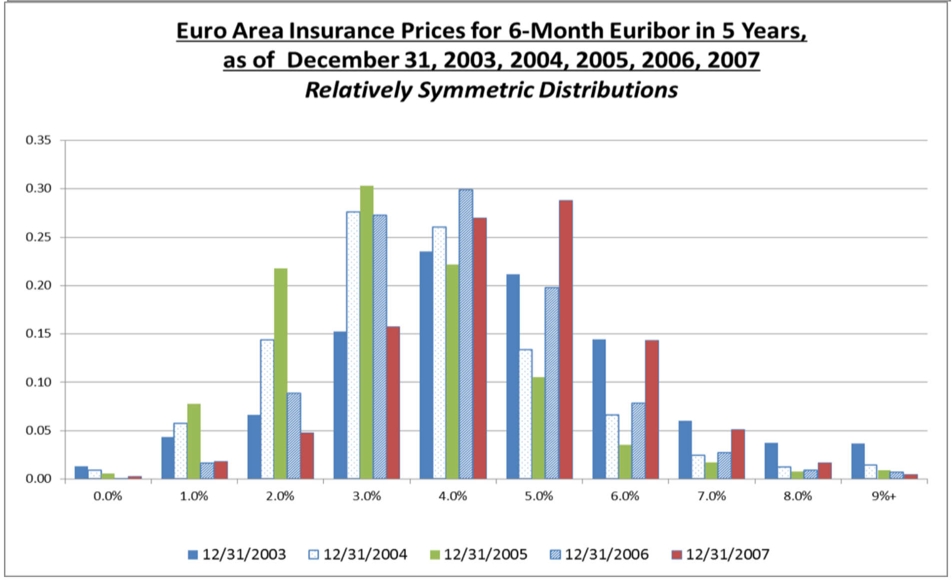
Of course, since forward values are equal at the same date, present values are also equal.

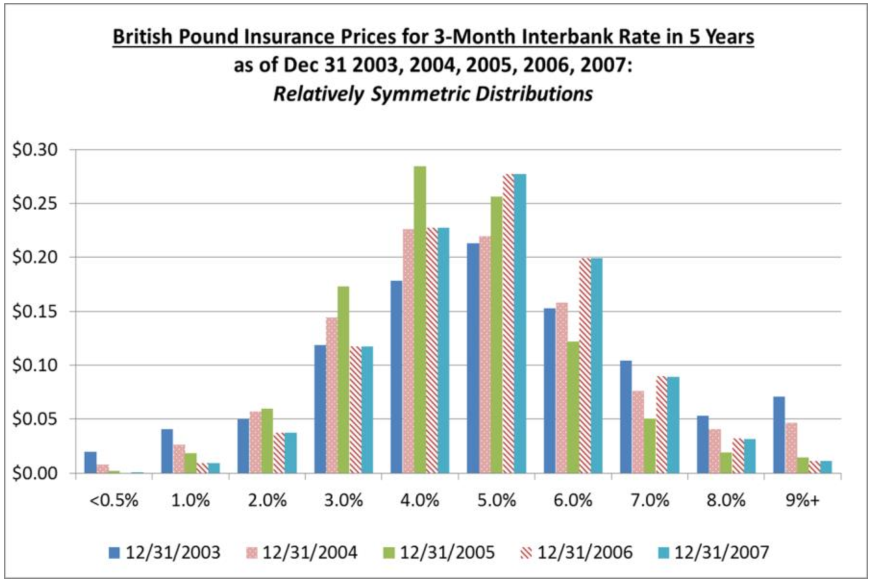
Q.E.D.

[1] Do note that there is a macro inconsistency in applying this approach with RNDs linear in rates where the {a,b} coefficients change from rate range to rate range, as would be realistic. With overlapping triangles, this would give an RND for the 4% to 5% range that is different for the 3/4/5 butterfly than for the 4/5/6 butterfly. Thus, this Proposition's result is just an approximation that is for useful intuition about butterflies and digital options.

Insurance Price Distributions for 5 Years for USA, Eurozone, UK 2003-2007







III. Impact of Central Bank Policy Announcements on Interest Rate Insurance Prices for 3-Month LIBOR: 2008-2020

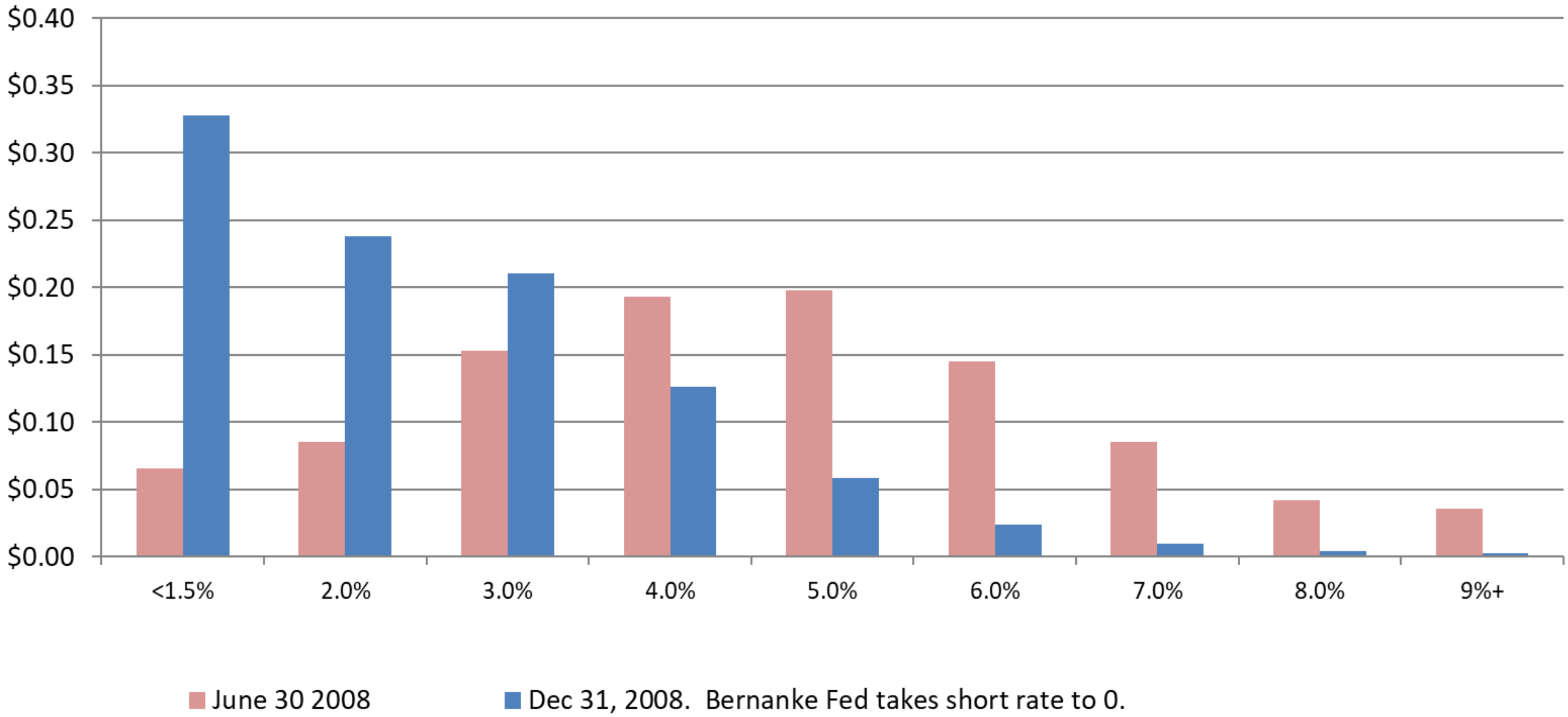
Major Federal Reserve Announcements 2008-2020

- **December 2008**. Cut rates to record lows in financial panic.
- **March 2009**: Will keep rates close to zero for "extended period." Stock market bottoms March 9th. Unemployment rate increases to peak of 10.0% in October 2009.
- **August 2011**: Budget impasse. Fed "will keep rates extremely low "at least until 2013."
- **September 2012**: Low "at least until 2015"
- **December 2012**: Will tie low rates to range in Unemployment (>6.5%), Inflation(<2%).
- **May/June 2013**: **May 22**: Given economic strength, Fed is seriously considering "tapering" asset purchases (QE3). **June 19:** Housing market is strong and supportive; tapering QE3 in 2nd half 2013.
- **Sept 18, 2013**: Fed announces "No tapering yet" and surprises markets.
- **Dec 18, 2013.** Bernanke Fed announces beginning of tapering, $10 billion/month.
- **March 19, 2014**. Yellen Fed indicates short rates may rise in 6 months after end of tapering, perhaps by mid-2015, earlier than markets expected.
- **April 30, 2014**. Job growth strong. Unemployment rate drops sharply: 6.7% to 6.3%.
- **October, 2014**. Unemployment at 5.9%. Yellen Fed ending asset purchases (QE).
- **March, 2015.** Unemployment at 5.5%, rapid job growth. Fed drops "patience" talk. "Dots" show that Fed members expect a slower ramping up of rates after liftoff.
- **December, 2015.** Fed "lifts off" and raises its policy rate 0.25%, first since Great Recession.
- **December 2017, 2018**. Fed has 5th rate hike, policy rate near 1.5%; 9th Increase to 2.5% in 12/18.
- **June 2019.** Trade War slowing global growth, Fed indicates possible pivot, lower rates.
- **March 2020**. Powell's Fed takes the short rate to zero amid "Coronavirus Pandemic." Massive QE.

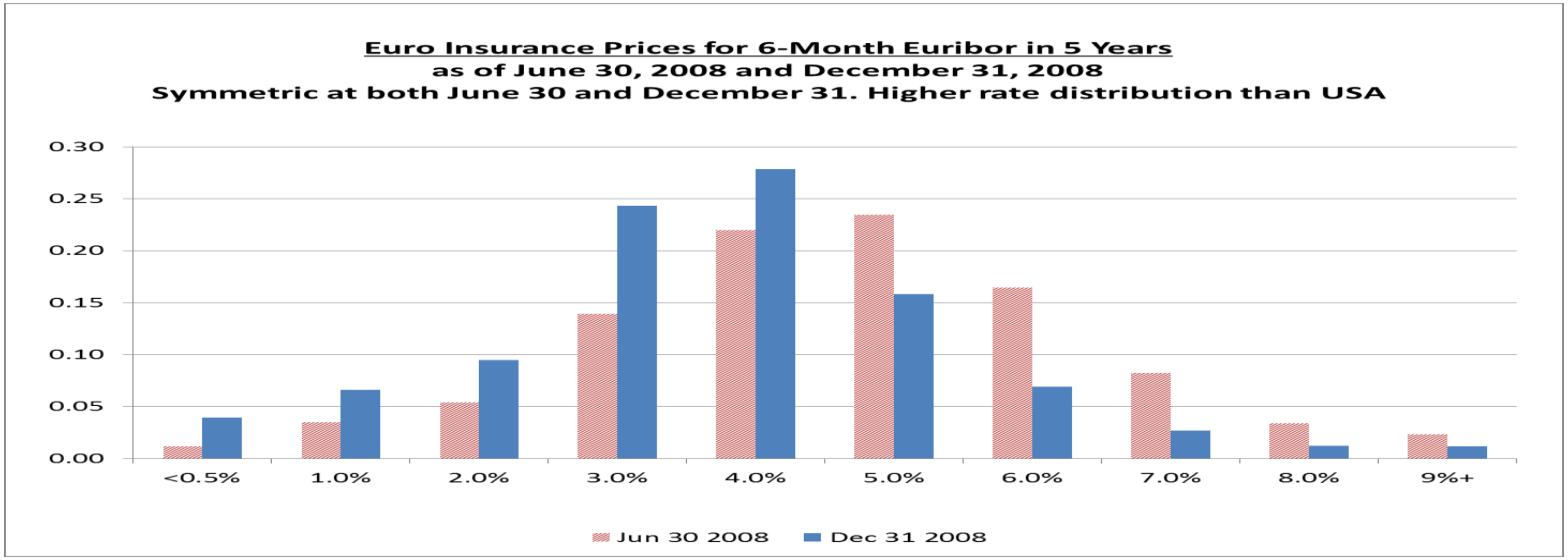
Financial Panic/Great Recession of 2008-2009

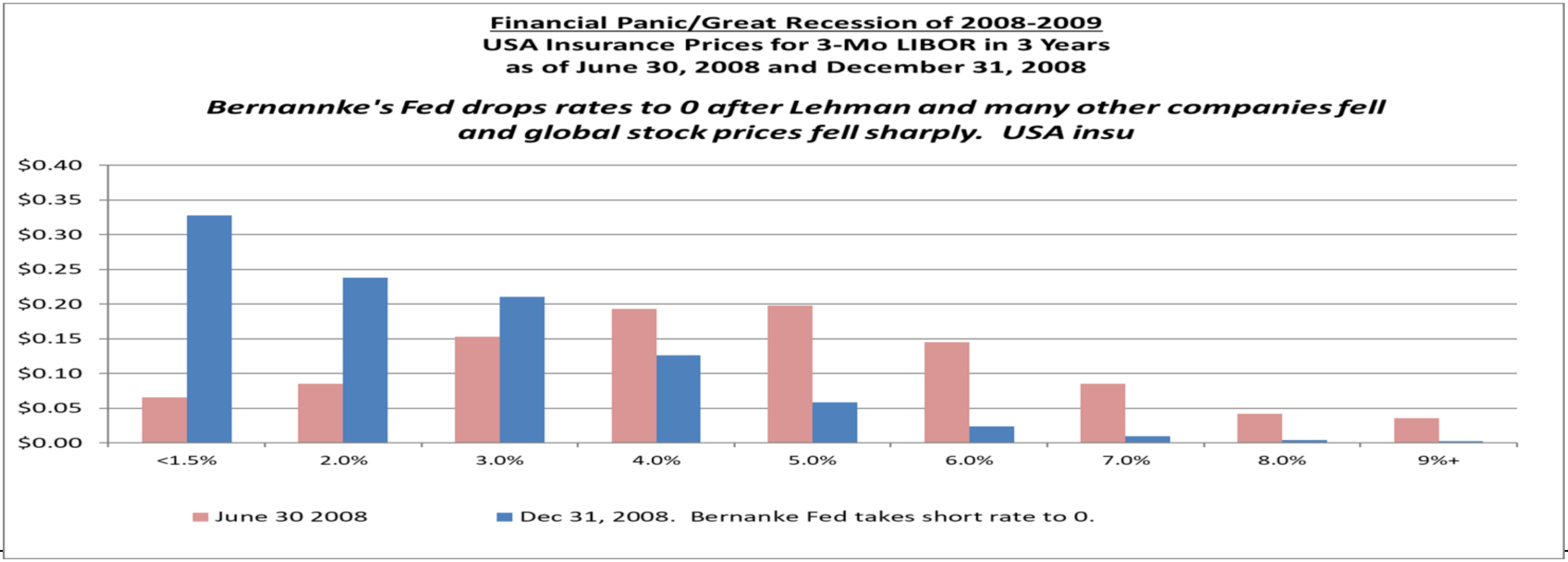
USA Insurance Prices for 3-Mo LIBOR in 3 Years as of June 30, 2008 and December 31, 2008

Bernanke's "Fed drops rates to 0 after Lehman and many companies fell and global stock prices plunged. USA distribution of state prices changed from symmetric to strongly positively skewed (concentrated near zero rate).



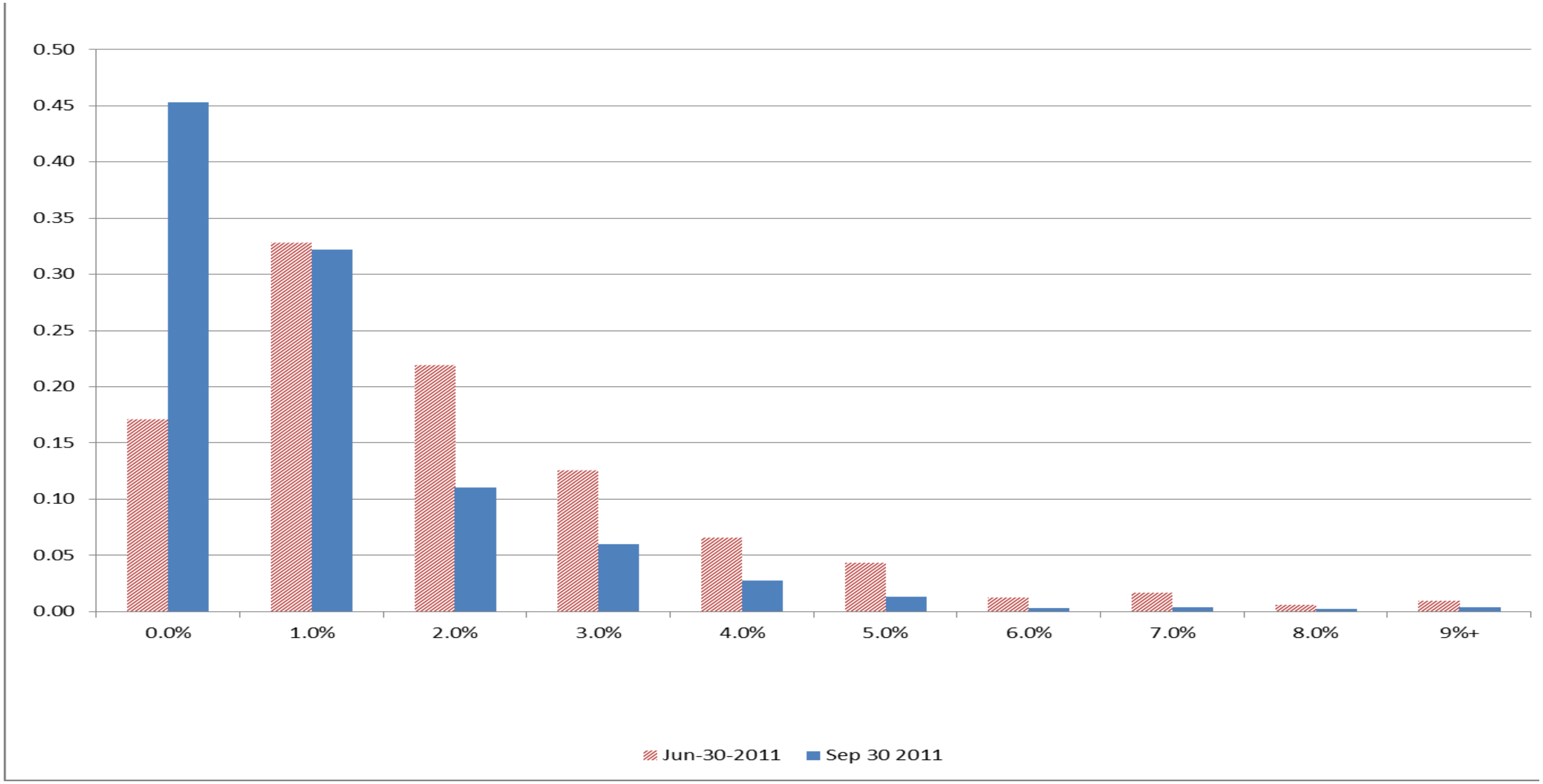
Dec 2008: Euro Area Rate Distribution Unaffected by USA problems



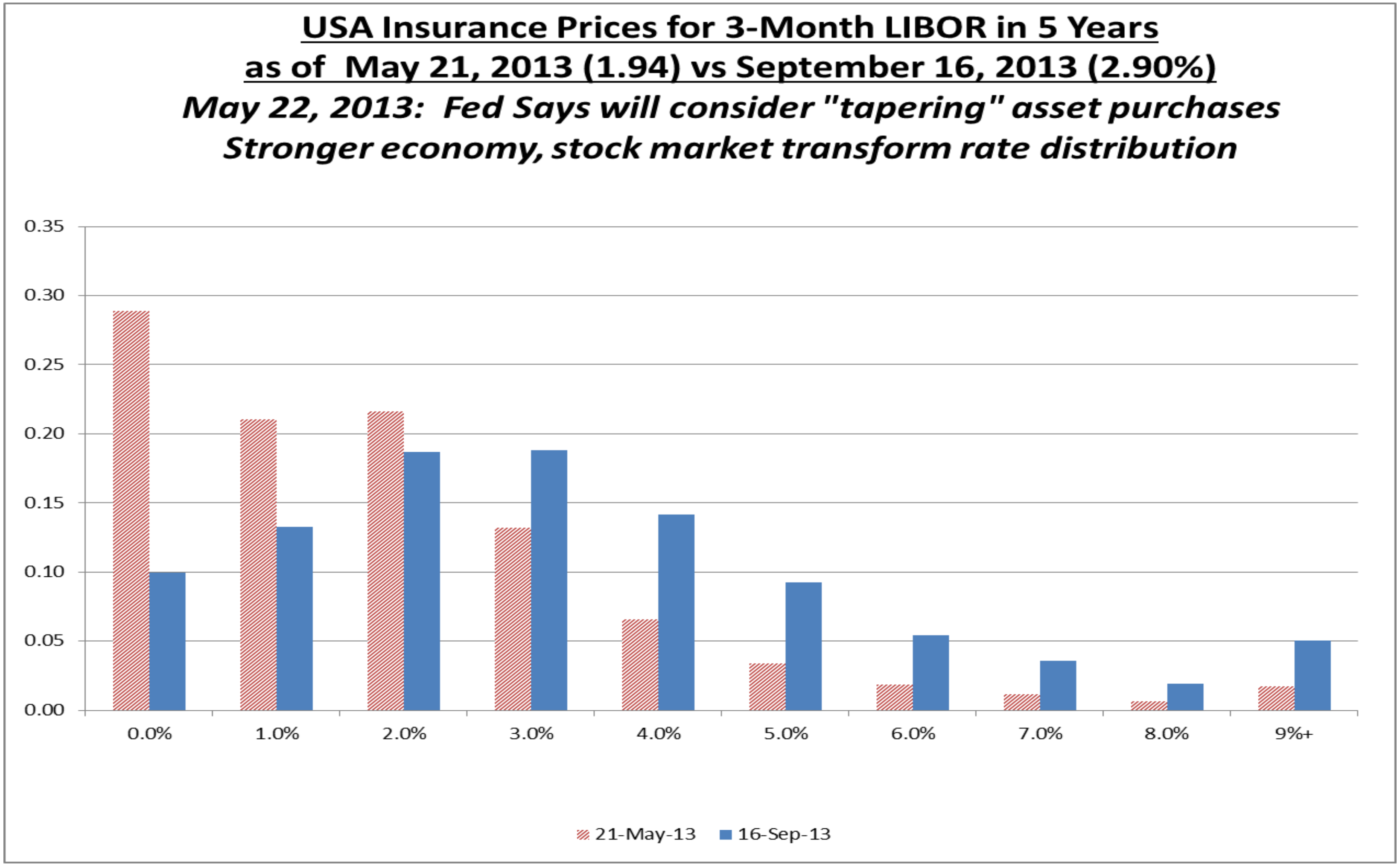


Market panic during 2011 budget impasse causes Fed to commit to low rates for 2.5 years. Specificity and long time commitment hammer down the 3-year interest rate insurance distribution.

June 30, 2011 and September 30, 2011 Distributions:



Summer 2013 Tapering Announcements: Stronger economy shifts distribution towards symmetry



IV. Relation of Insurance Prices to True Probabilities

Betas of Nominal Bonds Change Sign, Which Changes The Bias

True Probabilities vs.
Insurance Prices or "Risk Neutral Probabilities"

Insurance prices or "risk neutral probabilities" differ from true, objective probabilities, because investors price assets higher for those that pay off most when times are bad (negative beta), as they are portfolio diversifiers. Thus, their insurance prices (risk neutral probabilities) exceed their true probabilities.

Payoffs for states that correspond to good economies have positive betas and will have lower insurance prices to provide fair risk premiums, so their insurance prices will underestimate the true probabilities.

State Price/Probability Ratios should be highest for highest marginal utility states, which are those with lowest real consumption

- $$\max_{\{c_{ts}^k\}} \mathcal{L} = u_0^k(c_0^k) + \Sigma_t \Sigma_{s \in S_t} \pi_{ts}^k u^k(c_{ts}^k, t) + \lambda^k [W_0^k - c_0^k - \Sigma_t \Sigma_s \phi_{ts} c_{ts}^k]$$

$$\frac{\partial \mathcal{L}}{\partial c_0^k} = u_0'^k - \lambda^k = 0 \quad \Rightarrow \lambda^k = u_0'^k$$

Define: $u_{ts}'^k \equiv \partial u^k(c_{ts}^k, t)/\partial c_{ts}^k$, evaluated at c_{ts}^k

$$\frac{\partial \mathcal{L}}{\partial c_{ts}^k} = \pi_{ts}^k u_{ts}'^k - \lambda^k \phi_{ts} = 0$$

$$\Rightarrow \quad \phi_{ts} = \frac{\pi_{ts}^k u_{ts}'^k}{u_0'^k}$$ = price of \$1 in time-state ts

$$\Rightarrow \quad \frac{u_{ts}'^k}{u_0'^k} = \frac{\phi_{ts}}{\pi_{ts}^k} \Rightarrow \frac{1}{u_0'^k} \begin{pmatrix} u_{t_1 s_1}'^k \\ \vdots \\ u_{TS}'^k \end{pmatrix} = \begin{pmatrix} \phi_{t_1 s_1} / \pi_{t_1 s_1}^k \\ \vdots \\ \phi_{TS} / \pi_{TS}^k \end{pmatrix}$$

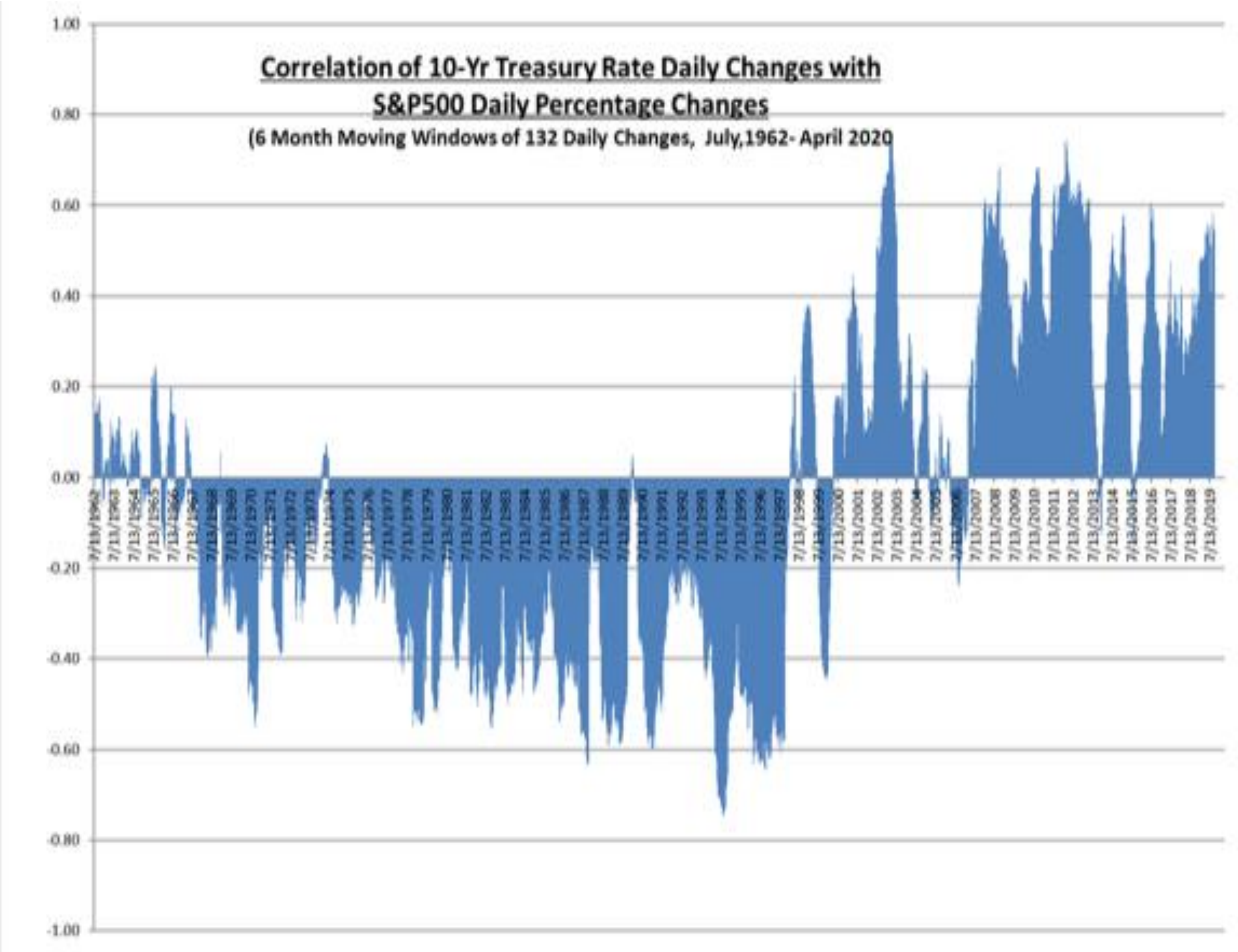
Nominal Bonds Have Betas That Change Signs, Changing Price/Probability

From Breeden, "Consumption, Production and Interest Rates, A Synthesis," Journal of Financial Economics, May 1986, pp. 32-33:

> "Since inflation is typically believed to be related to the growth rate of real consumption, the risk premium of the nominally riskless asset may be non-trivial. The relation of inflation to the real growth of the economy may be nonstationary… If a Phillips curve relates inflation and unemployment (pre-1973), then inflation is likely to be high when real consumption is high, resulting in a negative real consumption beta for the nominally riskless assets. … In contrast, recent experience (see Fama (1982)) has been that inflation is negatively related to real movements in the economy. If that were expected, then the real consumption betas for nominally riskless assets are positive, which results in equilibrium real returns on them that are in excess of those on purchasing power bonds."

More recently, Campbell, Sundarem and Viceira (CSV, 2017) have a very sensible model of changing correlations of inflation with the macroeconomy, based upon changing Federal Reserve policy response functions. Both Breeden's (1986) and CSV's (2017) results show that the state prices for interest rates will be biased estimates of probabilities of interest rates, with the direction and extent of the biases (some positive, some negative) depending upon the sign and magnitude of the consumption beta for nominal bonds, which changes over time. Positive consumption beta securities will have lower prices, and state prices will be biased low estimates of true probabilities. Negative beta securities, such as $1 payoffs if rates are below 1% (hedges of a bad economy), will have high prices and their state prices will be biased high as estimates of true probabilities.

When rates are high, is marginal utility high or low? Depends on the time period.



This graph shows the dramatic switch from negative to positive in 1999/2000 in the correlation between changes in the 10-year interest rate and moves in the S&P 500.

This switch in correlation reflects a shift from supply-oriented inflation concerns in the 1970s and 1980s to inflation concerns dominated more by demand issues.

The beta of long-term bond returns versus stock returns and the economy thus shifted from positive to negative. The fair risk premium on long-term bonds should have shifted from positive to negative, as long-term bonds became excellent hedges for risks of a bad economy.

Assuming power utility (CRRA) and lognormally distributed consumption, we get a simple formula for state price to probability ratios:

(Note: gts is the annualized growth rate to time-state ts, and μ is its mean):

$$\log\left(\frac{\phi_{ts}^{*}}{\pi_{ts}}\right) = \gamma\left[\mu_t - g_{ts} - \frac{1}{2}\gamma\sigma_c^2\right]t \qquad (19)$$

As expected, higher growth states for consumption have lower $\left(\frac{\phi_{ts}^{*}}{\pi_{ts}}\right)$ ratios. One could input different estimates of relative risk aversion and different states' growth rates and consumption volatility into the eq. 19 and compute the estimated log of the risk neutral probability to the true probability.

Illustration of True Probabilities Related to Risk Neutral Probabilities

True probability = K*Risk Neutral x exp(Gamma*(gts - mu))

Assumes: CRRA-Lognormal real growth model

Real Growth on Nominal Rate: 1998 to 2011 Data				
Intercept	-3.71	(t= -2.2)		
Slope	1.42	(t= 3.8)		
MuCgrow	3			
		Relative Risk Aversion (Gamma)		
Nominal	Real	2	4	8
Rate	Growth	Ratio of True Probability to Risk Neutral*		
1	-2.29	0.90	0.81	0.65
2	-0.87	0.93	0.86	0.73
3	0.55	0.95	0.91	0.82
4	1.97	0.98	0.96	0.92
5	3.39	1.01	1.02	1.03
6	4.81	1.04	1.08	1.16
7	6.23	1.07	1.14	1.29
8	7.65	1.10	1.20	1.45
9	9.07	1.13	1.27	1.63
10	10.49	1.16	1.35	1.82

*=Up to a scalar multiple

Real Growth on Nominal Rate: 1977 to 1997 Data				
Intercept	4.11	(t= 3.2)		
Slope	-0.12	(t= -0.8)		
MuCgrowt	3			
		Relative Risk Aversion (Gamma)		
Nominal	Real	2	4	8
Rate	Growth	Ratio of True Probability to Risk Neutral*		
1	3.99	1.02	1.04	1.08
2	3.87	1.02	1.04	1.07
3	3.75	1.02	1.03	1.06
4	3.63	1.01	1.03	1.05
5	3.51	1.01	1.02	1.04
6	3.39	1.01	1.02	1.03
7	3.27	1.01	1.01	1.02
8	3.15	1.00	1.01	1.01
9	3.03	1.00	1.00	1.00
10	2.91	1.00	1.00	0.99
11	2.79	1.00	0.99	0.98
12	2.67	0.99	0.99	0.97
13	2.55	0.99	0.98	0.96
14	2.43	0.99	0.98	0.96
15	2.31	0.99	0.97	0.95
16	2.19	0.98	0.97	0.94

V. Interest Rate Insurance Prices for Euribor During the Sovereign Debt Crisis 2010-2012 and the Bounceback

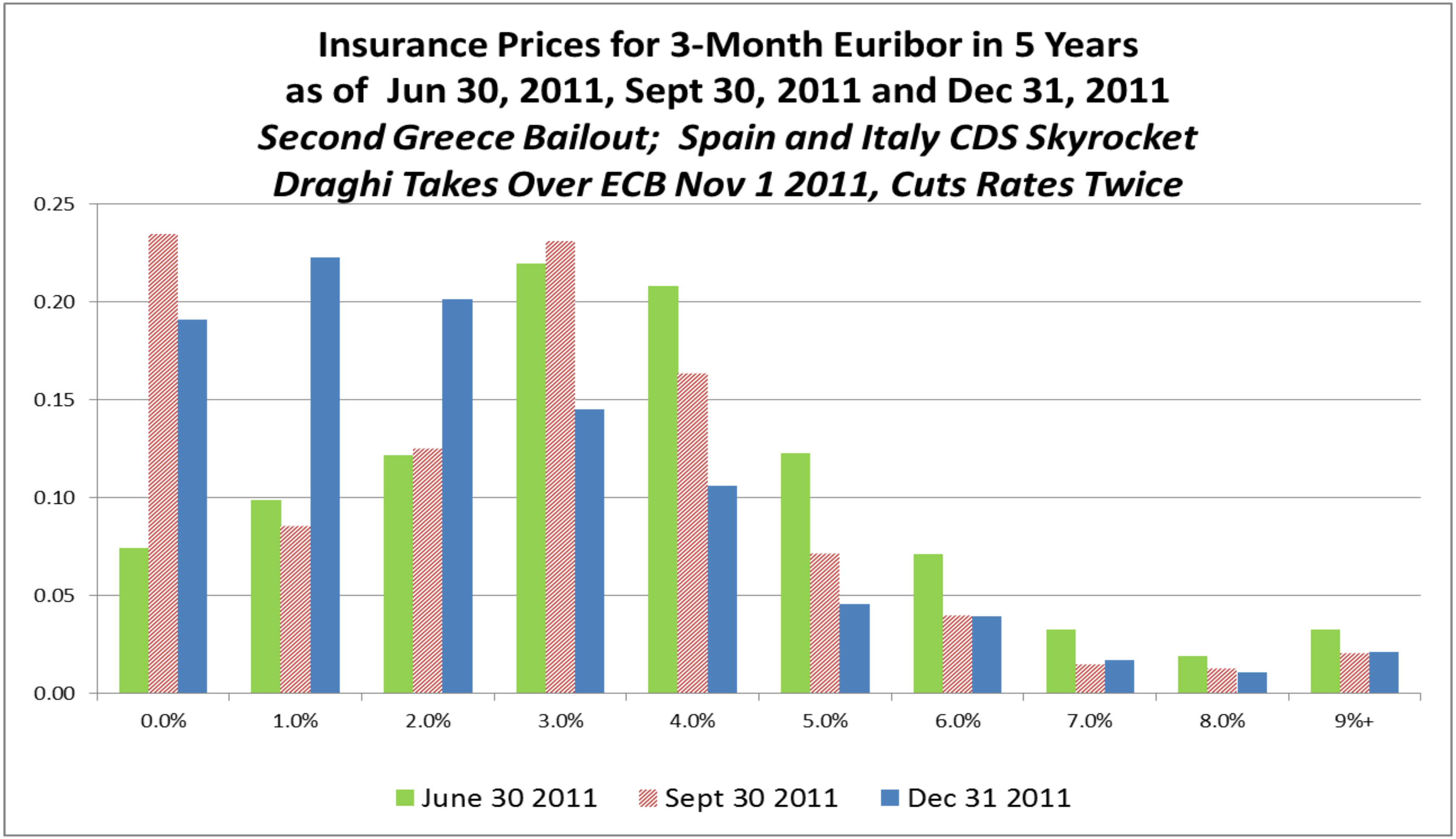
Key Events in the European Sovereign Debt Crisis

European Central Bank 2010-2020

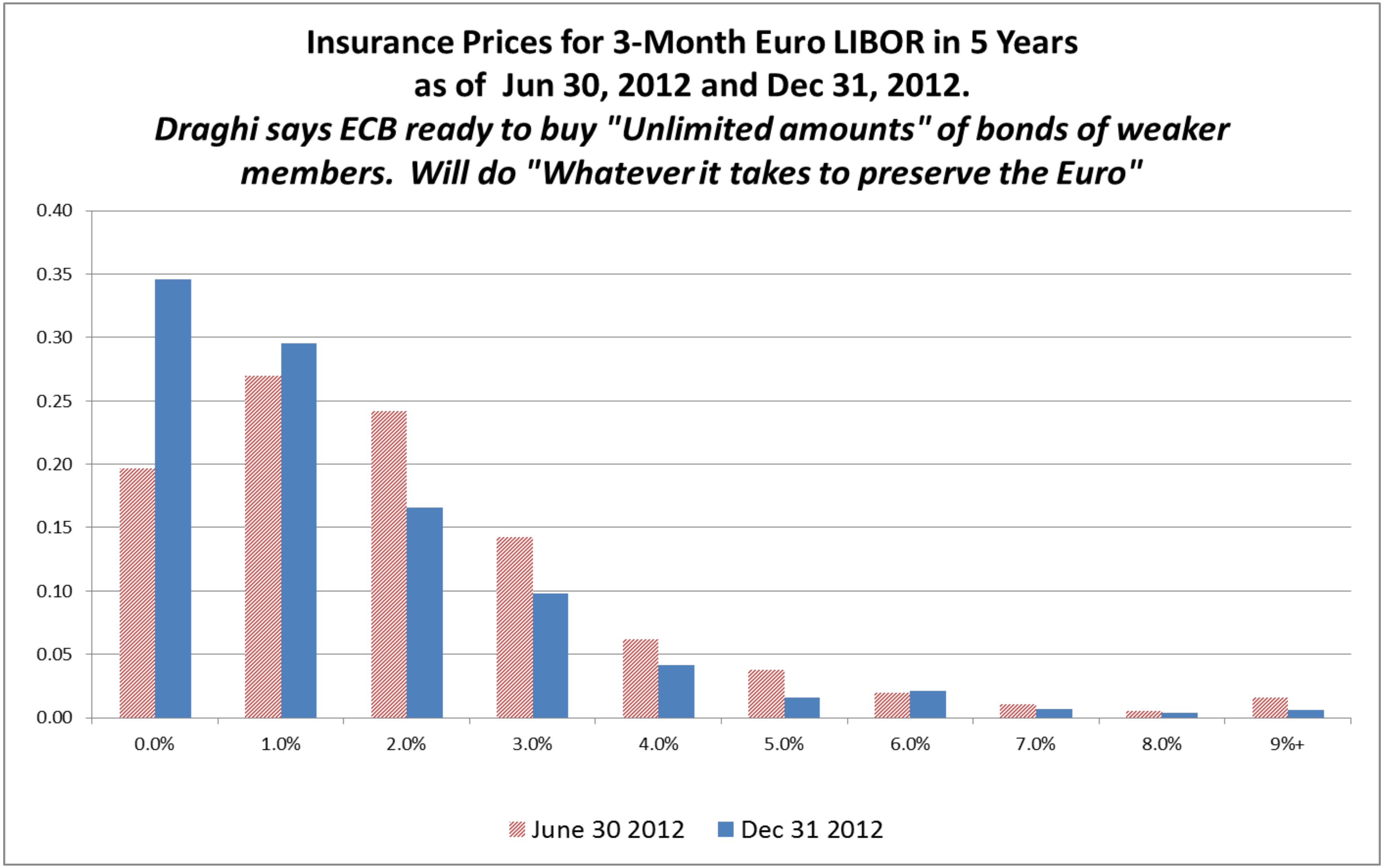
Sources: BBC,Reuters

- January-May 2010. Greek deficit revised upward from 3.7% to 12.7%. "Severe irregularities" in accounting. EU agrees to $30 billion, then $110 billion bailout of Greece. Ireland bailed out in November 2010.
- July-August 2011: Talk of Greek exit from Euro. Second bailout agreed. EC President Barroso: sovereign debt crisis spreading. Spain, Italy yields surge.
- November 1, 2011: Mario Draghi takes over European Central Bank from Jean-Claude Trichet. Draghi cuts rates twice quickly.
- September, 2012: ECB ready to buy "unlimited amounts" of bonds of weaker member countries. Draghi ECB will do "whatever it takes to preserve the Euro." "…and believe me, it will be enough."
- May/June 2013: U.S.Fed considers "tapering" asset purchases, as economy strengthens. Long term interest rates move up sharply.
- June-October, 2014: European economies weak, inflation expectations lower. Draghi cuts rates twice to 0.05%. Announces QE, buying ABS, possibly from Italy and Spain, up to 1 trillion Euro.
- January-March 2015: Draghi of ECB announces on January 22nd "Quantitative Easing" by massive asset purchases. Began QE March 9, 2015.
- 2018: Draghi ECB plans tapering and removal of "Quantitative Easing" asset purchases.
- June 2019: Draghi ECB plans continued QE, given global and Euro Area weakness and uncertainties from Brexit and trade wars.
- March 2020: Coronavirus pandemic leads to further rate reductions and QE.

2011 Sovereign Debt Crisis: Draghi ECB cuts rates sharply. Massive shift in Euribor interest rate distribution to positive skewness like U.S.



Draghi Rescues the Euro in 2012 with "Whatever it takes…"



VII. What are markets saying now in 2020 during the "Coronavirus Pandemic?"

Coronavirus Pandemic as of July 31, 2020

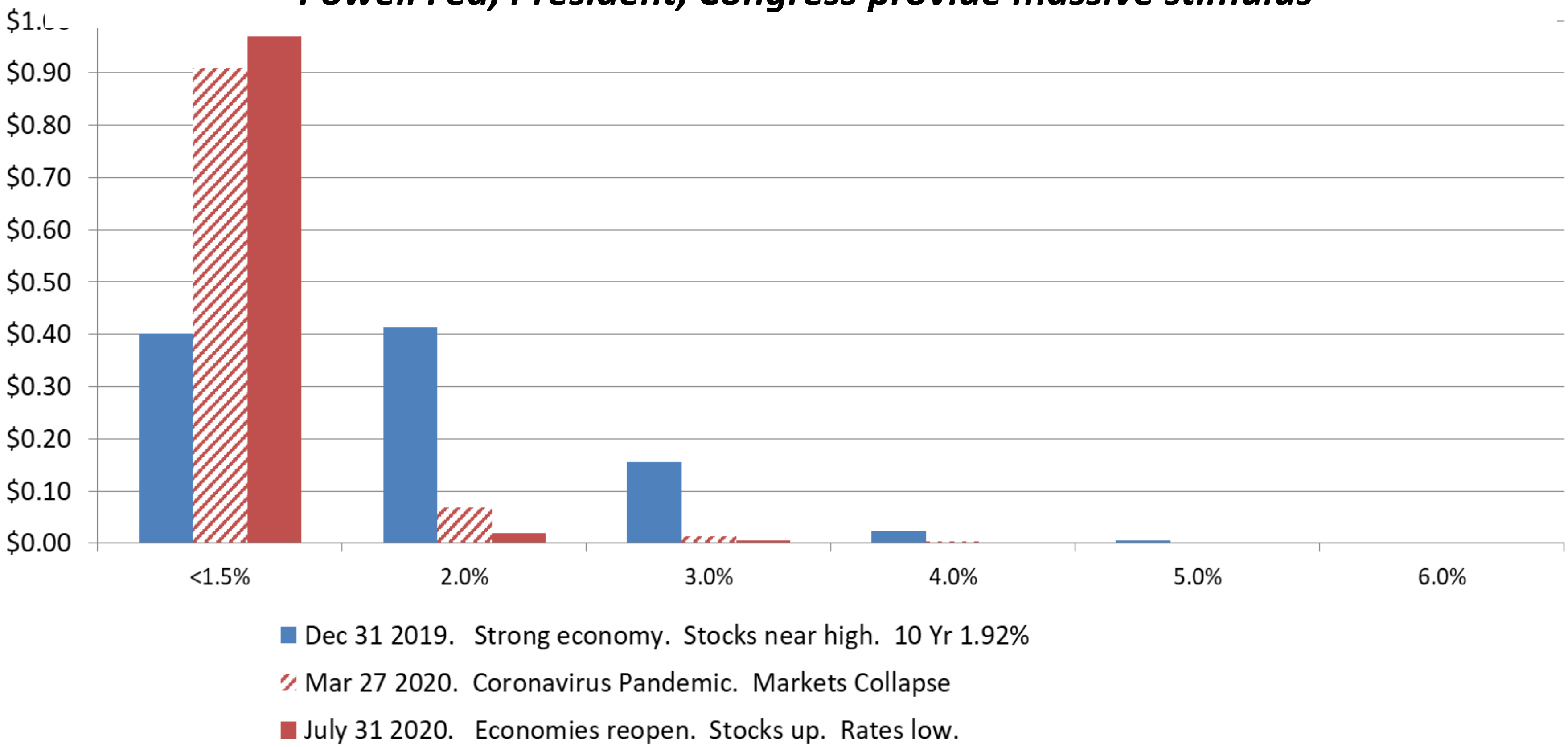
USA Insurance Prices for 3-Mo LIBOR in 3 Years

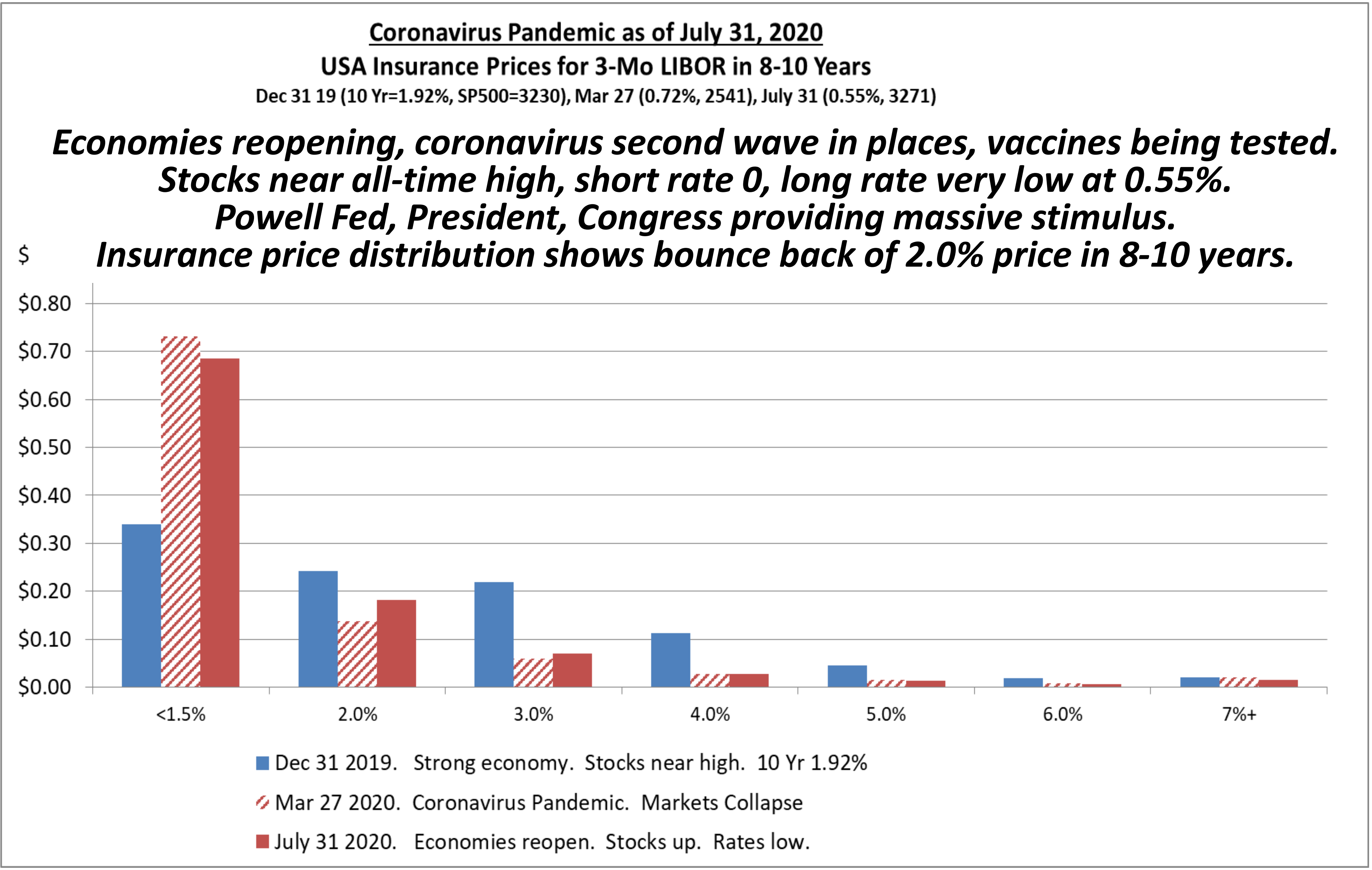
Dec 31 19 (10 Yr=1.92%, SP500=3230), Mar 27 (0.72%, 2541), July 31 (0.55%, 3271)

Economies reopening, coronavirus 2nd wave in some places, vaccines being tested.

Stock markets strong, rates very low.

Powell Fed, President, Congress provide massive stimulus





Coronavirus Pandemic as of July 31, 2020
USA Insurance Prices for 3-Mo LIBOR in 8-10 Years
Dec 31 19 (10 Yr=1.92%, SP500=3230), Mar 27 (0.72%, 2541), July 31 (0.55%, 3271)
Economies reopening, coronavirus second wave in places, vaccines being tested.
Stocks near all-time high, short rate 0, long rate very low at 0.55%.
Powell Fed, President, Congress providing massive stimulus.
Insurance price distribution shows bounce back of 2.0% price in 8-10 years.
$
$0.80
$0.70
$0.60
$0.50
$0.40
$0.30
$0.20
$0.10
$0.00
<1.5%
2.0%
3.0%
4.0%
5.0%
6.0%
7%+
Dec 31 2019. Strong economy. Stocks near high. 10 Yr 1.92%
Mar 27 2020. Coronavirus Pandemic. Markets Collapse
July 31 2020. Economies reopen. Stocks up. Rates low.

III. State Prices for the Stock Market From Prices of S&P 500 Options

Stripping A Zero Coupon Riskless Bond to Create Lottery Tickets (Insurance Payments) on the S&P 500 Return

A financial institution buys $100 million of Treasury bills maturing in 1 year.

The institution then “strips” the Tbill payoffs into 7 lottery tickets A1 to A7:

	State Name
Lottery ticket A1 pays $1.00 if SP500 <-12.5% in 1 year, zero otherwise.	Left tail
Lottery ticket A2 pays $1.00 if SP500 is between -7.51% and -12.5%	-10%
Lottery ticket A3 pays $1.00 if SP500 is between -2.51% and -7.5%	- 5%
Lottery ticket A4 pays $1.00 if SP500 is between -2.5% and +2.49%	0%
Lottery ticket A5 pays $1.00 if SP500 is between +2.5% and +7.49%	+ 5%
Lottery ticket A6 pays $1.00 if SP500 is between +7.5% and +12.49%	+10%
Lottery ticket A7 pays $1.00 if SP500 return is >+12.5%	Right tail

The institution could sell 100 million of each lottery ticket and pay off as promised.

Illustrative Example: Marginal Utility and Price/Probability Ratios. Arrow Securities as Building Blocks

State Prices vs. Probabilities for Various S&P500 Stock Returns and Real GDP Growth

Arrow Security	S&P 500 Return	State Description	S&P 500 Index	Projected Real GDP Growth	True Probability	State Price	"Risk Neutral Probability" Normalized State Price	Risk Neutral Prob/ True Probability State Price/ Probablility	Call Payoffs X=2600	Put Payoffs X=2400
A1	< -12.5%	Left Tail	2000	-2.5%	0.08	0.16	0.16	2.1	0	400
A2	-12.5% to -7.5%	-10.0%	2250	-1.0%	0.10	0.15	0.15	1.5	0	150
A3	-7.5% to -2.5%	-5.0%	2375	0.5%	0.12	0.14	0.14	1.2	0	25
A4	-2.5% to +2.5%	0.0%	2500	1.5%	0.20	0.18	0.19	0.9	0	0
A5	+2.5% to +7.5%	5.0%	2625	2.5%	0.25	0.20	0.21	0.8	25	0
A6	+7.5% to +12.5%	10.0%	2750	3.5%	0.15	0.09	0.09	0.6	150	0
A7	>12.5%	Right Tail	3000	4.5%	0.10	0.05	0.05	0.5	400	0
	Total =				1.00	0.97	1.00		$ 38.50	$ 90.00

Using Butterfly Spreads of Traded Equity Option Prices to Find Market-Implied Insurance or "State" Prices

1. Get Bloomberg's "implied volatilities by moneyness" to compute option prices for a cross-section of strike prices that are 80% to 120% of the current level of the SP500 and have maturities of 1 month to 24 months. Bloomberg's implied volatilities are estimated from many traded prices.

2. Compute the time series of costs of butterfly spreads of option prices and "risk neutral prices" per the Breeden-Litzenberger Method (1978, 2013).

3. Time series data covers 2005-2020, which covers (1) the Great Recession of 2008/2009, (2) the European Sovereign Debt Crisis of 2011/2012, (3) the China stock market crash in August 2015, (4) the UK Brexit vote in June 2016, (5) the Trump election and presidency from November 2016, including the increase in interest rates, the sizeable tax cut, the trade wars with China and Mexico and the Coronavirus Pandemic of 2020.

Bloomberg's Great Recession Calculations of Annualized Percentage Implied Volatilities by "Moneyness" from Option Prices on the S&P500.

Volatilities soar in 2008/early 2009, then fall back in late 2009 recovery.

Moneyness=S/X		Implied Volatilities for 1-Month Options					Implied Volatilities for 6-Month Options					Implied Volatilities for 12-Month Options				
SPX Index Date	SPX Price	80%	90%	100% ATM	110%	120%	80%	90%	100% ATM	110%	120%	80%	90%	100% ATM	110%	120%
12/29/2006	1418.3	26.0	19.3	10.1	9.3	9.2	20.8	17.2	13.3	10.1	9.4	19.4	16.6	14.0	11.8	10.7
6/29/2007	1503.4	27.3	23.2	15.0	10.9	10.9	22.3	19.1	15.5	12.4	10.7	20.8	18.3	15.8	13.6	11.8
12/31/2007	1468.4	29.8	27.1	20.6	14.5	14.1	29.5	26.2	22.6	19.1	16.2	27.9	25.0	22.2	19.7	17.3
3/31/2008	1322.7	34.7	29.7	23.7	18.1	16.6	27.9	27.5	24.0	20.9	20.7	29.0	26.3	23.8	21.4	19.2
6/30/2008	1280.0	33.9	28.9	22.4	17.5	16.6	28.7	25.9	22.4	19.2	16.8	27.8	24.9	22.3	19.9	17.9
9/30/2008	1166.4	44.0	43.0	36.8	31.1	30.4	34.8	31.6	28.5	25.7	23.2	31.9	29.4	27.0	24.8	22.7
10/31/2008	968.8	66.3	60.9	51.4	42.9	39.5	51.2	46.8	42.7	38.9	35.4	45.0	42.1	39.4	36.8	34.4
11/28/2008	896.2	64.1	57.6	50.2	43.7	41.5	52.2	48.4	44.8	41.5	38.4	46.8	44.1	41.6	39.2	37.0
12/31/2008	903.3	46.7	41.6	34.6	29.2	27.3	44.2	40.6	37.2	33.9	30.9	41.6	38.8	36.3	33.9	31.7
1/30/2009	825.9	54.5	47.4	39.6	33.6	31.1	45.5	41.6	38.1	34.8	31.9	42.7	39.8	37.1	34.6	32.4
2/27/2009	735.1	55.0	47.5	41.0	35.7	32.3	45.7	42.0	38.6	35.5	32.9	42.3	39.5	36.9	34.6	32.5
3/31/2009	797.9	52.1	44.8	38.7	34.6	33.7	44.8	41.5	38.5	35.8	33.4	41.6	39.1	36.9	34.8	33.0
4/30/2009	872.8	47.4	39.0	32.6	28.7	29.7	40.0	36.7	33.6	31.0	28.7	38.1	35.7	33.5	31.4	29.6
6/30/2009	919.3	40.2	30.6	23.0	18.7	18.8	33.8	29.8	26.2	23.1	20.7	32.3	29.4	26.8	24.4	22.4
12/31/2009	1115.1	30.4	26.7	17.0	16.2	18.0	29.7	25.5	21.7	18.7	16.8	28.4	25.5	22.8	20.4	18.5

Note: Bloomberg also publishes these for 3, 18 and 24 months to maturity.

Bloomberg's Coronavirus Pandemic Calculations of Annualized Percentage Implied Volatilities by "Moneyness" from Option Prices on the S&P500.
Volatilities soar in March, 2020, then fall back considerably by July 31, 2020.

Moneyness=S/X		Implied Volatilities for 1-Month Options					Implied Volatilities for 6-Month Options					Implied Volatilities for 12-Month Options				
SPX Index	SPX	80%	90%	100%	110%	120%	80%	90%	100%	110%	120%	80%	90%	100%	110%	120%
Date	Price			ATM					ATM					ATM		
9/28/2018	2914.0	33.6	21.3	8.7	11.9	11.9	23.3	18.0	12.6	9.2	11.5	21.7	18.0	14.3	10.8	10.0
12/31/2018	2506.9	39.1	29.8	22.4	17.7	20.6	28.0	24.3	20.4	17.1	15.9	25.0	22.2	19.4	16.9	15.5
3/29/2019	2834.4	30.9	20.4	11.7	9.2	9.2	23.3	19.0	14.3	10.3	11.3	21.8	18.7	15.1	11.7	10.7
12/31/2019	3230.8	32.4	22.2	11.1	11.6	12.7	23.6	19.4	14.1	10.4	11.2	22.4	19.3	15.6	11.9	10.9
1/31/2020	3225.5	35.8	25.0	16.1	12.3	17.2	24.7	20.4	15.3	10.8	10.9	22.4	19.4	15.9	12.1	10.7
2/28/2020	2954.2	54.7	47.5	37.1	23.6	24.0	31.7	27.9	23.4	17.2	13.8	26.7	23.9	20.7	16.8	13.7
3/9/2020	2746.6	65.3	58.1	49.4	37.7	32.1	42.4	38.7	33.7	28.2	22.7	34.5	31.6	27.9	24.0	20.2
3/12/2020	2480.6	83.9	77.0	68.8	59.3	41.7	52.2	48.4	44.1	39.0	32.9	41.4	38.4	35.2	31.4	27.6
3/16/2020	2386.1	92.5	85.7	77.7	67.2	52.2	59.6	54.9	50.1	44.2	37.4	46.1	42.5	38.7	34.7	30.1
3/31/2020	2584.6	67.6	57.3	45.4	32.5	30.6	44.3	39.6	34.4	28.8	23.6	36.6	33.1	29.7	26.5	25.7
4/30/2020	2912.4	49.0	39.8	28.2	20.7	25.7	39.3	34.4	28.9	23.2	19.1	34.5	30.9	26.6	22.3	18.9
5/29/2020	3044.3	43.2	32.8	22.0	17.7	23.0	35.7	30.7	25.1	19.1	16.2	31.5	27.5	23.4	19.1	16.1
6/30/2020	3100.3	44.0	34.3	24.3	20.1	24.9	36.2	31.5	26.1	20.1	17.6	32.4	28.6	24.4	19.9	16.8
7/31/2020	3271.1	40.0	29.5	19.3	15.8	20.6	34.5	29.6	24.0	18.7	17.2	30.9	26.9	22.8	18.7	16.6

Note: Bloomberg also publishes these for 3, 18 and 24 months to maturity.

Note in 2005-2006: Low price paid for left tail insurance. High right tail. Did not see risk.

In Financial Panic of 2008/9: Surge in left tail (downside) prices to hedge risk.

2011 Surge due to Europe Sovereign Debt Crisis. 2013 strong, so Bernanke Fed tapered.

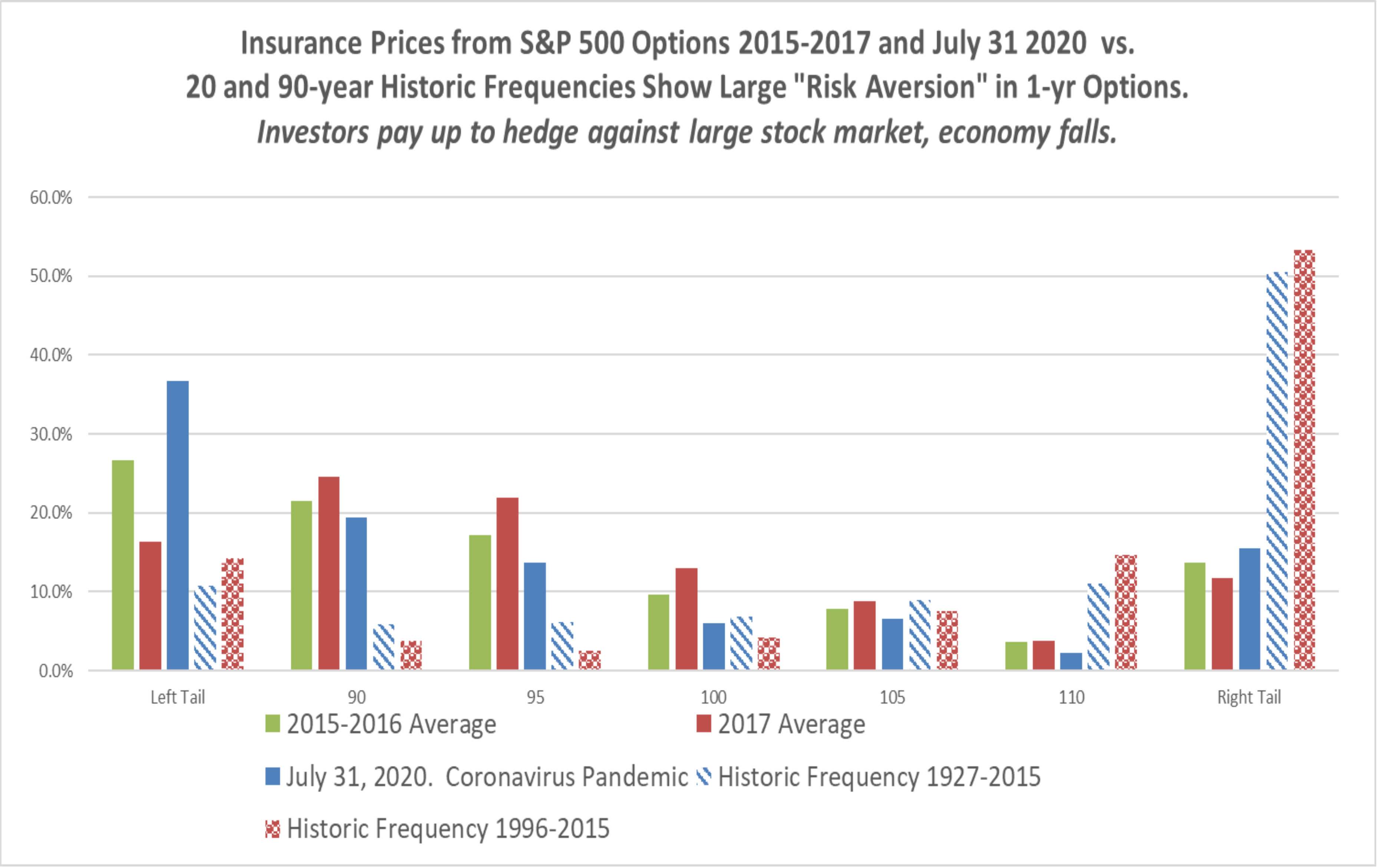
	S&P 500 Insurance Prices (Risk-neutral density) 2005-2013									12	Months
	Monthend Data from December 2004. Uses Breeden-Litzenberger (2014) technique									8/5/20 5:21 PM	
		$90%-$85 Puts				ATM		$110-$115 Calls			
	ATM	S&P 500	**Left Tail**	90	95	100	105	110	**Right Tail**	**Left Tail**	
Date	Implied σ	Spot Index	**Spread**	Butterfly	Butterfly	Butterfly	Butterfly	Butterfly	**Spread**	**-Right Tail**	
1/3/2005	14.8	1202.1	**14.0%**	11.8%	13.7%	13.8%	12.3%	8.4%	**26.0%**	**-12.0%**	
12/30/2005	14.3	1248.3	**11.3%**	15.4%	17.7%	15.5%	12.0%	7.8%	**20.3%**	**-9.0%**	
12/29/2006	14.0	1418.3	**10.1%**	15.9%	18.6%	15.6%	12.2%	7.7%	**19.9%**	**-9.9%**	
6/29/2007	15.8	1503.4	**14.9%**	15.8%	16.6%	13.2%	11.0%	6.9%	**21.7%**	**-6.8%**	
12/31/2007	22.2	1468.4	**32.7%**	13.1%	11.9%	8.5%	7.7%	5.0%	**21.1%**	**11.6%**	
3/31/2008	23.8	1322.7	**39.0%**	11.9%	10.6%	7.4%	6.9%	4.2%	**20.1%**	**18.9%**	
6/30/2008	22.3	1280.0	**34.6%**	13.1%	11.6%	8.5%	7.4%	4.8%	**20.0%**	**14.6%**	
9/30/2008	27.0	1166.4	**42.7%**	10.1%	9.1%	6.4%	6.3%	4.0%	**21.5%**	**21.2%**	
10/31/2008	39.4	968.8	**55.0%**	6.5%	6.1%	3.8%	4.5%	2.5%	**21.5%**	**33.5%**	
11/28/2008	41.6	896.2	**56.3%**	6.0%	5.7%	3.7%	4.2%	2.5%	**21.7%**	**34.6%**	
12/31/2008	36.3	903.3	**53.9%**	7.0%	6.5%	4.3%	4.7%	2.9%	**20.7%**	**33.2%**	
1/30/2009	37.1	825.9	**54.2%**	7.2%	6.4%	4.3%	4.6%	2.8%	**20.5%**	**33.7%**	
2/27/2009	36.9	735.1	**53.6%**	7.1%	6.4%	4.5%	4.6%	3.0%	**20.8%**	**32.8%**	
3/31/2009	36.9	797.9	**53.1%**	6.9%	6.3%	4.6%	4.6%	3.0%	**21.5%**	**31.6%**	
6/30/2009	26.8	919.3	**44.8%**	10.7%	9.2%	6.6%	6.0%	3.9%	**18.9%**	**25.9%**	
12/31/2009	22.8	1115.1	**38.6%**	13.0%	11.1%	8.1%	6.8%	4.3%	**18.1%**	**20.5%**	
12/31/2010	21.4	1257.6	**36.9%**	14.6%	12.2%	8.4%	7.1%	4.2%	**16.7%**	**20.2%**	
9/30/2011	30.8	1131.4	**50.4%**	9.4%	8.1%	5.3%	5.3%	3.0%	**18.4%**	**32.0%**	
12/30/2011	24.1	1257.6	**42.4%**	13.2%	10.8%	7.1%	6.3%	3.6%	**16.7%**	**25.8%**	
12/31/2012	18.7	1426.2	**31.9%**	17.1%	14.0%	10.0%	7.6%	4.2%	**15.2%**	**16.7%**	
12/31/2013	15.2	1848.4	**23.3%**	19.8%	17.3%	12.8%	8.7%	5.0%	**13.1%**	**10.2%**	

2015, August. China stock market crash dropped global markets.
2016 Brexit June 23, not much USA effect. Trump election calmed.
2017.Very low volatility. Sept Fed conference on Global Risk, Volatility.
2018. 4th Quarter: Trump wages China trade war, gov't shutdown. Tanks stocks.

	S&P 500 Insurance Prices (Risk-neutral density): 2014-2018								12	**Months**
8/5/20 5:39 PM		Monthend Data from December 2004. Uses Breeden-Litzenberger (2014) technique								
		$90%-$85 Puts				ATM		$110-$115 Calls		
Date	ATM Implied σ	S&P 500 Spot Index	**Left Tail Spread**	90 Butterfly	95 Butterfly	100 Butterfly	105 Butterfly	110 Butterfly	**Right Tail Spread**	**Left Tail -Right Tail**
12/31/2014	17.3	2,059	**27.0%**	21.0%	16.7%	9.8%	7.8%	3.9%	**13.8%**	**13.2%**
7/31/2015	15.3	2,104	**22.5%**	22.9%	18.9%	10.9%	8.3%	3.8%	**12.8%**	**9.7%**
8/31/2015	19.8	1,972	**34.0%**	18.1%	14.2%	7.9%	7.2%	3.3%	**15.3%**	**18.7%**
9/30/2015	20.2	1,920	**36.4%**	16.0%	13.1%	7.7%	7.2%	4.0%	**15.7%**	**20.8%**
12/31/2015	17.5	2,044	**27.5%**	21.1%	16.7%	8.9%	7.9%	3.6%	**14.3%**	**13.2%**
6/30/2016	17.0	2,099	**26.1%**	22.3%	17.5%	9.1%	7.9%	3.5%	**13.6%**	**12.5%**
10/31/2016	16.8	2,126	**25.2%**	22.6%	17.8%	9.1%	8.0%	3.5%	**13.9%**	**11.3%**
12/30/2016	16.4	2,239	**23.5%**	22.4%	18.1%	10.6%	8.1%	3.6%	**13.6%**	**9.9%**
6/30/2017	14.1	2,423	**17.4%**	24.3%	21.3%	12.4%	8.7%	3.9%	**12.0%**	**5.3%**
12/29/2017	13.6	2,674	**14.1%**	26.8%	23.8%	12.3%	8.6%	3.0%	**11.4%**	**2.6%**
1/29/2018	14.3	2,854	**18.0%**	23.8%	20.7%	13.0%	8.6%	3.8%	**12.1%**	**5.9%**
2/5/2018	20.0	2,649	**32.8%**	21.1%	15.4%	5.1%	7.2%	3.2%	**15.3%**	**17.5%**
2/28/2018	16.3	2,714	**22.8%**	23.5%	18.7%	10.2%	8.0%	3.9%	**13.0%**	**9.8%**
6/29/2018	15.8	2,718	**21.4%**	25.0%	19.9%	9.0%	8.2%	3.5%	**13.1%**	**8.3%**
9/28/2018	14.3	2,914	**16.6%**	25.5%	21.9%	12.1%	8.5%	3.4%	**11.8%**	**4.8%**
12/31/2018	19.4	2,507	**31.4%**	17.0%	13.8%	9.5%	7.6%	4.2%	**16.5%**	**14.9%**

2018 Q4: Trump wages China trade war. Gov't shutdown. Stocks tank.
2019: Some trade cooperation, stocks surge. Risk aversion, risk calms.
2020: March: Coronavirus pandemic, volatility, fear hit highest levels recorded.
2020: April-July, Huge stimulus, stocks strong. Rates very low. Vaccine hopes.
2020: July. Risk aversion calms from March 2020 extremes, but still very high.

	S&P 500 Insurance Prices (Risk-neutral density): 2018- 2020								**12**	**Months**
8/5/20 6:04 PM		Monthend Data from December 2004. Uses Breeden-Litzenberger (2014) technique								
		$90%-$85 Puts				ATM		$110-$115 Calls		
Date	ATM Implied σ	S&P 500 Spot Index	**Left Tail Spread**	90 Butterfly	95 Butterfly	100 Butterfly	105 Butterfly	110 Butterfly	**Right Tail Spread**	**Left Tail -Right Tail**
9/28/2018	14.3	2,914	**16.6%**	25.5%	21.9%	12.1%	8.5%	3.4%	**11.8%**	**4.8%**
12/31/2018	19.4	2,507	**31.4%**	17.0%	13.8%	9.5%	7.6%	4.2%	**16.5%**	**14.9%**
1/31/2019	16.4	2,704	**23.6%**	22.2%	18.0%	10.0%	8.2%	3.8%	**14.3%**	**9.3%**
4/30/2019	14.6	2,946	**17.1%**	25.6%	21.6%	11.8%	8.4%	3.2%	**12.3%**	**4.8%**
5/31/2019	16.9	2,752	**24.8%**	22.2%	17.6%	8.6%	8.1%	3.6%	**14.9%**	**9.9%**
6/28/2019	15.0	2,942	**18.4%**	25.1%	20.8%	11.0%	8.4%	3.3%	**13.1%**	**5.3%**
8/30/2019	17.3	2,926	**24.6%**	23.5%	18.0%	8.9%	7.8%	2.3%	**14.9%**	**9.7%**
12/31/2019	15.6	3,231	**19.6%**	25.7%	20.6%	9.8%	8.2%	2.4%	**13.7%**	**5.8%**
1/31/2020	15.9	3,226	**20.9%**	25.1%	20.0%	9.0%	8.2%	2.8%	**14.0%**	**7.0%**
2/28/2020	20.7	2,954	**35.8%**	18.6%	14.1%	4.1%	7.2%	3.0%	**17.2%**	**18.5%**
3/9/2020	27.9	2,747	**49.9%**	10.9%	9.4%	4.1%	5.8%	0.7%	**19.2%**	**30.7%**
3/12/2020	35.2	2,481	**56.9%**	7.8%	7.3%	0.8%	4.9%	1.7%	**20.7%**	**36.2%**
3/16/2020	38.7	2,386	**63.3%**	3.9%	5.8%	2.4%	4.5%	1.0%	**19.1%**	**44.1%**
3/31/2020	29.7	2,585	**40.9%**	16.6%	10.5%	5.3%	5.5%	2.5%	**18.6%**	**22.3%**
4/30/2020	26.6	2,912	**46.5%**	14.2%	10.8%	4.8%	5.9%	0.8%	**17.1%**	**29.4%**
5/29/2020	23.4	3,044	**40.1%**	17.9%	12.9%	4.8%	6.4%	2.2%	**15.7%**	**24.4%**
6/30/2020	24.4	3,100	**41.4%**	17.9%	12.7%	4.4%	6.2%	1.3%	**16.2%**	**25.2%**
7/31/2020	22.8	3,271	**36.7%**	19.4%	13.6%	5.9%	6.5%	2.3%	**15.5%**	**21.3%**


Insurance Prices from S&P 500 Options 2015-2017 and July 31 2020 vs.
20 and 90-year Historic Frequencies Show Large "Risk Aversion" in 1-yr Options.
Investors pay up to hedge against large stock market, economy falls.
60.0%
50.0%
40.0%
30.0%
20.0%
10.0%
0.0%
Left Tail
90
95
100
105
110
Right Tail
2015-2016 Average
2017 Average
July 31, 2020. Coronavirus Pandemic
Historic Frequency 1927-2015
Historic Frequency 1996-2015

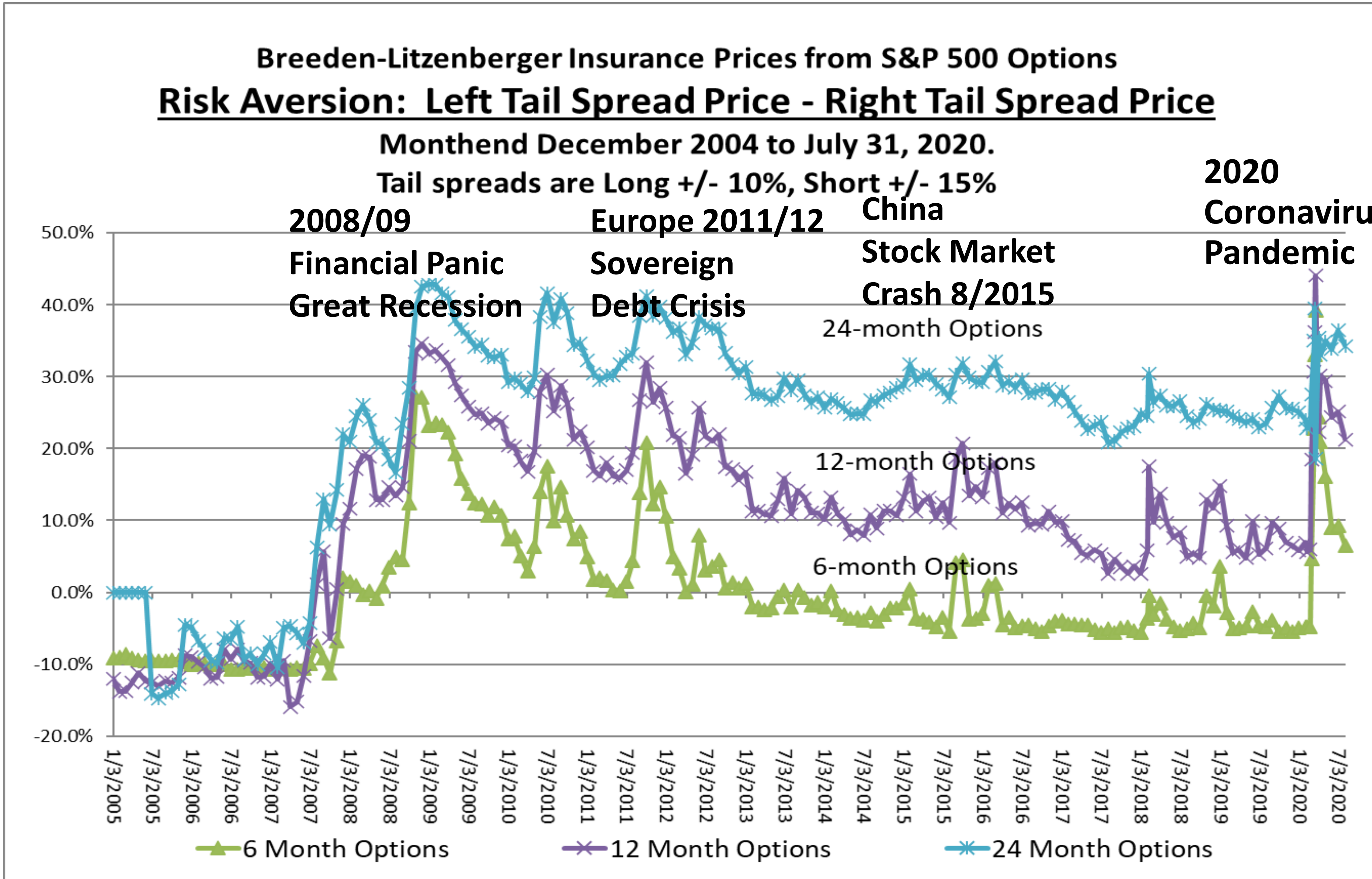
Risk Aversion Evident in Stock Market Insurance Costs from S&P 500 Options.

- In 2005-2006, Stock market insurance prices implicit in S&P500 options showed little risk aversion, as prices for "right tail" moves (stock prices up 12.5%+) were greater than for insurance against "left tail risks," falls of 12.5%

- From 2008-2020, prices of left tail insurance were higher than right tail upside bets, reflecting payment for hedges against sharp falls in stock prices and related poor economies. In extreme times such as the Great Recession, the Sovereign Debt Crisis and the China stock market crash, February 2018 correction, these price differentials were huge (e.g., 50%-20%=30%).

- Post August 2015 (China stock crash), risk aversion diminished and prices of downside tail risk dropped until a surge in Q4 2018, given Trump's USA-China trade war and the longest US gov't shutdown ever. And then in the first half of 2020, the Coronavirus Pandemic has taken risk aversion to some of the highest levels ever, similar to those in the Great Recession/Financial Panic of 2008/2009.

- Insurance prices for falling stock prices are substantially above those for rising stock prices, despite historical frequency distributions opposite.


Breeden-Litzenberger Insurance Prices from S&P 500 Options
Risk Aversion: Left Tail Spread Price - Right Tail Spread Price
Monthend December 2004 to July 31, 2020.
Tail spreads are Long +/- 10%, Short +/- 15%
2008/09 Financial Panic Great Recession
Europe 2011/12 Sovereign Debt Crisis
China Stock Market Crash 8/2015
2020 Coronavirus Pandemic
24-month Options
12-month Options
6-month Options
50.0%
40.0%
30.0%
20.0%
10.0%
0.0%
-10.0%
-20.0%
1/3/2005
7/3/2005
1/3/2006
7/3/2006
1/3/2007
7/3/2007
1/3/2008
7/3/2008
1/3/2009
7/3/2009
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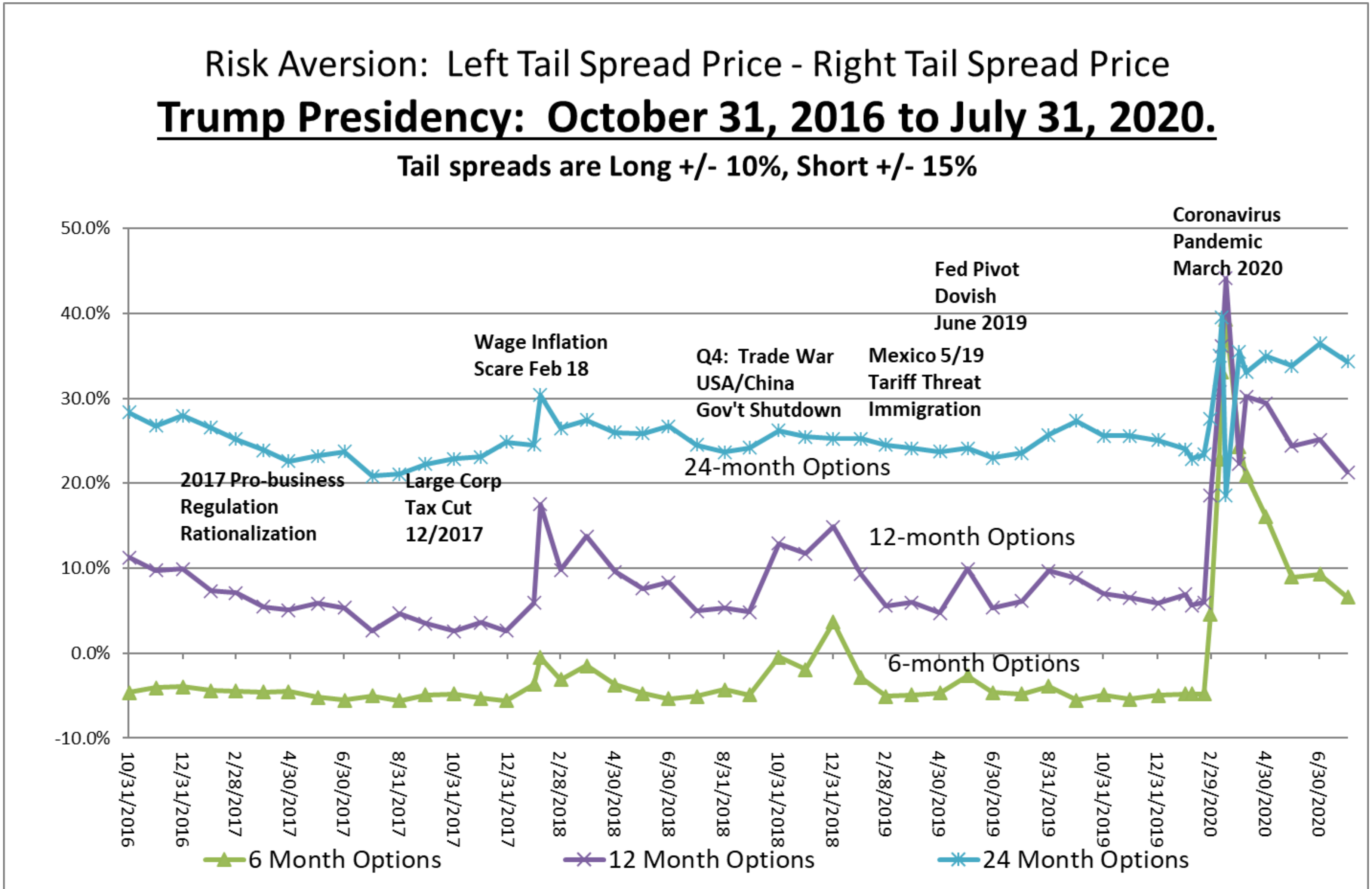
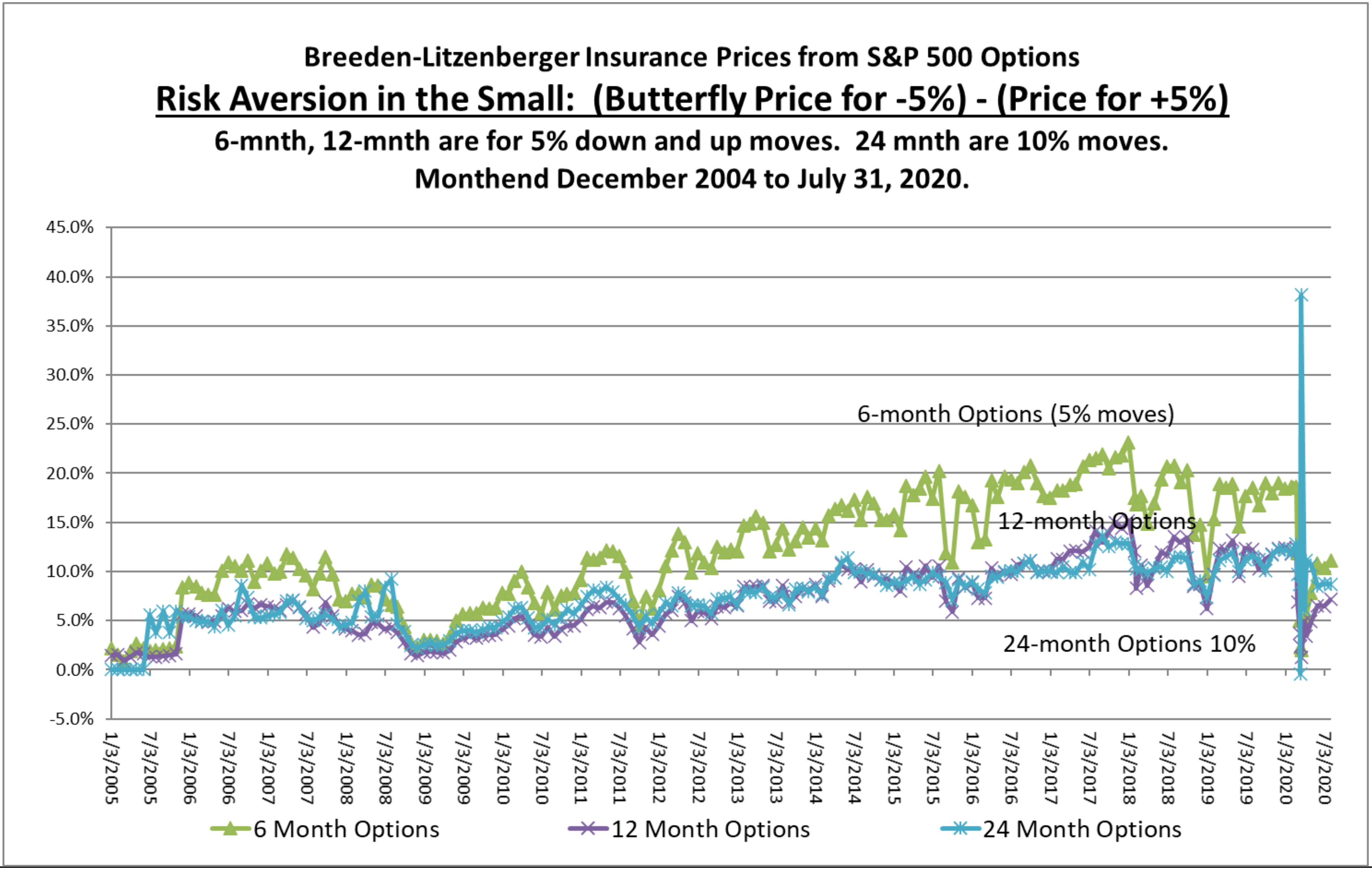
7/3/2020
6 Month Options
12 Month Options
24 Month Options



Risk Aversion: Left Tail Spread Price - Right Tail Spread Price
Trump Presidency: October 31, 2016 to July 31, 2020.
Tail spreads are Long +/- 10%, Short +/- 15%
50.0%
40.0%
30.0%
20.0%
10.0%
0.0%
-10.0%
Coronavirus
Pandemic
March 2020
Fed Pivot
Dovish
June 2019
Wage Inflation
Scare Feb 18
Q4: Trade War
USA/China
Gov't Shutdown
Mexico 5/19
Tariff Threat
Immigration
2017 Pro-business
Regulation
Rationalization
Large Corp
Tax Cut
12/2017
24-month Options
12-month Options
6-month Options
10/31/2016
12/31/2016
2/28/2017
4/30/2017
6/30/2017
8/31/2017
10/31/2017
12/31/2017
2/28/2018
4/30/2018
6/30/2018
8/31/2018
10/31/2018
12/31/2018
2/28/2019
4/30/2019
6/30/2019
8/31/2019
10/31/2019
12/31/2019
2/29/2020
4/30/2020
6/30/2020
6 Month Options
12 Month Options
24 Month Options



Breeden-Litzenberger Insurance Prices from S&P 500 Options
Risk Aversion in the Small: (Butterfly Price for -5%) - (Price for +5%)
6-mnth, 12-mnth are for 5% down and up moves. 24 mnth are 10% moves.
Monthend December 2004 to July 31, 2020.
45.0%
40.0%
35.0%
30.0%
25.0%
20.0%
15.0%
10.0%
5.0%
0.0%
-5.0%
6-month Options (5% moves)
12-month Options
24-month Options 10%
1/3/2005
7/3/2005
1/3/2006
7/3/2006
1/3/2007
7/3/2007
1/3/2008
7/3/2008
1/3/2009
7/3/2009
1/3/2010
7/3/2010
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1/3/2015
7/3/2015
1/3/2016
7/3/2016
1/3/2017
7/3/2017
1/3/2018
7/3/2018
1/3/2019
7/3/2019
1/3/2020
7/3/2020
6 Month Options
12 Month Options
24 Month Options

Does B-L Risk Aversion in Option Prices Forecast Future Stock Returns?

Campbell, Lettau-Ludvigson and others have shown that dividend yields forecast Future returns on stocks. High dividend yields precede high stock returns, as much as 7 years in advance. This makes some sense as high yields occur with low stock prices, which tend to be in recessions, when risks are high. So returns might well also be high.

Do the Breeden-Litzenberger risk aversion estimates predict future stock returns? Yes.
Do they do better or worse than dividend yield, one of the best predictors?
Mixed results. Bond options better short term, stock options worse.
Stock options better long-term, bond options similar to dividend yield.

Correlations of Forecast Variables with Future SP500 Stock Returns

Forecast Horizon	Dividend Yield Forecasts Shiller D/P' Overlapping corre	*Stock Options* Breeden-Litzenberger Stock Left-Rt Tail 2005-2019 Data	**Bond Options** Breeden-Litzenberger *Left Tail (R<1.5%) State Price* *LIBOR 3 Yr RND*	*LIBOR 5 Yr RND*	*LIBOR 8-10 Yr RND*
1 Year	37.6%	22.7%	43.5%	41.2%	38.8%
2 Year	51.2%	56.1%	61.8%	60.8%	60.5%
3 Year	49.7%	81.4%	73.5%	64.7%	67.1%
5 Year	70.4%	89.5%	65.4%	59.0%	68.6%
7 Year	64.2%	93.7%	75.4%	66.9%	72.4%

Does B-L Risk Aversion in Option Prices Forecast Future Stock Returns?

Do these Breeden-Litzenberger risk aversion estimates predict future stock returns? Yes.
Do they do better or worse than dividend yield, one of the best predictors?
Mixed results. Bond options better short term, stock options worse.
Stock options better long-term, bond options similar to dividend yield.

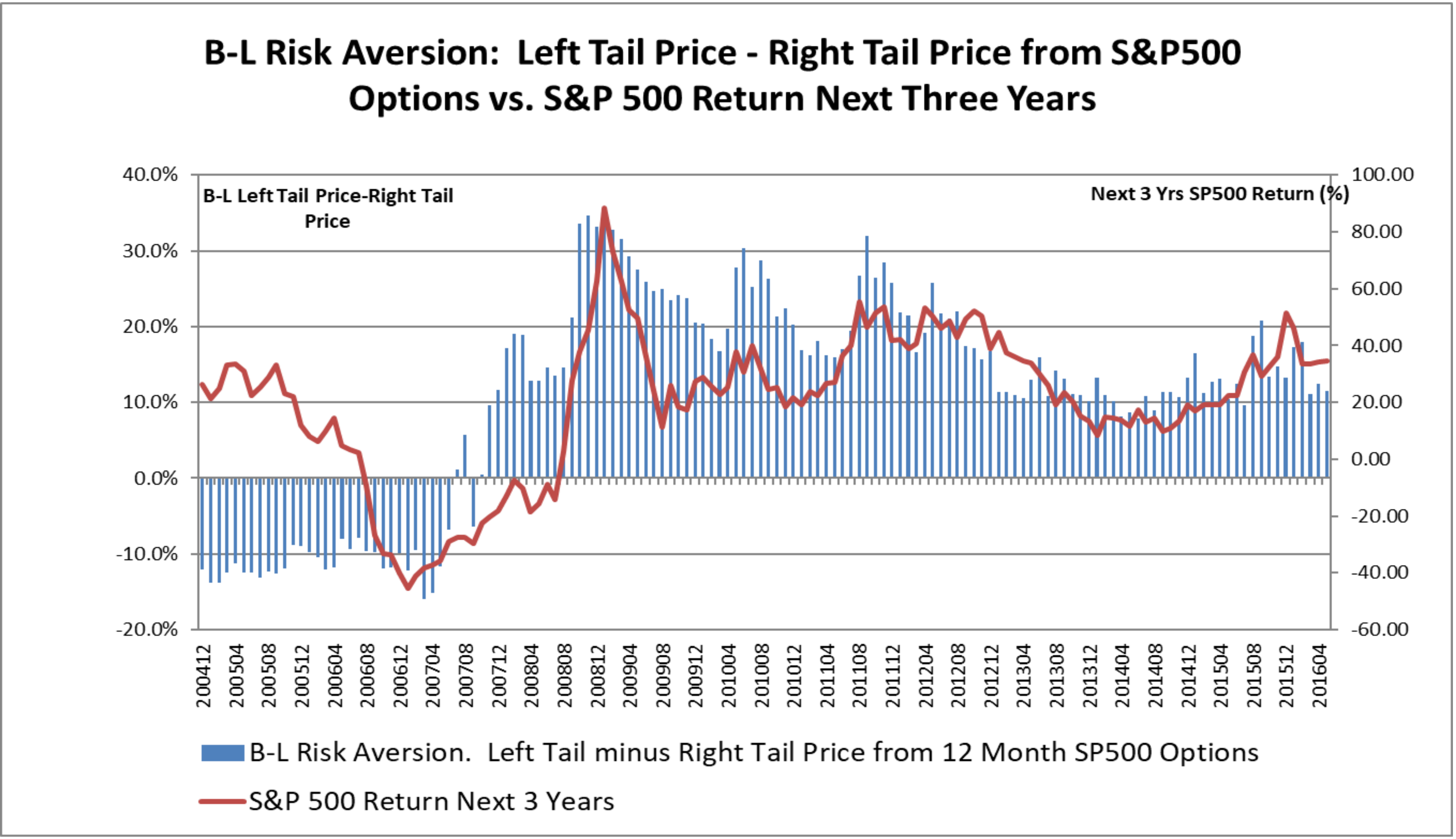
Due to the overlapping data of monthly rolling returns for long horizons, we compute t-statistics corrected for heteroscedasticity and autocorrelation (HAC). They show strong Performance of option-based state prices vs. S&P 500 dividend yield.

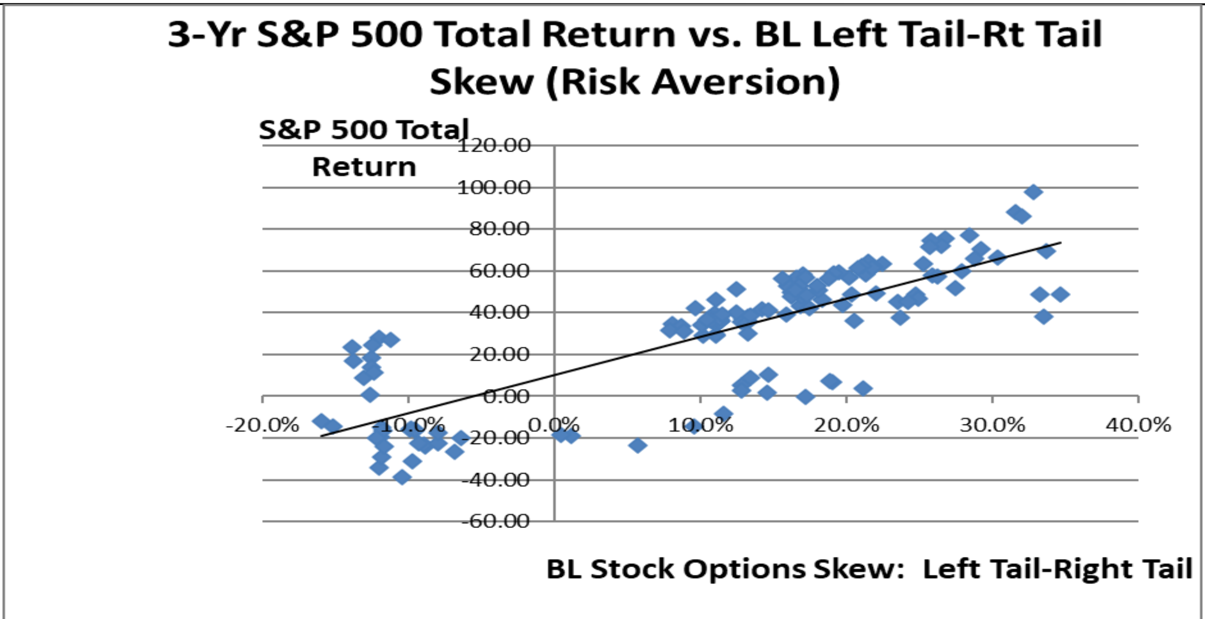
RSQ and t-Stats of Forecast Variables with Future SP500 Stock Returns

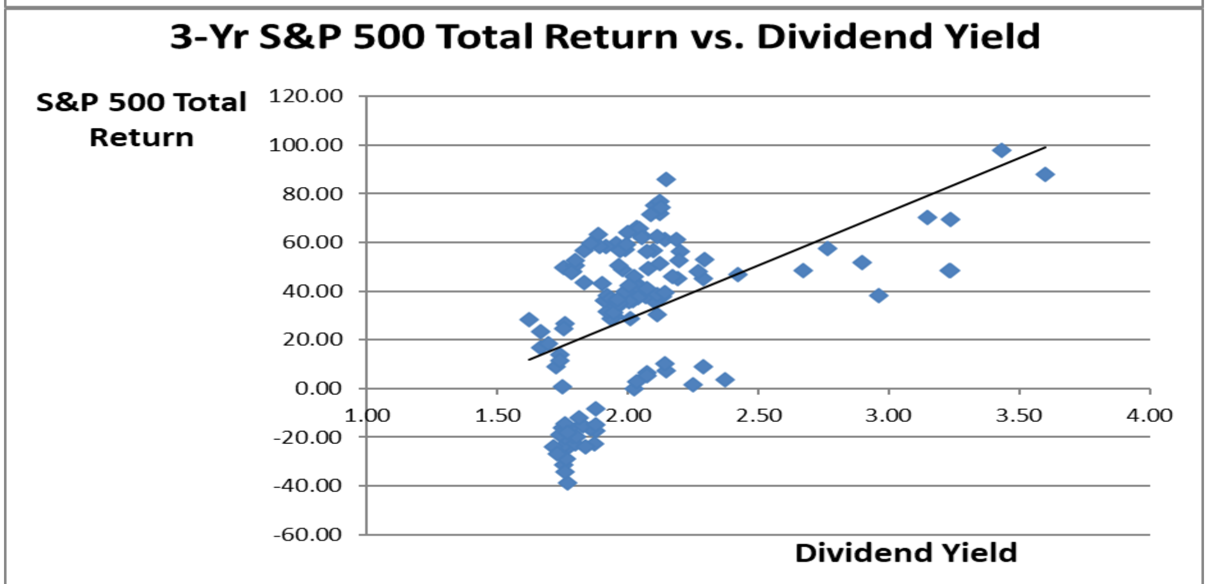
2005-2019 Data	Dividend		***Stock Options***				**Bond Options**			
	Yield		*Breeden-Litzenberger*		*Breeden-Litzenberger*					
Forecast	Forecasts				*Left Tail (R<1.5%) State Price*					
Horizon	Shiller D/P'		Stock Left-Rt Tail		*LIBOR*		*LIBOR*		*LIBOR*	
	RSQ	t(HAC)	RSQ	t(HAC)	*3 Yr RND*		*5 Yr RND*		*8-10 Yr RND*	
1 Year	12%	7.9	6%	2.1	22%	2.9	19%	2.5	17%	1.9
2 Year	24%	4.6	37%	3.1	42%	3.4	38%	3.2	36%	2.8
3 Year	24%	2.8	69%	7.2	54%	3.6	40%	2.9	39%	2.7
4 Year	30%	2.5	80%	13.3	50%	3.6	32%	2.3	31%	2.1
5 Year	44%	3.4	84%	25.7	47%	4.0	34%	2.9	33%	2.5
6 Year	48%	3.4	91%	26.1	44%	3.3	37%	2.8	49%	3.7

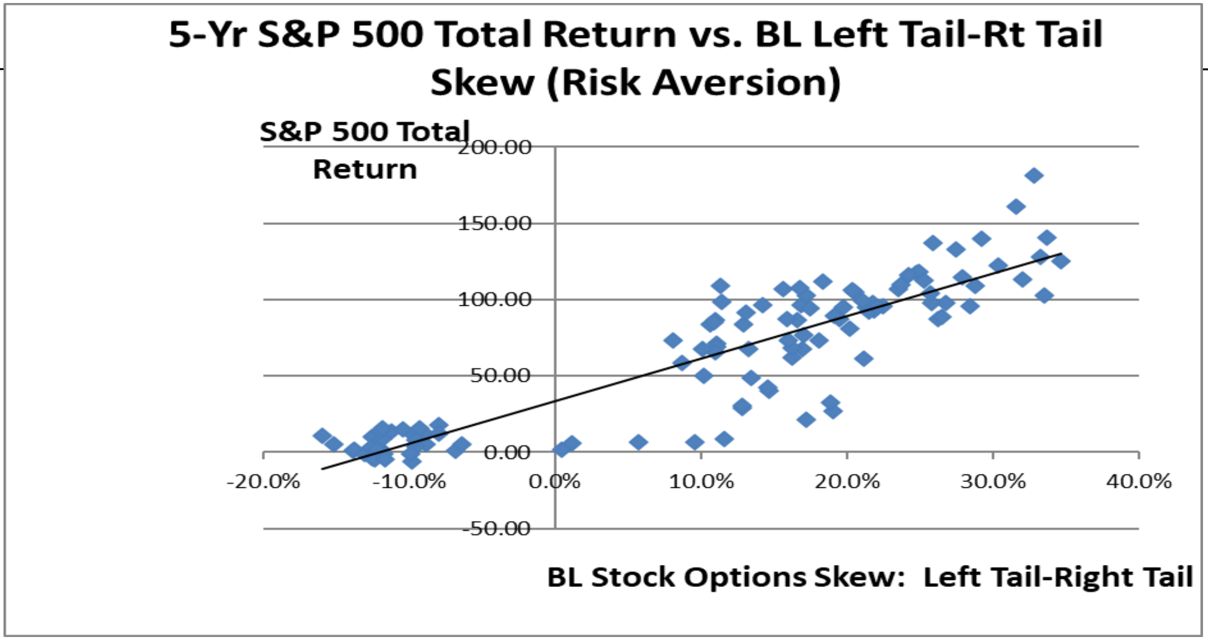
Preliminary Calculations. By Tingyan Jia, Stanford

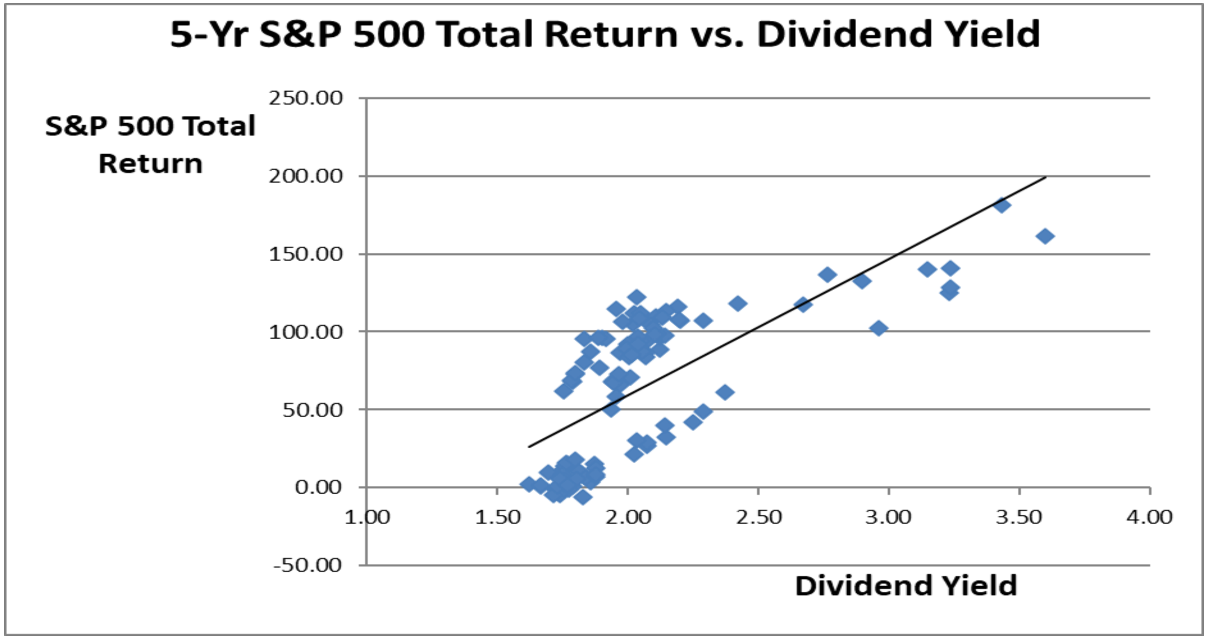
Graphs of B-L Stock Market "Risk Aversion" Estimates Vs. Future S&P 500 Stock Returns. Monthly data from 2004-2019.




3-Yr S&P 500 Total Return vs. BL Left Tail-Rt Tail Skew (Risk Aversion)
S&P 500 Total Return
120.00
100.00
80.00
60.00
40.00
20.00
0.00
-20.00
-40.00
-60.00
-20.0%
-10.0%
0.0%
10.0%
20.0%
30.0%
40.0%
BL Stock Options Skew: Left Tail-Right Tail


3-Yr S&P 500 Total Return vs. Dividend Yield
S&P 500 Total Return
120.00
100.00
80.00
60.00
40.00
20.00
0.00
-20.00
-40.00
-60.00
1.00
1.50
2.00
2.50
3.00
3.50
4.00
Dividend Yield


5-Yr S&P 500 Total Return vs. BL Left Tail-Rt Tail Skew (Risk Aversion)
S&P 500 Total Return
200.00
150.00
100.00
50.00
0.00
-50.00
-20.0%
-10.0%
0.0%
10.0%
20.0%
30.0%
40.0%
BL Stock Options Skew: Left Tail-Right Tail


5-Yr S&P 500 Total Return vs. Dividend Yield
S&P 500 Total Return
250.00
200.00
150.00
100.00
50.00
0.00
-50.00
1.00
1.50
2.00
2.50
3.00
3.50
4.00
Dividend Yield

Summary: Uses of Stock and Bond Insurance Prices from Options for Central Bank Policy Impacts and for Estimates of Risk Aversion that Forecast Stock Returns

- Using Breeden-Litzenberger butterfly spreads of time spreads of interest rate caps and floors gives **interest rate insurance prices.** These were shown to reflect major moves by the U.S. Fed, the European Central Bank and Bank of England in the Great Recession of 2008-2009, in the Sovereign Debt Crisis of 2011-2013 and in the Coronavirus Pandemic of 2020.

- Insurance prices implicit in **options on stock prices** show that prices paid for left tail risk (downside) vary considerably and increase substantially in times of higher risk and likely higher risk aversion. The spread between prices of downside tail risk protection and prices of large upside payoffs was shown to be a relatively good forecaster of future stock returns. Higher risk indicated by this spread is followed by higher returns, on average, which is sensible in equilibrium. For most horizons, this forecaster does better than dividend yield, using 2005-2019 data for options.

- The price of payoffs received if and only if interest rates are very low, 0% to 1.5%, is also shown to be a forecaster of future stock returns. Presumably, very low interest rates indicate great fears of recession or economic weakness. It is likely that risk aversion is higher than normal at those times, and lower with higher rates.