Central Bank Policy Impacts on the Distribution of State Prices for Future Interest Rates, 2003–2022

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KEY FINDINGS

- Identified combinations of butterfly spreads and tail spreads of interest rate caplets and floorlets can be used to extract discrete, nonparametric state prices ("risk neutral densities") from prices of interest rate caps and floors. A state's price reflects both the state's probability and the relative expected marginal utility of consumption in the state.
- The method presented shows that central banks in the USA, the Euro zone and the UK have dramatically affected not just levels of rates, but entire state price distributions. At major interventions, distributions have shifted from symmetry to skewness, and then back to relatively uniform distributions.
- Sign changes in correlations of interest rates with stock market moves do occur. They predictably affect the biases in using implied state prices to estimate objective probability distributions for interest rates. Biases should depend upon the consumption betas of the various states' prices.

ABSTRACT

In this article, we extend the 1978 Breeden–Litzenberger method of extracting state prices from option prices, showing how portfolios of butterfly spreads can be combined with right and left tail spreads to nonparametrically extract discrete state prices from option prices. We derive how those state prices should be biased estimates of true, objective probabilities. For interest rate options, we show that the biases can vary predictably over time (sometimes too high, sometimes too low), as the correlation of interest rates with consumption and wealth has changed signs over time. Consumption betas and proper risk premiums on bonds and of their state prices are at times predictably positive and at times predictably negative. We apply our technique to provide a brief 20-year history of central bank intervention impacts in the US, UK, and Eurozone from 2003 to 2022. Movements in state prices are quite large in the Financial Panic of 2008–2009, as well as in the European Sovereign Debt Crisis of 2010–2013, with Brexit and the Trump elections in 2016, and with the coronavirus pandemic in 2020–2021. Tapering in 2013 and 2022 and liftoffs in rates in 2015 and 2022 were shown to strongly shift state price distributions back toward the symmetry of 2003–2007. We show that central banks dramatically impacted entire state price distributions, not just levels of rates.

Since the 1990s, central banks have estimated state price distributions (also called option-implied risk-neutral probability distributions) for interest rates and inflation using the technique of Breeden and Litzenberger (B-L; 1978).[1] Central bank applications by the Bank of England are discussed in articles by Bahra (1996, 1997); Clews, Panigirtzoglou, and Proudman (2000); and Smith (2012). In the US, Malz (1995, 1997) of the Federal Reserve Board of New York and Durham (2007), and Kitsul and Wright (2013) of the Federal Reserve in Washington used the B-L technique. Kocherlakota's (2013) research group at the Federal Reserve Bank of Minneapolis used Shimko's (1993) interpolation method of applying the B-L formula to regularly estimate and publish risk neutral density functions and tail risks for many assets, such as stocks, crude oil, wheat, real estate, and foreign exchange. The European Central Bank (ECB; 2011) also used the technique to analyze movements in the financial crisis of 2008–2009, as did Figlewski and Birru (2012) and Zitzewitz (2009).

For many years, market participants used the Black-Scholes (1973) formula to estimate values for options on interest rates, using the assumption that the short-term interest rate itself was lognormally distributed. Others modeled rates with a normal distribution, using Vasicek's (1977) model, which permits negative interest rates. A typical Wall Street or Central Bank application estimated the volatility surface over time and across strike prices for an underlying asset, which then is used with the Black-Scholes or Vasicek formula to give the option pricing function. Volatility estimation is parameterized so as to give a thrice-continuous option pricing function, from which the second derivatives can be estimated and used as state prices and implied risk neutral densities. Many have written about alternative ways to estimate the volatility surface to replicate option prices, with Shimko's (1993) work as most significant. Figlewski (2018) provides a selective review of such works.

However, market prices are not constrained by Black-Scholes or Vasicek pricing functions and presumably reflect traders' true assessments of entire probability distributions for future interest rates, as well as the supply-and-demand equilibrium risk adjustments for hedges of those rates.[2] In this article, we extend the B-L technique and provide a tool to calculate discrete state prices that are nonparametric and may have any shape of distribution.[3] We show how the triangular payoffs of butterfly spreads of options aggregate in portfolios to trapezoidal payoffs, and how tail spreads of options can complete the portfolio and give riskless payoffs. We derive that when the risk neutral density is a linear function of interest rates within the range of the butterfly spread, the price of a digital ($1/0) option is equal to the price of the butterfly spread. For example, this means that the price of a $1 payoff if the short rate is between 3.5% and 4.5% is given by the cost of the 3/4/5 butterfly spread, a very useful result.

One shortcoming of some prior applications is the use of short-term options prices, as short maturities are most heavily traded. However, state price distributions for short rates in 3 months are often not very interesting, either with rates staying the same or up or down .25%. Our approach is different: we use long maturity interest rate cap and floor values fitted by Bloomberg from traded options prices.[4] By calculating the price difference between a longer maturity and a shorter maturity cap, a "caplet,"

[1] B-L built on Ross (1976), Cox and Ross (1976) and Merton (1973).

[2] Several respected authors such as Jackwerth and Rubinstein (1996), Aït-Sahalia, Wang and Yared (2001), Ross (2015) and Martin (2017) attempt to extract objective probabilities from option prices. However, as demonstrated by Carr and Yu (2012) and in this article (Results 3 & 4), such attempts cannot completely succeed in realistic economics due to proper risk adjustments needed to capture correlations with marginal utility movements.

[3] This differs from the nonparametric estimates of Ait-Sahalia and Lo (1998), Ilinski (2001), and Li and Zhao (2006), who estimated state price densities with continuous distributions.

[4] See Longstaff, Santa-Clara and Schwartz (2001) for valuation relationships for caps and floors with swaptions.

we create forward caps for the 3rd and 5th years, as well as years 8–10. We then use butterfly spreads of these forward caps to derive the arbitrage prices for triangular payoffs for various possible interest rate ranges and maturities of 3, 5 and 10 years.

We apply our method to Bloomberg data for the 20-year period from 2003 to 2022, using Bloomberg data for the US, the Eurozone, and the UK. The implied state prices from our technique are sensible and reflect key policy moves made by the Federal Reserve, the ECB, and the Bank of England. We analyze certain major moves with our new tool:

1. In the US Financial Panic of 2008/2009 (the "Great Recession"), real estate fell by 30% (50% in some Sunbelt areas such as California, Arizona, and Florida). Stock prices fell by more than 50%, and household net worth tumbled 25%. Credit risk and loan defaults surged. The US unemployment rate went from 4% in 2006 to 10% in 2009/2010, the peak level since the Great Depression of the 1930s. The US Federal Reserve Board, led by Chairman Ben Bernanke, reduced short-term interest rates to near zero in December 2008 in providing liquidity and stimulating the economy. Our technique shows that the state price distribution went from symmetry to a strong positive skewness. The Fed kept rates near zero for more than 7 years. The US economy steadily recovered, and the unemployment rate dropped to a postwar low of 3.5% in 2019.
2. Similarly, the ECB also dramatically provided liquidity and reduced rates during Europe's Sovereign Debt Crisis in the 2010–2013 period, with peaks in uncertainty in the summers of 2011 and 2012, as markets feared default in Greece (and possibly even Italy and Spain) would tear apart the Euro currency. ECB President Mario Draghi took bold action and said on July 26, 2012, that they would do "whatever it takes to preserve the Euro," continuing with "And, believe me, it will be enough" (*Financial Times* July 27, 2012)." Greece recovered impressively, and Spain's unemployment rate dropped from a peak of 26% down to 14%. Central Bank stimulus, with rate cuts and bond purchases (called "quantitative easing"), worked quite well in the Eurozone, transforming the state price distribution.
3. As US housing prices and stock prices strengthened, in 2013 Chair Bernanke announced plans to "taper" Fed asset purchases, which shook markets at first but then stabilized. Tapering started in early 2014. Fed Chair Janet Yellen followed with a first rate increase in December 2015, after strong job market growth and an unemployment rate down to 4.9%. From 2015 to 2019, the unemployment rate continued to drop to 3.5%, the lowest in 50 years, and the Fed raised rates nine times to a peak of nearly 2.25% for 3-month Treasury bills. State price distributions moved to a more symmetric shape.
4. In the UK in 2016, given a strong economy, the Bank of England, led by Governor Mark Carney, had begun to raise rates toward long-term averages. However, when Brexit was voted in on June 23, 2016, fears of slowing or contracting real GDP growth led the Bank of England to reduce rates to stimulate the UK economy. Within 2 months, we show that prices of payoffs for the "fear scenario" of very low rates (< 1.5%) increased by 40%, whereas state prices for 2% and 3% rates in 3 years decreased by 60%.
5. In the US in 2017, President Trump stimulated the economy with a large corporate tax cut from 35% to 21%. Interest rates rose, Fed Chair Yellen raised the short rate substantially, and the price of the low rate state dropped by 80%. Prices for 3% and 4% states jumped by 300% or more. When Trump started trade wars with China and Mexico, worries of a weakening economy sank stock prices and dropped rates in late 2018. In 2019, Fed Chair Jay

Powell then led rate cuts, attempting to avoid a recession. The price of the left tail, low rate state jumped from 7 cents to 40 cents per $1.00.

6. The Coronavirus Pandemic began with an outbreak in China in December 2019, which spread to Europe, the United States, Latin America, India and Africa in January–February 2020. In the USA, there was a massive surge in cases in March 2020, fear spiked, and stock markets tumbled. The VIX index volatility rose from 20% to 80%. US real GDP growth tumbled at an annual rate of over 30% in Q2 2020, as travel halted and consumers largely stopped shopping at retail stores. It was truly a global pandemic, causing deep recessions around the world in 2020. The US unemployment rate jumped from 3.5% to more than 14% in mid-2020. The US Fed under Chair Jay Powell immediately took the short rate back down to zero and began massive purchases of mortgages, Treasuries, and corporate bonds—a quantitative easing of several trillion dollars. The Fed's balance sheet ballooned from $4 trillion to $9 trillion in 2022. These moves showed up as major moves in state price distributions for interest rates, as we show.
7. In late 2021 and early 2022, with reopenings, the US economy strengthened, and inflation surged to the highest levels in 40 years. Chair Powell's Fed began tapering asset purchases in January 2022 and "lifted off" from zero rates in March. State price distributions shifted significantly again, this time to a relatively uniform distribution.

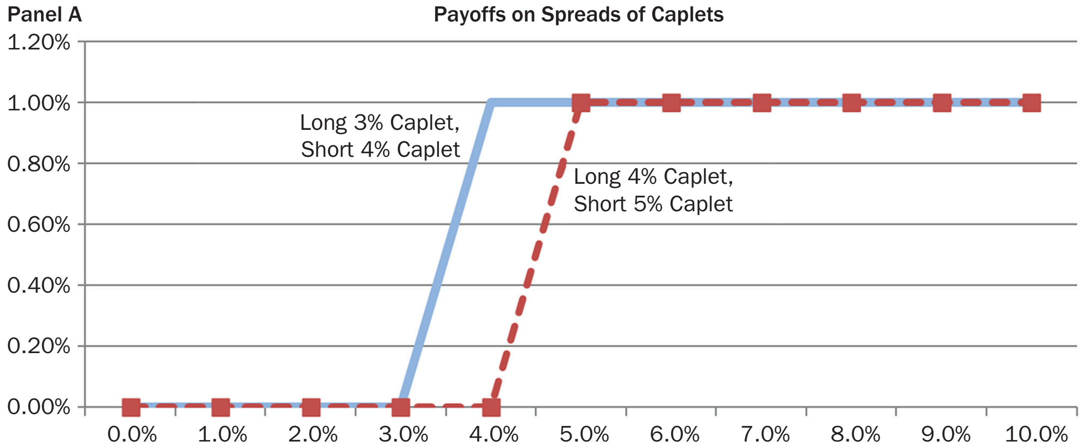
Our article flows as follows: The next section, Result 1, has a geometric presentation of our technique. The Result 2 section provides the proof that links butterfly spread costs to digital option values. The Result 3 section derives predicted biases of state prices as forecasts of objective probabilities. The Result 4 section shows changes in the sign of systematic risks of bonds, which should cause changes in risk premiums and biases of state prices as forecasts of true probabilities. The Result 5 section presents graphic analysis for 2003–2013 using our tool for US state price distributions implicit in option prices. The Euro Area Rates section presents corresponding graphic analysis for the Eurozone for 2003–2009. The subsequent section shows the dramatic moves in Eurozone state price distributions during the Sovereign Debt Crisis of 2010–2013. The next section covers the 2015–2019 period, when the US lifted off and raised rates. Brexit was passed, and President Trump regularly moved markets. The following section covers the coronavirus pandemic of 2020–2022, the massive stimulus by central banks around the world, the surge in global inflation, and the beginning of tapering and preparation for liftoff again. The final section provides concluding remarks.

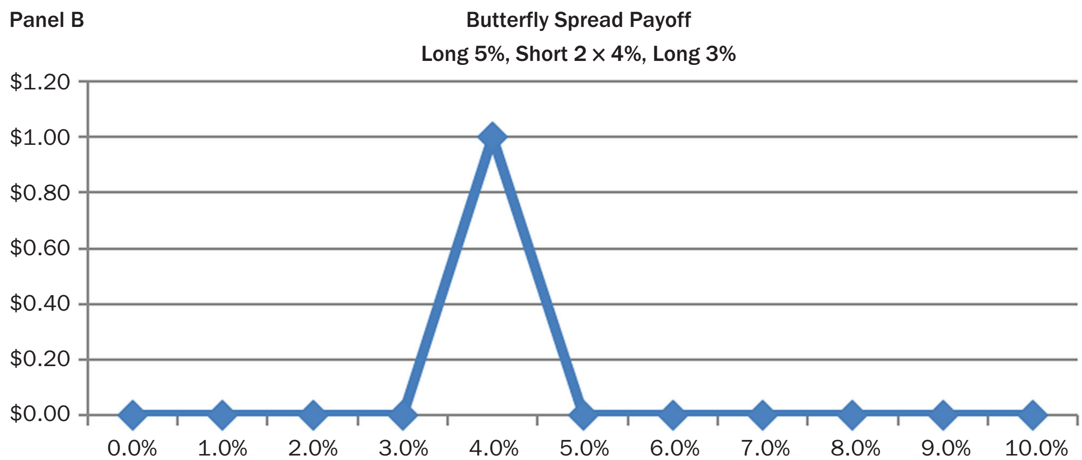
RESULT 1: TRIANGLES, TRAPEZOIDS AND TAIL SPREADS: DECOMPOSING OPTION PAYOFFS

In this section, we derive a method for finding the distribution of "state prices" (normalized to sum to 1.0) or, equivalently, the "risk neutral density" from option prices for an asset with a continuous distribution of payoffs. This follows the widely used B-L (1978) technique but considers assets with continuous payoffs rather than discrete ones. Although the technique is quite general and can be used with stocks and bonds and other assets, we use as an illustration interest rate caps and floors, which are options on interest rates. The distributions computed later in the article allow us to see central banks' policy impacts on the distributions of state prices for interest rates.

EXHIBIT 1

Spread and Butterfly Spread Payoffs





A 5-year cap or floor makes quarterly insurance payments over the 5-year period, based on what the short rate is each quarter. A typical cap pays off max [R-X,0], where R is the 3-month interest rate and X is the strike rate, so caps win increasingly as rates increase and bond prices fall, much like portfolios of put options on bond prices. Floors pay off max (X-R,0), so they win as rates drop and are like portfolios of call options on bond prices. A portfolio that is long a 5-year cap and short a 4-year cap for the same strike rate and notional principal would receive payments only in year 5, as the net cash flows for years 1–4 would all be zero. This portfolio is called a "caplet" or a "forward cap" for year 5. A cap is a portfolio of quarterly caplets. "Floorlets" are defined correspondingly as time spreads of floors with the same strike.

B-L (1978) demonstrated with discrete underlying asset prices that "butterfly spreads" (spreads of spreads) of options create unit payoffs in certain states of the world. If one allows the underlying state variable to have continuous values at the future payoff date (e.g., 5 years), then spreads of caplets have payoffs as in Exhibit 1A. The 3%–4%–5% butterfly spread produces a triangular payoff in Exhibit 1B that starts at zero for all rates below the lower strike rate (3%), then increases linearly and peaks at $1.00 at the middle strike rate (4%) and then declines linearly back to zero at the upper strike rate (5%) and remains zero for all higher rates.

Thus, the butterfly spread is a bet that pays off if rates are between the lower (3%) and upper (5%) rates of the spread, with a peak payoff of $1 in the middle (4%).

In equilibrium, informally stated, B-L (1978) showed that the value of such a payoff should depend upon the probability of being in that rate range, multiplied by the conditional expectation of the marginal utility of $1 of consumption if that state range occurs, relative to the marginal utility of $1 of consumption today. We call the cost of the 3%–4%–5% butterfly spread the "*state price* for a $1 payoff if the 3-month rate is 4% in 5 years." By arbitrage, it would have to be that in equilibrium, as these payoffs can be constructed by this cap portfolio for that cost. Thus, state prices reflect both probability and risk adjustments, in that payoffs that occur in bad economies, when consumption is low, have higher values because of higher marginal utilities then.

If one purchased a portfolio of seven butterfly spreads with strike rates centered on 2% to 8%, respectively, the butterfly spreads would have overlapping payoffs, as illustrated in Exhibit 2A, which gives a total payoff pattern that is a trapezoid, as shown in Exhibit 2B.

A riskless $1.00 payoff is created starting with the trapezoid of Exhibit 2B (a portfolio of butterfly spreads) and then "completing it" by adding on complementary tail spreads for the left and right tails. A spread that is long a 2% floorlet and short a 1% floorlet gives the complementary payoff pattern for the left tail, as indicated in Exhibit 2C. For the right tail, one uses a spread of caplets that is long an 8% and short a 9% to get the complementary tail payoff. Exhibit 2C shows the payoffs on the two portfolios of floors and caps that give the "tail spreads" for left and right tails. Exhibit 2D shows how these tail spreads combine with the trapezoid of butterfly spreads to give a riskless $1.00 payoff.

To summarize, we have shown that the portfolio of seven butterfly spreads (centered on strike rates from 2% to 8%), plus the left tail spread of floorlets (with strikes of 2% and 1%), plus the right tail spread of caplets (with strikes of 8% and 9%), gives a riskless payoff of $1.00. As a result, to prevent arbitrage, the cost of this "complete portfolio" must be the cost of a zero coupon bond for this maturity. We can then divide each component of the complete portfolio by the total and get fractions that are normalized state prices, or the risk neutral probability density.

To illustrate this, in Exhibit 3, we take the prices for 5-year and 4-year caps and floors on December 31, 2012, and compute the butterfly spreads of the time spreads between the 5-year and 4-year securities. As option trading is heavier and more liquid in cheaper options, we use floorlet prices for 1% to 3% butterflies, caplets for 7% and 8%, and an average of caplets and floorlet butterflies for 4% to 6% centers. We use the floorlet and caplet prices to compute the costs of the left and right tail spreads.

Thus, if the price of each component of our earlier "complete portfolio" of caplets and floorlets is divided by the riskless bond price (the sum of the portfolio's component prices), these normalized prices give the integrals of the "risk neutral density" over 1.0 percentage ranges centered on the midpoints of the butterflies, and the histogram of risk-neutral probabilities will sum to 1.0. The risk-neutral density for date T is given by the state price distribution for the butterfly spreads and tail spreads at T, normalized by dividing by the sum of those state prices for date T as in Exhibit 3. We use the phrases "risk-neutral density" and "state prices" synonymously, as they are proportional and have the identical shape as a distribution across rate levels.

RESULT 2: PRICES OF BUTTERFLY TRIANGULAR PAYOFFS EQUAL "DIGITAL OPTION" PRICES

In this brief section, we derive the very useful and intuitive result that, with a linear approximation for the risk neutral density, prices of butterfly spreads with triangular payoffs should be equal to the prices of "digital options" that have a rectangular $1.00 payoff in the middle of the same range. Within the range of rates for a butterfly

EXHIBIT 2

Butterfly Spread Payoff

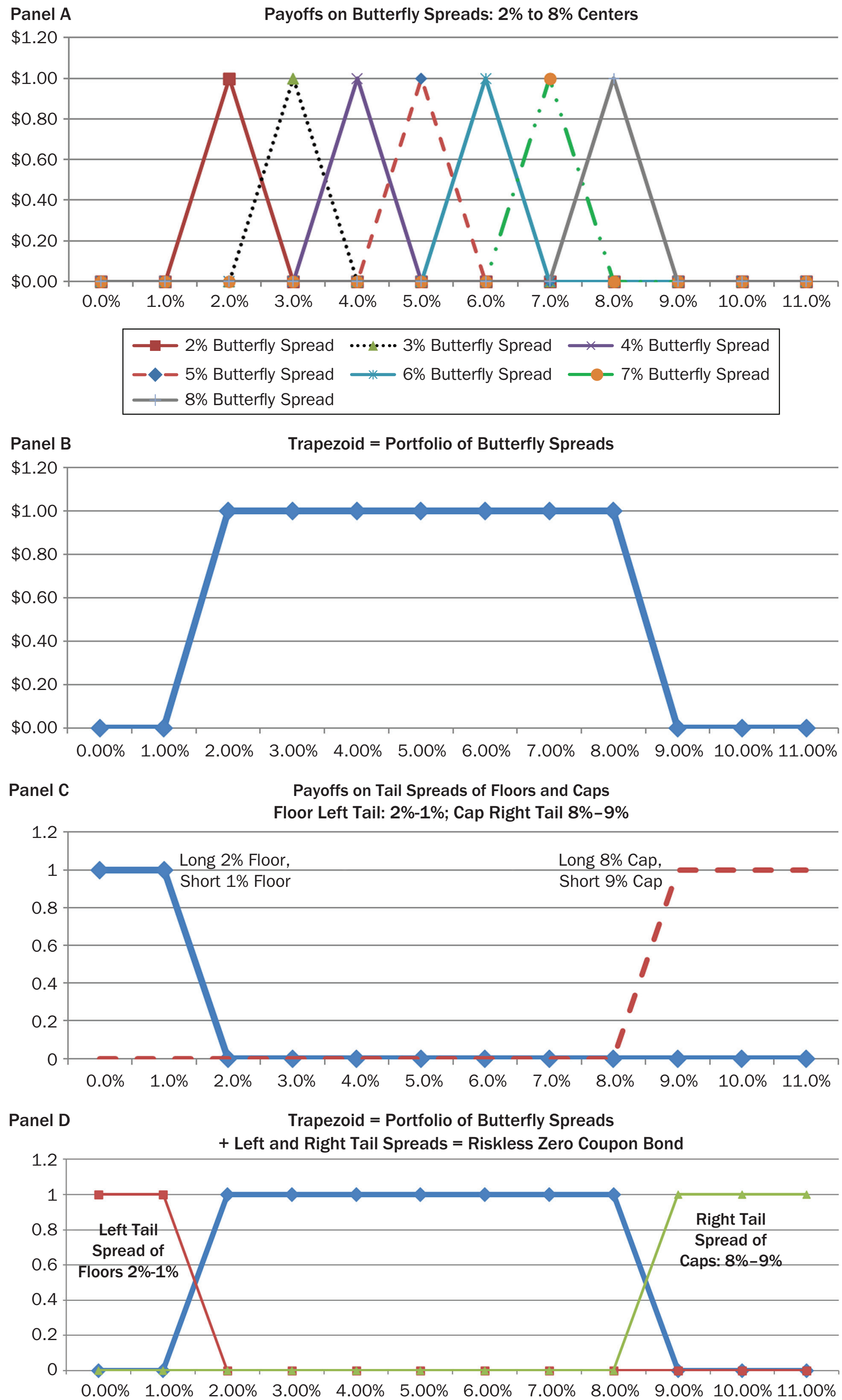


EXHIBIT 3

State Price Density Computed from Butterfly Spreads and Tail Spreads

	Spread Cost	"State Price Density" or "Risk-Neutral Probability"
"0%" = Left tail spread: Long 1%, Short 0% floorlet	$0.290	0.297
1% Butterfly spread (Long 0%, Short 2 1%, Long 2%)	$0.320	0.328
2% Butterfly spread (Long 1%, Short 2 2%, Long 3%)	$0.180	0.184
3% Butterfly spread	$0.080	0.082
4% Butterfly spread	$0.037	0.038
5% Butterfly spread	$0.028	0.028
6% Butterfly spread	$0.014	0.014
7% Butterfly spread	$0.007	0.007
8% Butterfly spread	$0.007	0.007
9%+ = Right tail spread: Long 8%, Short 9% caplet	$0.015	0.015
Total	$0.977	1.000

spread (e.g., from 3% to 5%), the risk–neutral density will not be constant, as the true probability density likely changes throughout the range, as do the conditional expected marginal utilities of $1.00. However, as a first approximation, let us assume that the risk neutral density changes *linearly* with the interest rate inside of the range. With that linear approximation, we can prove the following Proposition:

Proposition: The relationship between butterfly spread values and digital option values:

If the risk-neutral density (RND) is a linear function of the interest rate within the range of the butterfly strikes, then the value of a digital option that pays off $1.00 over the middle half of the range is equal to the value of the butterfly.

Proof: Let x be the interest rate, such that $x = c$ at the lower strike of the butterfly, $x = c + 1$ at the mid-point strike of the butterfly, and $x = c + 2$ at the high strike of the butterfly.

Assume that between c and $c + 2$ the risk-neutral density = RND = $a + b(x - c)$

The forward value of a digital option that pays off $1.00 between $x = c + 0.5$ and $x = c + 1.5$ is:

$$\int_{c+0.5}^{c+1.5} [a + b(x - c)] \cdot 1\, dx = a + b$$

The forward value of a butterfly is $\int_{c}^{c+1} \{[a + b(x - c)](x - c)\}\, dx + \int_{c+1}^{c+2} \{[a + b(x - c)](c + 2 - x)]\}\, dx$

$$= \frac{1}{3}bx^3 + \frac{1}{2}(a - 2bc)x^2 + (bc^2 - ac)x\,\Big|_{c}^{c+1} - \frac{1}{3}bx^3 + (bc + b - \frac{1}{2}a)x^2 + (2a - 2bc - bc^2 + ac)x\,\Big|_{c+1}^{c+2} = a + b$$

Of course, because forward values are equal at the same date, present values are also equal.

Q.E.D.

From the Proposition, under the assumption that the risk neutral density is linear in interest rates between 3% and 5%, the price of the 3%–4%–5% butterfly (with triangular payoffs) would be identical to the price of a claim that paid off $1.00 when interest rates were between 3.5% and 4.5% (digital payoffs). The 4%–5%–6% butterfly would have a value equal to that of a digital option that pays off between rates of 4.5% and 5.5%, and so on.

RESULT 3: STATE PRICES HAVE BIASES AS ESTIMATES OF "TRUE PROBABILITIES"

Although it is tempting to think of levels and shifts of state prices or risk neutral densities as predominantly affected by levels and shifts of the true probabilities of future interest rates, that is not necessarily (or even normally) true. Marginal utilities affect state prices, in addition to true probability shifts. In this section, we present the theoretical relationship between true probabilities and state prices, both in a state preference model and the special case of a constant relative risk aversion utility function, combined with lognormally distributed consumption.

The basic time-state preference model used is the same as used by B-L (1978). Their result was that each state price, ϕ_{ts}, should be proportional to its probability, π_{ts}, but multiplied by the marginal utility of the state, u'_{ts} divided by the marginal utility of $1 today:

$$\Rightarrow \phi_{ts} = \frac{\pi^k_{ts} u'^k_{ts}}{u'^k_0} = \text{price of \$1 in time-state ts.} \tag{1}$$

So if ordered from high to low, price/probability ratios are positively and monotonically related to marginal utilities, and negatively related to consumption across states. The risk-neutral probability for a state is higher, the higher the probability of the state and the higher the marginal utility in state (the lower the consumption in the state).

Our analysis has been for price/probability ratios for general states of the world. Can we say anything about prices of claims that pay off if the 3-month rate is, say, r_j, where $\{j = 1, \ldots N\}$ represents different possible interest rate levels at time t? Note that, in general, the interest rate could end up at the same level in a variety of different states. We have derived the following (proof available upon request):

$$\frac{\phi_{tr_j}*}{\pi_{tr_j}} = \frac{E\left[u'_{ts} \middle| r_j\right]}{E[u't]} \tag{2}$$

Thus, we see that the state price to true probability ratio for r_j is equal to the expected marginal utility of consumption in that state, conditional upon the interest rate being at the specified level, divided by the unconditional expected marginal utility of consumption at that date, time t. So if we are looking at butterfly spreads or digital options centered on $R = 2\%$, we need to compute the conditionally expected marginal utility of consumption, given that 2% rate.

To gain further specific, numerical insight into the potential fluctuations in risk neutral/true price to probability ratios, we make two further assumptions, one on preferences and one on distributions: (A1) constant relative risk aversion (CRRA) utility and (A2) lognormally distributed consumption.

A1: If we assume power utility:

$$\text{Let } u^k_t(c^k_{ts}) = \frac{e^{-\rho t}(c^k_{ts})^{1-\gamma}}{1-\gamma}; \text{ then } u'^k_{ts} = e^{-\rho t}(c^k_{ts})^{-\gamma} \text{ and } RRA = \frac{-u''c}{u'} = \gamma \tag{3}$$

Suppressing the k superscript:

$$\frac{\varphi^*_{ts}}{\pi_{ts}} = \frac{u'_{ts}}{E^k(\tilde{u}'_t)} = \frac{e^{-\rho t} c^{-\gamma}_{ts}}{E[e^{-\rho t}\tilde{c}^{-\gamma}_t]} = \frac{c^{-\gamma}_{ts}}{E[\tilde{c}^{-\gamma}_t]} \tag{4}$$

A2: If we assume consumption is lognormally distributed:

$$\text{Note: Lognormal } Y \equiv e^x \text{ where } x \sim N(\mu, \sigma^2) \Rightarrow E(\tilde{Y}) = e^{\mu + \frac{1}{2}\sigma^2} \tag{5}$$

$$\text{Assume lognormal } c_{ts} = c_0 e^{g_{ts} t}, \text{ where } \tilde{g}_t t \sim N(t\mu_t, \sigma_t^2 t) \Rightarrow -\gamma\tilde{g}_t t \sim N(-\gamma\mu_t t, \gamma^2\sigma_t^2 t) \tag{6}$$

$$\frac{\phi_{ts}^*}{\pi_{ts}} = \frac{c_{ts}^{-\gamma}}{E[\tilde{c}_t^{-\gamma}]} = \frac{c_0^{-\gamma} e^{-\gamma g_{ts} t}}{E[c_0^{-\gamma} e^{-\gamma \tilde{g} t}]} = \frac{e^{-\gamma g_{ts} t}}{E[e^{-\gamma \tilde{g} t}]} = \frac{e^{-\gamma g_{ts} t}}{e^{-\gamma\mu_t t + \frac{1}{2}\gamma^2\sigma_t^2 t}} \tag{7}$$

Taking logs of both sides and simplifying, we get

$$\log\left(\frac{\phi_{ts}^*}{\pi_{ts}}\right) = \gamma\left[\mu_t - g_{ts} - \frac{1}{2}\gamma\sigma_c^2\right]t \tag{8}$$

As expected, higher growth states for consumption have lower $\left(\frac{\phi_{ts}^*}{\pi_{ts}}\right)$ ratios. One could input different estimates of relative risk aversion and different states' growth rates and consumption volatility into Equation (8) and estimate the log of the risk neutral probability to the true probability.

Returning to the general case (not assumed lognormal or CRRA), Equation (2) shows that the state price (risk neutral probability) for a certain rate level will exceed (be less than) the true probability if marginal utility, conditional upon that rate level, exceeds (is less than) the unconditional expected marginal utility for that date. This would typically be true when real consumption growth is less (more) in the state than it is expected to be on average for that date.

RESULT 4: CHANGING REAL BETAS FOR NOMINAL BONDS 1962–2021 MEANS CHANGING BIASES OF INTEREST RATE STATE PRICES AS FORECASTS OF PROBABILITIES

In this section, we derive equilibrium relationships between state prices and true probabilities and show how changing betas of nominal bonds affect this relationship. The changing betas of nominal bonds were identified by Breeden (1986, 32–33):

> Since inflation is typically believed to be related to the growth rate of real consumption, the risk premium of the nominally riskless asset may be non-trivial. The relation of inflation to the real growth of the economy may be nonstationary. … If a Phillips curve relates inflation and unemployment (pre-1973), then inflation is likely to be high when real consumption is high, resulting in a negative real consumption beta for the nominally riskless assets. … In contrast, recent experience (see Fama 1982) has been that inflation is negatively related to real movements in the economy. If that were expected, then the real consumption betas for nominally riskless assets are positive, which results in equilibrium real returns on them that are in excess of those on purchasing power bonds.

Thirty years later, Campbell, Sundarem, and Viceira (2017) presented a very sensible model of changing correlations of inflation with the macroeconomy, based on changing Federal Reserve policy response functions. Both Breeden's (1986) and Campbell, Sundarem, and Viceira's (2017) results show that the state prices for interest rates will be biased estimates of probabilities of interest rates, with the direction and extent of the biases (some positive, some negative) depending on the sign and magnitude of the consumption beta for nominal bonds, which changes over time. Positive consumption beta securities will have lower prices, and state prices will be biased low estimates of true probabilities. Negative beta securities, such as $1 payoffs if rates are below 1% (hedges of a bad economy), will have high prices and their state prices will be biased high as estimates of true probabilities.

Next, let's consider issues related to the fact that our interest rate cap and floor prices are based on nominal interest rates, not real rates. Of course, marginal utilities are based on real consumption, not nominal consumption. The relation between inflation and real economic growth is an unstable one over time. In the big recessions in 1974–1975 and 1981–1982, we had high inflation and high nominal interest rates (giving negative realized bond returns) at times of recession (with stocks and real consumption negative), which would indicate positive real consumption and stock market betas for holders of long-term nominal bonds. We would characterize this as "supply-oriented inflation," as it was a situation of high inflation caused by constrained supplies of oil and grains.

In contrast, in recent years (2007–2021) of the Financial Panic of 2008/2009 and the steady recovery, followed by the coronavirus pandemic hits in 2020–2021, market participants were well aware of the very positive correlation of daily interest rate changes with the stock market. The logic is that aggregate demand issues are the dominant inflation risks, and higher stock prices are viewed as leading better economic growth, which would give higher inflation and interest rates. Thus, we have in these recent years 2000–2021 the negative real consumption and stock market betas for nominal long-term bonds, in contrast to the positive betas in the 1974–1975 and 1981–1982 recessions. This recent relationship is consistent with the "flight to quality" reactions that drive up nominally riskless bond prices on days when the stock market plummets.

To verify that the betas of nominal bonds have indeed changed signs over the years, as supply and demand inflation uncertainties alternate in volatility, we gathered daily data on nominal interest rates from the Federal Reserve's website for 1962 to December 2021. We also gathered daily index prices for the Standard & Poor's 500 from the Yahoo! Finance's website, which provides them back to 1950. Using windows of 3 months or 6 months of daily data, the graphic results are very similar, so only the 6-month window data are shown here. Exhibit 4 shows the moving window regression betas of daily changes in nominal rates with daily percentage changes in the S&P 500 from 1962 to 2021.

Betas for bond returns will be opposite in sign from those for rates. The negative correlations and betas for rates in the 1970s and 1980s correspond to positive betas for nominal long-term bonds relative to the S&P 500 and relative to real consumption growth. The positive correlations of rates with stock returns in 2000–2021 demonstrates the negative betas for long-term nominal bonds, which reflect their flight to quality appeal. Recent interest rate beta estimates averaged approximately 0.03, which would imply that a 10% increase in stock prices is associated with a 30 basis points move in the 10-year interest rate, which is not implausible.

The fact that the betas of interest rates and bond returns are changing over time leads us to conclude that the ratios of state prices (risk neutral probabilities) to true probabilities should also change, depending upon which regime we are in. If we are in a situation such as that of the 2007–2020 period, then low interest rates (0%, 1%, or 2%) are associated with low real growth forecasts and high conditional marginal utilities. The normalized state prices for those low-rate states should exceed true probabilities, and the reverse should be true for states with high interest rates (perhaps 6%–9% rates). If we return to a situation where high inflation is associated with very poor economic growth and poor stock returns (as in the 1970s and 1980s), then the pattern would reverse, and very high rate states would have high ratios of

EXHIBIT 4

Stock Market Regression Betas of Daily Changes in Nominal Rates

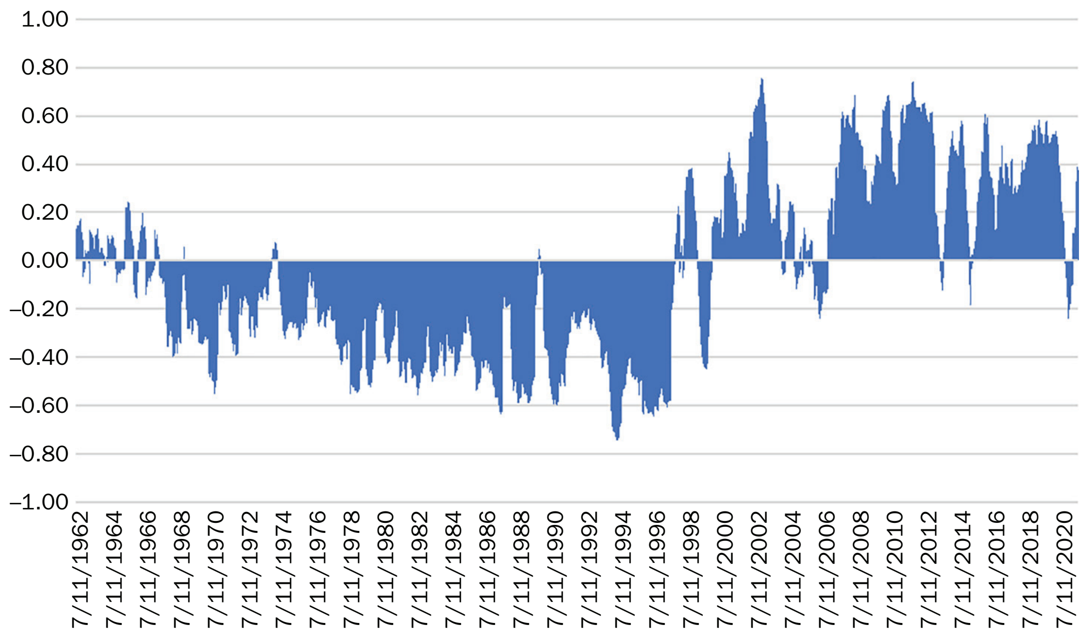


EXHIBIT 5

Illustration of True Probabilities Related to Risk Neutral Probabilities

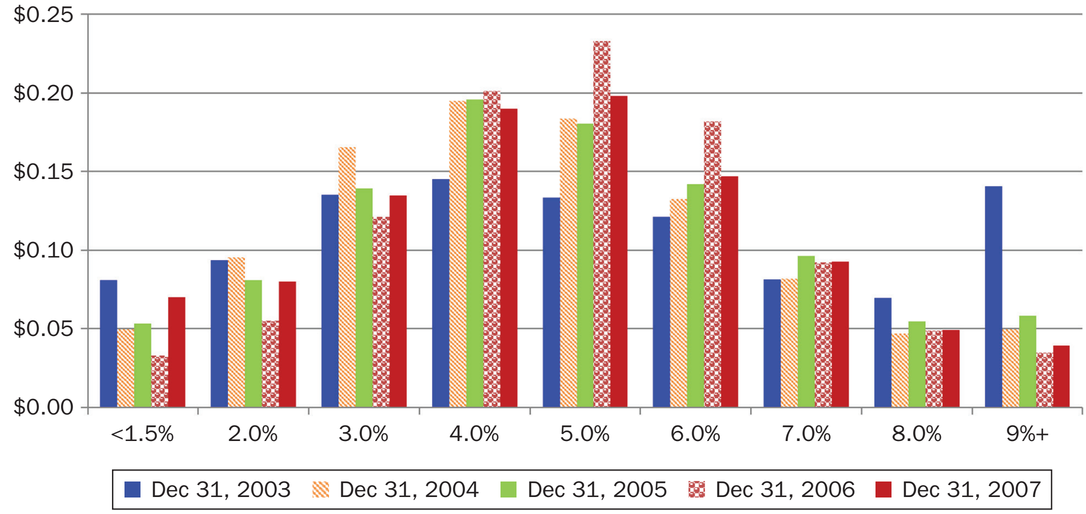
True probability = *K**Risk Neutral x exp(Gamma*(gts – mu))					Assumes: CRRA-Lognormal real growth model				
Real Growth on Nominal Rate: 1998 to 2011 Data					Real Growth on Nominal Rate: 1977 to 1997 Data				
Intercept	–3.71 (*t* = –2.2)				Intercept		4.11 (*t* = 3.2)		
Slope	1.42 (*t* = 3.8)				Slope		–0.12 (*t* = –0.8)		
MuCgrowth	3				MuCgrowth		3		
Relative Risk Aversion (Gamma)					**Relative Risk Aversion (Gamma)**				
		Ratio of True Probability to Risk Neutral*					**Ratio of True Probability to Risk Neutral***		
Nominal Rate	**Real Growth**	**2**	**4**	**8**	**Nominal Rate**	**Real Growth**	**2**	**4**	**8**
1	–2.29	0.90	0.81	0.65	1	3.99	1.02	1.04	1.08
2	–0.87	0.93	0.86	0.73	2	3.87	1.02	1.04	1.07
3	0.55	0.95	0.91	0.82	3	3.75	1.02	1.03	1.06
4	1.97	0.98	0.96	0.92	4	3.63	1.01	1.03	1.05
5	3.39	1.01	1.02	1.03	5	3.51	1.01	1.02	1.04
6	4.81	1.04	1.08	1.16	6	3.39	1.01	1.02	1.03
7	6.23	1.07	1.14	1.29	7	3.27	1.01	1.01	1.02
8	7.65	1.10	1.20	1.45	8	3.15	1.00	1.01	1.01
9	9.07	1.13	1.27	1.63	9	3.03	1.00	1.00	1.00
10	10.49	1.16	1.35	1.82	10	2.91	1.00	1.00	0.99

state prices to true probabilities. With the Russia–Ukraine war, this could be happening in 2022, reversing risks.

Exhibit 5 shows an illustration of how true probabilities should be related to state prices if one made the CRRA-Lognormal model assumptions, which yielded Equation (8). Two alternative regimes are estimated: the 1998–2011 recent period, in which nominal interest rates were positively correlated with real consumption growth, and

EXHIBIT 6

US State Prices for 2003 to 2007: Relatively Symmetric Distributions



the 1977–1997 period, in which interest rates were negatively related to stock returns and real consumption growth. The higher the amount of relative risk aversion, the higher the premium paid for insurance against low consumption growth. In those situations of low consumption growth, the relatively high-risk neutral probabilities must be scaled down to get proper estimates of true probabilities. For good scenarios that have high real consumption growth, risk neutral probabilities are low, because of low marginal utility, and must be scaled up more to properly estimate true probabilities.

Given the slightly negative relation of nominal rates to stock returns and real growth in the 1977–1997 period (in good part because of the correlation of high inflation and interest rates with the big recession in 1981–1982), Exhibit 5's second panel shows that the relation can reverse if that is the regime. Low interest rates were correlated with low inflation and higher real growth, low marginal utilities, and low risk neutral probabilities. These must be scaled up to properly to estimate true probabilities in this regime. With these complexities of changing real risks and risk premia of nominal bonds, the analyses of changes in the distributions for future interest rates in the following sections should most precisely be viewed in terms of changing state prices, which are what we uncover from prices of interest rate caps and floors.

RESULT 5: US CENTRAL BANK POLICY IMPACTS ON INTEREST RATE INSURANCE PRICES

In this section, we use the prices of interest rate caps and floors that were obtained from Bloomberg[5] to estimate the market's implied distribution of "state prices" (normalized to sum to 1.0) for possible future 3-month rates. Plotting this gives us the shape of the state price density implied from market prices. Exhibit 6 shows the densities implied from prices of interest rate caps and floors. Floors are used for rates 0% to 3%, as they best trace out the density for lower interest rates. Averages of cap and floor implied densities are used for rates from 4% to 6%, and cap densities are used for 7%, 8%, and 9% plus rates. Caps are most active for higher interest rates, as markets are more active for options out of the money, and

[5] We use Bloomberg's cap and floor prices that are re-fitted daily to traded market prices. Their volatility cube fits smooth out fluctuations due to bid/ask bounces.

therefore cheaper. We use year-end data for 2003–2007 for time spreads of 5-year and 4-year caps and floors, which means these distributions are for 3-month LIBOR from 4 to 5 years from the pricing date.

From Exhibit 6, we see that putting the information from caps and floors together gives one a relatively *symmetric* picture of risk-neutral probabilities for the 3-month LIBOR rate in 5 years, as seen from year-ends 2003–2007. The mode for long-term forecasts of 3-month LIBOR during these years centered around a projected 3-month LIBOR rate of about 4% to 5%, depending upon the year. In later sections, we examine in more detail the dramatic moves in state prices in response to Federal Reserve actions in the Financial Panic of 2008/2009 and in the European Sovereign Debt Crisis of 2010–2013, shifting to very positively skewed distributions.

Fed Policy Impacts: US 2008/09 Financial Panic, 2011 Obama Budget Crisis, 2013 Taper

Let us examine moves in the state price distribution in five major actions: (1) December 2008, when Bernanke's Fed took rates to zero; (2) in March/April 2009, when stock markets began to surge back; (3) during the US budget crisis in Q3 2011, when President Obama and the Republican Congress were unable to reach agreement; (4) in June 2013, when the Fed made clear its intent to begin tapering their purchases of securities (quantitative easing); and (5) December 2015, when Chair Yellen's Fed actually began raising rates (liftoff).

Major Policy Move 1: September 2008: Fall of Lehman and others

September 2008 was when Lehman Brothers filed for bankruptcy and many financial institutions were troubled (Wachovia, Merrill Lynch, Morgan Stanley) and did mergers or substantial capital raising. In response to the great fears and stock price drops around the globe in September to November 2008, the US Federal Reserve stepped in strongly in December 2008 to provide liquidity and reduce short-term rates to near-zero levels, and long-term rates dropped to lows for the prior 75 years (2.25%). Exhibit 7 shows the major shift in the state prices to lower rates between June 30, 2008, and December 2008. In addition, reflecting the true probabilities of lower rates in 5 years, these state prices likely also reflect changes in conditional consumption betas for these Arrow securities, with those at a low rate having increasingly negative consumption betas, whereas high-rate Arrow securities had more positive consumption betas.

Major Policy Move 2: March/April 2009 Stock Market Surges Back

The Fed announced in March 2009 that they were going to keep rates low "for an extended period of time." Exhibit 8, which compares state price distributions for LIBOR on March 31, 2009, and April 30, 2009, shows that the markets apparently felt that the strengthening economy would not allow the Fed to maintain rates this low for 3 or 5 years, as the state prices for rates then actually shifted toward higher rates. The stock market hit its low on March 9, 2009, and then began a very strong bull market that took stock prices higher by more than 75% in the remainder of 2009. Market participants apparently believed that an overheated economy might have higher inflation and require that rates be pushed up within 3 to 5 years.

Major Policy Move 3: August 2011 Budget Crisis

In early August, 2011, during the budget impasse between President Obama and the Republican Congress on raising the US Federal Debt Ceiling, the stock market

EXHIBIT 7

2008 Fall of Lehman. Bernanke's Fed Drove Short Rates to Zero

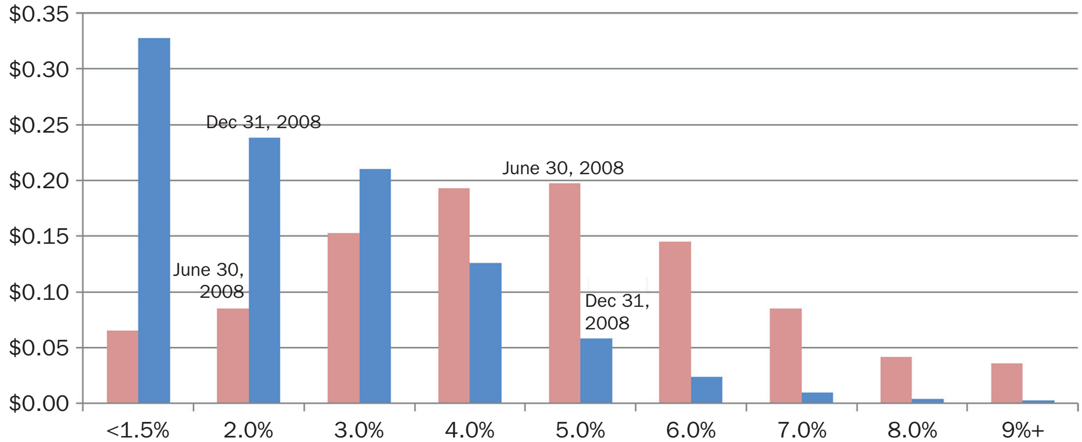
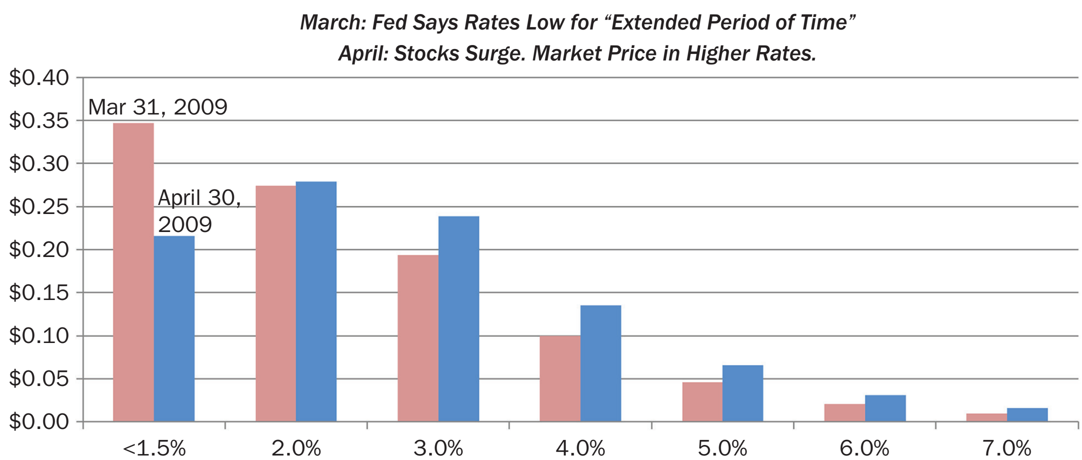


EXHIBIT 8

2009 Bottom of Great Recession, Turning Point

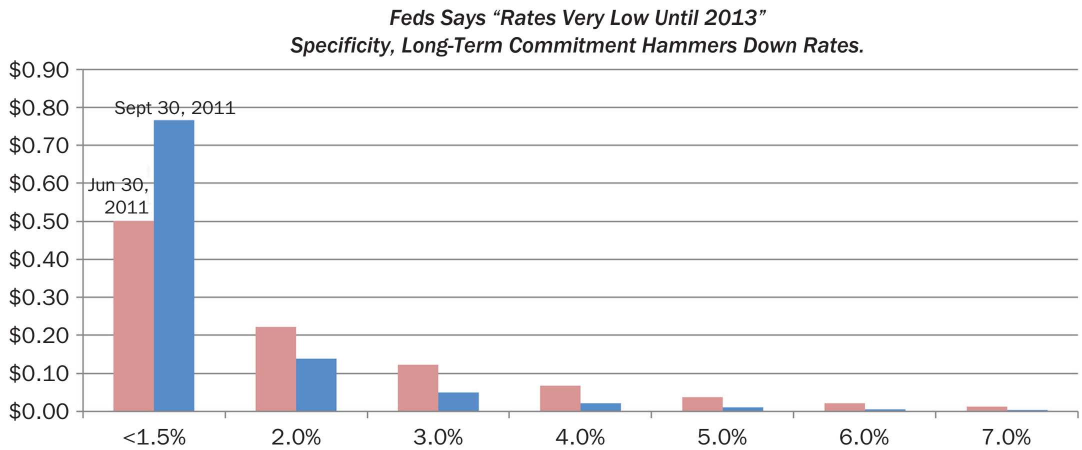


plummeted by 10% in 1 week. There was a huge flight to quality and drop in interest rates. In response, the Fed made a statement that surprised markets by specifically saying that they expected rates to main extremely low "at least through 2013" (i.e., for more than 2 years). This specificity and long time commitment of the Fed's interventions dramatically affected the expectations for LIBOR rates 3 to 5 years out. State prices for 1% Arrow securities increased 60%, whereas state prices for rates of 3% to 6% LIBOR dropped 50% from June 30 to September 30, 2011, as Exhibit 9 shows.

The effect of the August 2011 specificity and long time commitment was even more dramatic on the distribution for longer term 5-year forecasts for 3-month LIBOR (not shown). The gradually strengthening economy of 2009–2011 had shifted the state price distribution for LIBOR 5 years out to a relatively symmetric distribution, with low prices for 0% and 1% rates, and a mode of 4%, with highest probability densities from 3% to 5%, then tapering off gradually. After the Fed's announcement, the mode for rates out 5 years dropped from 3% to 4% to less than 1%, and the state price density shifted to very positive skewness. Clearly, the Fed's actions and the stock market fall during the budget impasse transformed the 2011 state price distribution.

EXHIBIT 9

August 2011: US Budget Crisis. President Obama versus Republicans



Major Policy Move 4: June, December 2013 Fed Considers, Implements Tapering Purchases

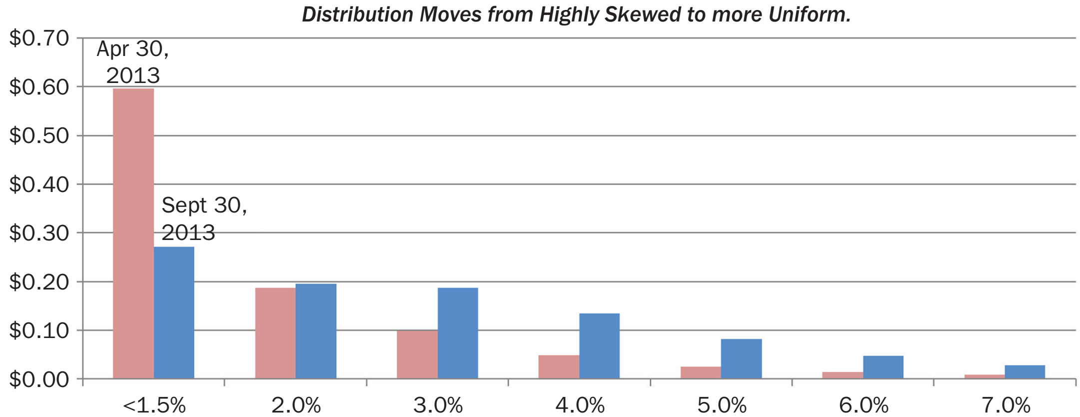
On May 22, 2013, Fed Chairman Ben Bernanke, testified before Congress that the Fed might taper its massive purchases of Treasury bonds and mortgages. Bernanke noted that the unemployment rate had dropped considerably. Over the next 6 days, the 10-year Treasury rate increased by 21 basis points, from 1.94% to 2.15%. A month later, on June 19, the Fed Chairman made even stronger statements, and markets around the world reacted sharply. Bernanke said tapering would likely begin "later this year" if growth picks up as the Fed projects, unemployment continues to decline and inflation moves closer to the Fed's 2% target. Bernanke said, "In particular, the housing sector, which has been a drag on growth since the crisis, is now obviously a support to growth." Rising home prices are increasing household wealth and strengthening consumer confidence and spending, he noted. (*The Wall Street Journal*, June 20, 2013). Both 3-year and 5-year state prices for US LIBOR moved sharply, as the 10-year Treasury rate moved 32 basis points from June 18 to June 21, 2013.

During the summer of 2013, the stock market, after falling sharply following Bernanke's June comments, recovered and moved back toward historic highs (1700+) on the S&P 500 index. Market participants appeared to believe that tapering would begin with at the Fed's meeting on September 18, perhaps with a monthly reduction of $10 billion of asset purchases. Interest rates drifted upward and touched 3.00% on the 10-year for the first time since the financial crisis, whereas the stock market regained its strength and the unemployment rate drifted lower. To see the cumulative impact of the Central Bank communications and market assessments of the economy, Exhibit 10 shows the 5-year state price distributions for May 21 (before Bernanke's first mention of tapering) and for September 16, prior to the Fed's September meeting. Both 3-year and 5-year distributions shifted quite dramatically over the summer to higher rates and to much more spread out and symmetric distributions, yet ones that now possessed visible right tail risk (9%+ LIBOR) for the 5-year forward scenario.

In the next section, we similarly present what happened in Europe during this same time. From 2010 to 2013, Europe dealt with sovereign debt crises in Greece, Ireland, Portugal, Spain, and Italy, so major policy actions were taken by the ECB at that time.

EXHIBIT 10

2013 Strong Economy Bernanke Announces "Tapering"



Euro Area Rates in 2003–2007 Growth, and in the Great Recession of 2008–09

In this section, we do a brief parallel analysis for interest rate state prices for the Euro Area. Prices for caps and floors for maturities from 2 to 10 years and strike rates from 1% to 9% for Euribor were obtained from Bloomberg's volatility cube fitted to market prices. Caps and floors are based on 6-month Euribor, paid semiannually. Butterfly spreads of time spreads of interest rate caps and floors are again used to compute prices of triangular payments (as in the Result 1 section) to arrive at state price densities for 6-month Euribor 5 years ahead. Exhibit 11 gives distributions at year-ends for the growth period from 2003 to 2007. The big picture looks quite similar to that of the US in Exhibit 6. The market's state prices for 6-month Euribor, 5 years hence, were approximately symmetric, with modal rates between 3% and 5%, depending upon the year.

In Exhibit 11, we see similar distributions of state prices for the Euro as for the dollar, with large uncertainties in 2003, evolving to tight distributions in 2006. However, note that 9%+ tail risk priced in in 2003 was much smaller in Europe than it was in the US.

ECB Major Policy Move 1: Cuts short rate from 5% to 1% in the Great Recession

In the Financial Panic of 2008–2009, the ECB cut short rates sharply from 5% to 1% to stimulate the European economy, as shown in Exhibit 12. However, as of December 31, 2008, after the fall of Lehman Brothers on September 15, the dramatic fall of stocks globally in October, and credit markets "seizing up" in November (*The New York Times,* November 21), European markets' implied state prices for 5 years out were quite different from those of the US. Exhibit 13 shows the Euro Area 5-year state price distribution on December 31, 2008, compared with the US distribution. We see a relatively symmetric distribution for Euribor 5 years hence, whereas the US dollar LIBOR was concentrated on very low short rates, but then had a very long tail with positive skewness. Apparently, at the end of 2008, there was less worry about Europe having a deep and lasting recession than for the US. Indeed, as shown in Exhibit 12, the ECB, led by Jean-Claude Trichet, actually *increased* rates in 2010 and early 2011, as stock prices had surged higher and economic recovery was expected. The US did not increase rates until December 2015.

EXHIBIT 11

Euro Area State Prices 2003–2007: Relatively Symmetric Distributions

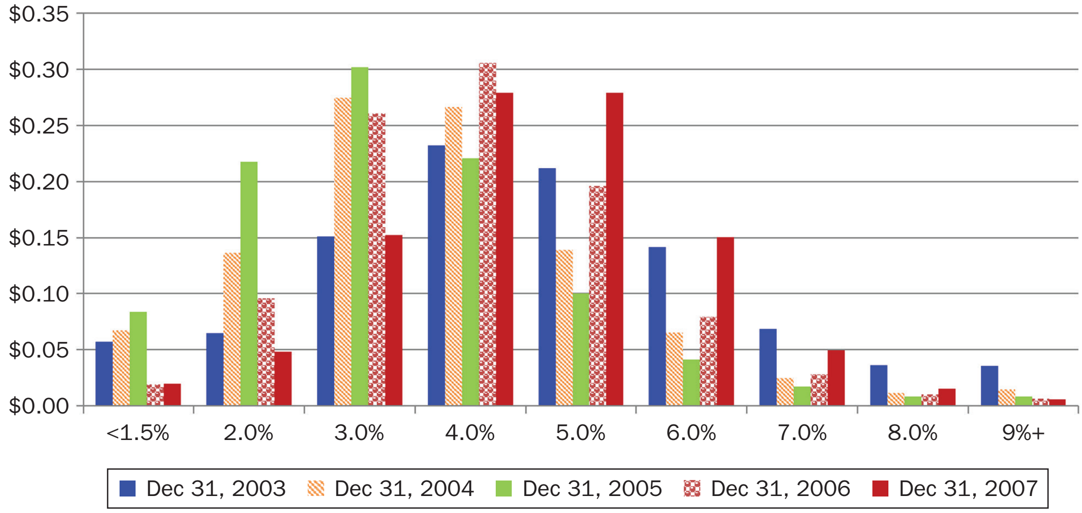
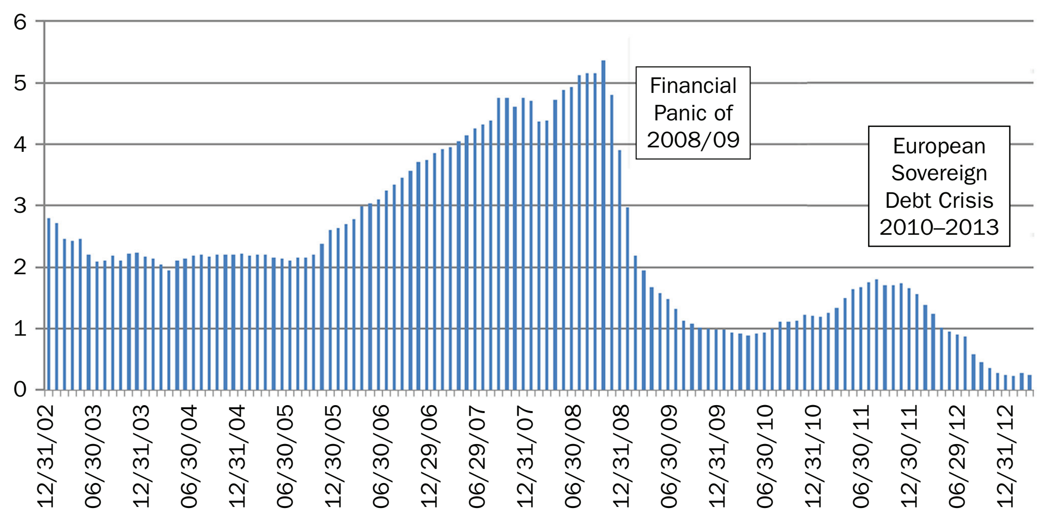


EXHIBIT 12

Six-Month EURIBOR Yield 2002–2013: Stimulus in Great Recession, Sovereign Debt Crisis



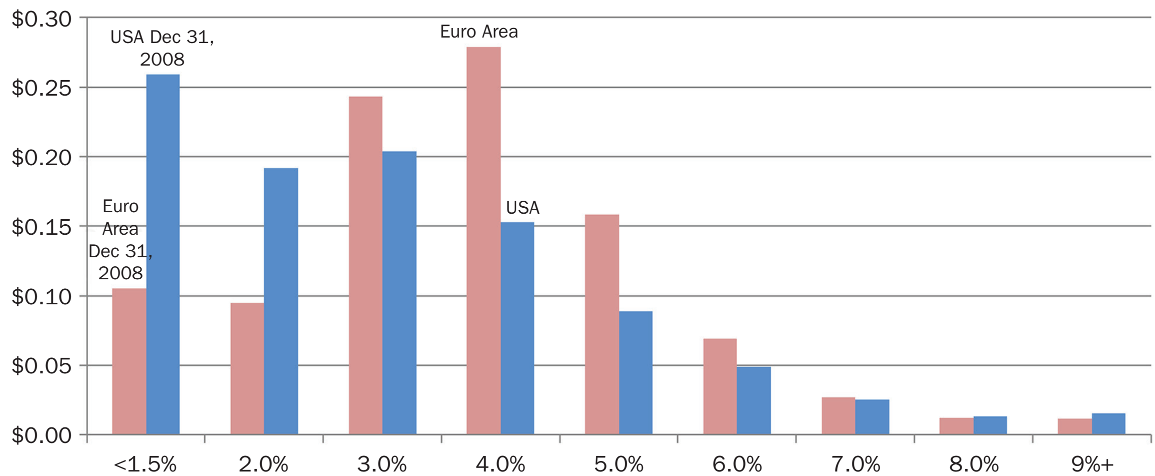
However, we see in the next section that the ensuing European Sovereign Debt Crisis of 2010–2013 caused the ECB under president Mario Draghi to provide highly stimulative policy responses, and market beliefs in 2013 had state price distributions that were quite similar in shape to those in the US, with very positive skewness starting from very low interest rates.

2010–2013: ECB POLICY IMPACTS IN THE EUROPEAN SOVEREIGN DEBT CRISIS

Using timelines provided by the BBC (June 13, 2012), Yahoo! (February 23, 2013), and a Reuters Special Report (March 2, 2013), we examine three key dates in the Sovereign Debt Crisis and the ECB's monetary stimulus, led by President Draghi: (1)

EXHIBIT 13

Euro Area versus US State Prices December 31, 2008: Symmetry versus Skew



Q3–Q4 of 2011, when Euro Area fears heightened greatly and Draghi took over and started cutting rates; (2) September 2012, when Draghi proclaimed that the ECB would do "whatever it takes" to save the Euro; and (3) Q1 2015, when the ECB began a massive "Quantitative Easing" program.

ECB Major Policy Move 1: Draghi Cuts Short Rate to Near 0 in Sovereign Debt Crisis

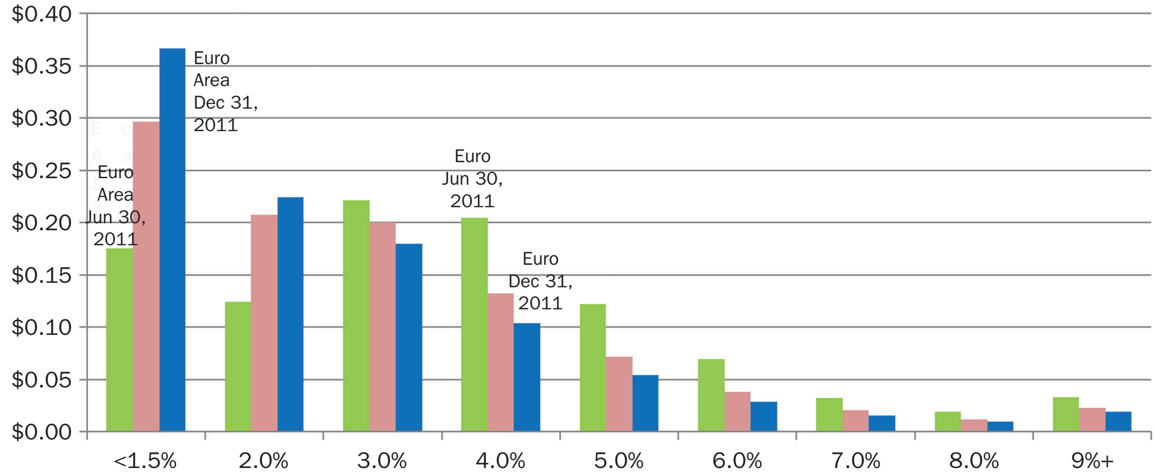
When Greece's budget deficit was restated from 3.7% to 12.7% in January 2010 and major irregularities found, their budget crisis ensued. The first Greece bailout was agreed in April 2010. Eurozone stock prices rebounded sharply (15%+) from mid-2010 to mid-2011. Then, in July 2011, the worries intensified again and there was widespread discussion of a possible Greek exit from the Euro. In August 2011, credit default swap insurance costs jumped for Italy and Spain, and their bonds' yields surged, as worries became acute that the debt crisis would spread to these larger countries, which had much greater amounts of debt. Eurozone stock prices plummeted by more than 25% from early 2011 peaks to the month-end lows on September 30, 2011. On November 1, 2011, Mario Draghi replaced Jean-Claude Trichet as president of the ECB and quickly cut short-term policy rates twice by the end of 2011.

Exhibit 14 gives the implied state prices as they moved through this tumultuous period (which also had the US's budget impasse in August 2011). Note that the prices of "left tail spreads" skyrocketed from June 30 to December 31, 2011, illustrating the fear that was in the minds of investors. When Draghi cut rates twice in late 2011, the state price distribution shifts left to much lower projected rates in 5 years and was transformed from a uniform distribution to a highly skewed distribution.

The tonic of lower rates worked initially in the Eurozone, much as it had in the US, and stock prices rebounded by about 20% from September 30, 2011, to March 31, 2012. But then the fears resurfaced that the larger economies of Italy and Spain would default on their debts and cause massive write downs for European and global banks. In just 2 months, from March 31 to May 31, 2012, Eurozone stock prices dropped 10%.

EXHIBIT 14

2011 Euro Area Sovereign Debt Crisis: Rate Cuts Transform Distribution



ECB Major Policy Move 2: July 2012, Draghi: "Will Do Whatever It Takes to Save the Euro."

In July 2012, the ECB again cut rates sharply. Then, on July 26, 2012, ECB President Draghi at a London business conference "dropped a bombshell" and surprised almost everyone by stating, "Within our mandate, the ECB is ready to do whatever it takes to preserve the Euro." He went on to say, "And believe me, it will be enough."[6] Exhibit 15 shows the significant shift in the 5-year Euribor state price distribution that occurred with Mario Draghi's strong ECB moves. State prices for interest rates from 0% to 2% increased substantially, whereas those for 3% and higher dropped significantly. Right tail risks for rates, already low, became much lower yet. Stock prices rather steadily marched upward from mid-2012 to early 2013 for the Eurozone.

The Euro stabilized, given President Draghi's bold ECB moves, Greece has recovered nicely and the Eurozone is intact in 2022, which indicates considerable success of central bank efforts in this very challenging time for the Euro and the Eurozone economies. And Mr. Draghi was the prime minister of Italy in 2022 …

2015–2019: YELLEN FED EXECUTES LIFTOFF. ECB QE

2016: Brexit Voted. Trump Elected. 2017 Corp Tax Cut. 2018–19 Trade Wars

The period from 2015 to 2019 was a time of steady economic growth and continuing drops in US and global unemployment. US unemployment hit 3.5% in 2019, a 50-year low. Fed Chair Janet Yellen led seven interest rate increases in 2015–2018, followed by two more under Fed Chair Jay Powell in 2018, as the US increased rates to provide dry powder to fight the next recession. This seemed quite wise when the coronavirus pandemic occurred in early 2020, as Jay Powell's Fed was able to drop rates sharply to stimulate the economy. The path of US interest rates from 2006 to December 2021 is given in Exhibit 16.

[6] See Front Page, *Financial TImes*, July 27, 2012.

EXHIBIT 15

2012 ECB President Draghi "Will do Whatever it Takes to Preserve the Euro"

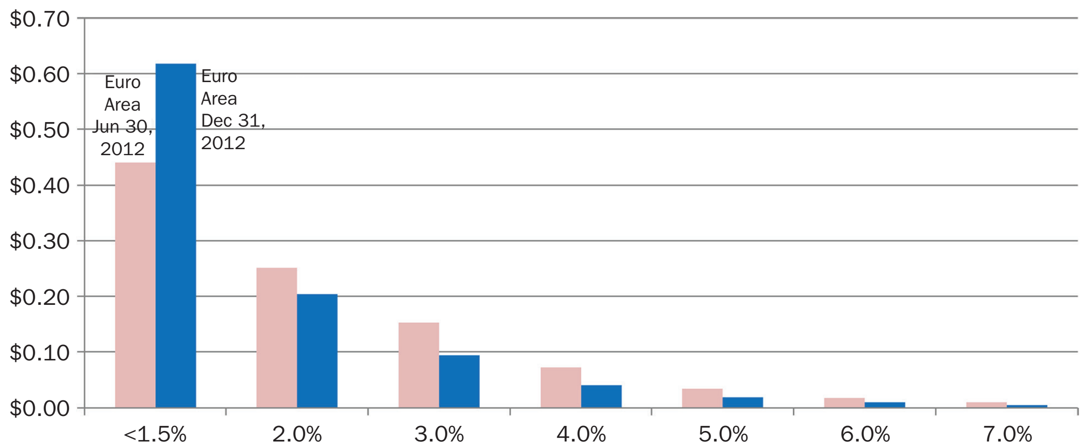
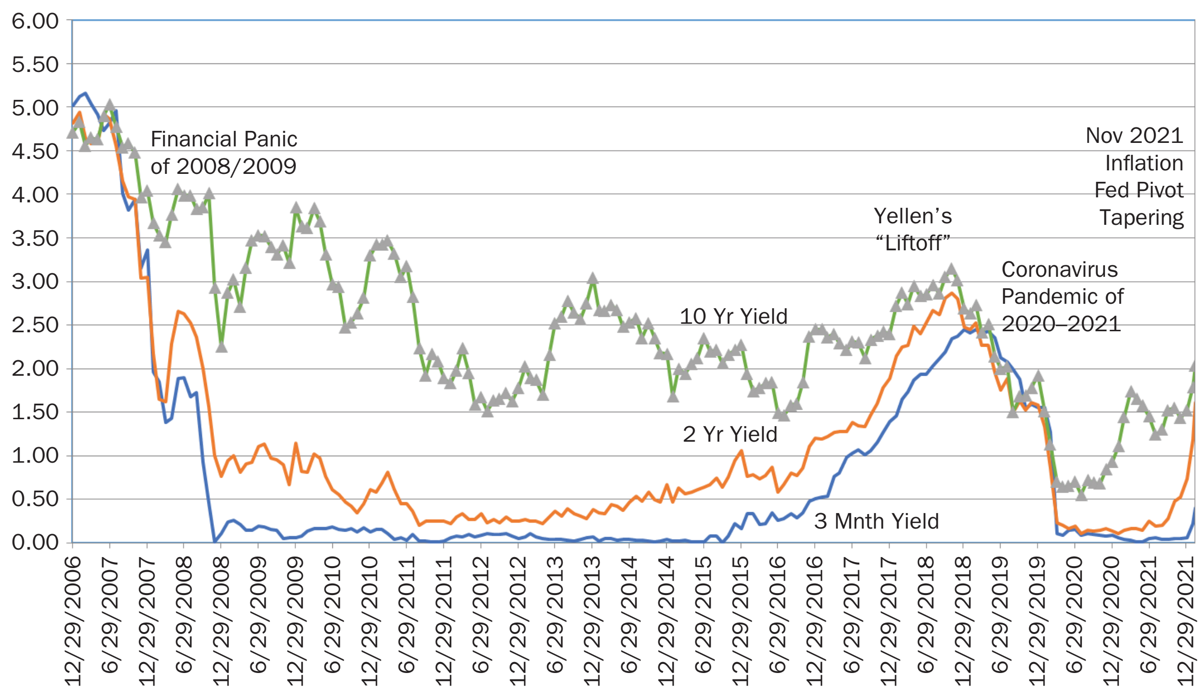


EXHIBIT 16

US Yields 2006 to February 2022: Fed Chair Yellen Executes Liftoff 2015–2018



In March 2015, Mario Draghi's ECB used massive asset purchases, "quantitative easing" (QE) to stimulate the European economies, and with apparent success. GDP growth was relatively strong and deflation fears seemed to diminish. Exhibit 17 shows the large move (for Europe) toward future normalization that was then built into interest rate option prices for the Eurozone.

On June 23, 2016, the UK voted for Brexit. Stock market prices held up well, but interest rates dropped sharply and the Bank of England prepared to stimulate the UK economy with lower rates, anticipating much lower growth. Exhibit 18 shows the moves in state prices in response to communications of rate cuts by Bank of England governor Mark Carney.

EXHIBIT 17

2015: Draghi's ECB Massive "Quantitative Easing" Stimulates Economy

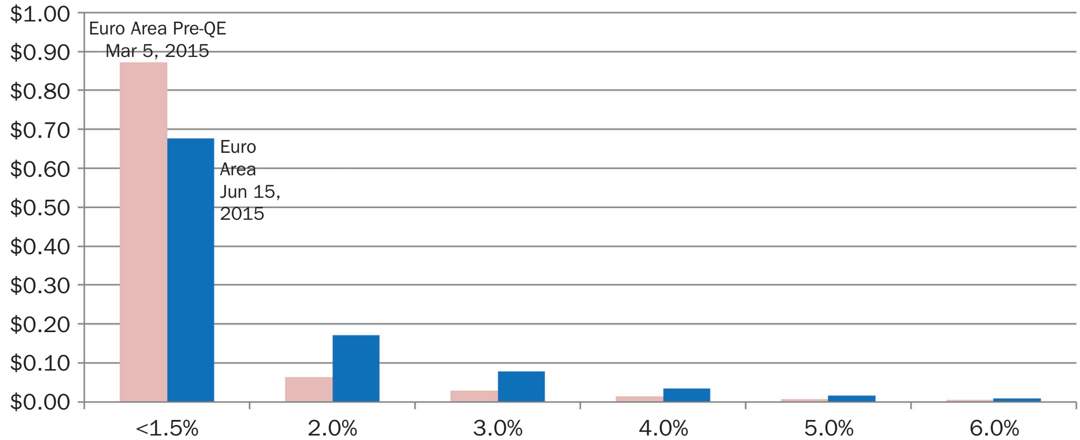
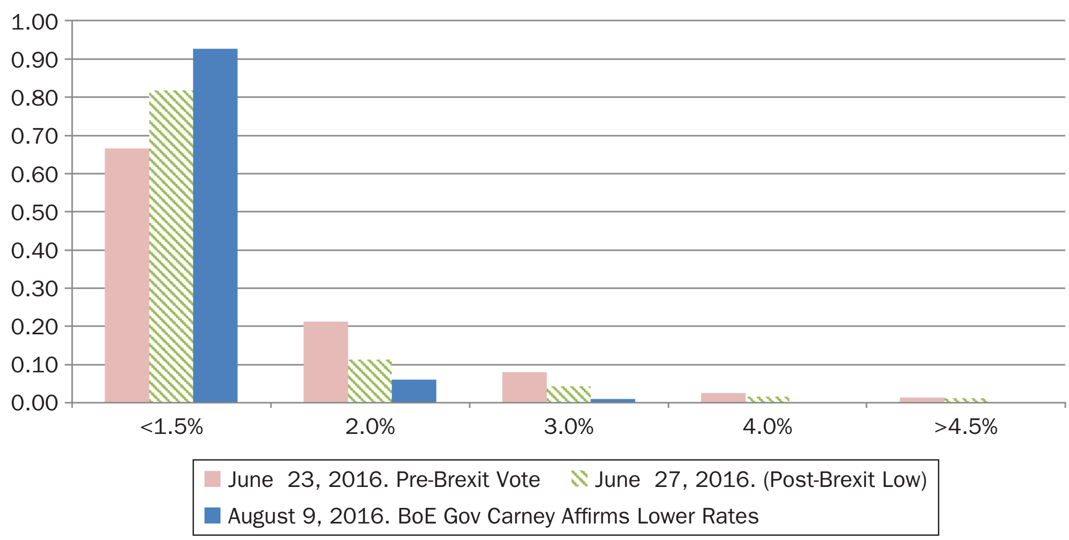


EXHIBIT 18

2016: After Brexit Vote, Governor Carney's Bank of England Stimulates UK



On November 8, 2016, Donald Trump was elected US president. He promised massive stimulus and signed a large corporate tax cut from 35% to 21% in December 2017. This raised inflation concerns, so Yellen's Fed continued to raise rates. Bond options priced in a higher rate distribution and a move toward more symmetric rate uncertainty (see Exhibit 19).

After Trump's large corporate tax cut passed in December 2017, and with Yellen's steady rate hikes, the US's distribution of state prices shifted quite dramatically toward symmetry and to a higher median rate in early 2018, as Exhibit 20 shows.

Given the tax cuts and stock market surges to new records, the US economy was very strong in 2017 and strong to Q3 2018. In Q4 2018, President Trump instigated tariffs and trade wars with China and Mexico, and some with Europe. Stock prices fell sharply (20%), and growth began to slow. The yield curve inverted in early 2019, and growth forecasts (Livingston/Philly Fed Survey) showed reduced growth expectations. After nine rate increases, Powell's US Fed began to reduce rates in mid-2019, given

EXHIBIT 19

2016: US Elected President Trump: Markets Expect Stimulus

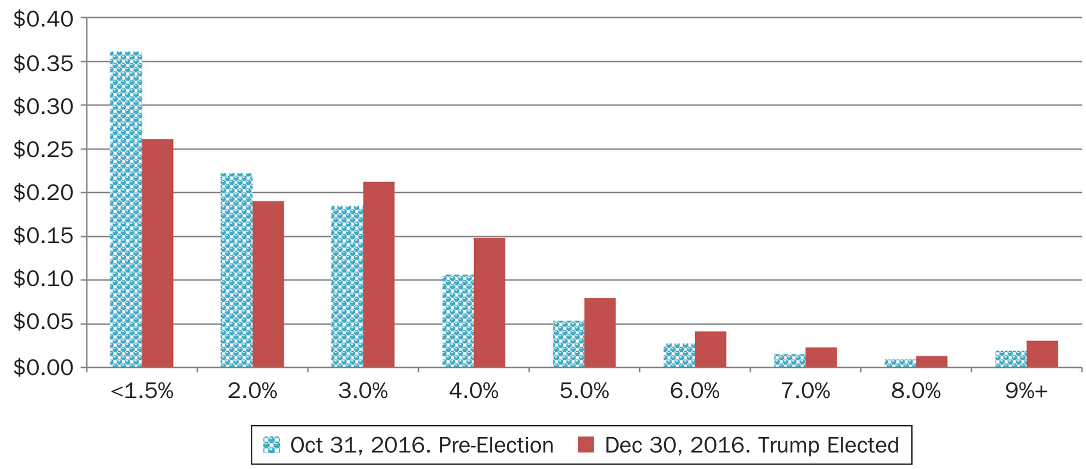
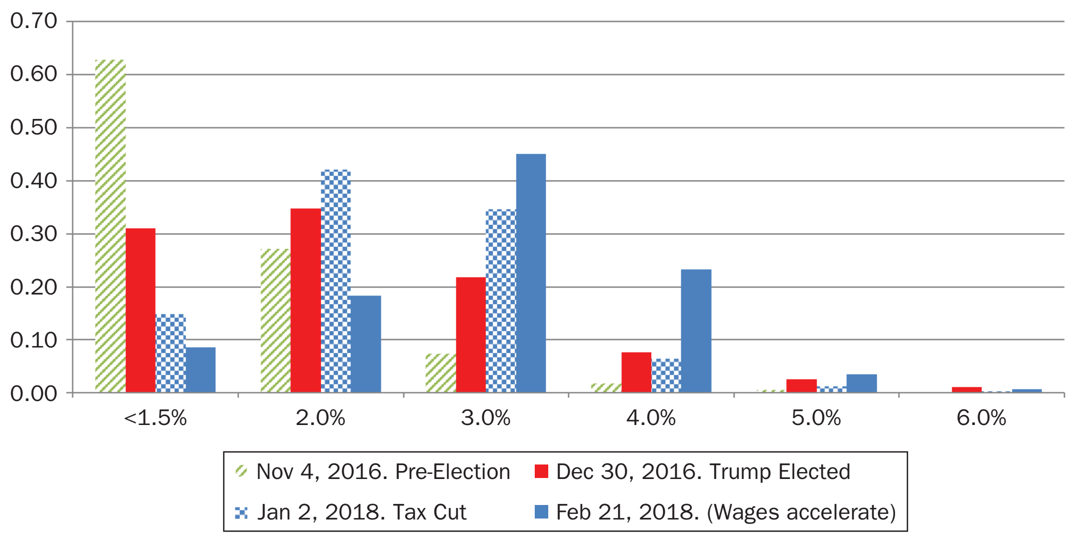


EXHIBIT 20

2016–2018 Trump Corporate Tax Cut Stimulates, Transforms Distribution, Skewness to Symmetry



fears of weakness. State prices at the end of 2018 and 2019 moved dramatically, showing large changes in rate expectations and risks, as in Exhibit 21.

2020–2022 CORONAVIRUS PANDEMIC, BRIEF HUGE GLOBAL RECESSIONS, AND RECOVERY

In December 2019, China reported an outbreak in Wuhan of the coronavirus, which was quickly identified as a serious virus that could kill many people. The virus spread to Europe, the United States, Latin America, India and Africa in January–February 2020. As cases surged in the US in March 2020, markets plunge. Real GDP dropped by largest-ever percentages in Q2 2020 for many countries (–30% in the US), and VIX surged to 80%. The US unemployment rate quickly increased from 3.5% to 19% in late 2020.

EXHIBIT 21

2018–2019: Trump Trade Wars, Govt Shutdown Slow Growth, Distribution Skews

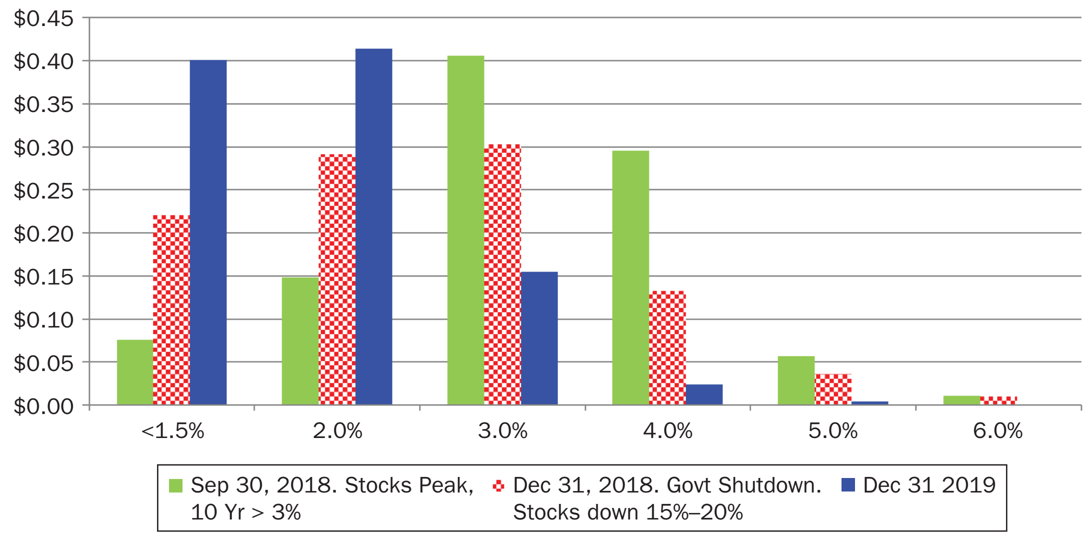
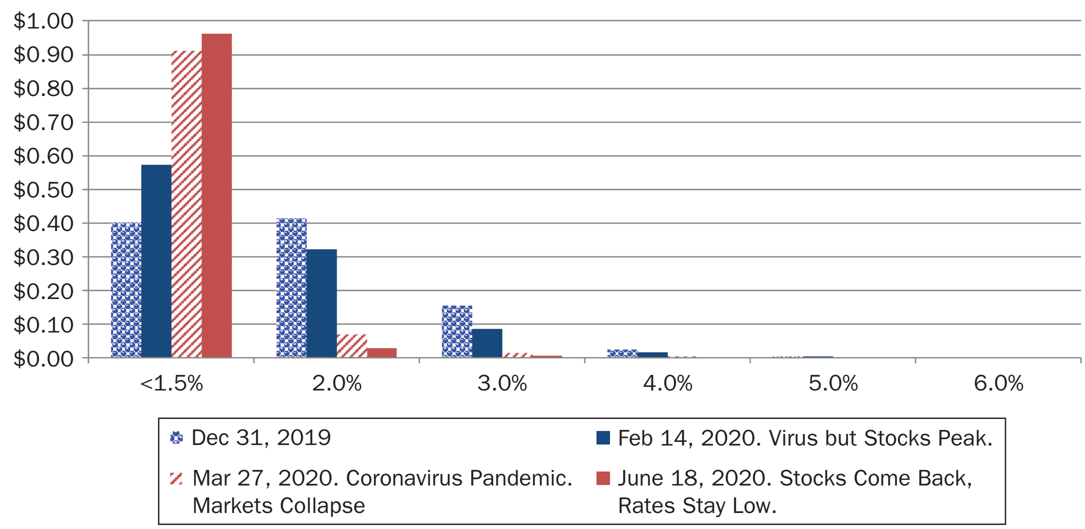


EXHIBIT 22

Coronavirus Pandemic of 2020–2022: Powell Fed Stimulus, Rate to Zero Massive QE, Skewed Distribution



President Trump and the US Congress agreed and made law a major $2 trillion of stimulus funds, replacing income for millions of people thrown out of work. Fed chair Jay Powell immediately led cuts in the short rate to zero again in 2020, as in 2008, and did massive, historic quantitative easing, providing liquidity through much of the economy. The 10-year Treasury rate dropped from 1.92% on December 31, 2019, to 0.71% in June 2020, after briefly dropping to 0.35% in March 2020. Chair Powell indicated that short rates would remain near zero through the end of 2021, which they did. Options markets moved sharply. Exhibit 22 shows the moves in the 3-year state prices, which saw huge surges in prices of bets that pay off with very low rates. The 10-year state price distribution also was significantly impacted in similar ways as the 3-year, but the long-term response was more muted, as markets expect a return to normal long term.

EXHIBIT 23

Coronavirus Pandemic of 2020–2022: Euro Area State Prices Show Little Dry Powder

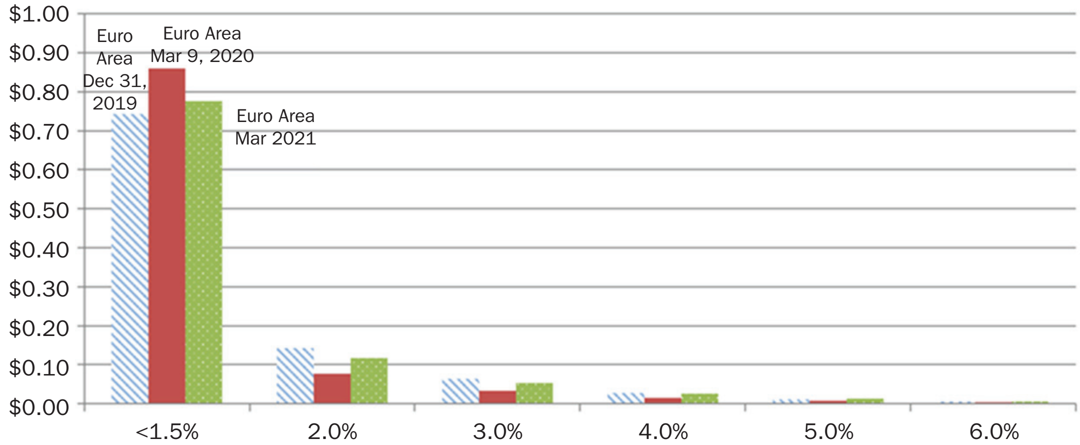
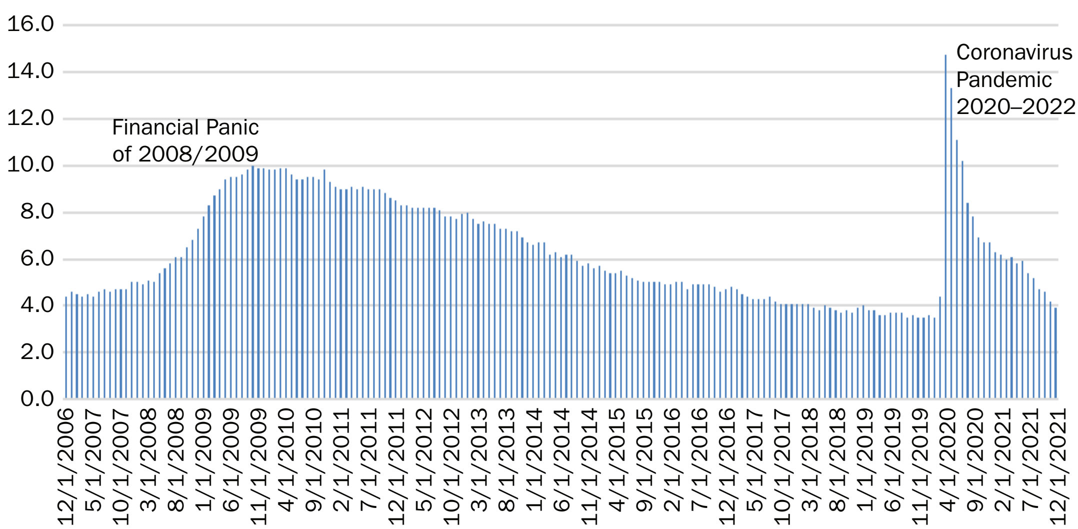


EXHIBIT 24

Coronavirus Pandemic of 2020–2022: US Unemployment Rate (%) Spikes and then Recovers



As the pandemic was global, economies in the Eurozone, the UK, and many others were affected. Euro area state prices for rates in 10 years shifted toward higher prices for low rates, likely because of both expected central bank policy and paying for insurance against bad times. However, as the ECB had not raised rates from their prior lows, they had no real "dry powder" on rates, so they used QE (massive bond purchases). Exhibit 23 shows the small moves in the 10-year Euro Area state price distribution. Moves in 3-year and 5-year distributions were even smaller.

As economies gradually reopened in 2021 and 2022, Exhibit 24 shows that the US unemployment rate fell to 3.9% in December 2021, nearing the pre-pandemic low of 3.5%. Many jobs went unfilled. Given the extremely tight job market in the US and other developed countries, and given supply chain bottlenecks, inflation soared. Exhibit 25 shows the surge in both "headline" and "core" inflation, both of which have moved up to levels of 7% and 5%, levels not seen for almost 40 years, since the early 1980s.

EXHIBIT 25

CPI Inflation Surges to 40-Year Highs (YOY%) in early 2022

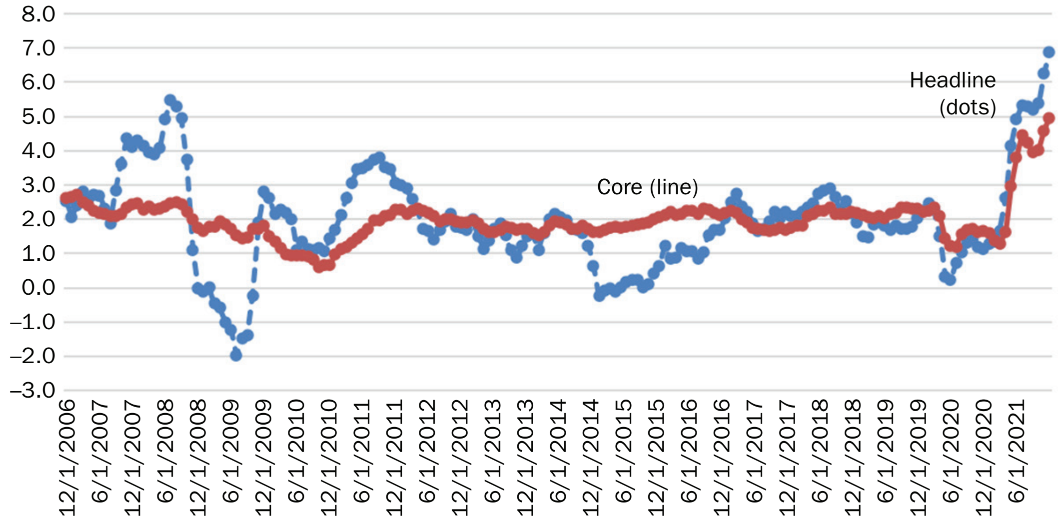
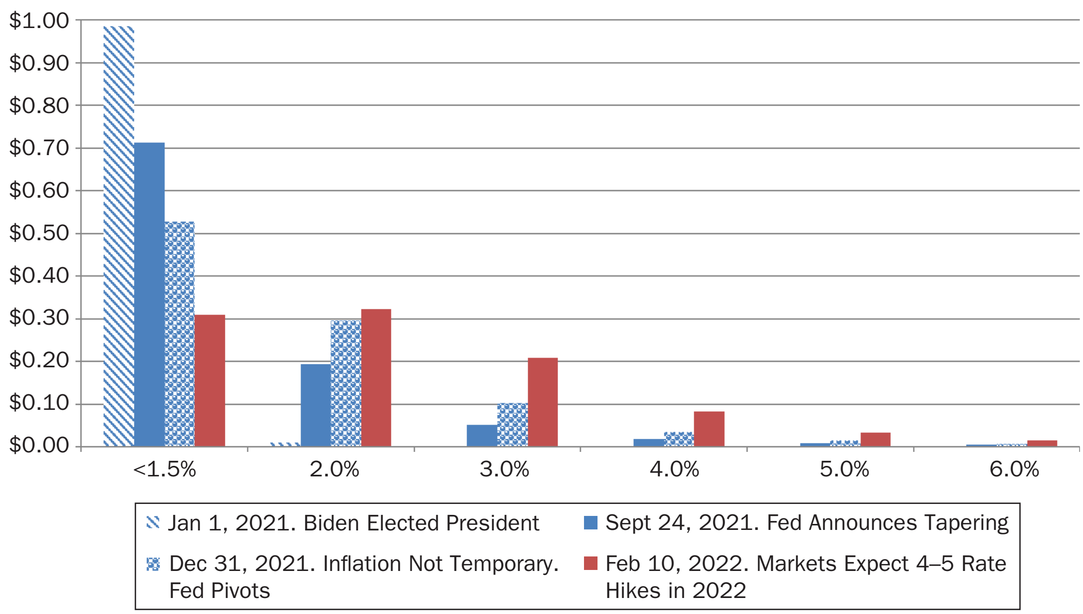


EXHIBIT 26

US State Prices 2021–2022: Powell Fed Pivots. Tapering. Liftoff in Rates. Inflation Fight.



Fed chair Jay Powell at first said that the inflation surge likely was “temporary,” but in late 2021 he admitted that it was no longer appropriate to say it is temporary, given supply chain bottlenecks, pervasive labor shortages, and the spread of inflation effects to many products. The Fed then “pivoted” in November 2021 from massive stimulus to its current status toward removal of stimulus, that is, tapering asset purchases rapidly in early 2022. Markets in February 2022 expected four or five rate increases in 2022, lifting off from the zero rates established in March 2020. Exhibit 26 shows the 3-year state price distribution in February 2022, a major transformation from a highly skewed distribution to a nearly uniform distribution. In the coming year, if the Fed raises rates, as is now expected, the distribution could go to an even more symmetric distribution.

EXHIBIT 27

Euro State Prices 2022: ECB President Lagarde Fights Inflation, Lifts Rates

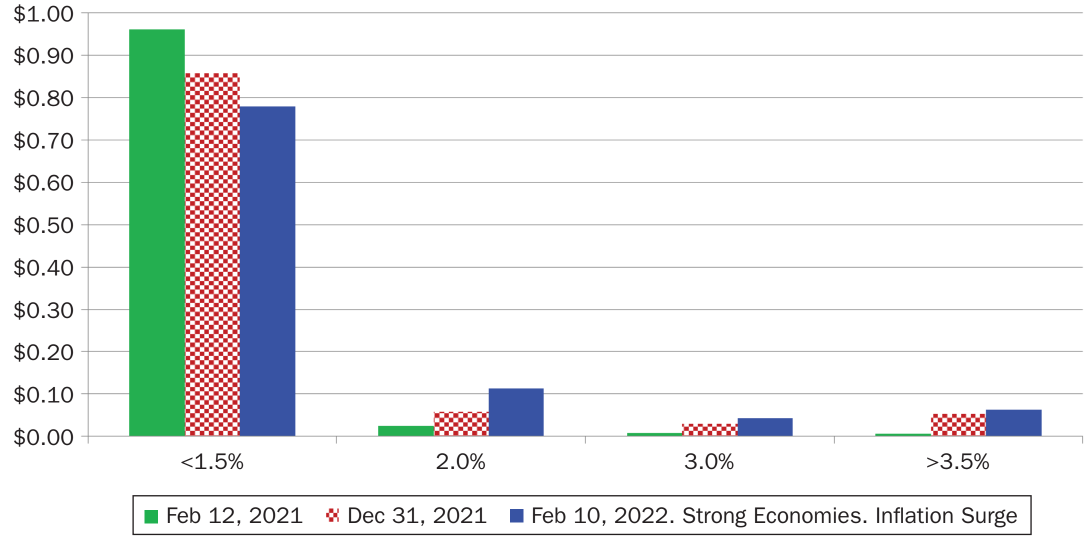
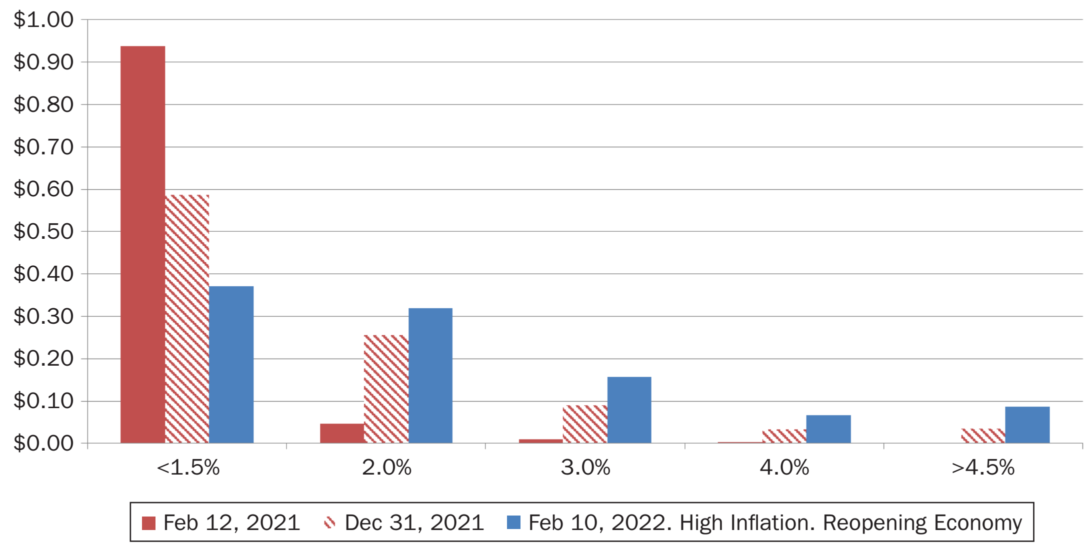


EXHIBIT 28

UK State Prices Late 2021–2022: Governor Bailey's BoE Lifts Rates Steadily



The Eurozone, which seemed totally anchored on zero or negative rates, finally had a notable change in its state price distribution in 2022. As inflation surged to near 40-year highs, over 5%, President Christine Lagarde's ECB began to implement tapering and to prepare for liftoff in rates. Exhibit 27 shows the 2021–2022 movements in the Eurozone's state price distribution. Markets in 2022 are paying up for securities that pay off at rates of 2%, 3%, and higher in the Eurozone in 5 years, quite a shift from the past 10 years, which seemed anchored below 1.5%.

In the UK, inflation surged to more than 7%, as in the US, and Prime Minister Boris Johnson reopened the economy. The Bank of England's governor Andrew Bailey led two increases in the short rate in back-to-back meetings for the first time in 2 decades. Exhibit 28 shows the flattening of the state price distribution in the UK, a dramatic shift in 2022.

CONCLUSION

In this article, we have extended the B-L (1978) method of extracting state prices from option prices, showing how the prices of butterfly spreads can be combined with right and left tail spreads to nonparametrically extract discrete state prices from option prices. We derived how those state prices should be biased estimates of true, objective probabilities, with biases changing over time. We applied our technique to the historic past 20 years of massive central bank interventions in the US, UK, and Eurozone. Movements in state prices are quite large in the Financial Panic of 2008–2009, as well as in the European Sovereign Debt Crisis of 2010–2013, with Brexit and the Trump elections in 2016, and the coronavirus pandemic in 2020–2021. Tapering in 2013 and 2022 and liftoffs in rates in 2015 and 2022 were shown to move state price distributions back toward their symmetry prior to 2007. Movements down in state prices by 40%–50% were fairly frequent, and movements up by more than 100% were also frequent.

We have shown that policy actions taken by the US Federal Reserve, the European Central Bank, and the Bank of England can and do affect the entire state price distribution for future interest rates, not just means and variances. The tools we develop are wholly arbitage based and nonparametric, so they do not rely on assumptions about the pricing function for actual market prices for options. These tools should be helpful to macroeconomists, policy makers, and market participants in measuring policy impacts. The state price distributions that we show are core pricing densities that underly the pricing of interest rate dependent securities and derivatives. Their movements can substantially affect the relative valuation of many such securities. Furthermore, the general technique for computing state prices implicit from option prices is applicable to the stock market, commodity options, foreign exchange and other assets, as well as interest rate options. Much more research remains to be done.

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