DUKE
THE FUQUA SCHOOL OF BUSINESS

Stock Market State Prices, B-L Skew and the Equity Risk Premium

Douglas T. Breeden*

November 3, 2023

Reference notes for UMass Amherst Isenberg School Talk at their hybrid conference on "Black-Merton-Scholes Option Pricing, a 50-Year Celebration and Looking Ahead."

***William W. Priest Professor of Finance and former Dean, Duke University Fuqua School of Business.** **Thanks to Robert Litzenberger, Robert Merton, John Cox, Stephen Ross and Robert Litterman for comments on earlier versions. Thanks to Jiwook Yoo, Jack Yan, Song Xiao, Tuo Yang, Gloria Zeng, Tingyan Jia and several prior RAs for their fine research assistance.**

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Developers of the Black-Scholes-Merton Option Pricing Formula

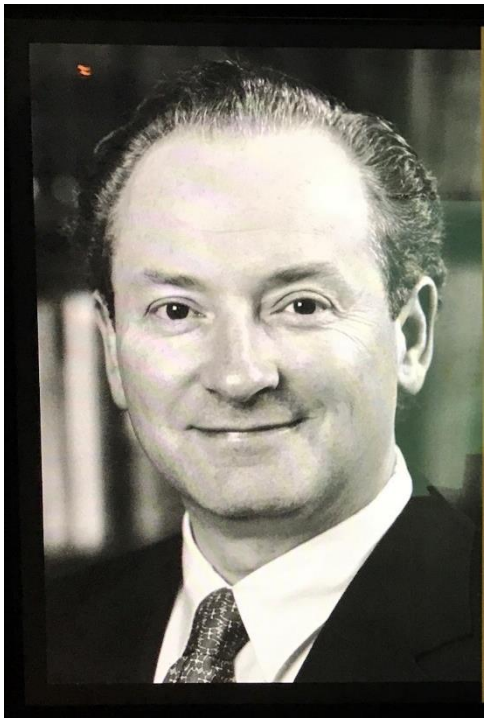
Nobel Laureates from MIT/Harvard (Merton) and MIT/Chicago/Stanford (Scholes)

Duke 20 Year Celebration 1993

Fischer Black
1938-1995

Nobel Museum Photos in Stockholm

Myron Scholes
1941-



Robert C. Merton
1944-

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1993 Duke Conference: 20 Years After Black-Scholes

Mark Rubinstein, Myron Scholes, Fischer Black, Robert Merton, Robert Whaley, Doug Breeden

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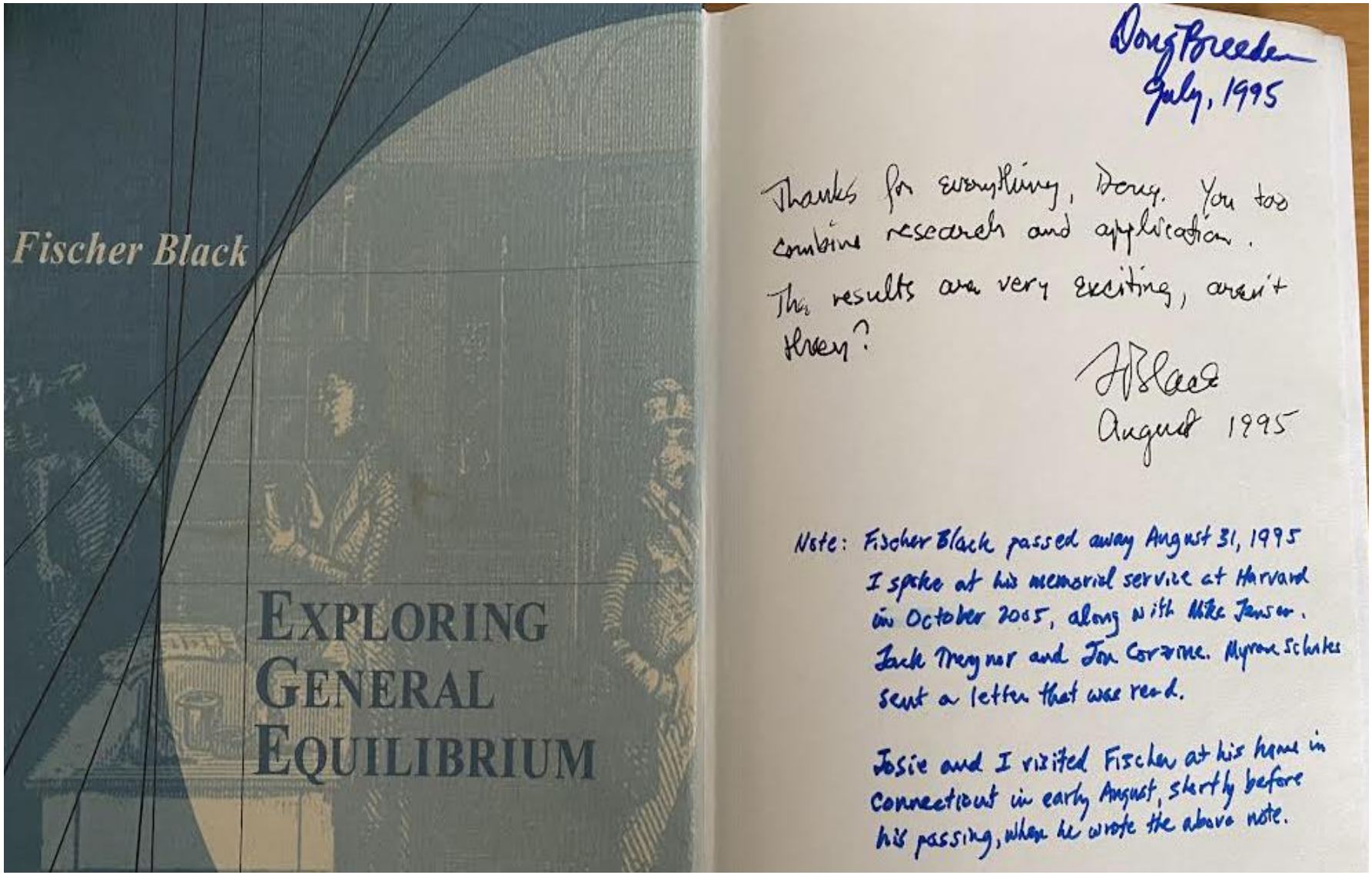
August 1995 Inscription and the Passing of Fischer Black



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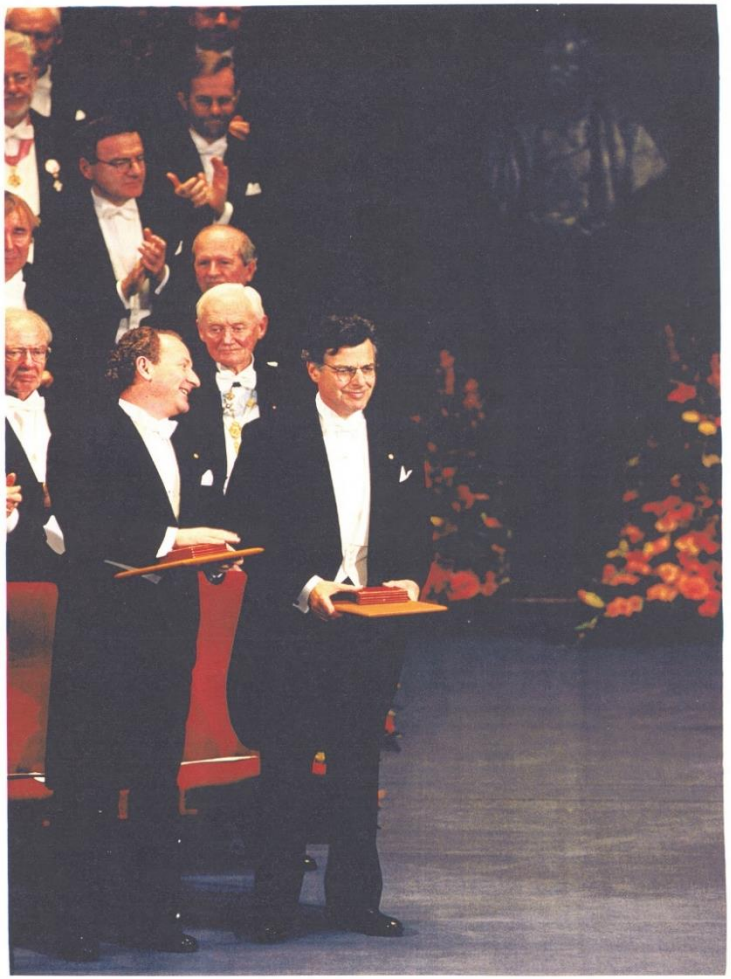
Attended 1997 Nobel Prize Ceremony in Stockholm, Sweden
Robert Merton and Myron Scholes (MIT/Harvard, MIT/Stanford)

Winning for their breakthroughs in option pricing/derivatives.
Breedens attended this, courtesy of Robert Merton, great friend and teacher



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Breeden party at the Kentucky Derby 1999. Fun in hard times!

Nobels Merton and Scholes and Stanford Prof. Mark Wolfson

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I. How to Find Interest Rate Insurance State Prices From Option Prices:

Ross (1976), Quarterly Journal of Economics article “Options and Efficiency”

Breeden and Litzenberger (1978) Journal of Business article, “Prices of State-Contingent Claims Implicit in Option Prices.”

Breeden and Litzenberger (2022) Journal of Fixed Income on “Central Bank Policy Impacts on the Distribution of Insurance Prices for Future Interest Rates: 2003-2022” gives the method for calculations in this talk. .

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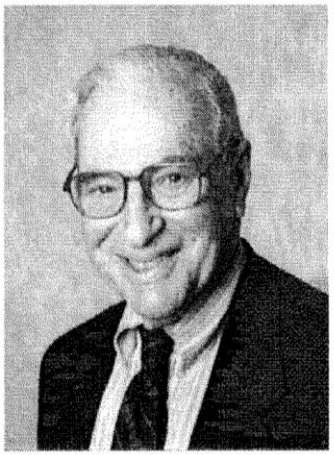
Developers of the Time-State Preference Model for Pricing More General Economic Risks Than CAPM

,Nobel Laureates from Harvard/Stanford (Arrow) and Berkeley (Debreu)

- Fundamental theoretical contributions to analysis of equilibrium under uncertainty by Kenneth Arrow and Gerard Debreu , e.g., "Existence of a competitive equilibrium for a competitive economy," 1954.



1921-2017

1921-2004

The Black-Scholes-Merton Option Pricing Model (1973) allowed Breeden and Litzenberger (1978) to implement the Arrow-Debreu time-state preference model and compute state prices.

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Breeden-Litzenberger Find Butterfly Spreads Create Insurance Payoffs

Journal of Business, October 1978

Call Option Payoffs

Asset Price	X=	2	3	4	5	6
1.00		0	0	0	0	0
2.00		0	0	0	0	0
3.00		1	0	0	0	0
4.00		2	1	0	0	0
5.00		3	2	1	0	0
6.00		4	3	2	1	0
7.00		5	4	3	2	1
8.00		6	5	4	3	2
9.00		7	6	5	4	3
10.00		8	7	6	5	4

Butterfly Spread Payoffs

Long 1 X=	2	3	4	5
Short 2 X=	3	4	5	6
Long 1 X=	4	5	6	7
	0	0	0	0
	0	0	0	0
	1	0	0	0
	0	1	0	0
	0	0	1	0
	0	0	0	1
	0	0	0	0

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More generally, B-L showed that 2nd derivatives of option pricing functions provide the pricing density.

Butterfly Spreads of Options and the State Price Density:

With Continuous Underlying Asset Price, but Discrete Exercise prices:

Butterfly spread: $\frac{[c(x-\Delta)-c(x)]-[c(x)-c(x+\Delta)]}{\Delta} = \frac{[c(x-\Delta)-2c(x)+c(x+\Delta)]}{\Delta}$

Values of derivative assets:

$$PV(f(\tilde{P})) = \int_{\tilde{P}} c_{xx}(x=\tilde{P}) \cdot f(\tilde{P}) d\tilde{P},$$

where $c(x,P)$ = price of European call option with exercise price x,

and c_{xx} is its second partial derivative with respect to x.

A similar formula holds with regard to European put formula, e.g.:

$$PV(f(\tilde{P})) = \int_{\tilde{P}} g_{xx}(x=P) \cdot f(P) dP.$$

These are pure arbitrage relations. Preferences and probabilities are reflected in c_{xx} and g_{xx}, but are not otherwise needed. Don't need homogeneous probability beliefs

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Douglas T. Breeden
University of Chicago

Robert H. Litzenberger
Stanford University

Prices of State-contingent Claims Implicit in Option Prices*

B-L derived that the price of $1.00 received if underlying price ends between Y_1 and Y_2 and the Black-Scholes formula holds is:

$$\Delta(Y_1, Y_2, T) = B(T)\{N[d_2(X = Y_1)] - N[d_2(X = Y_2)]\}. \qquad (7)$$

TABLE 2 **Values of the Cumulative Pricing Function and the Prices of Delta Securities: An Example***

Market Change in 1 Year (%)	1-Year Price Relatives (Y_1/M_0)	Cumulative Pricing Function [$G(Y_1/M_0)$]	Prices of Delta Securities [$\Delta(\ldots; t = 1)$]
−40	.6	93.7¢	
			3.0¢
−30	.7	90.7¢	
			9.0¢
−20	.8	81.7¢	
			15.7¢
−10	.9	66.0¢	
			18.9¢
0	1.0	47.1¢	
			17.3¢
+10	1.1	29.8¢	
			12.8¢
+20	1.2	17.1¢	
			8.1¢
+30	1.3	8.9¢	
			4.6¢
+40	1.4	4.3¢	
			2.3¢
+50	1.5	2.0¢	

* Parameters for this example are: $\sigma = .20$, $\delta = .04$, $r_1 = .06$, and $t = 1$ year.

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1978 Time-state prices for Arrow Securities Enabled by the Black-Scholes-Merton Formula

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Stanford University

Prices of State-contingent Claims Implicit in Option Prices*

TABLE 3 **Delta-Security Prices***

$\frac{Y_1}{M_0} - \frac{Y_2}{M_0}$	Time to Maturity									
	3 Mos.	6 Mos.	9 Mos.	1 Yr.	2 Yrs.	3 Yrs.	4 Yrs.	5 Yrs.	10 Yrs.	20 Yrs.
0–.1										.2¢
.1–.2									.3¢	.9
.2–.3							.1¢	.3¢	1.2	1.6
.3–.4					.1¢	.3¢	.7	1.2	2.5	1.9
.4–.5					.6	1.5	2.4	3.0	3.5	2.0
.5–.6			.2¢	.5¢	2.5	4.0	4.7	4.9	4.0	1.9
.6–.7		.6¢	1.7	3.0	6.1	6.8	6.7	6.4	4.2	1.8
.7–.8	1.2¢	5.0	7.6	9.0	9.9	9.0	8.0	7.1	4.2	1.7
.8–.9	13.1	16.6	16.5	15.7	12.4	10.1	8.5	7.3	4.0	1.6
.9–1.0	34.8	26.4	21.9	18.9	12.9	10.0	8.2	6.9	3.6	1.4
1.0–1.1	32.5	24.3	20.0	17.3	11.7	9.1	7.4	6.2	3.3	1.3
1.1–1.2	13.5	14.7	13.8	12.8	9.6	7.7	6.4	5.5	3.0	1.2
1.2–1.3	2.9	6.5	7.8	8.1	7.3	6.2	5.4	4.7	2.6	1.0
1.3–1.4	.4	2.2	3.7	4.6	5.3	4.9	4.4	3.9	2.3	.9
1.4–1.5		.6	1.5	2.3	3.7	3.7	3.5	3.2	2.0	.9
1.5–1.6				1.1	2.5	2.8	2.8	2.6	1.8	.8
1.6–1.7				.5	1.6	2.1	2.2	2.1	1.5	.7
1.7–1.8				.2	1.0	1.5	1.7	1.7	1.3	.6
1.9–2.0					.4	.8	1.0	1.1	1.0	.5
2.0–2.1					.2	.5	.8	.9	.9	.5
2.1–2.2					.1	.4	.6	.7	.8	.4
2.2–2.3					.1	.3	.4	.6	.7	.4
2.3–2.4					.1	.2	.3	.5	.6	.4
2.4–2.5						.1	.3	.4	.5	.3
2.5–2.6						.1	.2	.3	.5	.3
2.6–2.7						.1	.1	.2	.4	.3
2.7–2.8							.1	.2	.4	.3
2.8–2.9							.1	.1	.3	.2
2.9–3.0							.1	.1	.3	.2
3.0–3.1							.1	.1	.2	.2
3.1–3.2								.1	.2	.2
3.2–3.3								.1	.2	.2
3.3–3.4									.2	.2
3.4–3.5									.1	.2
3.5–3.6									.1	.1
3.6–3.7									.1	.1
3.7–3.8									.1	.1
3.8–3.9									.1	.1
3.9–4.0									.1	.1
4.0–4.1									.1	.1

* Assumptions for all maturities are: $r = .06$, $\delta = .04$, $\sigma = .20$.

Journal of Busine *State-contingent Claims in Option Prices*

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Quantifying the Nightmare Scenarios

JUSTIN WOLFERS
03/02/2009 | 12:13 pm

Dartmouth's Eric Zitzewitz is one of my favorite co-authors, and a whiz at tracking financial markets. And when he mentioned to me last week that a close look at the options markets told an interesting tale of fear, I asked him to share his observations. Here goes.

Quantifying the Nightmare Scenarios
By Eric Zitzewitz
A Guest Post

There's no shortage of fear about the economy. But just how fearful should we be? Perhaps financial markets can provide some guidance.

There's a neat mathematical trick, by which we can use option prices to quantify the probability of the stock market falling by various amounts. Breeden and Litzenberger (1978) show that by comparing the prices of options at adjacent strike prices, you can calculate the approximate value of securities that would pay $1 if the underlying stock traded in a certain range on expiry day.

> "Option prices suggest that there is a very real chance of, dare I write it, another Great Depression."

(Economists know these as Arrow-Debreu securities; they approximate what option traders call "butterfly spreads" when strike prices are close together.)

For example, using last Friday's options prices, we can calculate that it would cost 10 cents to buy a portfolio of options that pays $1 if the S&P 500 falls below 250 on December 18, 2010. If markets were risk-neutral (I'll come back to this), we could infer that the market thought there was a 10 percent probability that the value of U.S. stocks could fall to one-third their current value by the end of next year. Such a drop would leave the index down to one-sixth of its peak level in late 2007. By way of comparison, in the Great Depression the value of stocks fell to between one-sixth and one-seventh of their earlier values.

There's an important caveat to all this. Even when the market price of a bundle of options paying $1 if "the S&P 500 is below 250 in December 2010" costs 10 cents, we cannot infer that there's a 10 percent chance of this happening. Since this security would help hedge against extreme wealth losses, investors should be willing to pay an insurance premium. Furthermore, the investors buying these securities could be panicking and overpaying for them, and the more sanguine may be unable to offset this fear if their money is tied up elsewhere.

Regardless of whether it reflects risk aversion, panic, or a true probability, the 10 cent price being paid for a dollar of Depression insurance highlights the fears that are holding stocks down. Policymakers have been trying to reassure investors that they understand the risks of depression and will do what is needed to avoid them. These graphs provide a measure of how far they have to go in convincing us.

TAGS: depression, financial crisis, guest posts, recession, stock

Freakonomics article
March 2, 2009 (market bottom)
Uses Breeden-Litzenberger Method
To find market's insurance prices for
Possible falls and increases in the
S&P500 Stock Price Index

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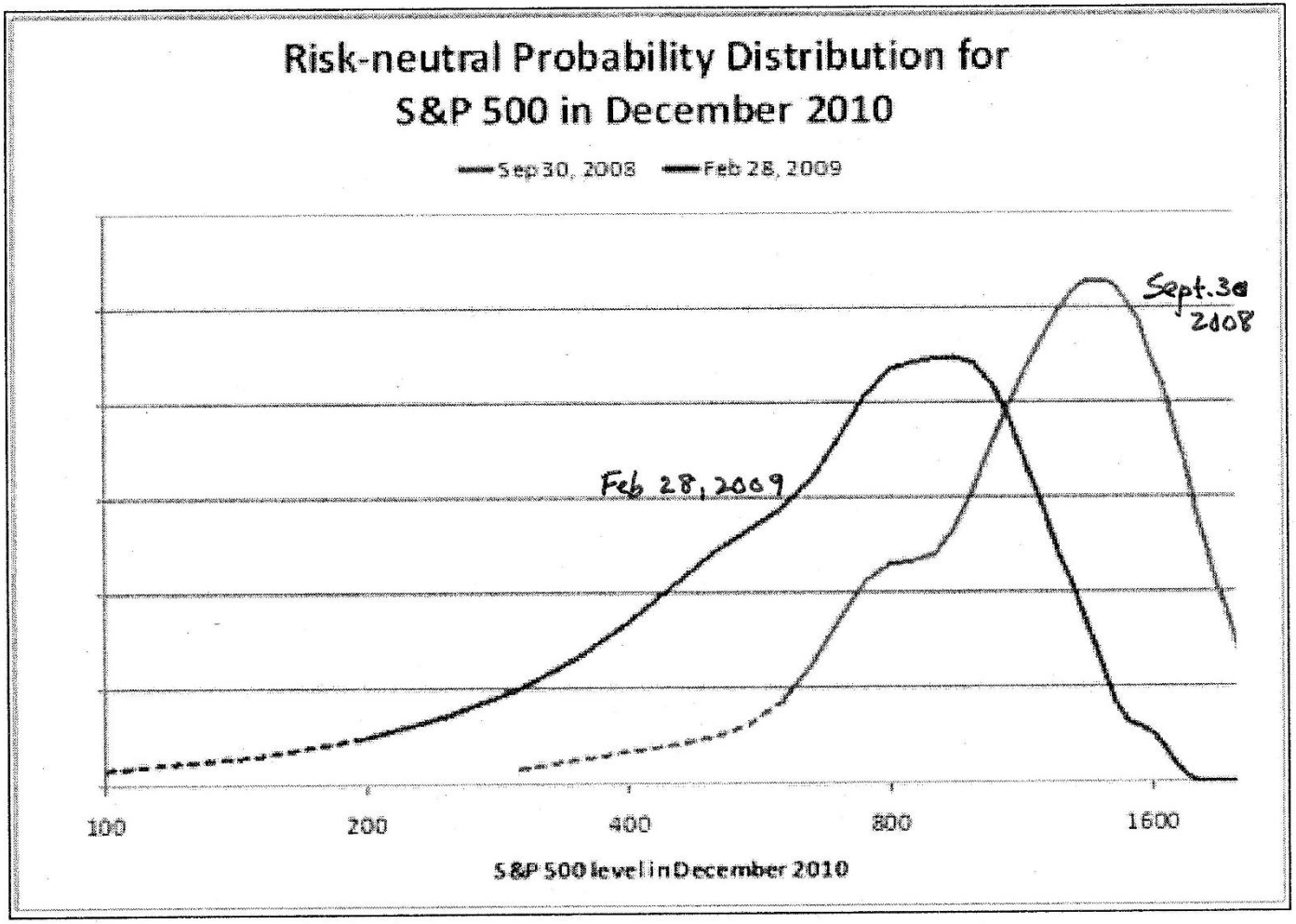
Freakonomics article: "Quantifying the Nightmare Scenarios"

Eric Zitzewitz (Dartmouth) Uses Breeden-Litzenberger 1978 Technique
In ***Freakonomics*** Blog by Justin Wolfers, March 2, 2009



9/30/2008:
S&P500= 1166
VIX = 39.4%

2/28/2009
S&P500 = 735
VIX = 46.4%

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The best securities market for pricing risk. Arrow's Complete Market

Underlying	Arrow's Dream Securities, a "Complete Market"													
Stock											Creating Portfolio Positiions			
Price	*Arrow Securities that pay $1 in one state of the world, 0 elsewhere*										Call	Call	Put	Put
SP&P 500	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	X=4	X=6	X=4	X=6
1	1	0	0	0	0	0	0	0	0	0	0	0	3	5
2	0	1	0	0	0	0	0	0	0	0	0	0	2	4
3	0	0	1	0	0	0	0	0	0	0	0	0	1	3
4	0	0	0	1	0	0	0	0	0	0	0	0	0	2
5	0	0	0	0	1	0	0	0	0	0	1	0	0	1
6	0	0	0	0	0	1	0	0	0	0	2	0	0	0
7	0	0	0	0	0	0	1	0	0	0	3	1	0	0
8	0	0	0	0	0	0	0	1	0	0	4	2	0	0
9	0	0	0	0	0	0	0	0	1	0	5	3	0	0
10	0	0	0	0	0	0	0	0	0	1	6	4	0	0

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Arrow Securities for S&P 500 States

Table of Payoffs on the Lottery Tickets for Different Returns on the S&P500 in 1 Year

				Lottery Ticket Payoffs for the S&P500 Return in 1 Year ($)							
S&P 500	State	S&P 500	True	Ticket ID							A1-A7
Return	Description	Index	Probability	A1	A2	A3	A4	A5	A6	A7	Portfolio
< -12.5%	Left Tail	2000	0.08	1	0	0	0	0	0	0	1
-12.5% to -7.5%	-10.0%	2250	0.10	0	1	0	0	0	0	0	1
-7.5% to -2.5%	-5.0%	2375	0.12	0	0	1	0	0	0	0	1
-2.5% to +2.5%	0.0%	2500	0.20	0	0	0	1	0	0	0	1
+2.5% to +7.5%	5.0%	2625	0.25	0	0	0	0	1	0	0	1
+7.5% to +12.5%	10.0%	2750	0.15	0	0	0	0	0	1	0	1
>12.5%	Right Tail	3000	0.10	0	0	0	0	0	0	1	1
Total =			1.00								

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True Probabilities vs.
State Prices or "Risk Neutral Probabilities"

State prices or "risk neutral probabilities" differ from true, objective probabilities, because investors price assets higher for those that pay off most when times are bad (negative beta). Thus, their insurance prices (risk neutral probabilities discounted) exceed their true probabilities.

States that correspond to good economies will have lower insurance prices, and their insurance prices will underestimate the true probabilities.

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II. Central Bank Policy Impacts on the Distribution of State Prices for Future Interest Rates

Source: Breeden and Litzenberger,

September 2022, 30th Anniversary Edition,

Journal of Fixed Income

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While I was "sleeping" (Business, Dean 1992-2007) …

Breeden-Litzenberger Method (1978) was used by Central Banks to find price distributions from option prices.

Probability distributions of future asset prices implied by option prices

1996 Bank of England Quarterly

By Bhupinder Bahra of the Bank's Monetary Instruments and Markets Division.

Introduction

Many monetary authorities routinely use the forward-looking information that is embedded in financial asset prices to help in formulating and implementing monetary policy. For example, they typically look at changes in the forward rate curve implied by government bond prices to assess changes in market perceptions of future short-term interest rates.(1) But, although implied forward rates are informative about the market's mean expectation for future interest rates, they tell us nothing about the range of expected outcomes around such estimates. For this, we can turn to options markets.

exercising it only if the price of the underlying asset lay above the strike price at that time.

Consider a set of European options on the same underlying asset, with the same time-to-maturity, but with different exercise prices. The prices of such options are related to the probabilities attached by the market to the possible values of the underlying security on the maturity date of the options. Intuitively, this can be seen by noting that the difference in the price of two options with adjacent exercise prices will reflect the value attached to the ability to exercise the options when the price of the underlying asset lies between the two exercise prices. This price difference in turn depends on the probability of the underlying asset price

The Breeden and Litzenberger approach

Breeden and Litzenberger (1978) derived a relationship linking the curvature of the call pricing function to the terminal RND function of the price of the underlying asset. In particular, they showed that the second partial derivative of the call pricing function with respect to the exercise price is directly proportional to the terminal RND function. Details about the derivation of the Breeden and Litzenberger result are given in Bahra (1996). The rest of this article focuses on how this result can be applied in order to estimate market RND functions for short-term interest rates in the future and how such RND functions can be used for policy analysis.

FEDERAL RESERVE BANK OF MINNEAPOLIS

BANKING AND POLICY STUDIES

Methodology for Estimating Risk Neutral Probability Density Functions

We estimate risk neutral probability density functions (RNPDs) for a variety of different asset classes using a variation of the technique developed by Shimko (1993). This procedure involves fitting a curve to the implied volatilities of a series of options and expressing the volatility as a function of the strike price. The implied volatilities are then translated into continuous call option prices, and the risk neutral distribution of the underlying asset is obtained through the Breeden-Litzenberger (1978) method.

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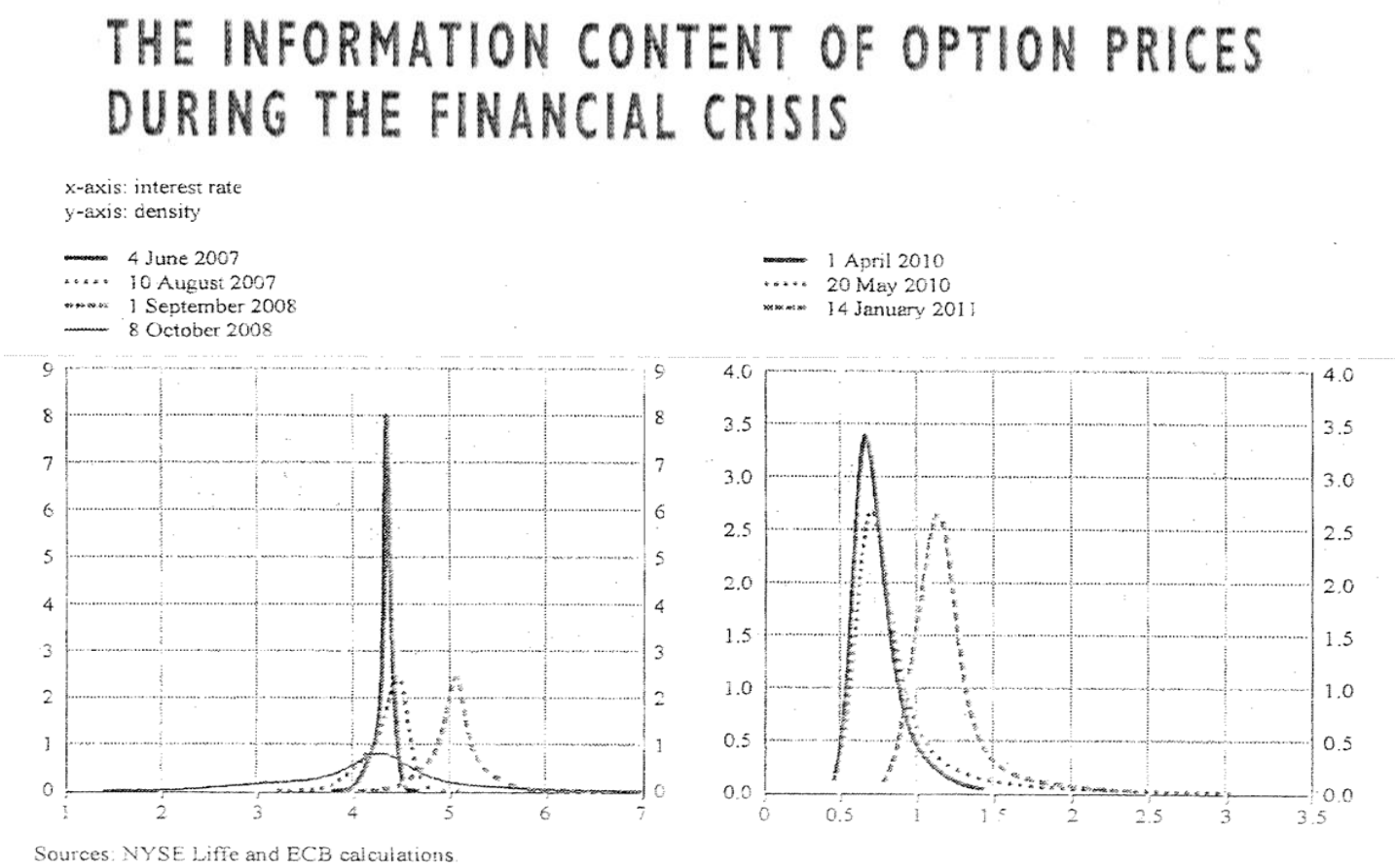
European Central Bank's Monthly Bulletin, February 2011, used the Breeden-Litzenberger 1978 method to estimate interest rate distributions for what Euribor will be in 3 Months:



Key Disadvantages of Many Approaches. Our Approach.

- 1. ***Short-term option prices used***.

Most options mature in 3 months to 18 months, as many markets only have active markets for those maturities. Often there are not options actively traded for a large number of standardized strike prices. We use interest rate caps and floors that have longer term maturities from 2 to 10 years.

- 2. ***Parametric vs. nonparametric approach.***

Applications often parameterize option prices with 3 or 4 parameters (mean, variance, skewness, kurtosis) and estimate implied volatility surfaces and entire risk-neutral densities. It is well-known among practitioners that these methods can be off significantly in estimating tail risks. For interest rate options, we use Bloomberg's volatility cube estimates of cap and floor prices, which are smoothly fitted from daily option market prices and give sensible insurance price distributions. In our approach for S&P 500 options, we use (nonparametric) traded option prices from Bloomberg, which give implied volatility smiles, smirks and skews that may be of any shape.

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Central Bank Policy Impacts on the Distribution of State Prices for Future Interest Rates, 2003–2022

Douglas T. Breeden and Robert H. Litzenberger

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KEY FINDINGS

- Identified combinations of butterfly spreads and tail spreads of interest rate caplets and floorlets can be used to extract discrete, nonparametric state prices ("risk neutral densities") from prices of interest rate caps and floors. A state's price reflects both the state's probability and the relative expected marginal utility of consumption in the state.
- The method presented shows that central banks in the USA, the Euro zone and the UK have dramatically affected not just levels of rates, but entire state price distributions. At major interventions, distributions have shifted from symmetry to skewness, and then back to relatively uniform distributions.
- Sign changes in correlations of interest rates with stock market moves do occur. They predictably affect the biases in using implied state prices to estimate objective probability distributions for interest rates. Biases should depend upon the consumption betas of the various states' prices.

ABSTRACT

In this article, we extend the 1978 Breeden–Litzenberger method of extracting state prices from option prices, showing how portfolios of butterfly spreads can be combined with right and left tail spreads to nonparametrically extract discrete state prices from option prices. We derive how those state prices should be biased estimates of true, objective probabilities. For interest rate options, we show that the biases can vary predictably over time (sometimes too high, sometimes too low), as the correlation of interest rates with consumption and wealth has changed signs over time. Consumption betas and proper risk premiums on bonds and of their state prices are at times predictably positive and at times predictably negative. We apply our technique to provide a brief 20-year history of central bank intervention impacts in the US, UK, and Eurozone from 2003 to 2022. Movements in state prices are quite large in the Financial Panic of 2008–2009, as well as in the European Sovereign Debt Crisis of 2010–2013, with Brexit and the Trump elections in 2016, and with the coronavirus pandemic in 2020–2021. Tapering in 2013 and 2022 and liftoffs in rates in 2015 and 2022 were shown to strongly shift state price distributions back toward the symmetry of 2003–2007. We show that central banks dramatically impacted entire state price distributions, not just levels of rates.

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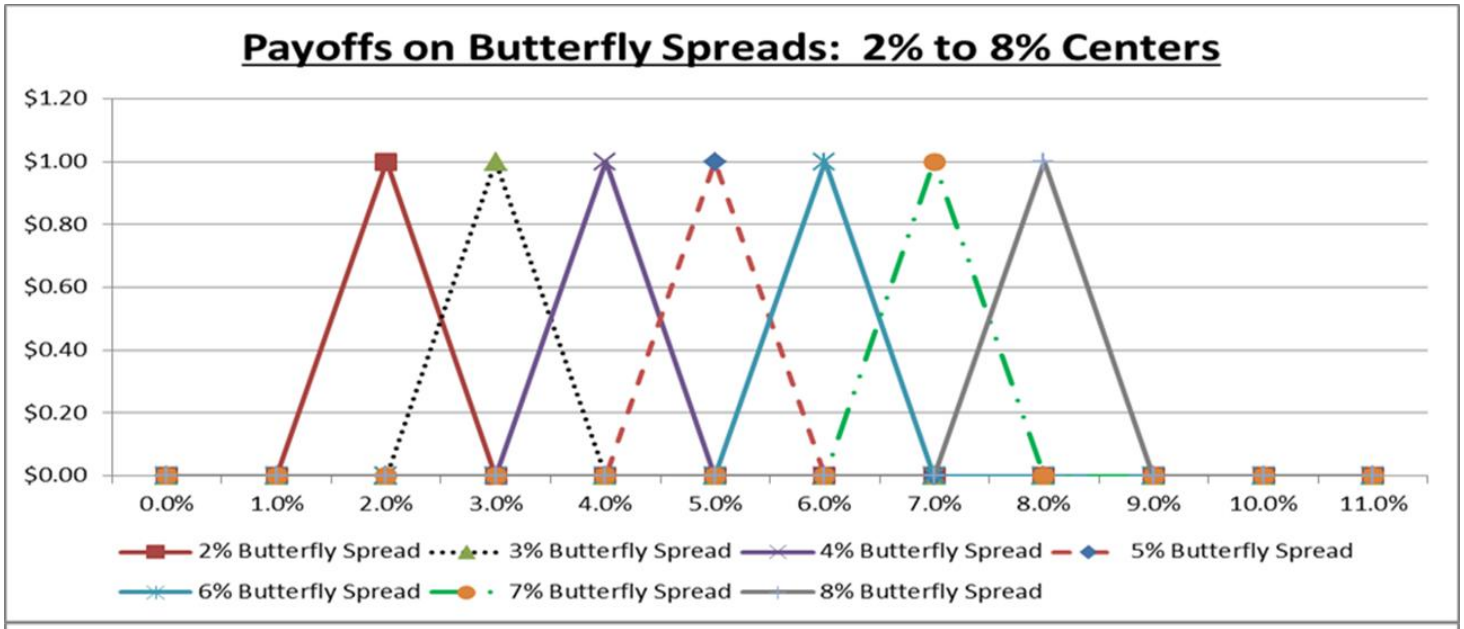

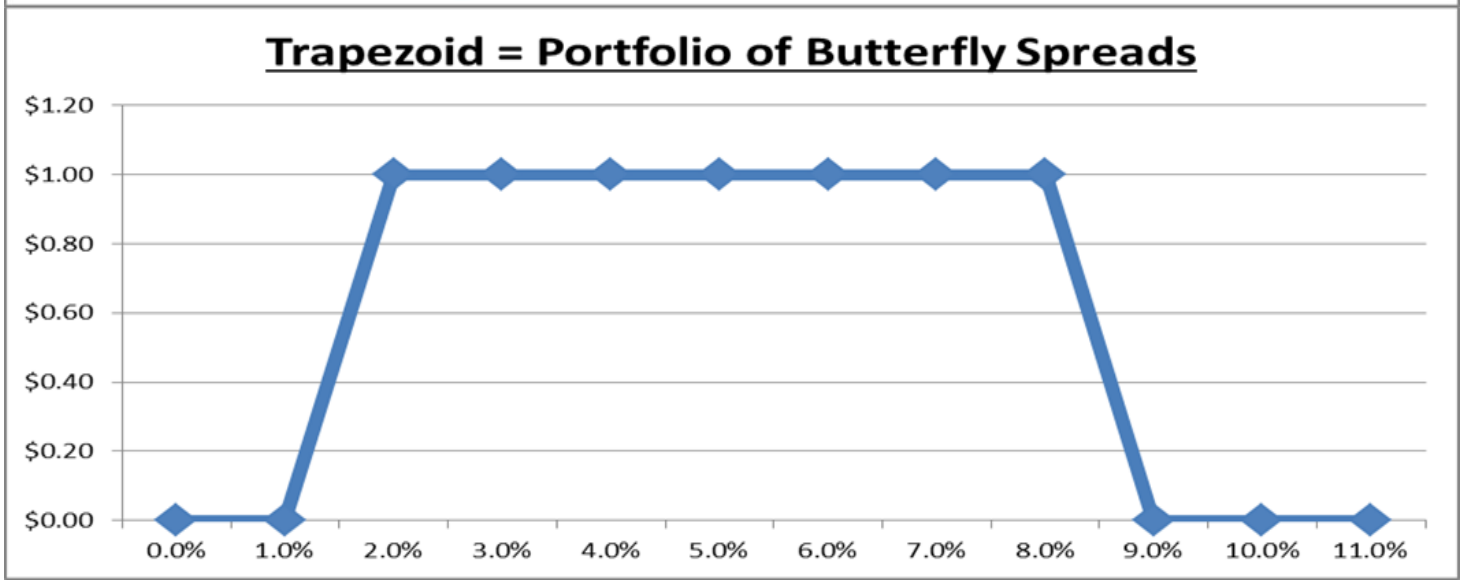

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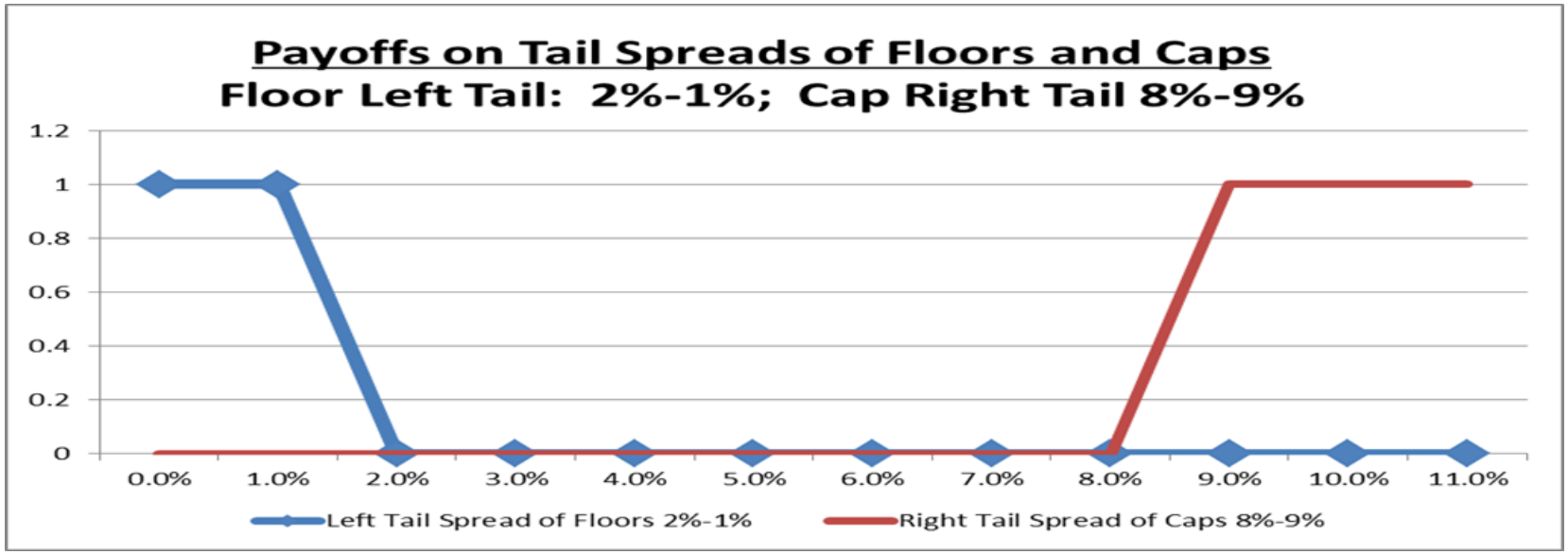

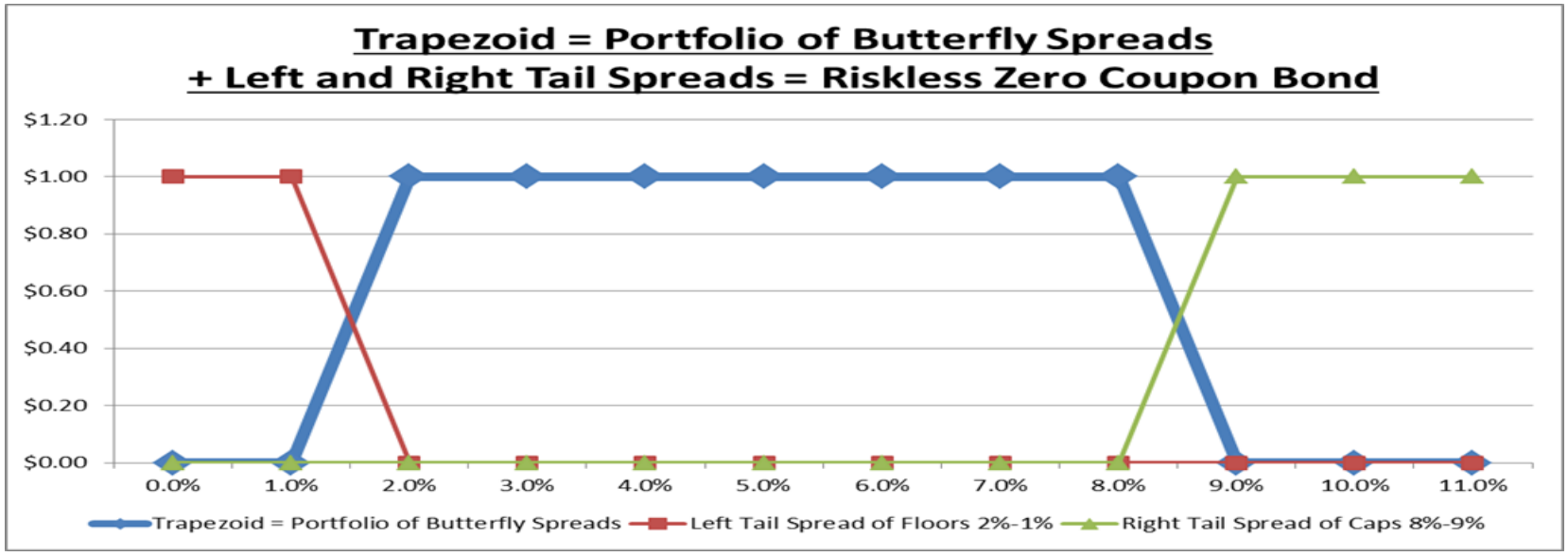

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Butterfly Spread and Tail Spread Costs and Risk Neutral Probabilites (Insurance Prices)

	Spread Cost	"Risk-Neutral Probability"
"0%" = Left tail spread: Long 1%, Short 0% floorlet	$0.290	0.297
1% Butterfly spread (Long 0%, Short 2 1%, Long 2%)	$0.320	0.328
2% Butterfly spread (Long 1%, Short 2 2%, Long 3%)	$0.180	0.184
3% Butterfly spread	$0.080	0.082
4% Butterfly spread	$0.037	0.038
5% Butterfly spread	$0.028	0.028
6% Butterfly spread	$0.014	0.014
7% Butterfly spread	$0.007	0.007
8% Butterfly spread	$0.007	0.007
9%+ = Right tail spread: Long 8%, Short 9% caplet	$0.015	0.015
Totals	$0.977	1.000

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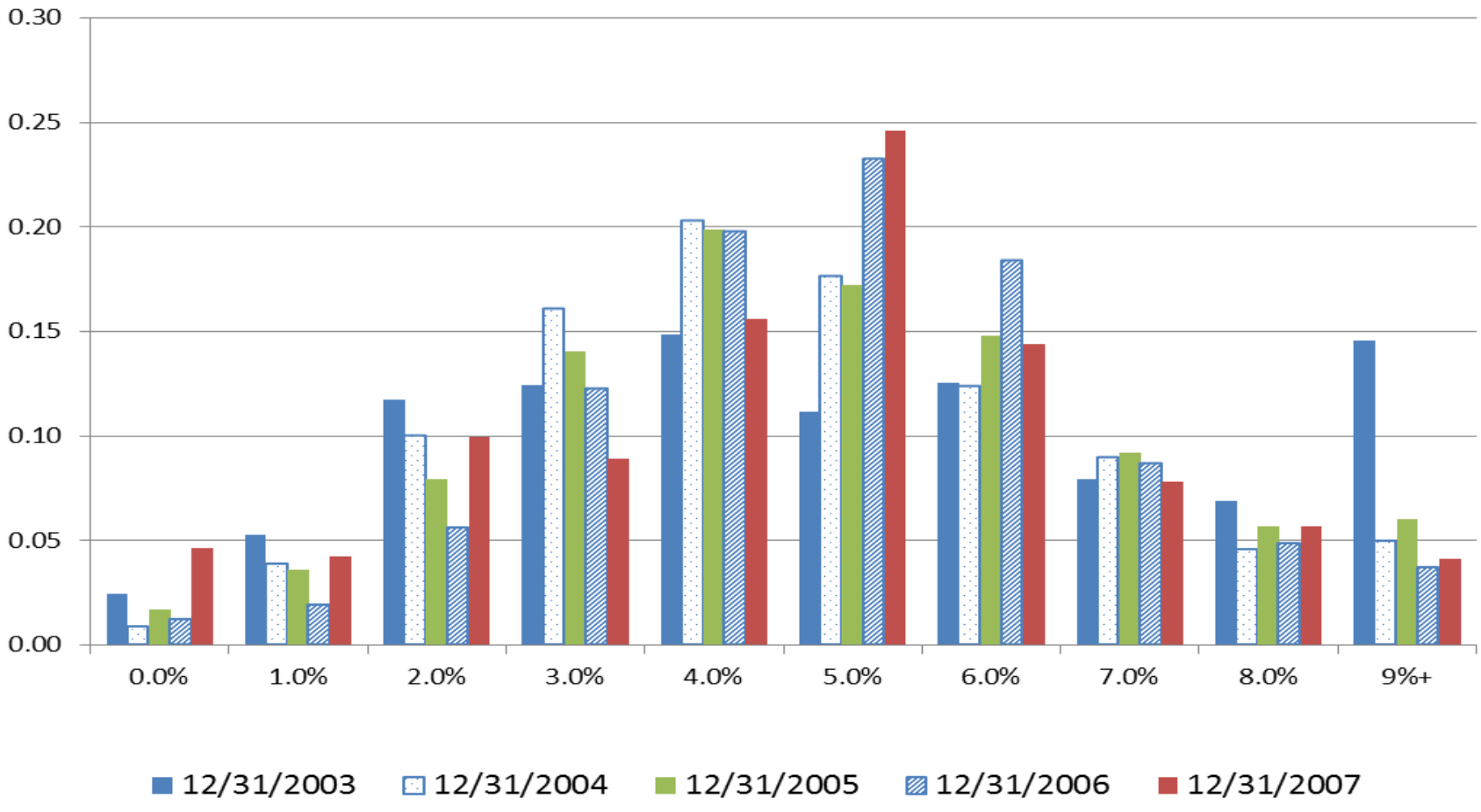
USA Insurance Prices for 3-Month LIBOR in 5 Years, as of December 31, 2003, 2004, 2005, 2006, 2007:

Relatively Symmetric Distributions



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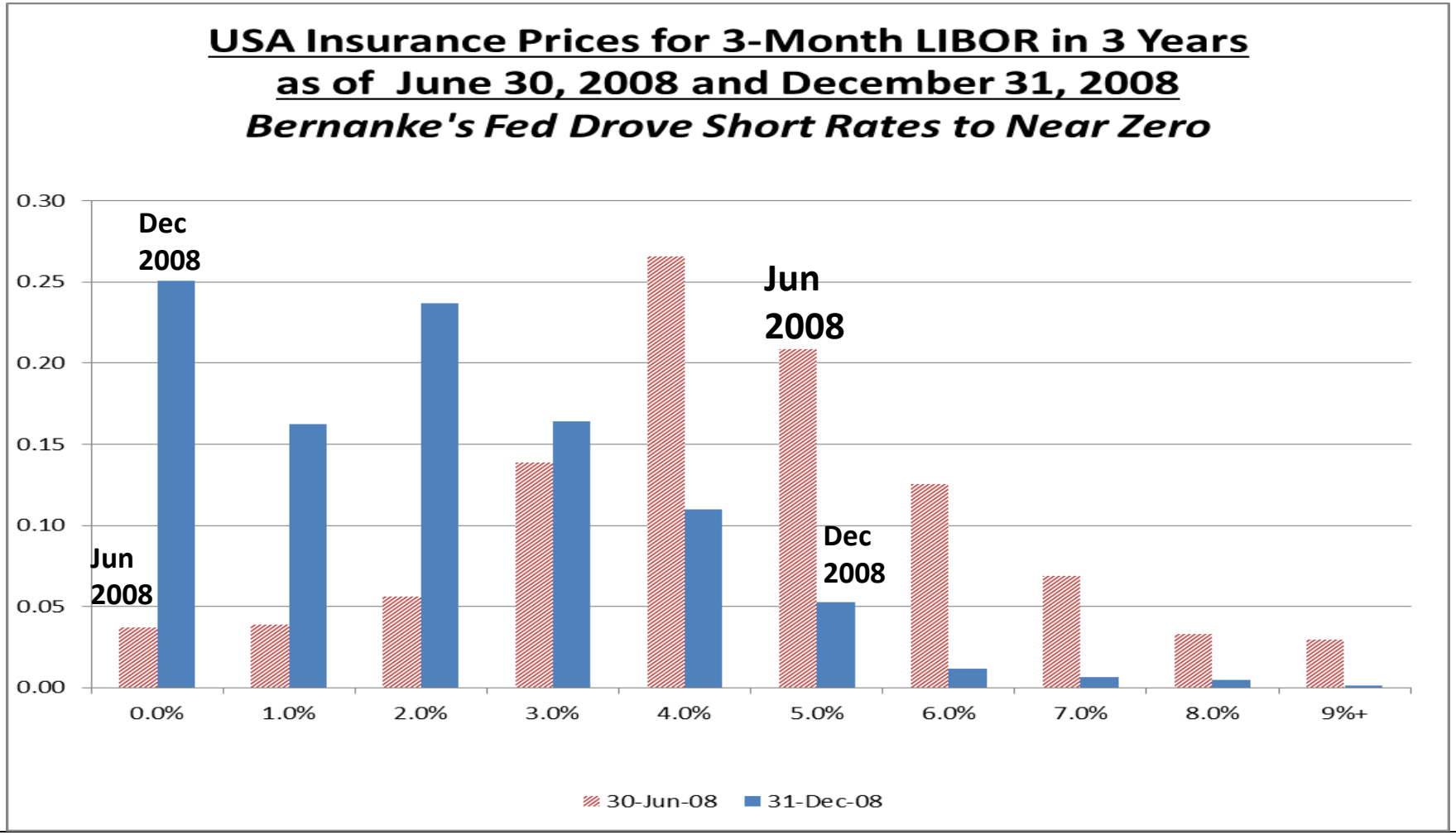
Dec 2008: U.S. Rate Distribution Transformed from Symmetric to Positive Skewness (Concentrated near zero, but long right tail)



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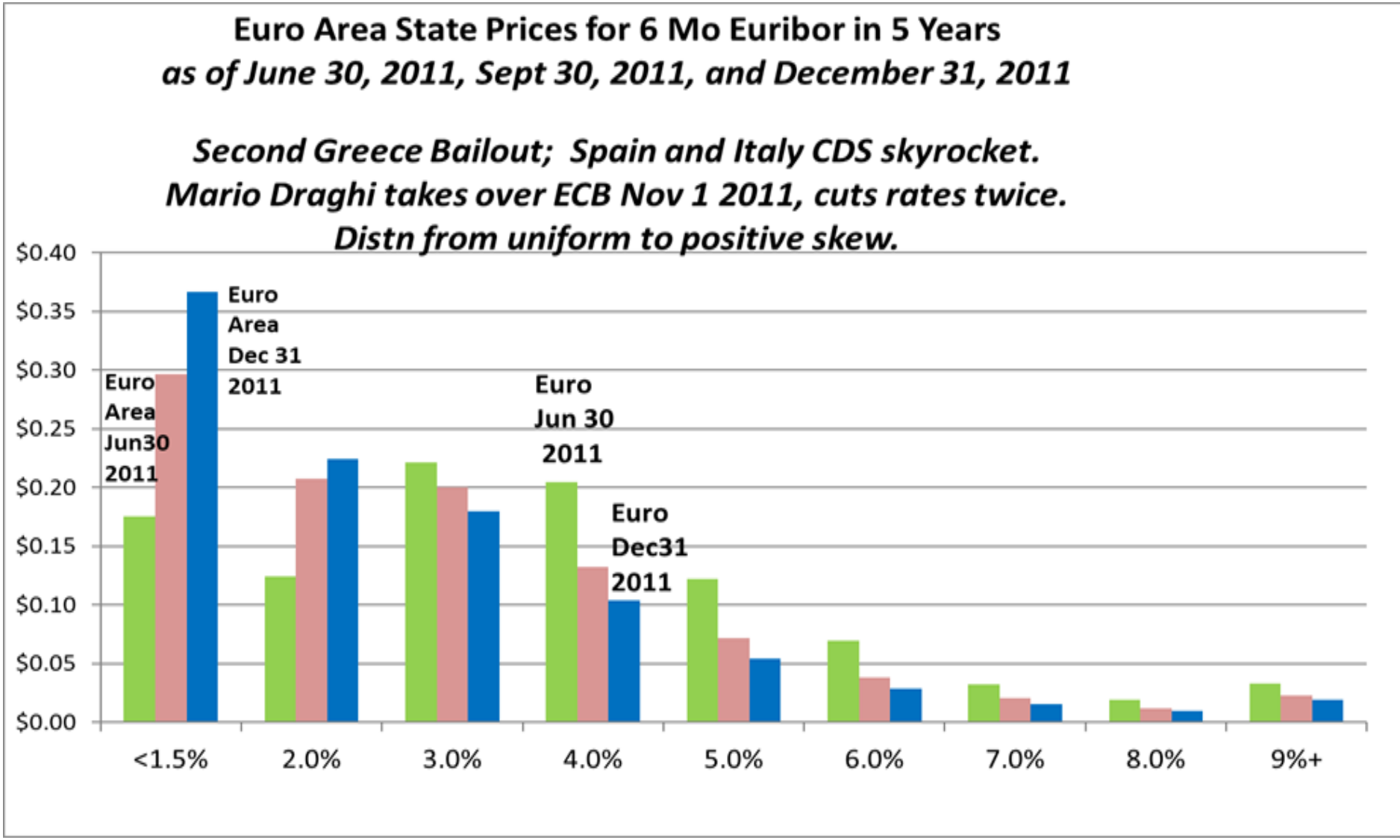

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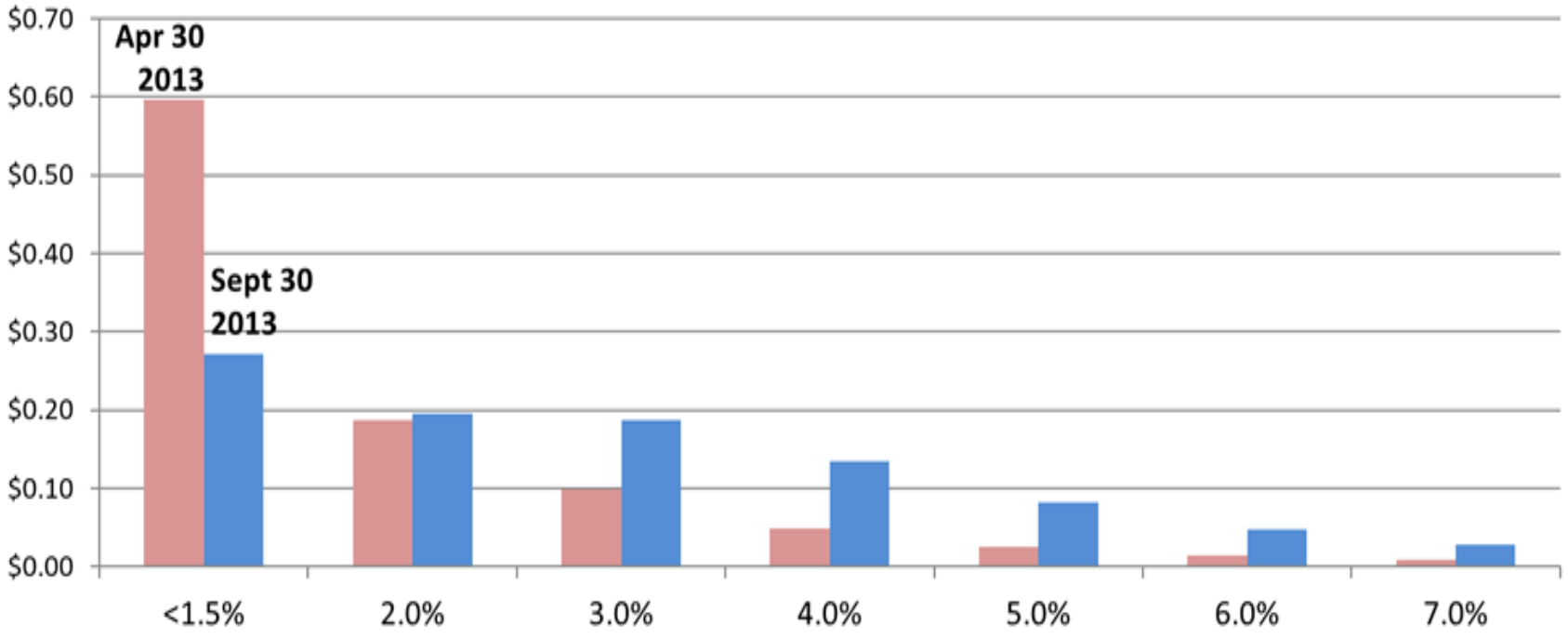
USA State Prices for 3-Mo LIBOR in 3 Years:

2013 Strong US economy: Fed Chair Bernanke talks and starts "tapering"

Rates surge. State price distribution moves from positive skew to uniform.



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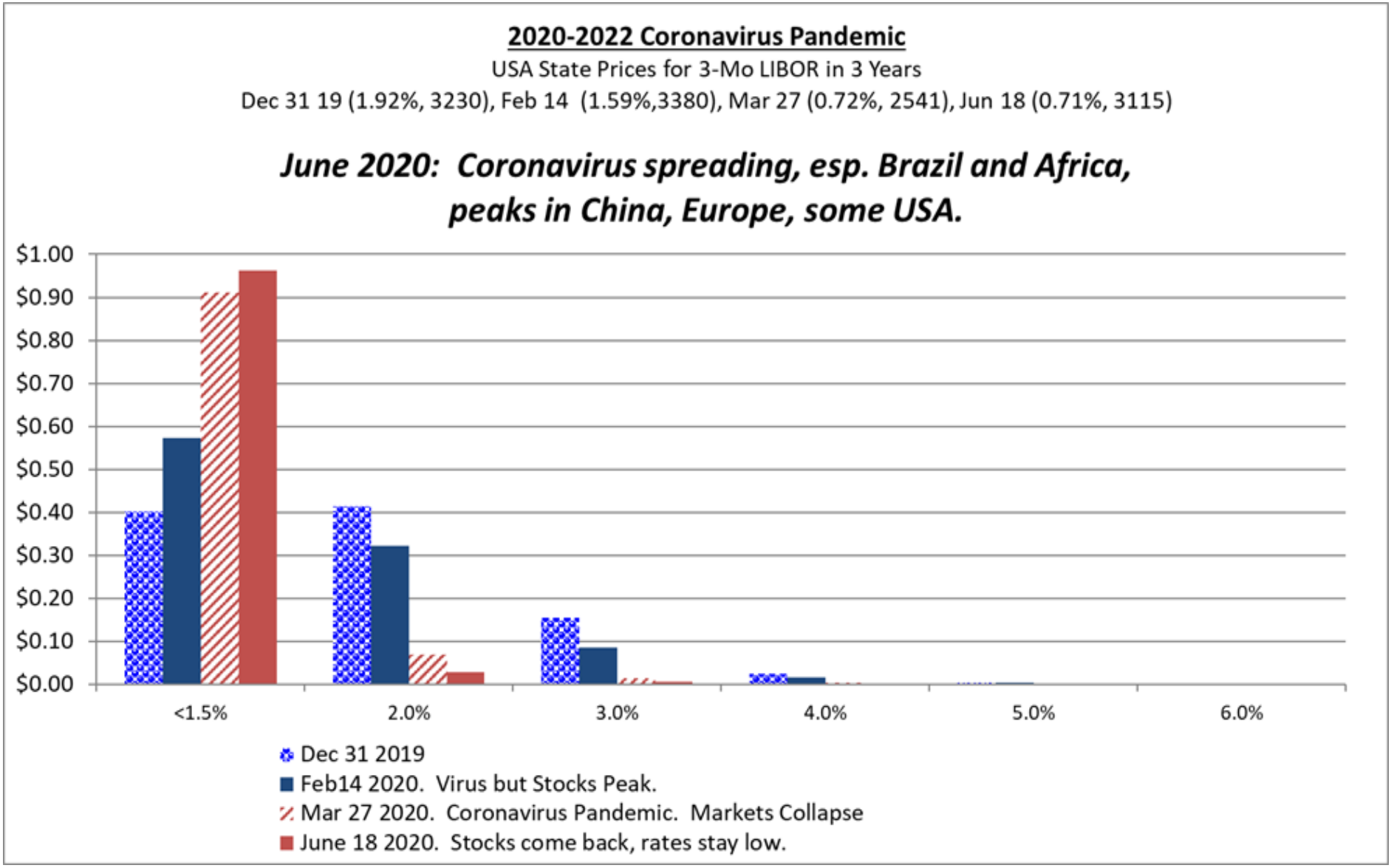

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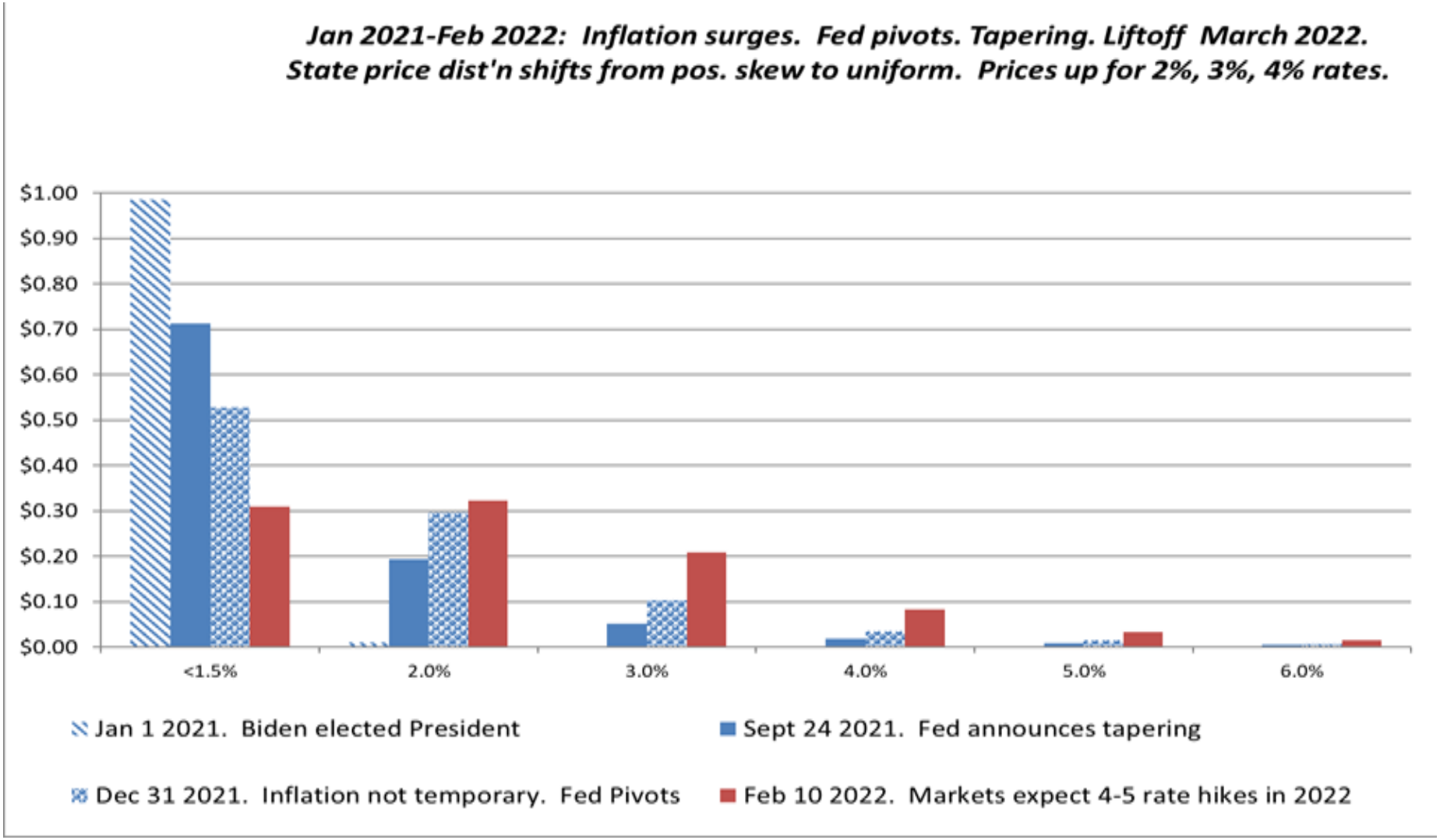
USA State Prices, January 2021-February 2022, Fed Pivot

For 3-month rate in 3 years.



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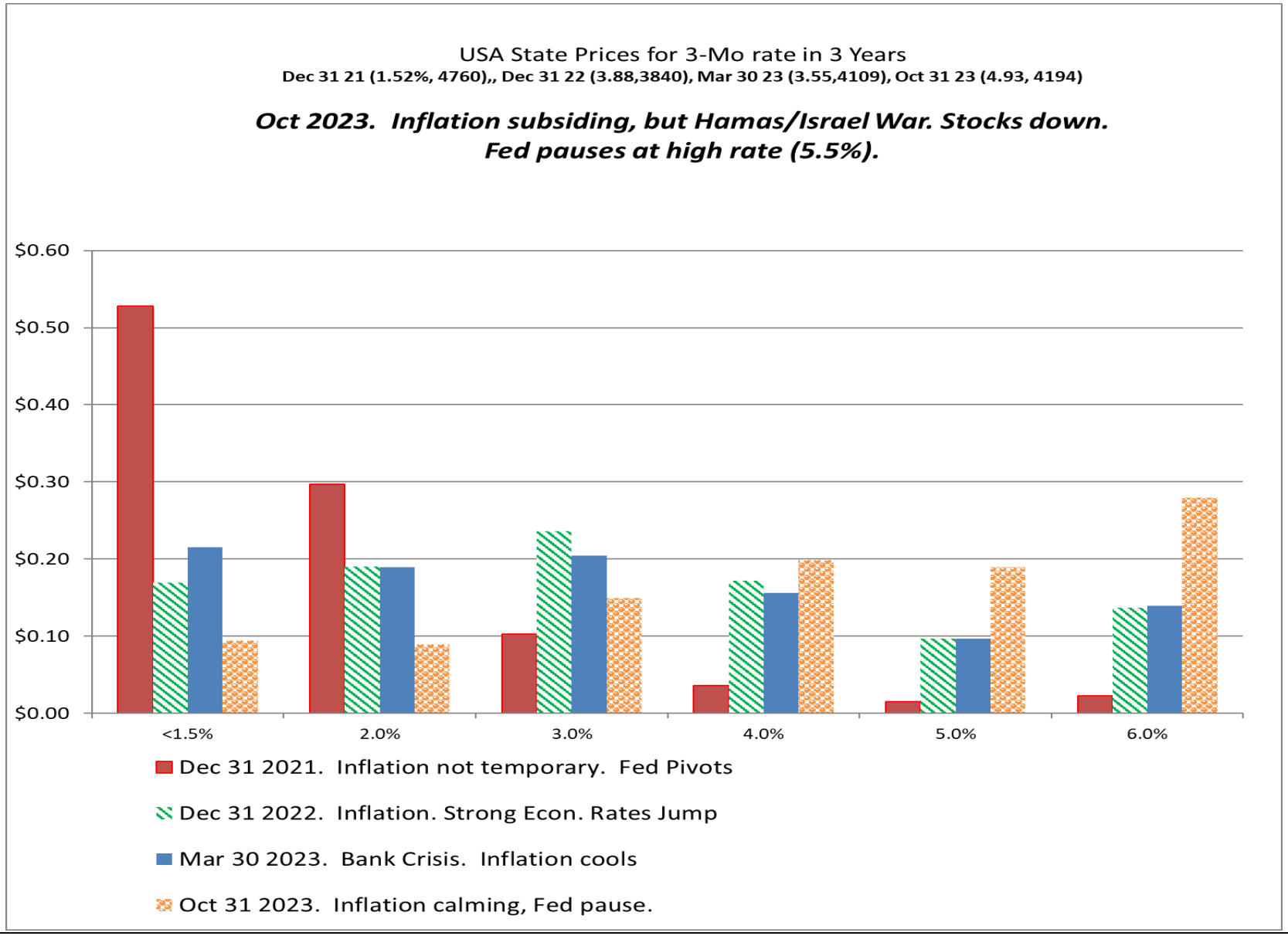
USA State Prices (Normalized), Dec 2021-October 31, 2023



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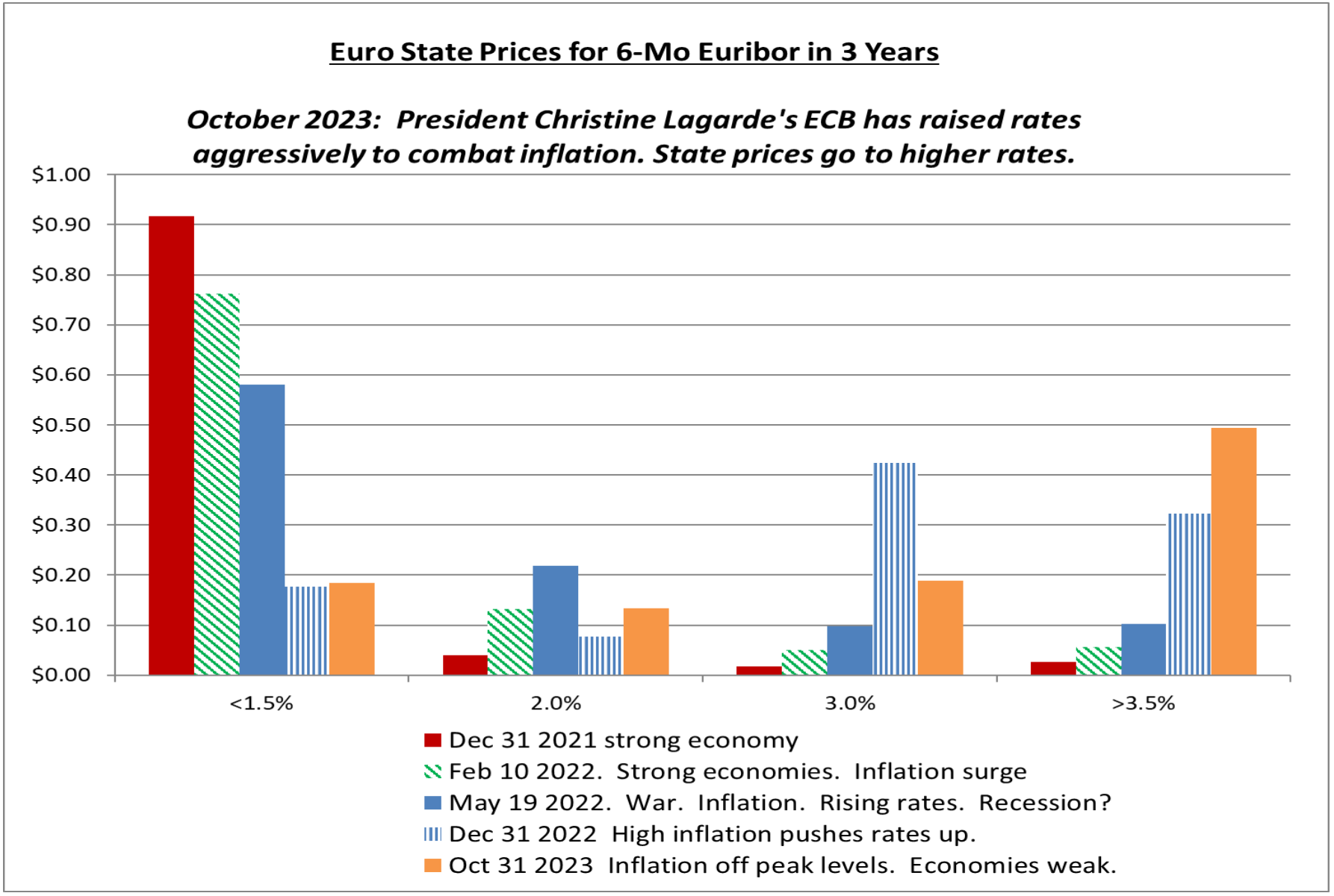
Euro Area State Prices (Normalized), Dec 2021-October 31, 2023



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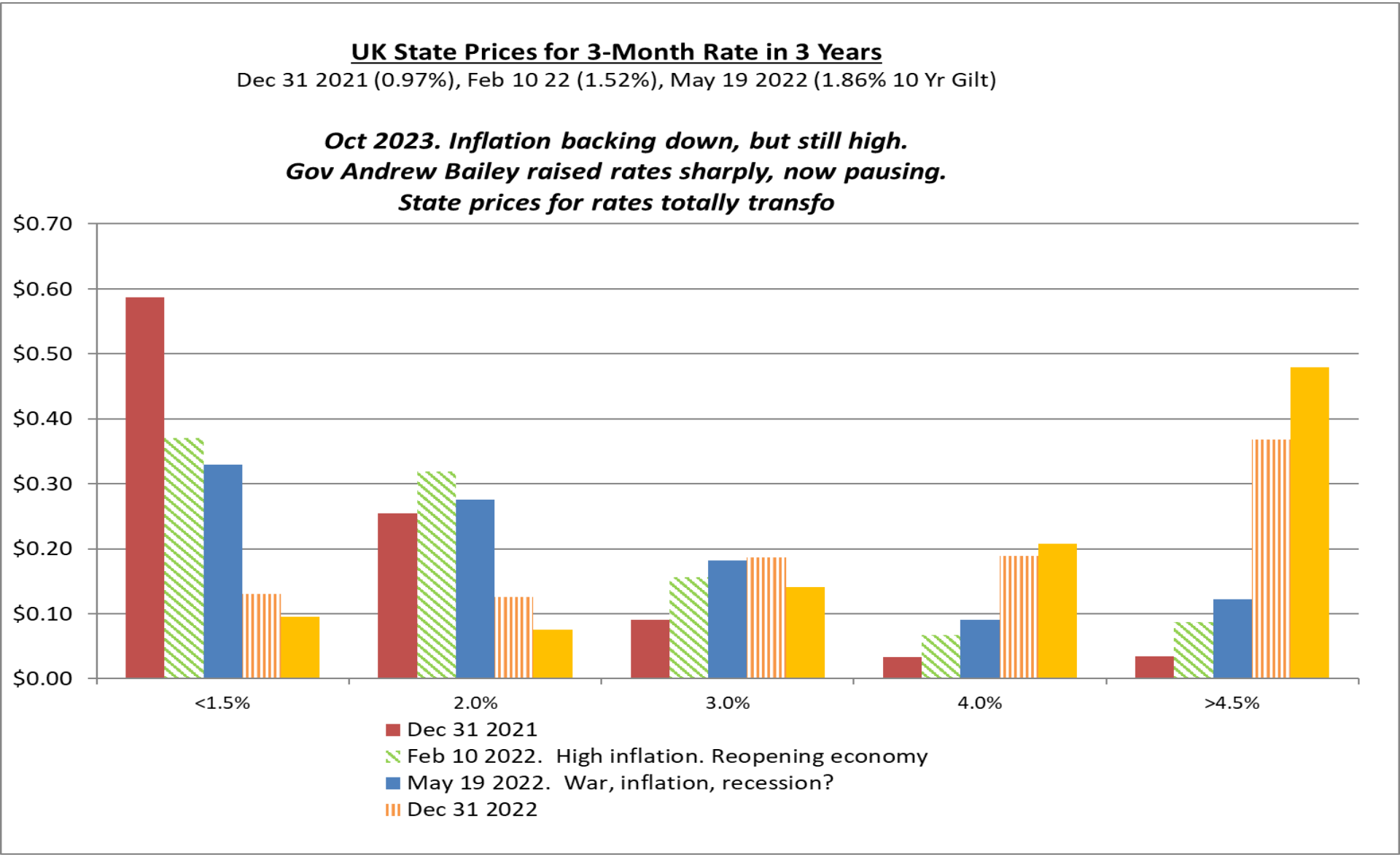
UK State Prices (Normalized), Dec 2021-October 31, 2023



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Summary: Uses of Stock and Bond State Prices from Options for Central Bank Policy Impacts and for Estimates of Risk Aversion that Forecast Stock Returns

- Using Breeden-Litzenberger butterfly spreads of time spreads of interest rate caps and floors gives **interest rate state prices.** We normalize these by dividing by their sum for each date, the riskless bond price, which gives "risk-neutral densities." These were shown to reflect major moves by the central banks in the Great Recession of 2008-2009, in the Sovereign Debt Crisis of 2011-2013 and in the Coronavirus Pandemic of 2020-2022.

- The price of payoffs received if and only if interest rates are very low, 0% to 1.5%, is later shown to be a forecaster of future stock returns. Presumably, very low interest rates indicate great fears of recession or economic weakness. It is likely that risk aversion is higher than normal then, and lower with higher rates.

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III. State Prices Implicit in Options on Stock Prices for the S&P500

"B-L Skew" is
Left Tail State Price – Right Tail State Price

= Price for Downside Protection – Price for Upside Potential

Used as a B-L measure of Risk Aversion

Breeden and Litzenberger (1978, 2022) technique.

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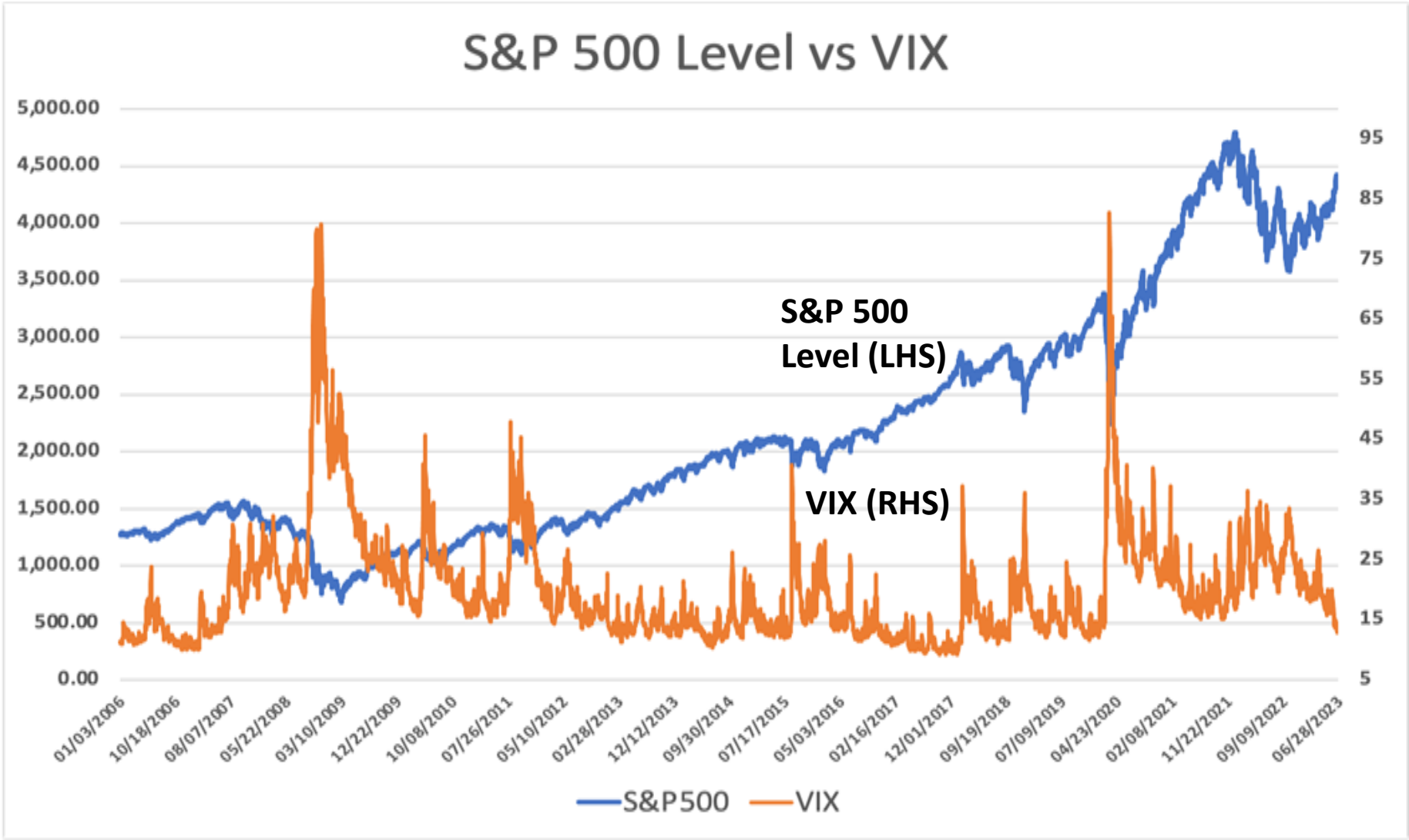
Sharp Drops in Stocks Are Correlated with Sharp Volatility Increases



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Economic Rationale for "Skew" in Implied Volatilities

John Cox of MIT (2016) has shown that the skew in implied volatilities is sensible, given that put options pay off most when stock prices fall sharply, which is when volatility surges, which amplifies the price gains on puts. This gives put options an even higher insurance value (negative beta) than if there were no correlation of volatility changes with the percentage changes in stock prices.

As the Black-Scholes formula (1973) assumes volatility is constant through time, this gives put options values that are greater than indicated by the Black-Scholes value computed with at-the-money option implied volatility. This leads to the higher implied volatilities for puts to fit actual prices and gives the skew.

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Bloomberg's Coronavirus Pandemic calculations of annualized percentage Implied Volatilities by "Moneyness" from Option Prices on the S&P 500. Volatilities soar in March 2020, fall back considerably by the end of 2021

Moneyness=S/X		Implied Volatilities for 1-Month Options					Implied Volatilities for 6-Month Options					I
SPX Index	SPX	80%	90%	100%	110%	120%	80%	90%	100%	110%	120%	
12/31/2019	3230.8	32.4	22.2	11.1	11.6	12.7	23.6	19.4	14.1	10.4	11.2	
1/31/2020	3225.5	35.8	25.0	16.1	12.3	17.2	24.7	20.4	15.3	10.8	10.9	
2/28/2020	2954.2	54.7	47.5	37.1	23.6	24.0	31.7	27.9	23.4	17.2	13.8	
3/9/2020	2746.6	65.3	58.1	49.4	37.7	32.1	42.4	38.7	33.7	28.2	22.7	
3/12/2020	2480.6	83.9	77.0	68.8	59.3	41.7	52.2	48.4	44.1	39.0	32.9	
3/16/2020	2386.1	92.5	85.7	77.7	67.2	52.2	59.6	54.9	50.1	44.2	37.4	
3/31/2020	2584.6	67.6	57.3	45.4	32.5	30.6	44.3	39.6	34.4	28.8	23.6	
4/30/2020	2912.4	49.0	39.8	28.2	20.7	25.7	39.3	34.4	28.9	23.2	19.1	
5/29/2020	3044.3	43.2	32.8	22.0	17.7	23.0	35.7	30.7	25.1	19.1	16.2	
6/30/2020	3100.3	44.0	34.3	24.3	20.1	24.9	36.2	31.5	26.1	20.1	17.6	
7/31/2020	3271.1	40.0	29.5	19.3	15.8	20.6	34.5	29.6	24.0	18.7	17.2	
8/31/2020	3500.3	44.7	31.9	19.3	17.3	23.1	35.3	29.8	24.0	19.3	17.7	
9/30/2020	3363.0	39.8	30.9	22.6	20.3	25.7	34.9	30.4	25.4	20.7	19.4	
10/30/2020	3270.0	50.9	41.4	32.6	22.6	25.8	37.0	32.3	27.1	21.5	19.0	
11/30/2020	3621.6	37.8	26.8	16.6	15.6	21.6	30.3	25.4	19.9	15.9	15.8	
12/31/2020	3756.1	40.3	28.7	17.3	15.6	21.0	31.0	25.8	20.0	15.5	14.9	
3/31/2021	3972.9	38.0	26.1	15.3	15.0	21.5	28.8	23.4	17.8	14.2	14.2	
6/30/2021	4297.5	36.9	24.2	11.2	11.8	13.2	27.0	21.5	15.7	12.2	12.3	
9/30/2021	4307.5	39.9	29.6	19.8	13.1	18.8	30.8	25.6	19.8	14.4	13.2	
12/31/2021	4766.2	37.2	24.2	12.5	12.4	14.4	29.2	23.6	17.7	13.4	13.0	

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B-L "Skew" (Lt tail price-Rt tail price) increased dramatically In the Coronavirus Pandemic then gradually normalized

S&P 500 Insurance Prices (Risk-neutral density): 2019- 2022									12	Months
10/12/22 11:54 AM	Monthend Data from December 2004. Uses Breeden-Litzenberger (2014) technique									
		$90%-$85 Puts				ATM		$110-$115 Calls		
Date	ATM Implied σ	S&P 500 Spot Index	**Left Tail Spread**	90 Butterfly	95 Butterfly	100 Butterfly	105 Butterfly	110 Butterfly	**Right Tail Spread**	**Left Tail -Right Tail**
12/31/2019	15.6	3,231	**19.6%**	25.7%	20.6%	9.8%	8.2%	2.4%	**13.7%**	**5.8%**
1/31/2020	15.9	3,226	**20.9%**	25.1%	20.0%	9.0%	8.2%	2.8%	**14.0%**	**7.0%**
2/28/2020	20.7	2,954	**35.8%**	18.6%	14.1%	4.1%	7.2%	3.0%	**17.2%**	**18.5%**
3/9/2020	27.9	2,747	**49.9%**	10.9%	9.4%	4.1%	5.8%	0.7%	**19.2%**	**30.7%**
3/16/2020	38.7	2,386	**63.3%**	3.9%	5.8%	2.4%	4.5%	1.0%	**19.1%**	**44.1%**
3/31/2020	29.7	2,585	**40.9%**	16.6%	10.5%	5.3%	5.5%	2.5%	**18.6%**	**22.3%**
4/30/2020	26.6	2,912	**46.5%**	14.2%	10.8%	4.8%	5.9%	0.8%	**17.1%**	**29.4%**
6/30/2020	24.4	3,100	**41.4%**	17.9%	12.7%	4.4%	6.2%	1.3%	**16.2%**	**25.2%**
9/30/2020	24.0	3,363	**38.0%**	18.3%	12.8%	5.7%	6.4%	1.9%	**17.0%**	**21.1%**
12/31/2020	20.5	3,756	**31.3%**	21.0%	15.1%	9.2%	6.9%	1.5%	**14.9%**	**16.4%**
1/29/2021	24.2	3,714	**39.5%**	18.2%	12.7%	5.4%	6.3%	1.5%	**16.4%**	**23.1%**
3/31/2021	18.8	3,973	**27.1%**	22.2%	16.4%	10.2%	7.3%	2.9%	**14.0%**	**13.2%**
6/30/2021	17.4	4,298	**24.1%**	23.6%	17.9%	11.6%	7.5%	2.3%	**13.1%**	**11.0%**
9/30/2021	20.8	4,308	**32.8%**	21.7%	15.4%	6.0%	6.9%	2.3%	**15.0%**	**17.8%**
11/30/2021	21.6	4,567	**34.6%**	20.9%	14.7%	6.0%	6.7%	2.1%	**15.0%**	**19.7%**
12/31/2021	19.3	4,766	**27.0%**	24.4%	17.2%	8.3%	7.1%	0.4%	**15.7%**	**11.3%**

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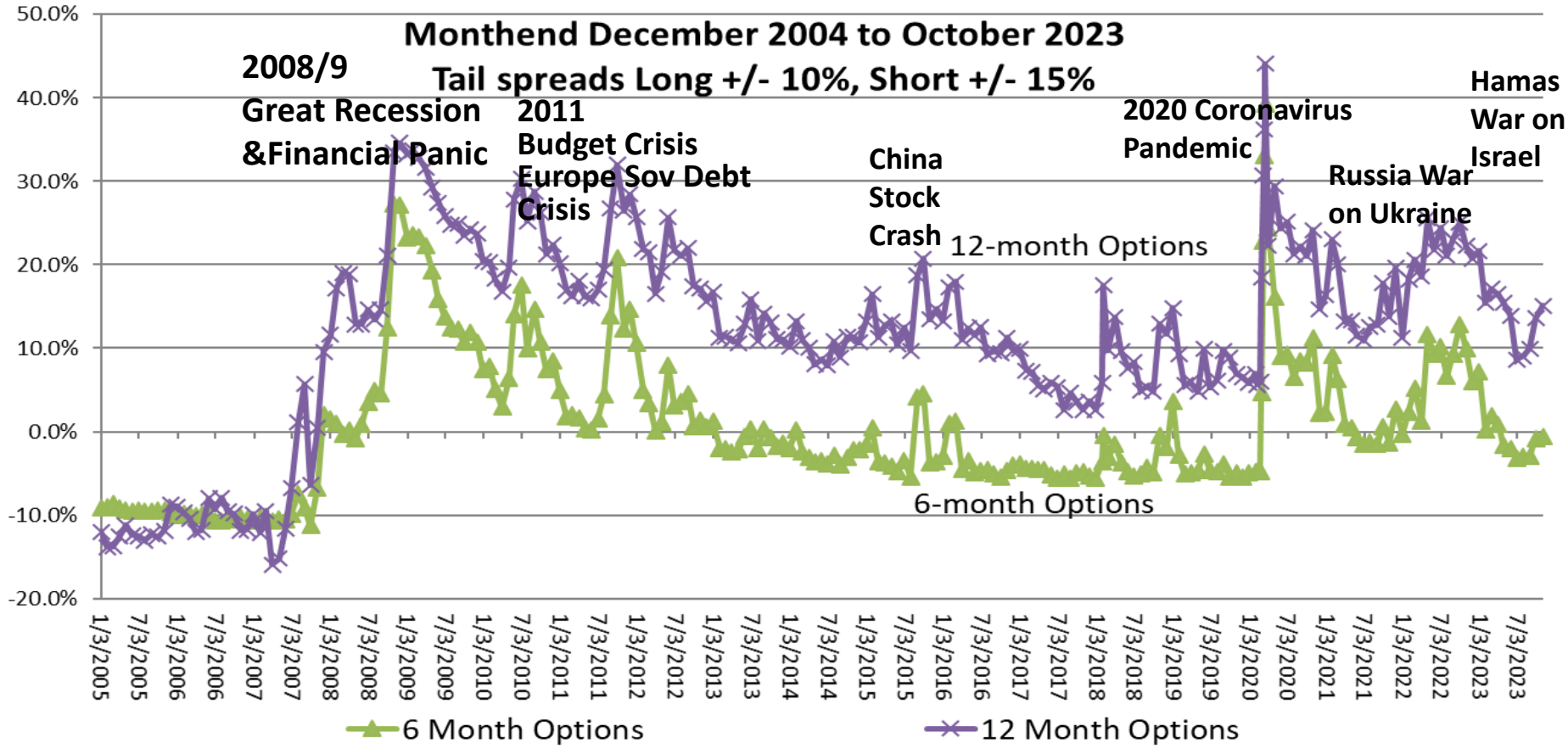
B-L "Skew" = Left Tail Spread Price - Right Tail Spread Price
= Price to Hedge Downside-Price of Upside Opportunity
BL Skew likely reflects degrees of risk and risk aversion



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IV. Decomposing the Equity Risk Premium Using Normalized State Prices and Historical Return Frequencies

Note: This research was presented in 2017 with data available at that time at the US Federal Reserve conference in Washington, DC on "Global Risk, Uncertainty and Volatility."

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Equilibrium Price/Probability Ratios

In time-state preference and in CRRA-Lognormal model:

Source: Breeden (1977 dissertation, 1986 Journal of Financial Economics)

$$\Rightarrow \phi_{ts} = \frac{\pi_{ts} u_t'(C_{ts})}{u_0'(C_0)} \qquad and \qquad \frac{\phi_{ts}}{\pi_{ts}} = \frac{u_t'(C_{ts})}{u_0'(C_0)}$$

$$\frac{\phi^*_{tr_j}}{\pi_{tr_j}} = \frac{E\left[\tilde{u}'_{ts} \middle| r_j\right]}{E[\tilde{u}'_t]}$$

$$\log\left(\frac{\phi^*_{ts}}{\pi_{ts}}\right) = \gamma\left[\mu_t - g_{ts} - \frac{1}{2}\gamma\sigma_c^2\right]t$$

Thus, we see that the risk-neutral probability to true probability ratio at the optimum for r_j is equal to the expected marginal utility of consumption, conditional upon the interest rate being at the specified level, divided by the unconditional expected marginal utility of consumption at time

As expected, higher growth states for consumption have lower $\left(\frac{\phi^*_{ts}}{\pi_{ts}}\right)$ ratios.

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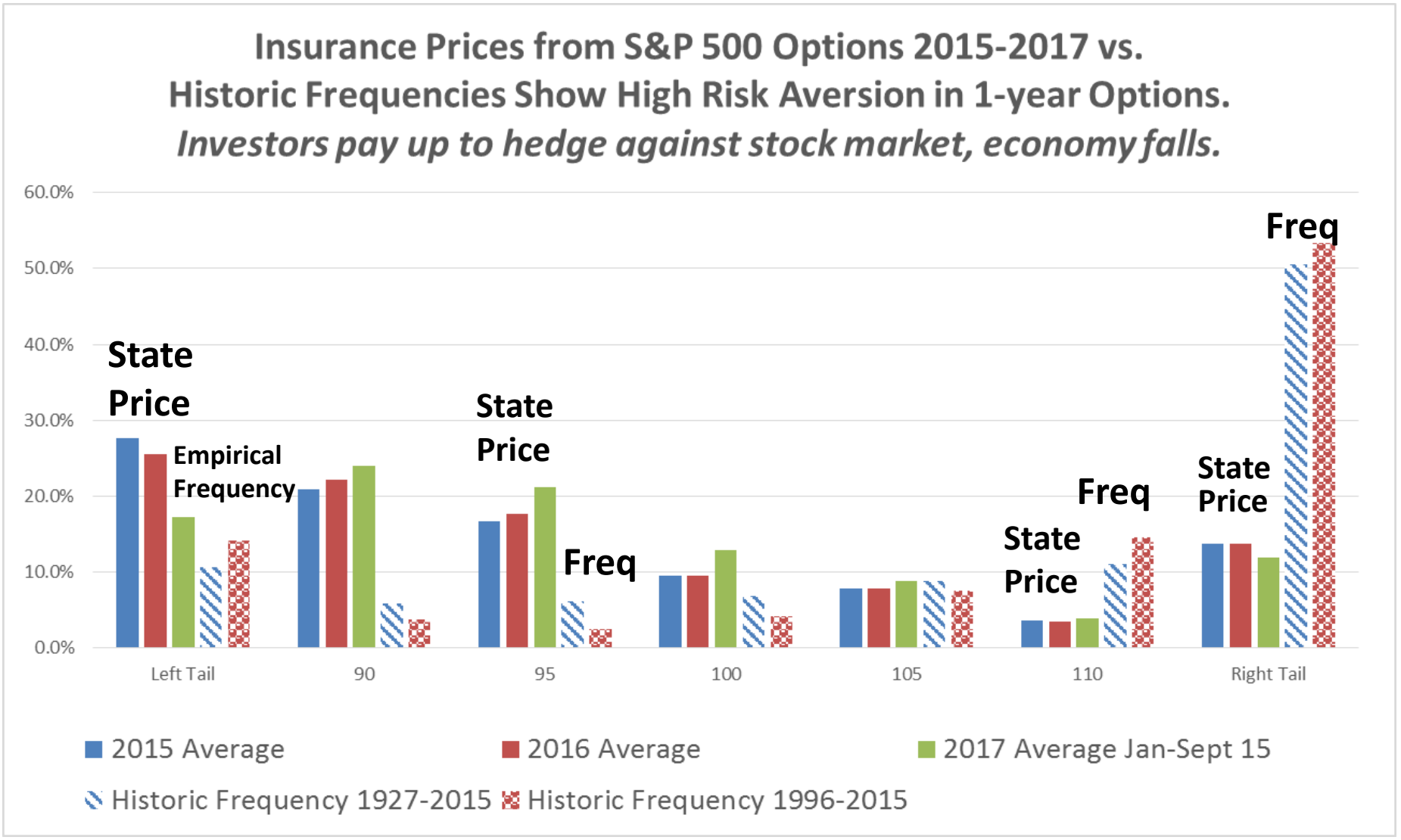
Presentation at 2017 Federal Reserve Conference in Washington DC on "Global Risk, Uncertainty and Volatility."



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Decomposing the Equity Risk Premium
Using B-L Normalized State Price/Frequency Multiples

	S&P 500 Return Ranges in 12 Months						
Low	-9999	-12.5	-7.5	-2.5	2.5	7.5	12.5
High	-12.51	-7.51	-2.51	2.49	7.49	12.49	9999
2015-2016 Data	*<=12.5%*	*-10%*	*-5%*	*0%*	*+5%*	*+10%*	*>12.5%*
Price/Frequency	*2.82*	*5.81*	*3.09*	*1.66*	*0.87*	*0.28*	*0.26*

This data shows that investors are pricing risk of -10% +/- falls very highly (in relation to probability/frequency), even more than the left tail of -20% or so. Also, insuring the -5% stock market scenario is priced very highly. So, much of the equity risk premium appears to be earned by payoffs in the modest to large downside scenarios, but not so much in the extreme downside scenarios. Pricing of large upside moves seems very cheap in relation to their historical frequencies. These results are consistent with the results in the excellent paper by Beason and Schreindorfer, JPE 2022.

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State Prices, Frequencies and Conditional Marginal Utilities

Complete Sample: 2005 to June 2023

		S&P 500 Insurance Prices (Risk-neutral density).						Option:	TTM
11/2/23 5:24 PM		Monthend Data from December 2004. Uses Breeden-Litzenberger (2014) tec						12	Months
		$90%-$85 Puts			ATM		$110-$115 Calls		
	ATM	S&P 500	**Left Tail**	90	95	100	105	110	**Right Tail**	**Left Tail**
Date	Implied σ	Spot Index	**Spread**	Butterfly	Butterfly	Butterfly	Butterfly	Butterfly	**Spread**	**-Right Tail**
Average Price			**28.3%**	**17.9%**	**15.3%**	**10.0%**	**8.1%**	**4.1%**	**16.4%**	**100.0%**
Average Frequency			**8.8%**	**5.0%**	**6.3%**	**6.2%**	**9.7%**	**13.1%**	**50.8%**	**100.0%**
Average Multiple (Yearly)			**3.20**	**3.79**	**2.44**	**1.60**	**0.83**	**0.31**	**0.32**	

$$\frac{\phi^*_{tr_j}}{\pi_{tr_j}} = \frac{E\left[\tilde{u}'_{ts} \middle| r_j\right]}{E[\tilde{u}'_t]}$$

Optimality Condition implies that the normalized Price to probability ratio should equal the expected marginal utility of consumption, conditional upon the state considered, divided by the average marginal utility for that date.

So, conditional upon being in the left tail, marginal utility is Normally 3.2 times the average marginal utility across all states.

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Expected Returns and Betas on Arrow Securities.

Since Arrow Securities each have a payoff of $1.00, their
Expected dollar payoffs are their state probabilities. Therefore, the
Expected return on any Arrow Security is its probability divided by its cost,
Which is the inverse of the "multiples" in the previous table.

The right tail spread should go up over 200% as 0.32 goes to 1.00
And the left tail spread should drop from 3.2 to 1.0, down about 2/3.

The right tail spread has a very big positive beta, the left tail spread a very big negative beta.

Arrow Security Betas (Daily Data, Average of 2005-2022 Annual Data)

		111%	105%	100%	95%	91%	87%	83%	
	Left Tail	$90.00	$95.00	$100.00	$105.00	$110.00	$115.00	$120.00	Right Tail Spread
Average Beta	*-20.1*	*-6.6*	*-0.9*	*3.0*	*5.1*	*6.8*	*4.7*	*4.0*	*3.8*

Expected Returns on Arrow Securities

Minimum Return	-9999	-12.5	-7.5	-2.5	2.5	7.5	12.5
Maximum Return	-12.5	-7.51	-2.51	2.49	7.49	12.49	9999
Average Arrow Return =	-69%	-72%	-59%	-38%	20%	224%	209%

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IV. Does B-L Skew Reflect Risk and Risk Aversion and Predict Subsequent Stock Market Returns?

"B-L Skew" is
Left Tail State Price – Right Tail State Price

= Price for Downside Protection – Price for Upside Potential

Used as a B-L measure of Risk Aversion

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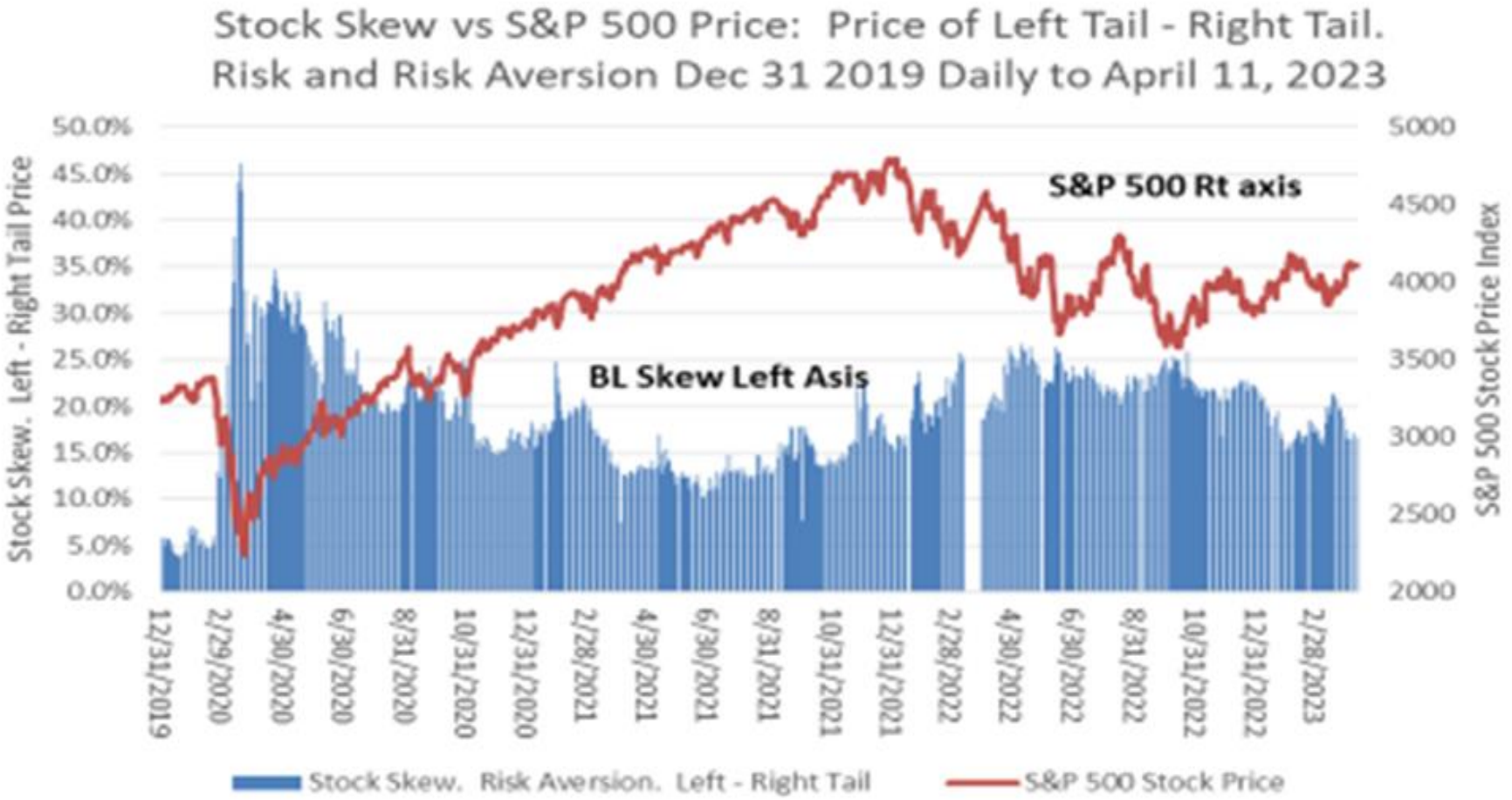
Anxiety/Risk Aversion was high early 2021, but subsided. Strong 2021

2022 High Anxiety with Russia invading Ukraine, surges in oil, inflation, rates

Stocks fell 20% in 2022. In 2023, Bank Crisis in March, rates drop. Stocks up 6%



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Does B-L Risk Aversion in Option Prices Forecast Future Stock Returns?

Cochrane, Lettau-Ludvigson and others have shown that dividend yields forecast Future returns on stocks. High dividend yields precede high stock returns, as much as 7 years in advance. This makes some sense as high yields occur with low stock prices, which tend to be in recessions, when risks are high. So returns might well also be high.

Do the Breeden-Litzenberger risk aversion estimates predict future stock returns? Yes.
Do they do better or worse than dividend yield, one of the best predictors? Better.
Bond options better short term, stock options worse for 1 year, better 2 years plus.
Stock options much better long-term, bond options similar to dividend yield.

Correlations of Forecast Variables with Future SP500 Stock Returns

	Dividend	*Stock Options*		Bond Options	
	Yield	*Breeden-Litzenberger*	*Breeden-Litzenberger*		
Forecast	Forecasts		*Left Tail (R<1.5%) State Price*		
Horizon	Shiller D/P'	Stock Left-Rt Tail	*LIBOR*	*LIBOR*	*LIBOR*
	Overlapping corre	2005-2019 Data	*3 Yr RND*	*5 Yr RND*	*8-10 Yr RND*
1 Year	37.6%	22.7%	43.5%	41.2%	38.8%
2 Year	51.2%	56.1%	61.8%	60.8%	60.5%
3 Year	49.7%	81.4%	73.5%	64.7%	67.1%
5 Year	70.4%	89.5%	65.4%	59.0%	68.6%
7 Year	64.2%	93.7%	75.4%	66.9%	72.4%

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Does B-L Risk Aversion in Option Prices Forecast Future Stock Returns?

Do these Breeden-Litzenberger risk aversion estimates predict future stock returns? Yes.
Do they do better or worse than dividend yield, one of the best predictors? Better.
Bond options better short term, stock options worse for 1 year, better 2 years plus.
Stock options much better long-term, bond options similar to dividend yield.

Due to the overlapping data of monthly rolling returns for long horizons, we compute t-statistics corrected for heteroscedasticity and autocorrelation (HAC). They show strong Performance of option-based state prices vs. S&P 500 dividend yield.

RSQ and t-Stats of Forecast Variables with Future SP500 Stock Returns

2005-2019 Data	Dividend		***Stock Options***				**Bond Options**			
	Yield		*Breeden-Litzenberger*		*Breeden-Litzenberger*					
Forecast	Forecasts				***Left Tail (R<1.5%) State Price***					
Horizon	Shiller D/P'		Stock Left-Rt Tail		*LIBOR*		*LIBOR*		*LIBOR*	
	RSQ	t(HAC)	RSQ	t(HAC)	*3 Yr RND*		*5 Yr RND*		*8-10 Yr RND*	
1 Year	12%	7.9	6%	2.1	22%	2.9	19%	2.5	17%	1.9
2 Year	24%	4.6	37%	3.1	42%	3.4	38%	3.2	36%	2.8
3 Year	24%	2.8	69%	7.2	54%	3.6	40%	2.9	39%	2.7
4 Year	30%	2.5	80%	13.3	50%	3.6	32%	2.3	31%	2.1
5 Year	44%	3.4	84%	25.7	47%	4.0	34%	2.9	33%	2.5
6 Year	48%	3.4	91%	26.1	44%	3.3	37%	2.8	49%	3.7

Preliminary Calculations. By Tingyan Jia, Stanford

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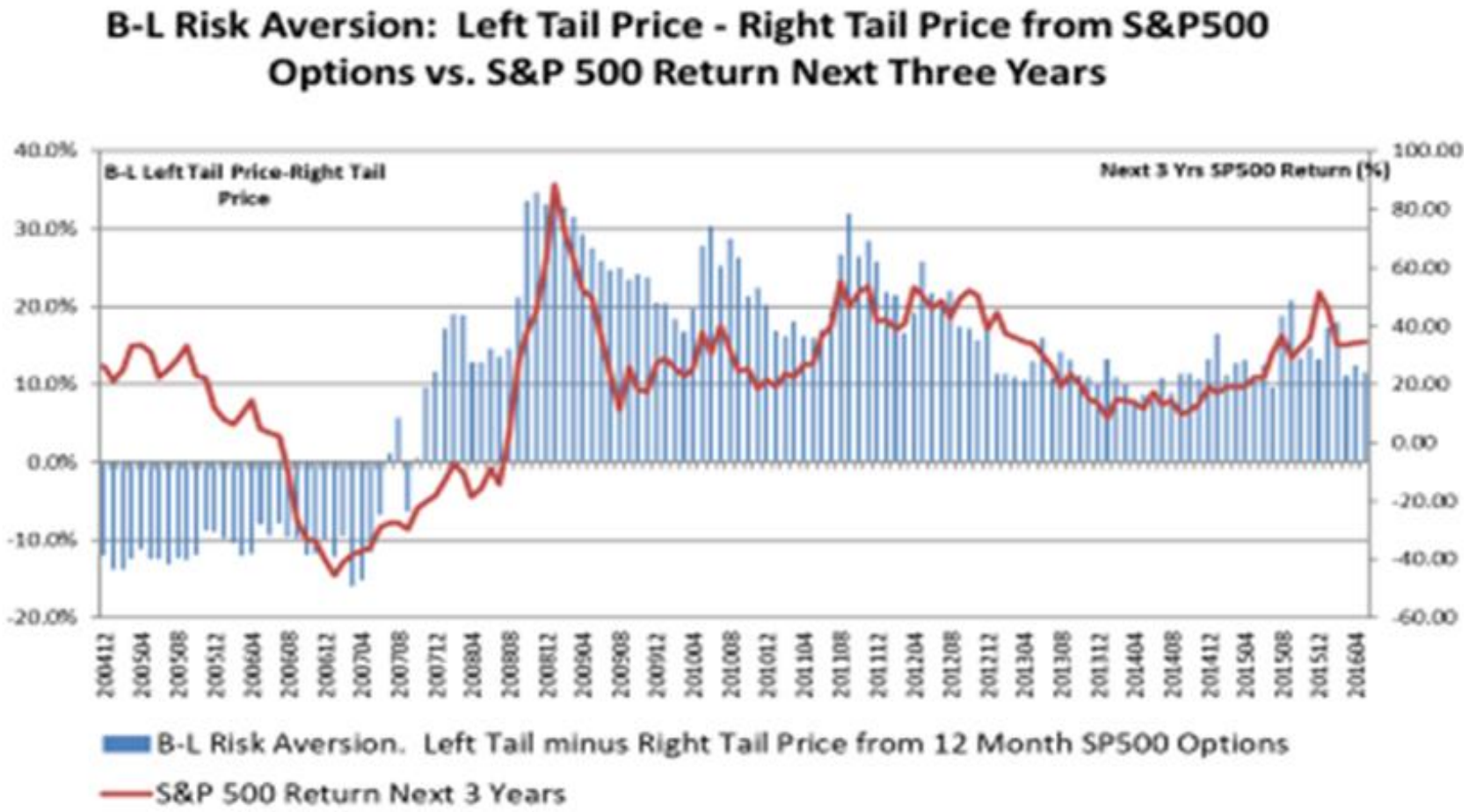
Graphs of B-L Stock Market "Risk Aversion" Estimates Vs. Future S&P 500 Stock Returns. Monthly data from 2004-2019.



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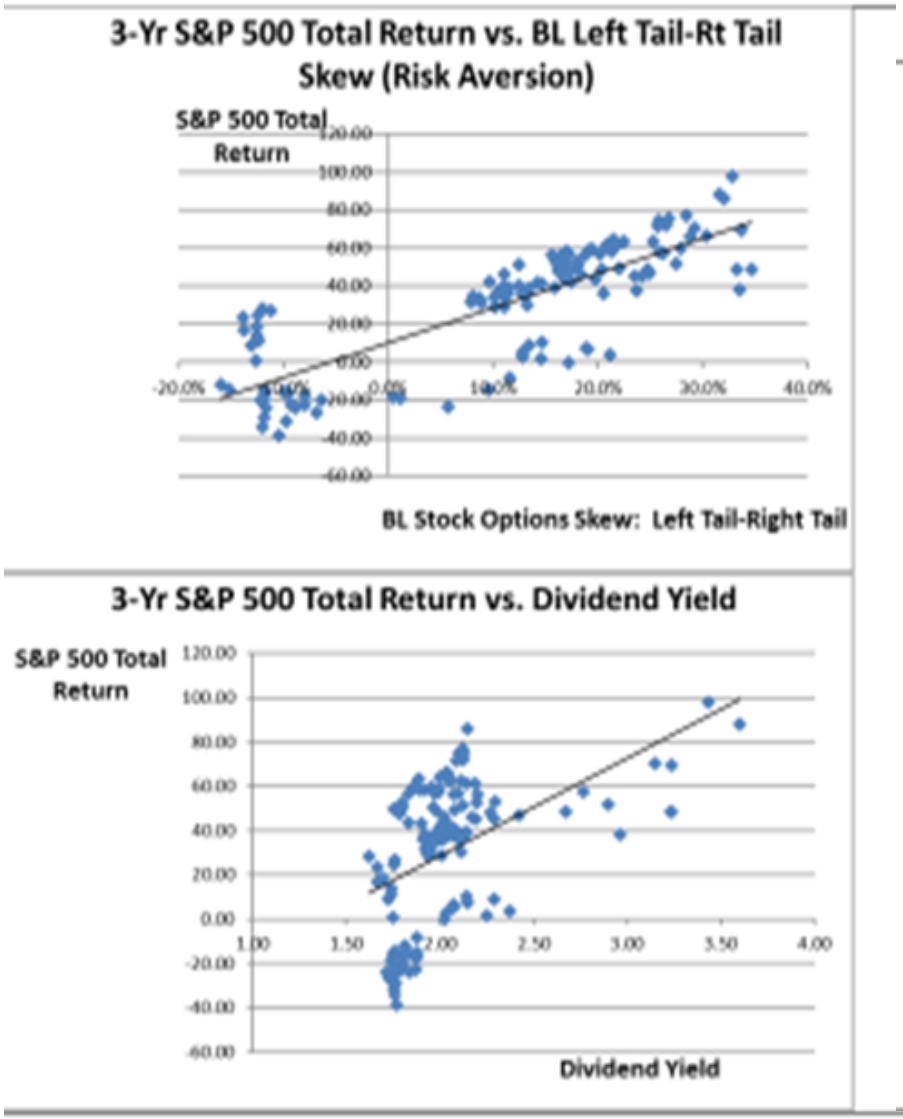

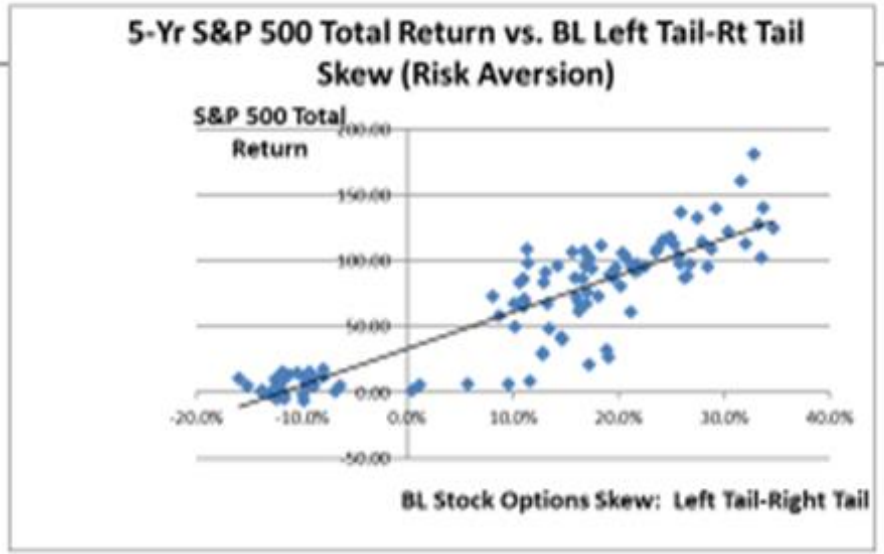

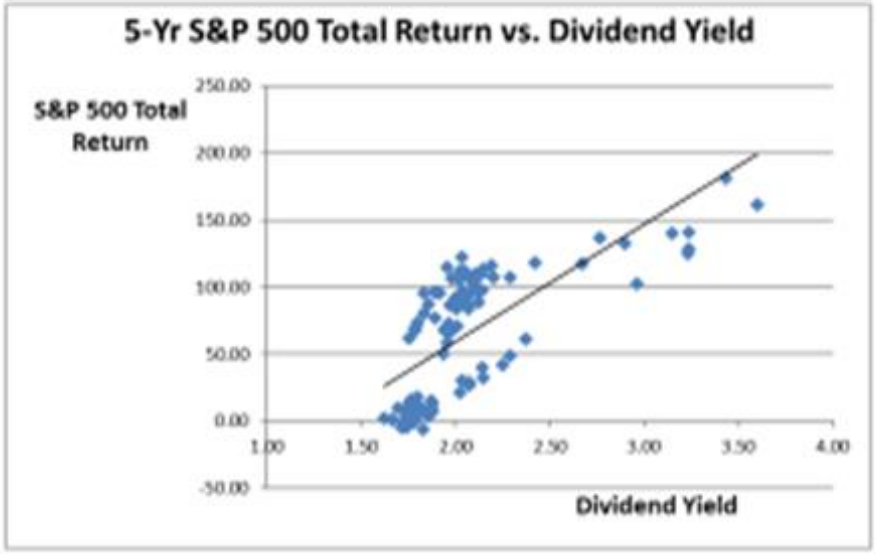

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Summary: Uses of Stock and Bond Insurance Prices from Options for Central Bank Policy Impacts and for Estimates of Risk Aversion that Forecast Stock Returns

- Using Breeden-Litzenberger butterfly spreads of time spreads of interest rate caps and floors gives **interest rate insurance prices.** These were shown to reflect major moves by the central banks in the Great Recession of 2008-2009, in the Sovereign Debt Crisis of 2011-2013 and in the Coronavirus Pandemic of 2020-2022.

- Insurance prices implicit in **options on stock prices** show that prices paid for left tail risk (downside) increase substantially in times of higher risk and higher risk aversion. The spread between prices of downside tail risk protection and prices of large upside payoffs was shown to be a good forecaster of stock returns. Higher risk is followed by higher returns, on average, which is sensible in equilibrium. For most horizons, this forecaster does better than dividend yield, using 2005-2019 data for options.

- The price of payoffs received if and only if interest rates are very low, 0% to 1.5%, is also shown to be a forecaster of future stock returns. Presumably, very low interest rates indicate great fears of recession or economic weakness. It is likely that risk aversion is higher than normal at those times, and lower with higher rates.

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Data Appendix

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Implied Volatilities by Moneyness Skyrocketed in 2008/9 Panic

Moneyness=S/X		Implied Volatilities for 1-Month Options					Implied Volatilities for 6-Month Options				
SPX Index Date	SPX Price	80%	90%	100% ATM	110%	120%	80%	90%	100% ATM	110%	120%
12/29/2006	1418.3	26.0	19.3	10.1	9.3	9.2	20.8	17.2	13.3	10.1	9.4
6/29/2007	1503.4	27.3	23.2	15.0	10.9	10.9	22.3	19.1	15.5	12.4	10.7
12/31/2007	1468.4	29.8	27.1	20.6	14.5	14.1	29.5	26.2	22.6	19.1	16.2
3/31/2008	1322.7	34.7	29.7	23.7	18.1	16.6	27.9	27.5	24.0	20.9	20.7
6/30/2008	1280.0	33.9	28.9	22.4	17.5	16.6	28.7	25.9	22.4	19.2	16.8
9/30/2008	1166.4	44.0	43.0	36.8	31.1	30.4	34.8	31.6	28.5	25.7	23.2
10/31/2008	968.8	66.3	60.9	51.4	42.9	39.5	51.2	46.8	42.7	38.9	35.4
11/28/2008	896.2	64.1	57.6	50.2	43.7	41.5	52.2	48.4	44.8	41.5	38.4
12/31/2008	903.3	46.7	41.6	34.6	29.2	27.3	44.2	40.6	37.2	33.9	30.9
1/30/2009	825.9	54.5	47.4	39.6	33.6	31.1	45.5	41.6	38.1	34.8	31.9
2/27/2009	735.1	55.0	47.5	41.0	35.7	32.3	45.7	42.0	38.6	35.5	32.9
3/31/2009	797.9	52.1	44.8	38.7	34.6	33.7	44.8	41.5	38.5	35.8	33.4
6/30/2009	919.3	40.2	30.6	23.0	18.7	18.8	33.8	29.8	26.2	23.1	20.7
12/31/2009	1115.1	30.4	26.7	17.0	16.2	18.0	29.7	25.5	21.7	18.7	16.8
6/30/2010	1030.7	41.3	37.7	29.1	22.0	21.0	38.7	33.7	29.1	25.0	21.6
12/31/2010	1257.6	26.0	23.8	15.2	13.5	13.4	28.3	24.0	20.0	16.8	15.0
6/30/2011	1320.6	24.4	23.3	13.6	13.7	13.9	26.1	21.9	17.8	14.4	12.8
9/30/2011	1131.4	48.4	45.5	36.9	28.8	26.7	41.6	36.9	32.6	28.6	25.2
12/30/2011	1257.6	30.1	28.3	20.3	16.0	16.2	32.7	28.0	23.6	19.7	16.8
12/31/2012	1426.2	25.3	24.7	16.1	14.0	14.2	25.6	21.4	17.3	14.0	12.8
12/31/2013	1848.4	19.3	19.5	11.2	12.0	12.0	22.1	17.6	13.7	11.2	11.3

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Note: 2005-2006: Low price paid for left tail insurance. High right tail. Investors must have been feeling pretty good about prospects.

S&P 500 Insurance Prices (Risk-neutral density) 2005-2013

12 Months

Monthend Data from December 2004. Uses Breeden-Litzenberger (2014) technique

11/2/23 12:07 PM

		$90%-$85 Puts				ATM		$110-$115 Calls		
	ATM	S&P 500	**Left Tail**	90	95	100	105	110	**Right Tail**	**Left Tail**
Date	Implied σ	Spot Index	**Spread**	Butterfly	Butterfly	Butterfly	Butterfly	Butterfly	**Spread**	**-Right Tail**
1/3/2005	14.8	1202.1	**14.0%**	11.8%	13.7%	13.8%	12.3%	8.4%	**26.0%**	**-12.0%**
12/30/2005	14.3	1248.3	**11.3%**	15.4%	17.7%	15.5%	12.0%	7.8%	**20.3%**	**-9.0%**
12/29/2006	14.0	1418.3	**10.1%**	15.9%	18.6%	15.6%	12.2%	7.7%	**19.9%**	**-9.9%**
6/29/2007	15.8	1503.4	**14.9%**	15.8%	16.6%	13.2%	11.0%	6.9%	**21.7%**	**-6.8%**
12/31/2007	22.2	1468.4	**32.7%**	13.1%	11.9%	8.5%	7.7%	5.0%	**21.1%**	**11.6%**
3/31/2008	23.8	1322.7	**39.0%**	11.9%	10.6%	7.4%	6.9%	4.2%	**20.1%**	**18.9%**
6/30/2008	22.3	1280.0	**34.6%**	13.1%	11.6%	8.5%	7.4%	4.8%	**20.0%**	**14.6%**
9/30/2008	27.0	1166.4	**42.7%**	10.1%	9.1%	6.4%	6.3%	4.0%	**21.5%**	**21.2%**
10/31/2008	39.4	968.8	**55.0%**	6.5%	6.1%	3.8%	4.5%	2.5%	**21.5%**	**33.5%**
11/28/2008	41.6	896.2	**56.3%**	6.0%	5.7%	3.7%	4.2%	2.5%	**21.7%**	**34.6%**
12/31/2008	36.3	903.3	**53.9%**	7.0%	6.5%	4.3%	4.7%	2.9%	**20.7%**	**33.2%**
1/30/2009	37.1	825.9	**54.2%**	7.2%	6.4%	4.3%	4.6%	2.8%	**20.5%**	**33.7%**
2/27/2009	36.9	735.1	**53.6%**	7.1%	6.4%	4.5%	4.6%	3.0%	**20.8%**	**32.8%**
3/31/2009	36.9	797.9	**53.1%**	6.9%	6.3%	4.6%	4.6%	3.0%	**21.5%**	**31.6%**
6/30/2009	26.8	919.3	**44.8%**	10.7%	9.2%	6.6%	6.0%	3.9%	**18.9%**	**25.9%**
12/31/2009	22.8	1115.1	**38.6%**	13.0%	11.1%	8.1%	6.8%	4.3%	**18.1%**	**20.5%**
12/31/2010	21.4	1257.6	**36.9%**	14.6%	12.2%	8.4%	7.1%	4.2%	**16.7%**	**20.2%**
9/30/2011	30.8	1131.4	**50.4%**	9.4%	8.1%	5.3%	5.3%	3.0%	**18.4%**	**32.0%**
12/30/2011	24.1	1257.6	**42.4%**	13.2%	10.8%	7.1%	6.3%	3.6%	**16.7%**	**25.8%**
12/31/2012	18.7	1426.2	**31.9%**	17.1%	14.0%	10.0%	7.6%	4.2%	**15.2%**	**16.7%**
12/31/2013	15.2	1848.4	**23.3%**	19.8%	17.3%	12.8%	8.7%	5.0%	**13.1%**	**10.2%**

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Implied Volatilites by Moneyness

from 12/31/2021 to October 31, 2023. Source: Bloomberg

Moneyness=S/X		Implied Volatilities for 1-Month Options					Implied Volatilities for 6-Month Options					Implied Volatilities for 12-Month Options				
SPX Index	SPX	80%	90%	100%	110%	120%	80%	90%	100%	110%	120%	80%	90%	100%	110%	120%
12/31/2021	4766.2	37.2	24.2	12.5	12.4	14.4	29.2	23.6	17.7	13.4	13.0	26.7	23.6	19.3	15.3	14.4
1/31/2022	4515.6	41.3	30.2	19.8	16.5	20.6	31.1	26.0	20.5	15.7	14.7	28.5	24.9	21.1	17.1	15.0
2/28/2022	4373.9	45.5	34.5	24.5	17.5	18.7	32.4	27.6	22.7	17.5	15.8	29.2	25.8	22.2	18.4	15.9
3/31/2022	4530.4	38.3	26.8	17.2	13.5	17.8	30.6	25.7	20.2	15.0	14.0	28.6	25.0	21.1	17.3	15.1
4/29/2022	4131.9	48.5	36.5	27.7	19.6	21.2	34.8	30.0	25.5	20.6	17.1	31.1	27.8	24.5	20.9	17.9
6/30/2022	3785.4	41.2	32.3	26.4	21.8	23.8	33.2	29.1	25.2	21.4	19.3	31.5	27.8	24.8	21.9	19.6
8/31/2022	3955.0	40.0	30.9	23.3	18.6	20.9	32.1	28.0	23.9	20.0	17.3	30.6	27.5	24.3	21.0	18.5
9/30/2022	3585.6	45.4	34.6	28.1	22.6	24.5	35.1	30.6	26.7	23.1	20.7	32.4	29.3	26.3	23.4	21.1
12/30/2022	3839.5	34.3	24.4	19.8	16.7	20.7	29.5	26.3	23.0	19.4	17.0	28.9	26.4	23.5	20.7	18.2
2/28/2023	3970.2	35.5	25.1	19.2	15.3	20.2	27.4	23.4	19.5	15.4	13.8	26.4	23.5	20.2	16.7	14.3
3/17/2023	***3916.6***	***40.4***	***30.1***	***22.1***	***16.3***	***18.6***	***31.6***	***27.0***	***22.2***	***17.6***	***14.9***	***29.0***	***25.8***	***22.1***	***18.3***	***15.4***
3/31/2023	4109.3	34.5	24.1	16.2	13.9	17.8	28.1	23.8	19.2	14.8	13.6	26.7	23.5	20.0	16.2	13.9
6/30/2023	4450.4	34.6	21.5	10.9	12.4	18.1	24.6	19.4	14.1	10.7	10.8	23.4	19.9	15.7	12.4	11.0
7/31/2023	4589.0	32.1	20.2	11.0	11.4	16.1	24.7	19.4	14.2	10.8	10.7	23.6	20.0	16.0	12.5	11.1
8/31/2023	4507.7	33.6	21.1	11.5	12.4	17.4	24.7	19.5	14.7	11.0	11.0	23.7	20.2	16.5	13.0	11.4
9/29/2023	4288.1	35.5	23.1	15.4	12.1	14.6	26.2	21.3	16.8	12.9	11.9	24.4	21.2	17.8	14.6	12.5
10/31/2023	4193.8	34.9	23.6	16.0	13.4	20.1	26.2	21.7	17.4	13.4	12.1	24.7	21.6	18.5	15.3	12.9

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Ukraine War (2/22), Inflation, Interest Rate Hikes Raise Risk Aversion in 2022
In 2023, Fear Recedes, USA Economy Strong, Stocks Rise Until September.
Hamas-Israel War (10/23) Elevates Risk Aversion

	S&P 500 Insurance Prices (Risk-neutral density): 2019- 2022								**12**	**Months**
11/2/23 1:21 PM	Monthend Data from December 2004. Uses Breeden-Litzenberger (2014) technique									
		$90%-$85 Puts				ATM		$110-$115 Calls		
Date	ATM Implied σ	S&P 500 Spot Index	**Left Tail Spread**	90 Butterfly	95 Butterfly	100 Butterfly	105 Butterfly	110 Butterfly	**Right Tail Spread**	**Left Tail -Right Tail**
12/31/2021	19.3	4,766	**27.0%**	24.4%	17.2%	8.3%	7.1%	0.4%	**15.7%**	**11.3%**
1/31/2022	21.1	4,516	**33.7%**	20.4%	14.7%	6.2%	6.9%	2.7%	**15.5%**	**18.3%**
2/28/2022	22.2	4,374	**37.1%**	18.1%	13.3%	5.3%	6.7%	2.8%	**16.5%**	**20.6%**
3/31/2022	21.1	4,530	**34.1%**	19.8%	14.4%	6.8%	6.9%	2.5%	**15.5%**	**18.6%**
4/29/2022	24.5	4,132	**43.1%**	14.0%	11.2%	4.5%	6.4%	3.4%	**17.5%**	**25.5%**
6/30/2022	24.8	3,785	**41.4%**	13.5%	10.8%	6.1%	6.4%	4.8%	**17.0%**	**24.4%**
8/31/2022	24.3	3,955	**41.1%**	14.6%	11.3%	5.6%	6.4%	2.9%	**18.1%**	**23.0%**
9/30/2022	26.3	3,586	**43.8%**	11.9%	9.8%	6.5%	6.1%	3.1%	**18.7%**	**25.1%**
12/30/2022	23.5	3,840	**40.7%**	13.1%	11.0%	6.5%	6.7%	3.0%	**19.0%**	**21.7%**
3/31/2023	20.0	4,109	**32.5%**	20.2%	15.1%	6.2%	7.2%	2.8%	**16.0%**	**16.5%**
5/31/2023	18.9	4,180	**28.7%**	23.7%	17.1%	5.7%	7.4%	2.8%	**14.8%**	**13.9%**
6/30/2023	15.7	4,450	**21.6%**	23.4%	19.2%	13.2%	7.9%	1.8%	**12.9%**	**8.6%**
7/31/2023	16.0	4,589	**22.0%**	23.9%	19.2%	11.3%	8.0%	2.6%	**13.0%**	**9.0%**
8/31/2023	16.5	4,508	**23.6%**	23.4%	18.5%	9.7%	8.0%	3.2%	**13.6%**	**10.0%**
9/29/2023	17.8	4,288	**28.4%**	20.1%	16.0%	9.3%	7.8%	3.5%	**14.8%**	**13.6%**
10/31/2023	18.5	4,194	**30.7%**	18.9%	15.1%	8.2%	7.7%	3.8%	**15.6%**	**15.1%**

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S&P 500 Insurance Prices (Risk-neutral density). **6 Month Averages** **Option: TTM**

7/25/23 12:16 PM — Monthend Data from December 2004. Uses Breeden-Litzenberger (2014) tec — **12 Months**

			$90%-$85 Puts			ATM		$110-$115 Calls		
Date	ATM Implied σ	S&P 500 Spot Index	**Left Tail Spread**	90 Butterfly	95 Butterfly	100 Butterfly	105 Butterfly	110 Butterfly	**Right Tail Spread**	**Left Tail -Right Tail**
6/30/2005	14.0	1184.2	**12.5%**	11.8%	14.2%	15.6%	12.8%	7.8%	**25.2%**	**-12.7%**
12/30/2005	14.3	1231.4	**12.6%**	13.0%	15.2%	15.6%	12.3%	7.6%	**23.8%**	**-11.3%**
6/30/2006	13.9	1284.4	**10.7%**	15.1%	17.7%	15.4%	12.4%	7.9%	**20.9%**	**-10.2%**
12/29/2006	13.9	1352.2	**9.8%**	15.7%	18.6%	15.9%	12.3%	7.7%	**20.0%**	**-10.1%**
6/29/2007	14.1	1463.7	**10.3%**	15.8%	18.5%	15.3%	12.2%	5.8%	**22.2%**	**-11.9%**
12/31/2007	20.3	1492.5	**26.2%**	14.9%	13.6%	10.1%	8.6%	4.1%	**22.5%**	**3.7%**
6/30/2008	22.2	1349.6	**35.6%**	12.9%	11.5%	8.4%	7.4%	4.6%	**19.7%**	**15.9%**
12/31/2008	31.4	1080.8	**46.1%**	9.3%	8.4%	5.9%	5.8%	3.6%	**21.0%**	**25.1%**
6/30/2009	33.2	845.0	**50.5%**	8.2%	7.3%	5.2%	5.1%	3.3%	**20.4%**	**30.1%**
12/31/2009	25.2	1052.0	**42.3%**	11.6%	9.9%	7.2%	6.3%	4.1%	**18.7%**	**23.6%**
6/30/2010	23.5	1109.1	**39.5%**	13.2%	11.1%	8.0%	6.7%	4.1%	**17.3%**	**22.2%**
12/31/2010	24.2	1152.3	**41.3%**	13.0%	10.8%	7.4%	6.5%	3.8%	**17.3%**	**24.0%**
6/30/2011	19.0	1328.1	**31.9%**	17.0%	14.1%	9.8%	7.6%	4.4%	**15.2%**	**16.7%**
12/30/2011	25.4	1233.4	**43.5%**	12.4%	10.3%	6.9%	6.2%	3.5%	**17.0%**	**26.5%**
6/29/2012	20.7	1359.5	**36.0%**	16.4%	13.2%	8.7%	7.0%	3.7%	**15.0%**	**21.1%**
12/31/2012	19.5	1413.5	**33.6%**	16.5%	13.5%	9.6%	7.4%	4.2%	**15.2%**	**18.3%**
6/28/2013	16.1	1569.4	**25.7%**	19.0%	16.3%	12.3%	8.3%	4.9%	**13.5%**	**12.2%**
12/31/2013	16.0	1735.2	**25.3%**	19.0%	16.4%	12.4%	8.4%	5.0%	**13.5%**	**11.7%**
6/30/2014	15.0	1880.4	**22.6%**	20.9%	18.1%	12.4%	8.6%	4.5%	**12.8%**	**9.8%**
12/31/2014	16.0	2008.5	**24.4%**	21.2%	17.5%	11.5%	8.2%	4.0%	**13.3%**	**11.1%**
6/30/2015	16.5	2070.6	**25.9%**	21.9%	17.5%	10.1%	7.9%	3.7%	**13.1%**	**12.8%**
12/31/2015	17.9	2033.3	**29.3%**	20.0%	16.0%	9.1%	7.7%	3.7%	**14.2%**	**15.1%**
6/30/2016	17.6	2032.2	**27.9%**	21.2%	16.8%	8.9%	7.7%	3.5%	**14.1%**	**13.8%**
12/30/2016	16.3	2179.4	**23.3%**	23.1%	18.5%	10.2%	8.1%	3.5%	**13.4%**	**9.9%**
6/30/2017	14.2	2370.8	**18.2%**	23.6%	20.6%	12.7%	8.7%	4.0%	**12.2%**	**6.0%**
12/29/2017	13.4	2559.6	**14.5%**	25.5%	23.1%	13.4%	8.8%	3.5%	**11.3%**	**3.3%**
6/29/2018	16.0	2713.3	**22.3%**	23.4%	19.0%	10.3%	8.2%	3.6%	**13.1%**	**9.2%**
12/31/2018	16.2	2768.4	**22.6%**	22.4%	18.7%	11.1%	8.1%	3.6%	**13.5%**	**9.1%**
6/28/2019	15.5	2827.1	**20.3%**	24.1%	19.9%	10.5%	8.3%	3.4%	**13.5%**	**6.8%**
12/31/2019	16.2	3048.8	**21.4%**	25.1%	19.7%	9.4%	8.0%	2.4%	**14.1%**	**7.3%**
6/30/2020	23.4	2970.2	**37.6%**	18.4%	13.5%	5.4%	6.6%	2.1%	**16.5%**	**21.1%**
12/31/2020	22.6	3463.7	**35.9%**	19.1%	13.6%	7.0%	6.6%	2.0%	**15.9%**	**20.0%**
6/30/2021	19.9	4030.2	**29.7%**	21.4%	15.8%	9.4%	7.1%	2.3%	**14.4%**	**15.4%**
12/31/2021	19.5	4527.3	**28.9%**	23.0%	16.6%	8.3%	7.1%	1.9%	**14.3%**	**14.6%**
6/30/2022	22.9	4244.9	**38.2%**	16.9%	12.7%	5.9%	6.6%	3.0%	**16.7%**	**21.5%**
12/30/2022	24.1	3910.4	**40.6%**	13.8%	11.1%	6.9%	6.5%	2.9%	**18.3%**	**22.3%**
6/20/2023	19.1	4149.0	**30.3%**	21.1%	15.9%	7.5%	7.3%	2.4%	**15.4%**	**14.9%**
Averages	**19.2**	**2135.8**	**28.3%**	**17.9%**	**15.3%**	**10.0%**	**8.1%**	**4.1%**	**16.4%**	**11.9%**

Source: Breeden (2017), Talk on "Global Risk, Uncertainty and Volatility" at the Federal Reserve Board conference in Washington D.C., September 2017. Updated

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	50-Year Trailing Frequency Distribution					6 Month Averages		
Low	-9999.00	-12.50	-7.50	-2.50	2.50	7.50	12.50	Checksum
High	-12.51	-7.51	-2.51	2.49	7.49	12.49	9999.00	
6/30/2005	7.5%	5.8%	7.0%	7.5%	9.6%	11.5%	51.1%	100.0%
12/30/2005	7.5%	5.8%	7.0%	7.5%	10.0%	11.8%	50.3%	100.0%
6/30/2006	7.5%	5.8%	7.0%	7.5%	10.2%	12.5%	49.5%	100.0%
12/29/2006	7.5%	5.8%	7.0%	7.5%	10.3%	13.1%	48.8%	100.0%
6/29/2007	7.5%	5.8%	6.9%	7.3%	9.9%	13.2%	49.4%	100.0%
12/31/2007	7.5%	5.8%	6.7%	6.9%	9.7%	13.2%	50.2%	100.0%
6/30/2008	7.5%	5.7%	6.4%	6.7%	9.8%	13.3%	50.5%	100.0%
12/31/2008	7.8%	5.9%	6.7%	6.3%	9.7%	13.2%	50.3%	100.0%
6/30/2009	8.8%	6.0%	6.7%	6.3%	9.7%	13.2%	49.4%	100.0%
12/31/2009	9.6%	6.0%	6.8%	6.3%	9.7%	13.2%	48.4%	100.0%
6/30/2010	9.7%	6.0%	6.8%	6.3%	9.3%	13.2%	48.8%	100.0%
12/31/2010	9.7%	6.0%	6.6%	5.7%	9.3%	13.3%	49.5%	100.0%
6/30/2011	9.7%	6.0%	6.3%	5.3%	9.3%	13.5%	49.8%	100.0%
12/30/2011	9.7%	6.0%	6.3%	5.4%	9.3%	13.6%	49.7%	100.0%
6/29/2012	9.7%	6.0%	6.3%	5.7%	9.6%	13.8%	48.9%	100.0%
12/31/2012	9.4%	5.6%	6.3%	5.8%	9.8%	14.0%	49.1%	100.0%
6/28/2013	9.2%	5.2%	6.2%	5.6%	9.8%	13.9%	50.1%	100.0%
12/31/2013	9.2%	5.2%	6.2%	5.5%	9.8%	13.8%	50.3%	100.0%
6/30/2014	9.2%	5.2%	6.2%	5.5%	9.8%	13.8%	50.3%	100.0%
12/31/2014	9.2%	5.2%	6.2%	5.5%	9.8%	13.8%	50.3%	100.0%
6/30/2015	9.2%	5.2%	6.2%	5.5%	9.8%	13.8%	50.4%	100.0%
12/31/2015	9.2%	5.2%	6.2%	5.7%	9.8%	13.7%	50.3%	100.0%
6/30/2016	9.2%	5.2%	6.3%	6.3%	9.9%	12.9%	50.3%	100.0%
12/30/2016	9.2%	4.9%	6.3%	6.4%	9.9%	13.0%	50.4%	100.0%
6/30/2017	9.2%	4.3%	6.2%	6.2%	9.8%	13.3%	50.9%	100.0%
12/29/2017	9.2%	4.3%	6.2%	6.2%	9.5%	13.2%	51.5%	100.0%
6/29/2018	9.2%	4.3%	6.2%	6.2%	9.3%	13.0%	51.9%	100.0%
12/31/2018	9.2%	4.3%	6.2%	6.2%	8.9%	12.8%	52.4%	100.0%
6/28/2019	9.2%	4.3%	6.3%	6.3%	9.3%	12.5%	52.1%	100.0%
12/31/2019	9.2%	4.3%	6.1%	6.1%	9.7%	12.6%	52.1%	100.0%
6/30/2020	8.9%	3.9%	5.8%	6.1%	9.8%	12.8%	52.7%	100.0%
12/31/2020	8.5%	3.5%	5.7%	6.2%	9.8%	13.2%	53.1%	100.0%
6/30/2021	8.5%	3.3%	5.5%	6.2%	9.7%	13.2%	53.6%	100.0%
12/31/2021	8.5%	3.3%	5.5%	6.2%	9.7%	13.1%	53.7%	100.0%
6/30/2022	8.5%	3.3%	5.5%	6.3%	9.6%	12.8%	54.0%	100.0%
12/30/2022	8.6%	3.6%	5.6%	6.5%	9.5%	12.5%	53.6%	100.0%
6/20/2023	9.0%	4.0%	5.9%	6.4%	9.5%	12.4%	52.9%	100.0%
Average	8.8%	5.0%	6.3%	6.2%	9.7%	13.1%	50.8%	100.0%

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Price/Frequency Multiples: Conditional Marginal Utilities							6 Month Averages		
Low	Average	Average	-9999	-12.5	-7.5	-2.5	2.5	7.5	12.5
High	VIX	S&P500	-12.51	-7.51	-2.51	2.49	7.49	12.49	9999
6/30/2005	14.0	1184.2	1.67	2.02	2.03	2.08	1.34	0.68	0.49
12/30/2005	14.3	1231.4	1.67	2.23	2.17	2.07	1.23	0.64	0.47
6/30/2006	13.9	1284.4	1.43	2.58	2.53	2.05	1.22	0.63	0.42
12/29/2006	13.9	1352.2	1.31	2.69	2.66	2.12	1.20	0.59	0.41
6/29/2007	14.1	1463.7	1.37	2.71	2.67	2.09	1.23	0.44	0.45
12/31/2007	20.3	1492.5	3.49	2.56	2.03	1.46	0.89	0.31	0.45
6/30/2008	22.2	1349.6	4.74	2.27	1.79	1.25	0.75	0.35	0.39
12/31/2008	31.4	1080.8	5.89	1.57	1.26	0.93	0.59	0.27	0.42
6/30/2009	33.2	845.0	5.77	1.37	1.10	0.82	0.53	0.25	0.41
12/31/2009	25.2	1052.0	4.41	1.93	1.46	1.14	0.65	0.31	0.39
6/30/2010	23.5	1109.1	4.09	2.20	1.63	1.27	0.72	0.31	0.35
12/31/2010	24.2	1152.3	4.27	2.16	1.64	1.30	0.70	0.29	0.35
6/30/2011	19.0	1328.1	3.30	2.83	2.22	1.84	0.81	0.32	0.31
12/30/2011	25.4	1233.4	4.50	2.07	1.63	1.28	0.66	0.26	0.34
6/29/2012	20.7	1359.5	3.73	2.75	2.08	1.53	0.73	0.27	0.31
12/31/2012	19.5	1413.5	3.58	2.97	2.14	1.65	0.75	0.30	0.31
6/28/2013	16.1	1569.4	2.80	3.68	2.62	2.19	0.85	0.35	0.27
12/31/2013	16.0	1735.2	2.76	3.68	2.66	2.25	0.86	0.36	0.27
6/30/2014	15.0	1880.4	2.47	4.05	2.94	2.26	0.87	0.33	0.25
12/31/2014	16.0	2008.5	2.66	4.10	2.84	2.08	0.83	0.29	0.26
6/30/2015	16.5	2070.6	2.82	4.23	2.83	1.83	0.80	0.27	0.26
12/31/2015	17.9	2033.3	3.20	3.87	2.60	1.59	0.79	0.27	0.28
6/30/2016	17.6	2032.2	3.04	4.11	2.67	1.42	0.79	0.27	0.28
12/30/2016	16.3	2179.4	2.54	4.72	2.92	1.60	0.82	0.27	0.27
6/30/2017	14.2	2370.8	1.99	5.45	3.33	2.04	0.89	0.30	0.24
12/29/2017	13.4	2559.6	1.58	5.88	3.74	2.17	0.92	0.27	0.22
6/29/2018	16.0	2713.3	2.43	5.40	3.09	1.67	0.88	0.28	0.25
12/31/2018	16.2	2768.4	2.47	5.17	3.03	1.80	0.91	0.28	0.26
6/28/2019	15.5	2827.1	2.21	5.57	3.14	1.67	0.90	0.27	0.26
12/31/2019	16.2	3048.8	2.33	5.82	3.26	1.55	0.83	0.19	0.27
6/30/2020	23.4	2970.2	4.20	4.69	2.32	0.89	0.67	0.16	0.31
12/31/2020	22.6	3463.7	4.22	5.41	2.38	1.13	0.67	0.15	0.30
6/30/2021	19.9	4030.2	3.50	6.42	2.86	1.53	0.73	0.17	0.27
12/31/2021	19.5	4527.3	3.40	6.89	3.01	1.35	0.73	0.14	0.27
6/30/2022	22.9	4244.9	4.49	5.07	2.31	0.94	0.69	0.24	0.31
12/30/2022	24.1	3910.4	4.70	3.78	1.96	1.07	0.68	0.23	0.34
6/20/2023	19.1	4149.0	3.37	5.32	2.72	1.17	0.77	0.19	0.29
Average	19.2	2135.8	3.20	3.79	2.44	1.60	0.83	0.31	0.32

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	Expected Returns on Arrow Securities						6 Month Averages		
Low	Average	Average	-9999	-12.5	-7.5	-2.5	2.5	7.5	12.5
High	VIX	S&P500	-12.5	-7.51	-2.51	2.49	7.49	12.49	9999
6/30/2005	14.0	1184.2	-40%	-50%	-51%	-52%	-25%	47%	102%
12/30/2005	14.3	1231.4	-40%	-55%	-54%	-52%	-19%	56%	111%
6/30/2006	13.9	1284.4	-30%	-61%	-60%	-51%	-18%	59%	136%
12/29/2006	13.9	1352.2	-24%	-63%	-62%	-53%	-16%	70%	144%
6/29/2007	14.1	1463.7	-27%	-63%	-63%	-52%	-19%	127%	123%
12/31/2007	20.3	1492.5	-71%	-61%	-51%	-31%	13%	221%	123%
6/30/2008	22.2	1349.6	-79%	-56%	-44%	-20%	33%	188%	157%
12/31/2008	31.4	1080.8	-83%	-36%	-21%	8%	69%	271%	139%
6/30/2009	33.2	845.0	-83%	-27%	-9%	22%	90%	301%	142%
12/31/2009	25.2	1052.0	-77%	-48%	-32%	-12%	53%	225%	159%
6/30/2010	23.5	1109.1	-76%	-55%	-39%	-21%	39%	218%	182%
12/31/2010	24.2	1152.3	-77%	-54%	-39%	-23%	44%	250%	186%
6/30/2011	19.0	1328.1	-70%	-65%	-55%	-46%	23%	208%	228%
12/30/2011	25.4	1233.4	-78%	-52%	-39%	-22%	51%	283%	191%
6/29/2012	20.7	1359.5	-73%	-64%	-52%	-35%	37%	270%	227%
12/31/2012	19.5	1413.5	-72%	-66%	-53%	-39%	34%	231%	223%
6/28/2013	16.1	1569.4	-64%	-73%	-62%	-54%	18%	186%	271%
12/31/2013	16.0	1735.2	-64%	-73%	-62%	-56%	17%	179%	272%
6/30/2014	15.0	1880.4	-59%	-75%	-66%	-56%	14%	204%	293%
12/31/2014	16.0	2008.5	-62%	-76%	-65%	-52%	20%	243%	278%
6/30/2015	16.5	2070.6	-65%	-76%	-65%	-45%	25%	273%	284%
12/31/2015	17.9	2033.3	-69%	-74%	-62%	-37%	27%	273%	253%
6/30/2016	17.6	2032.2	-67%	-76%	-63%	-30%	27%	273%	258%
12/30/2016	16.3	2179.4	-61%	-79%	-66%	-37%	22%	270%	276%
6/30/2017	14.2	2370.8	-50%	-82%	-70%	-51%	12%	235%	318%
12/29/2017	13.4	2559.6	-37%	-83%	-73%	-54%	8%	274%	358%
6/29/2018	16.0	2713.3	-59%	-81%	-68%	-40%	13%	261%	295%
12/31/2018	16.2	2768.4	-59%	-81%	-67%	-44%	10%	259%	288%
6/28/2019	15.5	2827.1	-55%	-82%	-68%	-40%	12%	270%	286%
12/31/2019	16.2	3048.8	-57%	-83%	-69%	-36%	21%	437%	270%
6/30/2020	23.4	2970.2	-76%	-79%	-57%	12%	50%	509%	220%
12/31/2020	22.6	3463.7	-76%	-82%	-58%	-12%	50%	553%	234%
6/30/2021	19.9	4030.2	-71%	-84%	-65%	-34%	37%	478%	273%
12/31/2021	19.5	4527.3	-71%	-85%	-67%	-26%	37%	593%	276%
6/30/2022	22.9	4244.9	-78%	-80%	-57%	7%	45%	322%	224%
12/30/2022	24.1	3910.4	-79%	-74%	-49%	-6%	47%	338%	193%
6/20/2023	19.1	4149.0	-70%	-81%	-63%	-14%	30%	422%	243%
Average	0.2	0.1	-69%	-72%	-59%	-38%	20%	224%	209%

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Daily Arrow Betas

Time Perio	Left Tail Sp	111% $90.00	105% $95.00	100% $100.00	95% $105.00	91% $110.00	87% $115.00	83% $120.00	Right Tail Spread
2005	-19.5	-8.2	-2.9	2.5	9.0	5.6	0.3	5.5	5.5
2006	-29.8	-12.6	-2.8	2.9	6.4	8.6	4.1	6.5	4.4
2007	-25.6	-7.7	-1.5	4.8	4.7	15.5	-2.8	*0.0*	3.5
2008	-2.3	3.2	3.3	3.1	2.9	2.8	2.8	2.5	5.8
2009	-4.8	1.2	2.5	1.8	0.1	2.3	2.8	3.0	2.9
2010	-8.2	3.3	4.9	5.2	7.5	2.1	3.8	3.1	2.9
2011	-7.2	3.5	5.1	5.8	3.7	3.2	3.5	3.2	3.1
2012	-20.6	-4.7	0.1	4.6	6.4	7.1	8.3	4.3	3.2
2013	-31.1	-17.3	-9.0	-2.7	3.0	5.8	7.0	7.0	4.8
2014	-28.3	-9.3	-0.9	5.8	8.4	9.1	8.1	4.5	3.5
2015	-15.4	3.7	7.7	9.6	6.4	8.5	-1.7	5.2	3.4
2016	-22.3	-2.5	3.4	8.9	9.4	5.6	6.1	1.1	3.2
2017	-40.8	-24.6	-14.5	1.8	7.9	11.0	14.4	6.0	3.5
2018	-21.1	-1.8	10.8	9.5	7.8	8.4	0.2	3.0	2.9
2019	-47.6	-30.8	-18.4	-11.3	0.8	10.9	12.6	6.5	4.1
2020	-4.9	1.8	4.8	2.7	4.9	2.8	2.2	2.2	3.0
2021	-28.9	-21.1	-12.3	-4.4	0.5	9.3	9.1	4.3	5.2
2022	-3.2	5.5	2.8	3.5	1.5	3.3	3.6	3.4	2.9
								*	
Average	-20.1	-6.6	-0.9	3.0	5.1	6.8	4.7	4.0	3.8

*$120 Excludes 2007 Outlier of -79.9

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