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Stock Market Insurance Prices, BL Skew, Conditional Marginal Utilities and the Equity Risk Premium

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I. Introduction

Option prices contain information about implicit state prices. In their recent article, Breeden and Litzenberger (B-L, 2022) demonstrated how **option prices in bond markets** from interest rate cap and floor price data can be used to identify the impacts of central bank policies on the distribution of state prices for future interest rates. They examined the massive central bank interventions during the 20-year period from 2003 to 2022 for the USA, the Eurozone and the United Kingdom and their nonparametric technique identified substantial impacts. In this article, we focus on the information from **option prices on equities**, specifically the S&P 500 index for US stock prices and likely fluctuations in the equity risk premium at different times.

First, we look at insurance prices (or Arrow (1965)-Debreu(1959) "state prices") for payoffs for different levels of equity market returns (down 5%, 10%, 20% and up 5%, 10% and 20%) , showing how they move in times of major stress, as well as in times of relatively steady economic growth. We show that the movements in state prices are consistent with plausible changes in risk aversion. We define a measure of risk aversion that we call the "Breeden-Litzenberger Skew," (BL Skew) as the price being paid for a $1 payoff in the left tail (large down markets) less the price of a $1.00 payoff in the right tail, which pays off when stock prices surge. We show that the movements in the 12-month BL Skew are much greater than the movements in the skew of the 12-month implied volatilities (IV), computed as the 80% moneyness IV less the 120% moneyness IV. We hypothesize that when the BL Skew is high, risk aversion is high and prices of assets with positive systematic risk are relatively low, giving high risk premiums. We show that subsequent equity returns are highly correlated with the BL Skew for periods from 3-10 years.

We show comparisons of the forecasting performance of the BL Skew with nine other identified predictors, including (1) dividend yield, (2) VIX Volatility Index, (3) the

Implied Volatility Skew from options for 1 month and 12 months, (4) Graham and Harvey's (G&H) CFO Survey forecast medians for 10-year equity returns, and for the Equity Risk Premium over 10-year bond returns, (5) TIPs yields, which Graham and Harvey found had the strongest predictive power for the CFOs' Equity Risk Premium forecasts, (6) Damodoran's discounted cash flow based forecast for 10-year equity returns and the Implied Equity Risk Premium, (7) Yardeni's "Fed Model" of the 12-month forward earnings yield gap over the 10-year yield, (8) Breeden-Litzenberger-Jia's (2015) "CAPE Yield Gap" or and (9) Shiller's enhancement, the "Excess CAPE Yield," published online from 2022 to the present.

II. State Prices Implied from Option Volatilities by Moneyness: History, Analysis.

Nobel Laureates Kenneth Arrow (Harvard) and Gerard Debreu (UC Berkeley), provided the fundamental motivation for our work on state prices. Arrow dreamed of a "complete market" of securities, where one could insure all important risks with trading in some very simple securities that paid $1.00 in different states of the world, on different dates, i.e., in different "time-states" or "economic scenarios." If one had the prices of these securities, one could immediately calculate fair prices for basic derivatives such as calls and puts, as well as values of different capital investments, where payoffs can vary dramatically across economic scenarios. This approach is quite general and does not assume normal or lognormal returns, so it can be used with securities that have highly nonlinear payoffs, and positive or negative convexity and skewness.

In this article, we assume that there are 7 different states of the world defined as in Figure 1, based upon the 1-year return on the S&P 500. The cutpoints for state definitions were chosen based on the availability of Bloomberg's implied volatility pricing by "moneyness," (option strike price divided by current S&P index), which normally runs from 80%, 90%, 100%, 110% and 120%. We interpolate for 85%, 95%, 105% and 115% between these IVs observed by Bloomberg from actual option market prices. These Arrow securities are much like the "supershares" of Hakansson (1976):

Figure 1

Insurance Payments Based on the S&P 500 Return

A financial institution buys $100 million of Treasury bills maturing in 1 year.
The institution then "strips" the riskless Tbill payoffs into 7 pieces, A1 to A7:

	State Name
Security A1 pays $1.00 if SP500 <-12.5% in 1 year, zero otherwise.	Left tail
Security A2 pays $1.00 if SP500 is between -7.51% and -12.5%	-10%
Security A3 pays $1.00 if SP500 is between -2.51% and -7.5%	- 5%
Security A4 pays $1.00 if SP500 is between -2.5% and +2.49%	0%
Security A5 pays $1.00 if SP500 is between +2.5% and +7.49%	+ 5%
Security A6 pays $1.00 if SP500 is between +7.5% and +12.49%	+10%
Security A7 pays $1.00 if SP500 return is >+12.5%	Right tail

The institution could sell 100 million of each security and pay off as promised.
We call their prices "insurance prices," for if you wish to hedge a loss of $100,000 that occurs when stocks are down 10%, you buy 100,000 of Security A2, the -10%.

Figure 2 illustrates Arrow's dream securities for his complete market.

Stephen Ross's (1976, Yale) classic QJE article on "Options and Efficiency" provided great insights into the spanning properties of call and put options, which improve the economic efficiency of capital markets. His work was profoundly impactful and inspired the Breeden-Litzenberger 1978 solution for state prices. These values allowed the implementation of the time-state preference model of Arrow and Debreu. Securities and derivatives with all sorts of non-normal, nonlinear cash flows could be valued with this state price density. With these prices, one could quickly compute the "risk neutral density" of Cox and Ross (1976), just by normalizing time-state prices by dividing by the price of a riskless security for the maturity examined.

Figure 2

Arrow's Dream Securities, a "Complete Market"											
State=	***Arrow Securities that pay $1 in one state of the world,***							**Creating Portfolio Positiions**			
Stock	***and 0 otherwise***							**Call**	**Call**	**Put**	**Put**
Price	**A1**	**A2**	**A3**	**A4**	**A5**	**A6**	**A7**	**X=4**	**X=6**	**X=4**	**X=6**
1	**1**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**3**	**5**
2	**0**	**1**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**2**	**4**
3	**0**	**0**	**1**	**0**	**0**	**0**	**0**	**0**	**0**	**1**	**3**
4	**0**	**0**	**0**	**1**	**0**	**0**	**0**	**0**	**0**	**0**	**2**
5	**0**	**0**	**0**	**0**	**1**	**0**	**0**	**1**	**0**	**0**	**1**
6	**0**	**0**	**0**	**0**	**0**	**1**	**0**	**2**	**0**	**0**	**0**
7	**0**	**0**	**0**	**0**	**0**	**0**	**1**	**3**	**1**	**0**	**0**

Several researchers demonstrated the importance of marginal utility in asset pricing, including Hirshleifer (1970), Rubinstein (1974, 1980), Lucas (1974,1978), Nielsen (1976), Grauer-Litzenberger (1976) /and Breeden-Litzenberger (1978, 2022). Assets that pay off most when marginal utility is highest are worth the most. If payoffs are when marginal utility is low, then the asset is worth less. Negative betas have higher prices and lower returns. Positive betas have lower prices and higher returns in equilibrium. Sounds pretty simple now.

From B-L (2022, eq. 8), we have that in the power utility, lognormal consumption model, the state price/probability ratio is negatively related to the growth of consumption in the state's deviation from unconditionally expected growth, g_{ts}-μ_{ts}:

$$\log\left(\frac{\phi_{ts}^{*}}{\pi_{ts}}\right) = \gamma\left[\mu_t - g_{ts} - \frac{1}{2}\gamma\sigma_c^2\right]t \qquad (8)$$

States with higher than expected growth have lower ex ante price/probability ratios, which implies higher expected future stock returns (equity risk premiums) in those states.

Let us go to the actual historic data and show the state prices implicit in option prices and examine how they behave. The constructive method used starts with downloads from Bloomberg of "implied percentage volatilities by moneyness" for the S&P 500. Figures 3 shows the time series of this data for 1-month, 6-month and 12-month options during four very interesting and wild times that had sharp contractions—2008/9 (the Great Recession and Financial Panic), 2011/2012, the European Sovereign Debt Crisis and USA Budget Crisis, the 2020 Coronavirus Pandemic, and the 2022-2024 surge in inflation and interest rates.

Option trading volume is higher on cheaper options than on expensive options, as traders seek the biggest bang for $1.00 of equity, and omegas (elasticity of option value w.r.t. underlying asset value) are higher on the cheap options. The cheapest options are out-of-the-money (OTM) calls and puts, so the 80% moneyness (downside bets) implied volatilities in Figure 3 are computed from prices of out-of-the money puts, which pay off when stock prices drop 20% or more from 100 to 80 or less. Similarly, for the 120% moneyness, the cheapest options that bet on price increases of 20% or more are the call options, so these are OTM call option implied volatilities. The difference between the 80% implied volatility and the 120% implied volatility is called the "IV skew." Note that the IV skews shown are all positive, as out-of-the-money puts have higher implied volatilities than do out-of-the money calls.

Figure 3

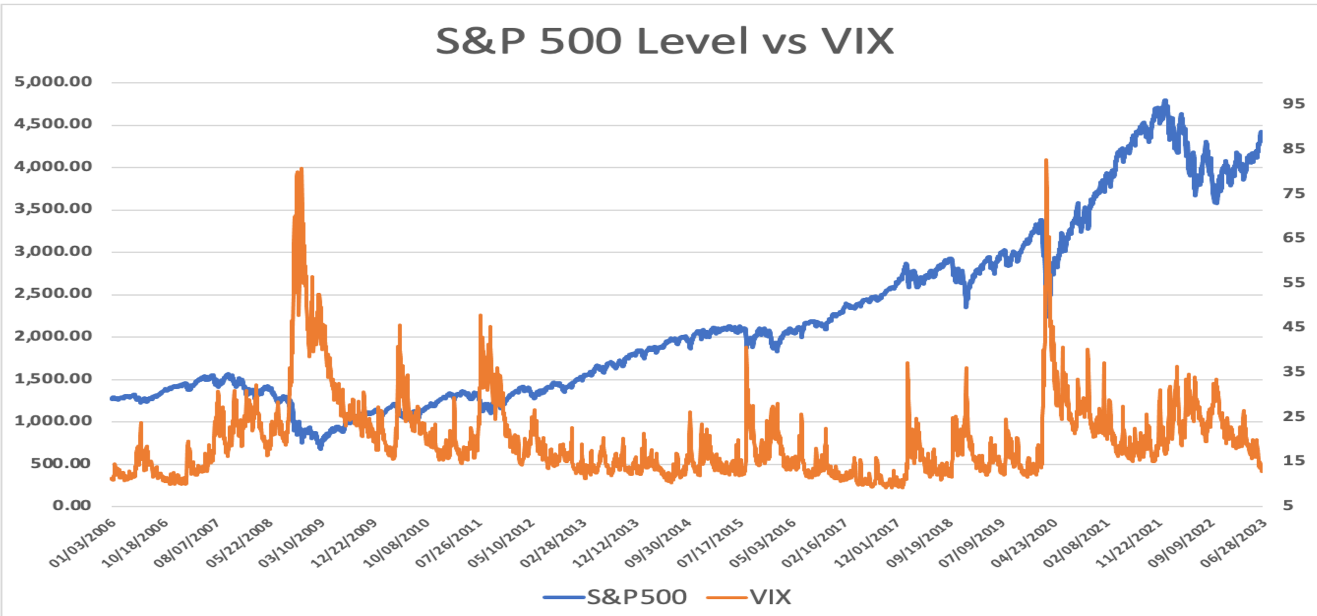
Bloomberg's Implied Volatilities from S&P 500 Option Prices by "Moneyness"

2008/2009 Financial Panic, 2011 European Sovereign Debt Crisis, 2015 China Stock Market Crash, 2020/2021 Coronavirus Pandemic

Moneyness=S/X		Implied Volatilities for 1-Month Options					Implied Volatilities for 6-Month Options					Implied Volatilities for 12-Month Options					Volatility
SPX Index	SPX	80%	90%	100%	110%	120%	80%	90%	100%	110%	120%	80%	90%	100%	110%	120%	Skew
Date	Price			ATM					ATM					ATM			80%-120%
USA Financial Panic of 2008/2009																	
12/29/2006	1418.3	26.0	19.3	10.1	9.3	9.2	20.8	17.2	13.3	10.1	9.4	19.4	16.6	14.0	11.8	10.7	**8.8**
12/31/2007	1468.4	29.8	27.1	20.6	14.5	14.1	29.5	26.2	22.6	19.1	16.2	27.9	25.0	22.2	19.7	17.3	**10.6**
6/30/2008	1280.0	33.9	28.9	22.4	17.5	16.6	28.7	25.9	22.4	19.2	16.8	27.8	24.9	22.3	19.9	17.9	**9.9**
10/31/2008	968.8	66.3	60.9	51.4	42.9	39.5	51.2	46.8	42.7	38.9	35.4	45.0	42.1	39.4	36.8	34.4	**10.6**
12/31/2008	903.3	46.7	41.6	34.6	29.2	27.3	44.2	40.6	37.2	33.9	30.9	41.6	38.8	36.3	33.9	31.7	**9.9**
2/27/2009	735.1	55.0	47.5	41.0	35.7	32.3	45.7	42.0	38.6	35.5	32.9	42.3	39.5	36.9	34.6	32.5	**9.7**
6/30/2009	919.3	40.2	30.6	23.0	18.7	18.8	33.8	29.8	26.2	23.1	20.7	32.3	29.4	26.8	24.4	22.4	**9.9**
12/31/2009	1115.1	30.4	26.7	17.0	16.2	18.0	29.7	25.5	21.7	18.7	16.8	28.4	25.5	22.8	20.4	18.5	**9.9**
Europe's Sovereign Debt Crisis and USA Budget Impasse (Summer 2011)																	
6/30/2011	1320.6	24.4	23.3	13.6	13.7	13.9	26.1	21.9	17.8	14.4	12.8	25.8	22.5	19.4	16.8	15.0	**10.8**
7/29/2011	1292.3	32.7	31.3	21.6	15.9	16.1	28.6	24.2	20.0	16.4	14.4	27.1	23.8	20.7	18.0	15.8	**11.3**
8/31/2011	1218.9	37.8	36.9	27.4	20.8	20.6	35.7	30.8	26.1	22.1	19.3	32.3	28.6	25.3	22.4	20.0	**12.3**
9/30/2011	1131.4	48.4	45.5	36.9	28.8	26.7	41.6	36.9	32.6	28.6	25.2	37.6	34.1	30.8	27.8	25.1	**12.5**
10/31/2011	1253.3	35.4	33.3	24.5	18.6	18.7	33.9	29.0	24.5	20.6	17.7	31.5	27.8	24.5	21.5	18.9	**12.6**
12/30/2011	1257.6	30.1	28.3	20.3	16.0	16.2	32.7	28.0	23.6	19.7	16.8	31.0	27.4	24.1	21.2	18.6	**12.3**
1/31/2012	1312.4	25.2	24.8	16.2	15.0	15.1	29.1	24.3	19.9	16.1	14.0	28.2	24.6	21.2	18.2	15.8	**12.4**
Global Coronavirus Pandemic of 2020-2021																	
12/31/2019	3230.8	32.4	22.2	11.1	11.6	12.7	23.6	19.4	14.1	10.4	11.2	22.4	19.3	15.6	11.9	10.9	**11.5**
2/28/2020	2954.2	54.7	47.5	37.1	23.6	24.0	31.7	27.9	23.4	17.2	13.8	26.7	23.9	20.7	16.8	13.7	**12.9**
3/16/2020	2386.1	92.5	85.7	77.7	67.2	52.2	59.6	54.9	50.1	44.2	37.4	46.1	42.5	38.7	34.7	30.1	**16.1**
3/31/2020	2584.6	67.6	57.3	45.4	32.5	30.6	44.3	39.6	34.4	28.8	23.6	36.6	33.1	29.7	26.5	25.7	**10.9**
4/30/2020	2912.4	49.0	39.8	28.2	20.7	25.7	39.3	34.4	28.9	23.2	19.1	34.5	30.9	26.6	22.3	18.9	**15.6**
6/30/2020	3100.3	44.0	34.3	24.3	20.1	24.9	36.2	31.5	26.1	20.1	17.6	32.4	28.6	24.4	19.9	16.8	**15.6**
12/31/2020	3756.1	40.3	28.7	17.3	15.6	21.0	31.0	25.8	20.0	15.5	14.9	28.6	24.8	20.5	16.8	15.4	**13.2**
Global Inflation and Interest Rate Surge 2022-2024																	
12/31/2021	4766.2	37.2	24.2	12.5	12.4	14.4	29.2	23.6	17.7	13.4	13.0	26.7	23.6	19.3	15.3	14.4	**12.3**
9/30/2022	3585.6	45.4	34.6	28.1	22.6	24.5	35.1	30.6	26.7	23.1	20.7	32.4	29.3	26.3	23.4	21.1	**11.4**
12/30/2022	3839.5	34.3	24.4	19.8	16.7	20.7	29.5	26.3	23.0	19.4	17.0	28.9	26.4	23.5	20.7	18.2	**10.7**
12/29/2023	4769.8	33.7	20.8	11.0	12.4	14.8	24.0	18.9	14.1	11.0	11.0	22.9	19.5	15.8	12.7	11.2	**11.7**
6/28/2024	5460.5	33.2	20.5	10.6	12.2	20.6	23.1	18.1	13.5	10.7	10.8	21.9	18.6	15.0	12.2	10.9	**11.1**

Volatility often surges when stock prices plummet, as shown in Figure 4, which makes put options even more valuable during sharp declines in prices:

Figure 4
Plunges in Stocks Are Correlated with Sharp Volatility Increases: 2006-2024



John Cox of MIT (2016) has shown that the skew in implied volatilities is sensible, given that put options pay off most when stock prices fall sharply, which is when volatility surges, which amplifies the price gains on puts. This gives put options an even higher insurance value (negative beta) than if there were no correlation of volatility changes with the percentage changes in stock prices. As the Black-Scholes formula (1973) assumes volatility is constant through time, this gives put options value that is greater than indicated by the Black-Scholes value computed with at-the-money option implied volatility. This leads to the higher implied volatilities for puts to fit actual prices, which gives the skew.

To compute state prices using the B-L insights, our procedure is as follows: (1) use the implied volatilities by moneyness downloaded from Bloomberg to compute call and put option prices for various strikes from 80% to 120% moneyness. We interpolate between observed implied volatilities (such as 115% moneyness being an average of 110% and 120%), but we do not interpolate outside of the observed data from Bloomberg. In working at Goldman Sachs as Chief Risk Officer in the late 1990s (including the 1998 turbulence), Bob Litzenberger observed that extrapolations into the tails were often greatly in error in fitting actual market prices. Thus, based upon Litzenberger's observation, in this article we do not do extrapolations beyond observed implied volatilities from traded options. We want our estimates to be as nonparametric as possible, while still having enough cross-sectional observations to do interesting cross-sectional state analysis. (Note that this differs from Beason-Schreindorfer and others who use both "interpolations and extrapolations" to get their complete state price distributions.) After having the implied volatilities by moneyness, we use the Breeden-Litzenberger (2022) method for continuous price distributions. It consists of butterfly spreads and tail spreads to estimate state prices for payoffs in the various intervals for returns, as described by the state definitions in Figure 2. Figure 5 shows how their approach looks with our state price distribution for 1-year equity returns. State prices are computed from the butterfly spreads of options costs, plus the costs of tail spreads for the left and right tails, normalized to sum to 1.0 (giving a "risk neutral density").

Figure 5

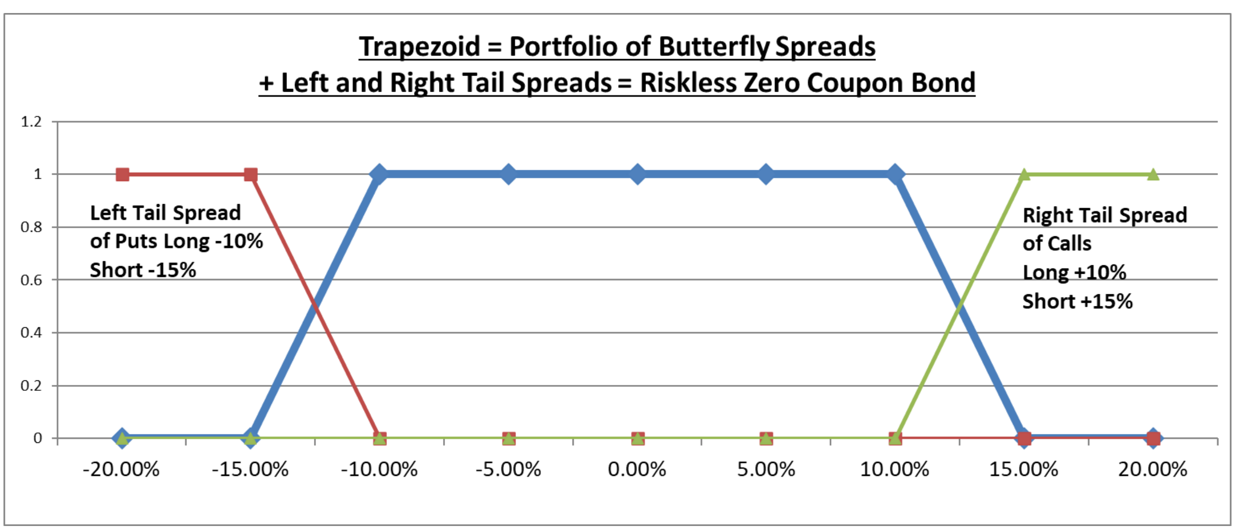
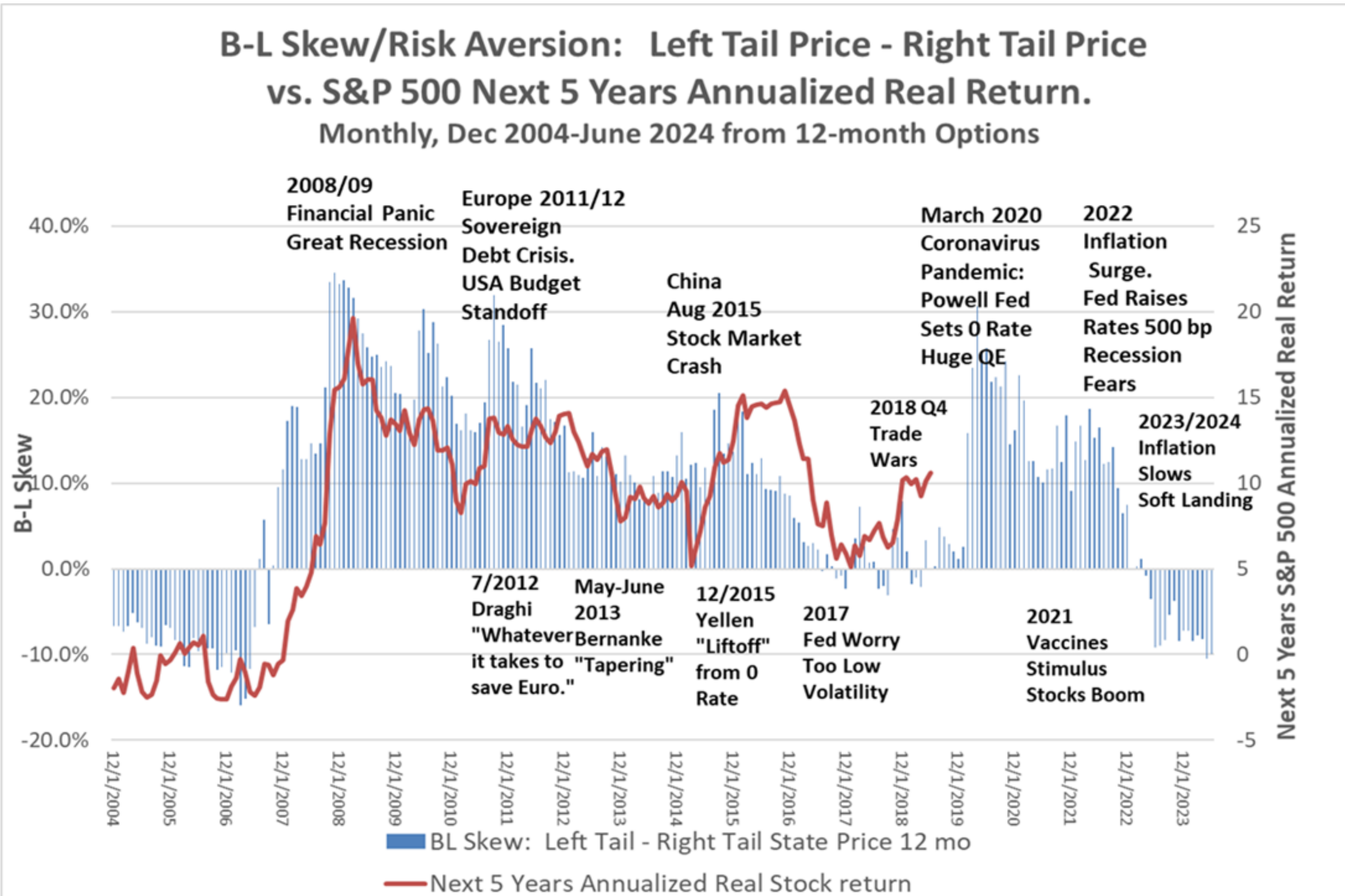


Figure 6

		S&P 500 Insurance Prices (Risk-neutral density).								7/25/24 9:20 PM		
		Monthend Data from December 2004. Uses Breeden-Litzenberger (2022) technique									TTM	12 month
					$90%-$85 Puts			ATM			$110-$115 Calls	B-L Skew
	ATM	S&P 500	Treasury Yields		**Left Tail**	90	95	100	105	110	**Right Tail**	**Left Tail**
Date	Implied σ	Spot Index	3 mnth	2 Year	**Spread**	Butterfly	Butterfly	Butterfly	Butterfly	Butterfly	**Spread**	**-Right Tail**
USA Financial Panic of 2008/2009												
12/29/2006	14.0	**1418.3**	**5.02**	**4.82**	**10.1%**	15.9%	18.6%	15.6%	12.2%	7.7%	**19.9%**	**-9.9%**
12/31/2007	22.2	**1468.4**	**3.36**	**3.05**	**32.7%**	13.1%	11.9%	8.5%	7.7%	5.0%	**21.1%**	**11.6%**
6/30/2008	22.3	**1280.0**	**1.90**	**2.63**	**34.6%**	13.1%	11.6%	8.5%	7.4%	4.8%	**20.0%**	**14.6%**
10/31/2008	39.4	**968.8**	**0.46**	**1.56**	**55.0%**	6.5%	6.1%	3.8%	4.5%	2.5%	**21.5%**	**33.5%**
12/31/2008	36.3	**903.3**	**0.11**	**0.76**	**53.9%**	7.0%	6.5%	4.3%	4.7%	2.9%	**20.7%**	**33.2%**
2/27/2009	36.9	**735.1**	**0.26**	**1.00**	**53.6%**	7.1%	6.4%	4.5%	4.6%	3.0%	**20.8%**	**32.8%**
6/30/2009	26.8	**919.3**	**0.19**	**1.11**	**44.8%**	10.7%	9.2%	6.6%	6.0%	3.9%	**18.9%**	**25.9%**
12/31/2009	22.8	**1115.1**	**0.06**	**1.14**	**38.6%**	13.0%	11.1%	8.1%	6.8%	4.3%	**18.1%**	**20.5%**
Europe's Sovereign Debt Crisis and USA Budget Impasse (Summer 2011)												
6/30/2011	19.4	**1320.6**	**0.03**	**0.45**	**32.5%**	16.8%	13.7%	9.9%	7.4%	4.3%	**15.4%**	**17.0%**
7/29/2011	20.7	**1292.3**	**0.10**	**0.36**	**35.7%**	15.4%	12.7%	8.7%	7.2%	4.1%	**16.3%**	**19.5%**
8/31/2011	25.3	**1218.9**	**0.02**	**0.20**	**43.4%**	12.6%	10.2%	7.2%	6.1%	3.6%	**16.7%**	**26.7%**
9/30/2011	30.8	**1131.4**	**0.02**	**0.25**	**50.4%**	9.4%	8.1%	5.3%	5.3%	3.0%	**18.4%**	**32.0%**
10/31/2011	24.5	**1253.3**	**0.01**	**0.25**	**43.0%**	12.9%	10.6%	7.1%	6.3%	3.6%	**16.5%**	**26.5%**
12/30/2011	24.1	**1257.6**	**0.02**	**0.25**	**42.4%**	13.2%	10.8%	7.1%	6.3%	3.6%	**16.7%**	**25.8%**
1/31/2012	21.2	**1312.4**	**0.06**	**0.22**	**37.3%**	15.7%	12.6%	8.3%	6.9%	3.7%	**15.4%**	**21.8%**
Global Coronavirus Pandemic of 2020-2021												
12/31/2019	15.6	**3230.8**	**1.55**	**1.58**	**16.6%**	24.5%	21.2%	10.7%	8.9%	2.8%	**15.4%**	**1.2%**
2/28/2020	20.7	**2954.2**	**1.27**	**0.86**	**33.9%**	18.7%	14.4%	4.4%	7.4%	3.1%	**18.0%**	**15.9%**
3/16/2020	38.7	**2386.1**	**0.24**	**0.36**	**63.2%**	3.9%	5.8%	2.4%	4.6%	1.0%	**19.2%**	**44.0%**
3/31/2020	29.7	**2584.6**	**0.11**	**0.23**	**41.7%**	16.6%	10.4%	5.2%	5.5%	2.5%	**18.2%**	**23.5%**
4/30/2020	26.6	**2912.4**	**0.09**	**0.20**	**47.4%**	14.1%	10.7%	4.7%	5.8%	0.7%	**16.7%**	**30.6%**
6/30/2020	24.4	**3100.3**	**0.16**	**0.16**	**41.8%**	17.8%	12.6%	4.4%	6.2%	1.2%	**16.0%**	**25.8%**
12/31/2020	20.5	**3756.1**	**0.09**	**0.13**	**31.1%**	21.0%	15.1%	9.3%	6.9%	1.5%	**15.0%**	**16.1%**
Global Inflation and Interest Rate Surge 2022-2024												
12/31/2021	19.3	**4766.2**	**0.06**	**0.73**	**25.5%**	24.1%	17.5%	8.6%	7.4%	0.5%	16.4%	**9.1%**
9/30/2022	26.3	**3585.6**	**3.33**	**4.22**	**36.8%**	12.1%	10.5%	7.3%	6.9%	3.7%	22.7%	**14.2%**
12/30/2022	23.5	**3839.5**	**4.42**	**4.41**	**31.7%**	13.2%	11.8%	7.5%	7.7%	3.8%	24.2%	**7.5%**
12/29/2023	15.8	**4769.8**	**5.40**	**4.23**	**12.5%**	18.4%	19.2%	14.6%	10.7%	4.8%	19.8%	**-7.2%**
6/28/2024	15.0	**5460.5**	**5.48**	**4.71**	**10.2%**	17.2%	19.4%	16.0%	11.5%	5.5%	20.2%	**-9.9%**

Let us now look at the movements in implied volatilities (Figures 3 and 4) and our B-L insurance prices and skews in the major economic episodes in the past 20 years (Figure 6). Please note that Figure 7 below presents a complete monthly time series of the BL Skew, which is also helpful for understanding the following commentary.

Figure 7



The first panel in Figures 3 and 6 is from Dec 31, 2006-Dec 31, 2009, which covers the Financial Panic of 2008/9 (recession Dec 2007-June 2009) and the beginning of recovery after June 2009. As real estate prices began to tumble and subprime loans and commercial real estate sank, stock prices dropped by half from June 2007 to February 2009, and implied volatilities in options surged. MIT PhD Ben Bernanke's Fed dropped the Fedfunds rate to near zero in December 2008 to stimulate the economy.

Bank stocks tumbled more in January/February 2009, and the government stepped in and provided capital to stabilize Citibank. March 9, 2009, was the low for stock prices. Then in the last 9 months of 2009, stocks soared by 75% and 1-month at-the-money (ATM) implied volatilities declined dramatically from peaks of 50%+ to just 17% on 12/31/2009.

Implied volatilities showed a substantial skew in late 2008, with volatilities from out-of-the-money puts (80% moneyness) much higher than ATM options and higher than out-of-the money calls (120% moneyness). As stock prices recovered, the skew was reduced, but still present. Note that on 12/31/2009, a small "smile" emerged in the 1-month options, as 120% moneyness volatility was higher than the ATM. Note that for longer maturity options, 6-month and 12-month, the skew is still present, as out-of-the-money puts have significantly higher implied volatilities than ATM options and out-of-the-money calls.

Europe's Sovereign Debt Crisis began in 2010, when serious accounting irregularities were discovered in Greece's books, and skew surged. In 2011 and 2012 those concerns were acute and, while starting with the smaller countries, fears spread to the much larger economies of Italy and Spain. Simultaneously, in August 2011, the USA had a major budget crisis, and stocks fell 15% very rapidly, while President Obama fought with a Republican Congress. Bernanke's Fed provided very strong support with "Quantitative Easing" and statements that rates would stay near zero for more than 2 years, until the end of 2013 (which they did). In Europe, another MIT PhD, Mario Draghi, became President of the European Central Bank in November 2011 and immediately cut rates twice. When there were still fears that the Euro area would break up and Spain or Italy or Greece might depart, Draghi finally stated in London in July 2012 that they would do "whatever it takes to save the Euro," and added "and believe me, it will be enough." And it was, and Europe stabilized. Insurance prices tracked these fears and stabilization in Europe and the USA, with the BL Skew peaking at 32% in September 2011.

However, prices for left-tail insurance remained high all year in 2011 and 2012. In mid-2013, the US economy had strengthened so much that real estate prices were rising again, and Fed Chair Bernanke said the Fed was ready to start "tapering" QE stimulus. Markets briefly had a "taper tantrum" and fell (as they were hooked on the drug of zero rates), but then they quickly stabilized and resumed their march upward later in 2013. Markets than built in a tapering of Quantitative Easing intervention involving massive government purchases of bonds and, ultimately lifting rates off the "Zero Lower Bound" in the USA. Bernanke's Vice Chair, Janet Yellen of Berkeley, took over as Fed Chair in 2014, and kept rates at zero to overstimulate the economy in 2014 and 2015, then finally led the "liftoff" in rates in December 2015. Quarter point rate increases were done predictably through 2016 and 2017 until a total of 9 rate hikes were completed to get the Fedfunds rate back to 2.25%-2.5%. In 2016, Donald Trump was elected President, and expectations were for high stimulus and growth. As one who followed markets closely, it was said the markets had a "Trump put," for if stocks weakened too much, Trump would somehow stimulate the economy. Stocks did well under Trump, but faltered in late 2018, when his major trade wars with China and Mexico worried the markets. 2019 was pretty calm, but markets worried that a recession would come in 2020, given slowing global growth due to protectionism and tariffs.

Next up was the Coronavirus Pandemic in 2020-2021, which reached an extreme level of impact in March 2020. In Q2 2020 when the virus was surging, many businesses were closed or sharply hit (such as airlines and restaurants). as people stayed home and did not risk direct contacts as often as normal. Stock prices declined sharply (down 25%) in Q1 of 2020 when the pandemic hit the USA. As stock prices fell, 1-month implied volatilities surged from 11% to 77%. The BL Skew had a massive jump, from 1.2% in

December 2019, to 44.0% on March 16, 2020. The price paid for the left tail insurance payoffs went to 63.2%, the highest state price in our data set. Implied volatilities by moneyness developed a very large skew in March 2020, which was substantially reduced as stock prices surged with Fed stimulus in the remainder of 2020 and 2021.

Major countries and businesses worked closely together to develop vaccines for the virus. Vaccines were widely distributed in late 2020 and 2021, and the virus gradually became under control and businesses started returning towards normal. Profits were strong, and stock prices rebounded to record levels in 2021, supported by near-zero short term interest rates. The BL Skew tumbled in 2021, back to relatively normal levels.

Normalcy was relatively short-lived, as Russia invaded the Ukraine in late February 2022. Oil prices skyrocketed and inflation began to surge. With surging inflation (to 7%+, up from 2%), the Fed pivoted and started raising rates rapidly. As Figure 6 shows, the 1-month rate went from 0.06% at 12/31/21 to 4.42% at 12/30/22, and to 5.50% on 12/29/23, up over 500 basis points in 2022/2023. Stocks dropped over 20% in 2022, and long-term bonds did, too, given the surge in interest rates, reversing their normal negative correlation. However, the job market was still very strong and unemployment very low, so stocks rebounded more than 20% in 2023 to set new highs.

The BL Skew was fairly low in 2022, which was surprising, given the war in Ukraine and the massive increases in rates that the Fed implemented. The BL Skew peaked at 14% in 2022, a far cry from the surges to 30% and 40% in the prior three events. Unemployment remained below 4% most of the time, wealth was still high, and consumers spent well, using up some of their large savings from the Coronavirus Pandemic. Stock prices regularly hit new highs in 2023 and 2024, as inflation started declining rapidly back to normal, with PCE inflation in June 2024 back down to 2.5% from a 7%+ peak. The Fed has stated that they can reduce interest rates and plan to in late 2024 so as not to hold back the economy. These circumstances have rewarded and encouraged investors, as it seems a “soft landing” has been accomplished. Reflecting these successes, the BL Skew has actually turned negative at -9.9% on June 30, 2024. Investors are paying up for upside potential more than they are paying for downside protection. This seems a bit giddy and may lead to poor returns for investors going forward the next few years.

III. Option-Based Forecasts of Future Real Returns, Equity Risk Premiums

As noted, Figure 7 gives the complete time series picture of BL Skew, graphing its monthly history from Dec 31, 2004 to June 30, 2024. As we view this as reflecting changing risk and risk aversion of investors, we have overlayed the real stock market returns of the S&P 500 during the subsequent 5 years. Our hypothesis is that when investors are highly risk averse and paying up heavily for downside protection, fear is high and equity prices are relatively low to provide high returns for risk takers. Future returns should provide greater rewards at those times. At times of low volatility, risk premiums are lower (2005-2006, 2017, and 2023-2024). Graphically, there is a correlation between the BL Skew and real returns on stocks going forward 5 years. Our statistical analysis will demonstrate this.

Figure 8 shows a comparison of simple correlations of the VIX, Implied Volatility Skews, the BL Skew and subsequent real returns on stocks, comparing them to an update of John Cochrane’s (2008) U. Chicago demonstration of simple dividend yield’s correlation. For the IV skews, we look at both 1-month skews (which can move a lot), and 12-month skews, which are less volatile. Declines and increases of 20% or more in the S&P 500 are relatively rare over a 1-month period, so prices for the 1-month 20% OTM puts are surely low and IV may not be well-estimated. Thus, for the 1-month period, we also look at the 90/110 spread of implied volatilities, as there are more 10%

moves in a month and should be better estimated. Indeed the 90/110 skews are more correlated with future real returns than are the 80/120 skews for 1 month options. However, the 12-month skew addresses the larger moves over a full year, and those are most significantly correlated with future 5-year returns, with results that rival and even exceed those of dividend yield in longer-term returns, as Figure 8 shows.

Figure 8

Correlations of Indicators with Future Real Returns: 2005-June 2024 Data

Forecasting Real Returns: Correlations of Forecast Variables						
2005-2024 June Data	Cochrane		*Bloomberg Options Data*			*Stock Options*
Overlapping correlations	2008		*Implied Volatility by Moneyness*			*Breeden-*
	Dividend	VIX	*IV Skew*	*IV Skew*	*IV Skew*	*Litzenberger*
Forecast	Yield LTM		*1 month*	*1 month*	*12 months*	BL-Skew 12 mo
Horizon	Forecasts		*80/120*	*90/110*	*80/120*	Left Tail-Right Tail
3 Year Real Stock	41.2%	25.4%	-0.3%	8.0%	47.0%	63.2%
5 Year Real Stock	62.1%	41.5%	12.1%	24.9%	62.9%	80.5%
7 Year Real Stock	55.1%	45.7%	18.3%	36.6%	67.9%	86.5%
10 Yr Real Stocks	57.7%	59.2%	9.6%	37.9%	66.2%	91.8%
3,5,7,10 Avg	54.0%	42.9%	9.9%	26.8%	61.0%	80.5%
Statistics for the Estimates: R-Squareds and t(HAC)						
	RSQ t(HAC)	RSQ t(HAC)	RSQ t(HAC)	RSQ t(HAC)	RSQ t(HAC)	RSQ t(HAC)
3 Year Real Stock	17% 3.9	6% 2.4	0% 0	1% 0.8	22% 4.0	40% 5.8
5 Year Real Stock	39% 4.4	17% 4.1	1% 0.8	6% 1.8	40% 10.5	65% 13.1
7 Year Real Stock	34% 5.0	16% 3.6	0% -0.4	1% 0.7	25% 4.8	65% 10.0
10 Yr Real Stocks	33% 4.7	35% 4.3	1% 0.6	14% 3.0	44% 6.8	84% 18.5

For the forecast of longer-term real returns on stocks, (3,5,7 and 10 years out), we find that both the IV 12-month skew and the B-L Skew are more correlated with future real equity returns than is Cochrane's dividend yield, with average correlations of 54% for dividend yield, 61% for the 12-month IV Skew, and 80% for the BL Skew. The VIX itself has a lower average correlation with long-term returns at 43%. To estimate the statistical significance of these correlations, as the data are overlapping monthly data, we compute Newey-West t-statistics, which adjust for heteroscedasticity and autocorrelation (HAC t-stats). Dividend yields have t(HAC) of 2.4 to 4.3, the 12- month IV skew has t(HAC) highly significant at 4.0 to 10.5, and the BL Skew has t(HAC) from 5.8 on 3-year real returns to 18.5 on 10-year real returns. Thus, all of these are statistically highly significant, as expected. When investors pay up much more than normal for left tail downside protection vs right tail upside, it likely does reflect higher risk aversion, and subsequent real returns are significantly higher.

In Figure 9, we compare the 12-month B-L Skews in Figure 6 to the 12-month IV Skews in Figure 3. The B-L Skews move by larger amounts and in ways that are plausibly more related to likely risk and risk aversion, given stock price and volatility moves. This likely explains why the B-L Skew has a higher correlation with subsequent returns than do the Implied Volatility skews.

Figure 9

USA Financial Panic of 2008/2009				Europe's SDC, USA Budget Crisis				Coronavirus Pandemic				Inflation, Interest Rate Surge			
Date	S&P 500	BL Skew	IV Skew	Date	S&P 500	BL Skew	IV Skew	Date	S&P 500	BL Skew	IV Skew	Date	S&P 500	BL Skew	IV Skew
12/29/2006	1418.3	-9.9%	8.8	6/30/2011	1320.6	17.0%	10.8	12/31/2019	3230.8	1.2%	11.5	12/31/2021	4766.2	9.1%	12.3
12/31/2007	1468.4	11.6%	10.6	7/29/2011	1292.3	19.5%	11.3	2/28/2020	2954.2	15.9%	12.9	***9/30/2022***	***3585.6***	***14.2%***	***11.4***
6/30/2008	1280.0	14.6%	9.9	8/31/2011	1218.9	26.7%	12.3	***3/16/2020***	***2386.1***	***44.0%***	***16.1***	12/30/2022	3839.5	7.5%	10.7
10/31/2008	968.8	33.5%	10.6	***9/30/2011***	***1131.4***	***32.0%***	***12.5***	3/31/2020	2584.6	23.5%	10.9	12/29/2023	4769.8	-7.2%	11.7
12/31/2008	903.3	33.2%	9.9	10/31/2011	1253.3	26.5%	12.6	4/30/2020	2912.4	30.6%	15.6	6/28/2024	5460.5	-9.9%	11.1
2/27/2009	***735.1***	***32.8%***	***9.7***	12/30/2011	1257.6	25.8%	12.3	6/30/2020	3100.3	25.8%	15.6				
6/30/2009	919.3	25.9%	9.9	1/31/2012	1312.4	21.8%	12.4	12/31/2020	3756.1	16.1%	13.2				
12/31/2009	1115.1	20.5%	9.9												

Figures 10 and 11 compare the scatter plots of Cochrane's dividend yield with the Breeden-Litzenberger 12-month Skew in forecasting the next 5 years of real stock returns, using data from 2005-June 2024. The BL Skew has a much stronger relationship, reflecting prices paid for the tails of the distribution.

Figure 10

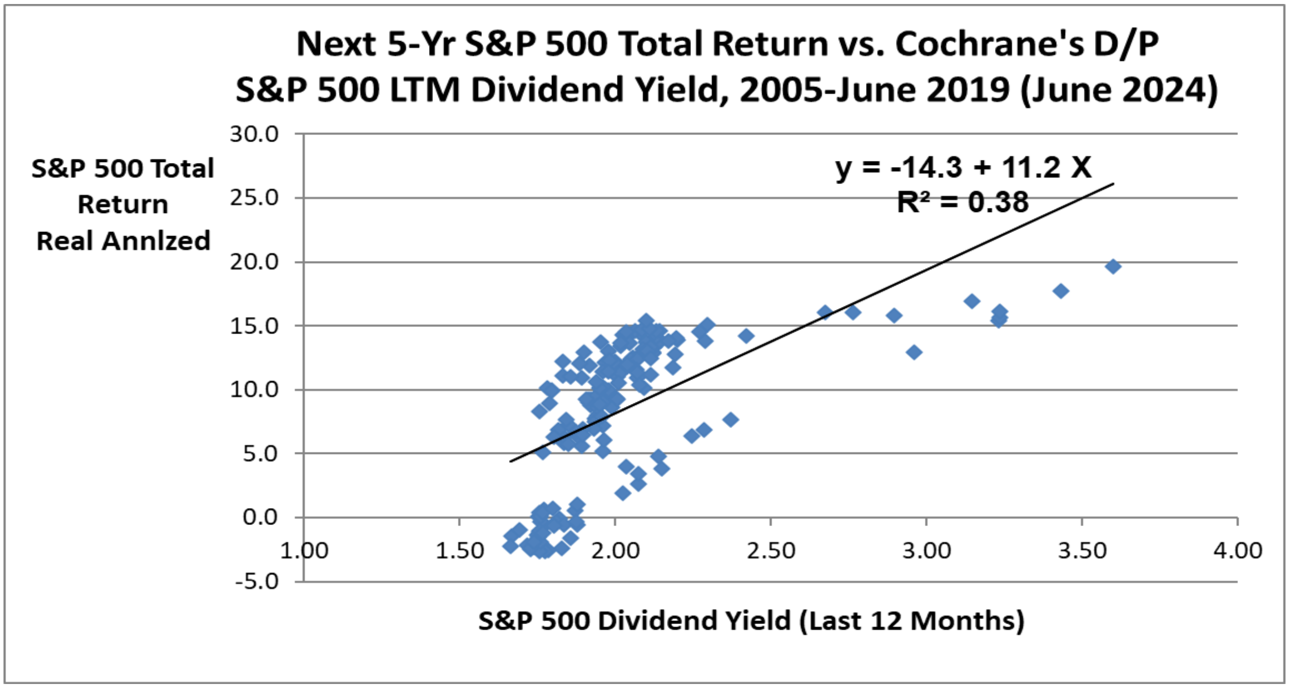


Figure 11

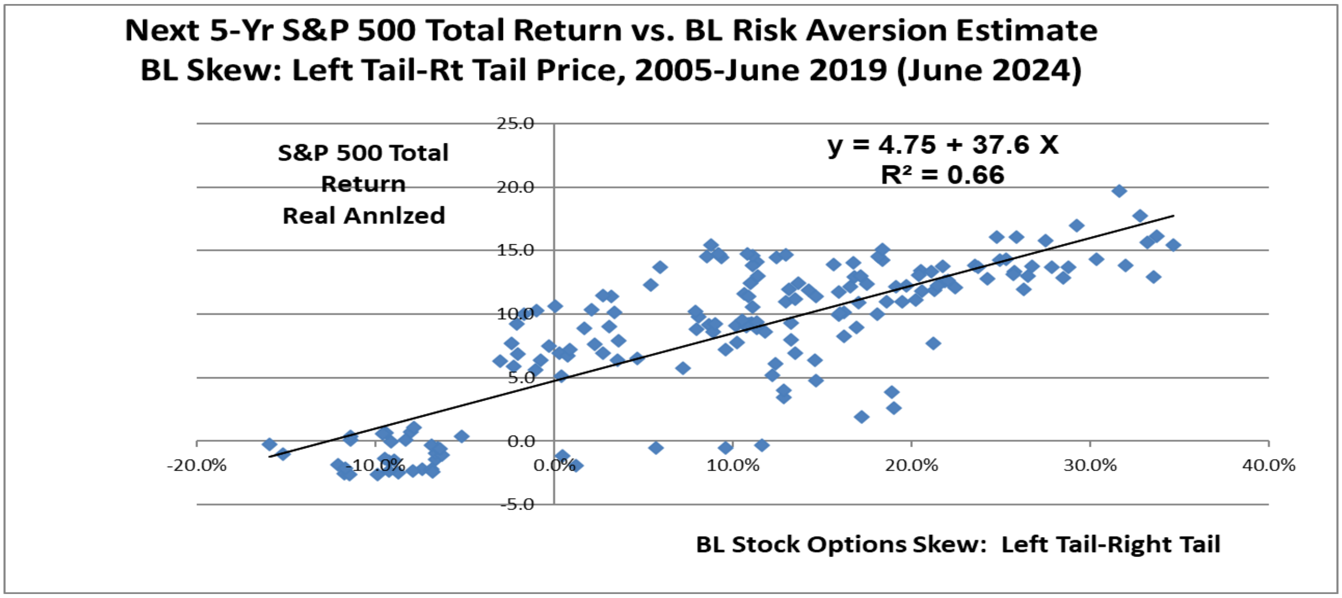
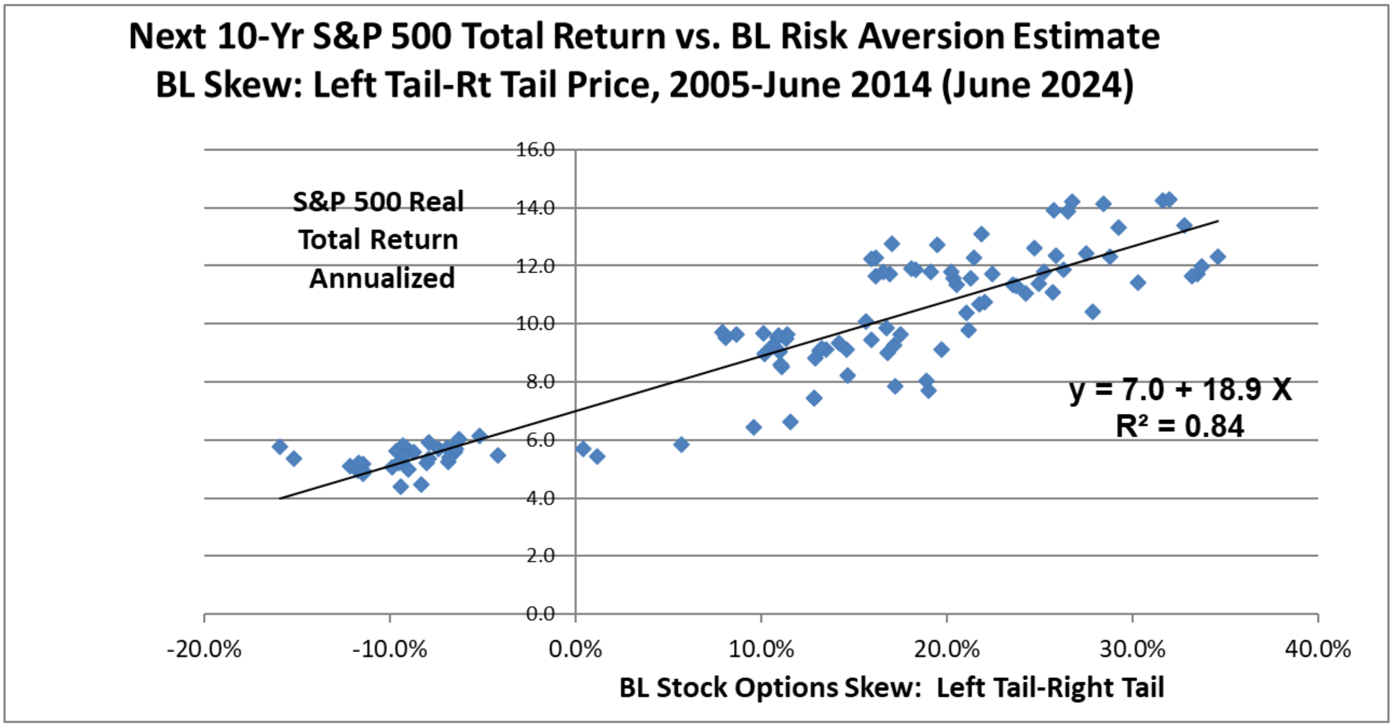


Figure 12 shows that forecast performance improves over longer time periods in this sample, as the 10-year fit is much stronger than the 5-year forecast's fit, with an R-squared of 0.84. HAC corrected t-statistic for the slope is 18.5, as shown in Figure 13.

Figure 12



IV. Comparison of Forecasts: Graham-Harvey CFO Survey, TIPS, Damodoran ERP, Yardeni, Shiller and B-L Jia Excess CAPE Yield

In this section, we examine some of the well-known forecasts of the Equity Risk Premium (Stock return less 10-year Bond Return). John Graham and Campbell Harvey of Duke survey 450 Chief Financial Officers and ask for their 1-year and 10-year forecasts of stock returns and for the excess return of stocks over 10-year bond returns. These forecasts have been done quarterly from 2000 to Q1 of 2024, missing three observations when these surveys were not conducted. Figures 13 and 14 show that Graham and Harvey's CFO Survey median forecasts of the Equity Risk Premium have a statistically significant relationship with future realized equity risk premiums, with t(HAC) statistics of 5.4 to 6.8, exceeding that of dividend yields, which go from 1.7 to 3.8 for 3-10 year forecasts.

As Graham and Harvey found that their equity risk premiums forecasted by CFOs were most related to TIPS yields (negatively), we also include TIPs yields as a forecast variable. The TIPS yield relationship has been that negative or low TIPS yields occur primarily in recessions (when the Fed has actively stimulated the economy by reducing rates). So when risk is high and risk premiums are high, TIPS yields are low, which gives TIPS a negative relation with future equity risk premiums. HAC t-statistics for TIPS are strong and similar to those for the Graham-Havey Survey forecasts of the equity risk premium.

Aswath Damodoran of NYU has a well-respected set of discounted cash flow based forecasts of the equity risk premium and of the expected total return on the S&P 500. Damodoran has beginning of year forecasts annually back to the 1960s. His monthly forecasts began in August, 2008. We use the monthly data, with monthly forecasts for 2005-July 2008 interpolated from the annual forecasts. Figures 13 and 14 show that Damodoran's cash flow based forecasts perform better than the CFO Survey medians of Graham and Harvey. Figure 14 shows average correlation with subsequent equity risk premiums for 3, 5, 7 and 10 year forecasts is 0.79 for Damodoran vs. 0.60 for the CFOs. Of course, Graham and Harvey are reporting the forecasts of the CFOs and are not modeling and forecasting in the ways they might think best.

Note that the BLSkew has an average correlation with the realized ERP that is in between Graham-Harvey and Damodoran at 0.70, while the IV Skew is 0.66 and dividend yield is 0.41. The BL Skew does better at forecasting future absolute annualized real returns on stocks, with an 0.80 correlation, but is lower in forecasting the long stocks, short bonds ERP. In contrast, but sensibly, the ERP forecasts of the

Graham-Harvey CFOs surveyed and of Damodoran have higher correlations forecasting realized equity risk premiums than they have for simple real total returns.

It is perhaps surprising that the estimated nominal total returns from both the CFO Survey and Damodoran's cash flow calculations have negative correlations with subsequent real returns. However, when "breakeven inflation" from TIPs at the forecast date is subtracted from Damodoran's 10-year return forecasts, the implied real return has a positive correlation with realized real returns over 5, 7 and 10-year horizons.

As a final set of forecasts, we examine 3 that are of the form: (E/P – R10), i.e., the earnings yield on stocks less the 10-year riskless rate. In the first forecast, we use one that Ed Yardeni, a popular business forecaster and student of James Tobin at Yale, has computed for many years. He computes what he calls the "Fed Model" as the forward 12-month earnings forecast for the S&P 500, divided by its current price, and then subtracts the 10-year riskless rate. In some versions he subtracts the nominal riskless rate, while in others he subtracts the (real) 10-TIPs yield. The second forecast of this type comes from the approach of Breeden, Litzenberger and Jia (2015 Part 2, Section 5), who liked the approach of Robert Shiller at Yale, using "CAPE" to smooth out cyclical earnings fluctuations by having a 10-year average of past earnings. They computed "CAPE-inverse minus the 10-year riskless rate" and found this "CAPE Yield Gap" to be plausibly related to changes in risk and changes in the unemployment rate, using Shiller's data back to 1924. The third approach, was by Robert Shiller himself. Following BLJ, Shiller did similar calculations to BLJ's CAPE-inverse – R10, but with the use of real returns and more careful consideration of inflation. Shiller starting publishing this in 2022 on his website as the "Excess CAPE Yield (ECY)." His graph of ECY versus equity risk premiums realized over the next 10 years is very impressive, demonstrating a very high correlation.

Figures 12 and 13 show the comparison of results from the various forecasters. For the realized Equity Risk Premium (Long stocks, short 10-year bonds), using Shiller's careful calculations of real returns, we have for the 2005-2024 period data:

Figure 13

Correlations of Indicators with Future Real Equity Returns: 2005-June 2024 Data

Forecasting Annualized Real Stock Returns (S&P 500)

	Next 10 Years	Next 10 Years: Correlation	Next 10 Years: t(HAC)		Average of 3,5,7,10 Year	Avg of 3, 5,7,10 Yrs: Correlation	Avg of 3, 5,7,10 Yrs: t(HAC)
1	B-L Skew 12mo Left Tail-Right	0.92	18.5	1	B-L Jia CAPE Yield Gap	0.84	12.4
2	B-L Jia CAPE Yield Gap	0.87	11.8	2	B-L Skew 12mo Left Tail-Right	0.81	11.8
3	Shiller Excess CAPE Yield	0.87	11.8	3	Shiller Excess CAPE Yield	0.79	10.0
4	Damodoran DCF ERP	0.80	9.5	4	Damodoran DCF ERP	0.77	9.2
5	Yardeni Fed Model Fwd12m	0.73	7.9	5	Yardeni Fed Model Fwd12m	0.75	8.2
6	IV Skew 12 month 80-120	0.66	6.8	6	IV Skew 12 month 80-120	0.61	6.5
7	VIX	0.59	4.9	7	TIPS 10 Year Real Yield	-0.56	-4.2
8	Dividend Yield Cochrane	0.58	4.7	8	Dividend Yield Cochrane	0.54	4.5
9	TIPS 10 Year Real Yield	-0.53	-3.5	9	VIX	0.43	3.6
10	CFOs ERP Graham-Harvey	0.49	4.0	10	CFOs ERP Graham-Harvey	0.41	3.4

Figure 14

Forecasting Annualized Real S&P 500 Stocks Minus Real LT Bonds (Equity Risk Premium)

	Next 10 Years	Next 10 Years: Correlation	Next 10 Years: t(HAC)		Average of 3, 5, 7, 10 Year	Avg 3, 5, 7, 10 Yr: Correlation	Avg 3, 5, 7, 10 Yr: t(HAC)
1	B-L Jia CAPE Yield Gap	0.92	13.4	1	B-L Jia CAPE Yield Gap	0.85	10.5
2	Shiller Excess CAPE Yield	0.91	12.3	2	Damodoran DCF ERP	0.79	9.0
3	Damodoran DCF ERP	0.87	11.2	3	TIPS 10 Year Real Yield	-0.75	7.0
4	Yardeni Fed Model Fwd12m	0.86	13.8	4	Yardeni Fed Model Fwd12m	0.74	8.9
5	B-L Skew 12mo Left Tail-Right	0.83	12.7	5	Shiller Excess CAPE Yield	0.73	7.7
6	TIPS 10 Year Real Yield	-0.77	-6.9	6	B-L Skew 12mo Left Tail-Right	0.70	8.3
7	CFOs ERP Graham-Harvey	0.65	6.8	7	IV Skew 12 month 80-120	0.66	8.2
8	IV Skew 12 month 80-120	0.63	7.8	8	CFOs ERP Graham-Harvey	0.60	5.5
9	Dividend Yield Cochrane	0.51	3.8	9	Dividend Yield Cochrane	0.41	3.0
10	VIX	0.39	3.1	10	VIX	0.29	3.1

V. **Price/Frequency Multiples and Conditional Marginal Utilities**

As B-L (1978) derived, insurance prices should in equilibrium be proportional to the product of the state probability and investors' marginal utilities, conditional upon the state examined. In their review article, Breeden, Litzenberger and Jia (2015, eq. 5) derived this as:

$$\Rightarrow \phi_{ts} = \frac{\pi_{ts} u_t'(C_{ts})}{u_0'(C_0)} \qquad \textit{and} \qquad \frac{\phi_{ts}}{\pi_{ts}} = \frac{u_t'(C_{ts})}{u_0'(C_0)} \qquad \text{(eq.1)}$$

A natural way to estimate likely probabilities of various 1-year percentage moves in stock prices is to examine historical frequencies observed over long periods of time for 1-year returns prior to the time examined. Figure 15 shows frequencies for S&P 500 stock market returns by 25-year and 50-year period, running from 1900 to June 2024:

Figure 15

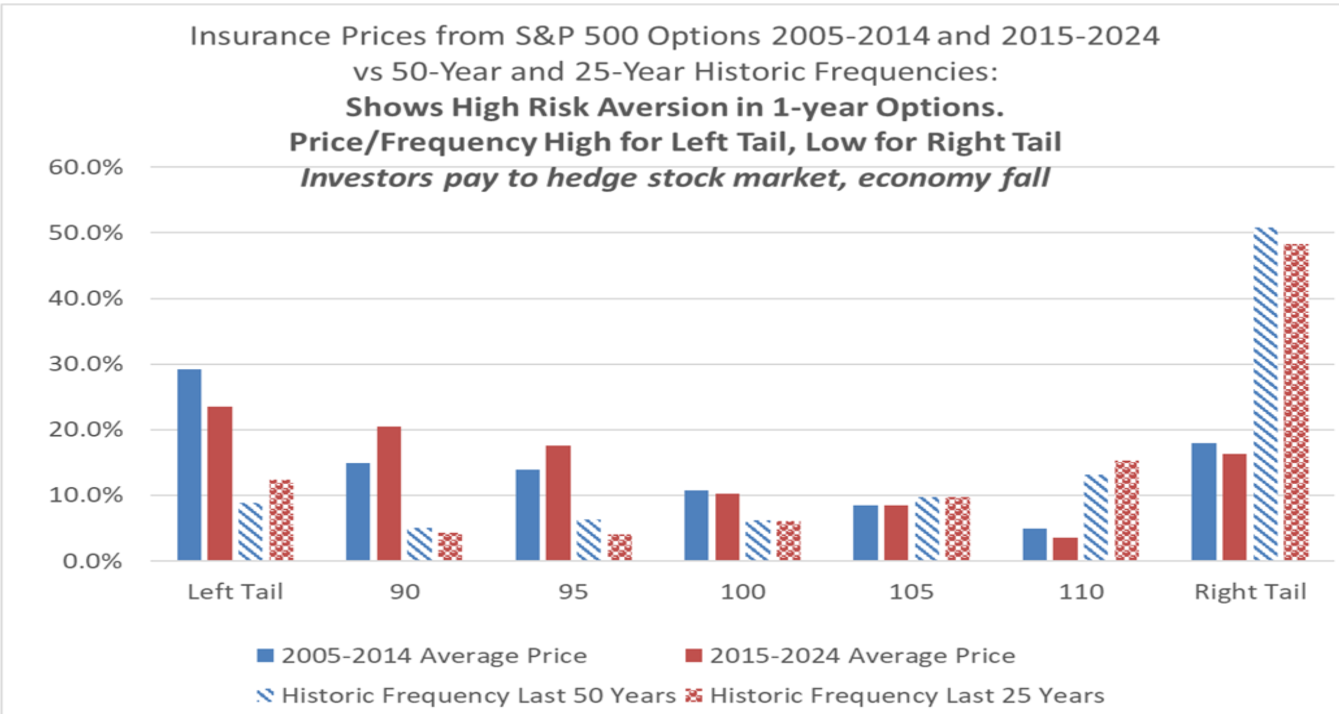
25-Year Trailing Frequency Distribution for 1-Year Returns on S&P 500 Stocks (%)

End of Range	Low	-9999	-12.5	-7.5	-2.5	2.5	7.5	12.5	
	High	-12.51	-7.51	-2.51	2.49	7.49	12.49	9999	Total
192512		9.7%	10.7%	8.3%	9.3%	10.7%	8.0%	43.3%	100.0%
195012		18.3%	8.3%	6.0%	5.0%	6.3%	8.0%	48.0%	100.0%
197512		7.0%	6.3%	6.3%	10.7%	9.7%	11.3%	48.7%	100.0%
200012		1.0%	3.0%	7.7%	5.0%	10.0%	11.7%	61.7%	100.0%
202406		12.3%	4.3%	4.0%	6.0%	9.7%	15.3%	48.3%	100.0%
Average 1900 to 2024		9.7%	6.5%	6.5%	7.2%	9.3%	10.9%	50.0%	100.0%
		50-Year Trailing Frequency Distribution of S&P 500 Returns							
195012		14.0%	9.5%	7.2%	7.2%	8.5%	8.0%	45.7%	100.0%
200012		4.0%	4.7%	7.0%	7.8%	9.8%	11.5%	55.2%	100.0%
202406		6.7%	3.7%	5.8%	5.5%	9.8%	13.5%	55.0%	100.0%

Note that the very large right tail (about 50% of all observations are for returns over 12.5%) is due to the fact that the S&P 500 mean total return was approximately 10% to 12% compounded annually over most of these time periods.

At a Federal Reserve conference in September 2017 on "Global Risk, Uncertainty and Volatility," using data available at the time through 2015, Breeden presented the same basic results as shown in the 2024 updated version (Figure 16), which shows frequency distributions vs. B-L S&P 500 insurance prices implicit in S&P 500 options:

<u>Figure 16</u>



In Figure 16, please note that for each state, the two left bars reflect Arrow state prices (first and second halves of the sample), while the two right bars both reflect historical frequencies of state occurrences (one for the last 50 years, and the other for the last 25 years). As equation 1 shows, state prices should reflect both probabilities and marginal utilities of consumption, conditional upon being in the various states. In the left tail, stock prices fall sharply, wealth (and likely consumption) is low, and marginal utility for a dollar is high. Thus, it is not surprising that state prices are much higher than probabilities and frequencies in down markets and the left tail. Insurance that pays off in those bad states sells for a premium, as they are diversifying. On the other hand, when stocks are up sharply, and the higher wealth normally gives higher consumption, marginal utility is lower, and state prices are well below probabilities. Securities that pay off in the up states have high betas and should have high expected excess returns, whereas the downside insurance ones have negative betas and negative excess returns, according to the CAPM. The empirical data displayed in Figure 16 strongly confirm risk aversion in the financial markets, as investors pay up so much for insurance, relative to its probability of payoff.

We can gain further insights by examining the Price/Frequency Ratio for each state of S&P 500 return. By just doing a little algebraic manipulation of equation 1, we see that the ratio of the normalized state price (Individual State Price/Sum of State Prices), divided by the state's probability, should equal the conditionally expected marginal utility in the state, divided by the expected marginal utility across all states for the date (1 year from now in this case). We have computed price/frequency ratios for every 6 month period and averaged them for each state, as shown in Figure 17. We have also computed the average price for each state and divided by the average frequency. The multiples found with both calculations are quite similar, as shown in the last two rows of Figure 17.

Figure 17

	Conditional Marginal Utilities: Average State Price to Frequency Ratios								
	Range	**Low**	**-9999**	**-12.5**	**-7.5**	**-2.5**	**2.5**	**7.5**	**12.5**
		High	**-12.51**	**-7.51**	**-2.51**	**2.49**	**7.49**	**12.49**	**9999**
Average Price 2005-2024			**26.4%**	**17.6%**	**15.6%**	**10.5%**	**8.4%**	**4.2%**	**17.1%**
Average Frequency 50 Year			**8.8%**	**5.0%**	**6.3%**	**6.2%**	**9.7%**	**13.1%**	**50.8%**
Implied Multiple of Price to Frequ			**3.01**	**3.50**	**2.48**	**1.68**	**0.87**	**0.32**	**0.34**
Average of Semiannual Multiples			**3.00**	**3.77**	**2.51**	**1.68**	**0.87**	**0.33**	**0.34**

Surprisingly, Figure 17 shows that risk aversion on average seems greater in the down 10% (price/prob=3.5) scenarios than in the extreme left tail scenario (down 12.5% or more where price/prob = 3.0). This shows that investors are, on average, displaying more “risk aversion in the medium/small risks” than “risk aversion for the large extreme moves.” This somewhat surprising result presented in 2017 at the Fed conference is similar to that found by Beason and Schreindorfer (2022) in their excellent article.

With this table, we can see a cardinal aspect to marginal utility, as we see that the marginal utility of $1.00 conditional upon being in the left tail state, is 3.0 times the average marginal utility across all states 1 year out. In contrast in the right tail, with a booming stock market, marginal utility is only about 1/3 the the average marginal utility for that date. So the marginal utility of $1.00 in the left tail is approximately 9 times as great as the marginal utility of $1.00, conditional upon being in the right tail of the distribution.

Those are averages over time. We can gain further insight by looking at the movements of these of these conditional marginal utilities through time as circumstances and economic performance change. Figure 18 shows marginal utilities in the 4 stressful economies we examined earlier, the 2008/9 Financial Panic, the 2011/12 Sovereign Debt Crisis, the 2020 Coronavirus Pandemic and the recent surge in inflation related in part to the Russia/Ukraine war and the Israeli/Hamas war. At times of great fear, as in October 2008 and in March 2020, the left tail risk was priced at about 7 times the average marginal utility, rather than just 3 times, as has been more normal. And at times when the markets have had surprising calm, as in 2023/2024 and 2017/2019 and 2005/2006, left tail prices (downside scenarios) came down to just 1.5 times the average marginal utility. Figure 19 shows the prices during some of these times the markets believed risk to be low. Note that in many cases of relative calm, the pricing of the less extreme risk coverage has a higher multiple than the tail scenario. This is not economically sensible, given that decreasing absolute risk aversion says marginal utility increases as wealth goes lower. This likely is evidence that the investor consensus (often wrong) was that probabilities were not so high for the extreme negative scenarios.

Figure 18

Conditional Marginal Utilities: State Price/Frequency Ratios

	Monthend Data from December 2004. Uses Breeden-Litzenberger (2022) te									TTM	12 mon
8/7/24 1:42 PM						$90%-$85 Puts		ATM			$110-$115 Ca
	ATM	S&P 500	Treasury Yields		Left Tail	90	95	100	105	110	Right Tail
Date	Implied σ	Spot Index	3 mnth	2 Year	Spread	Butterfly	Butterfly	Butterfly	Butterfly	Butterfly	Spread
USA Financial Panic of 2008/2009											
12/29/2006	14.0	**1418**	**5.02**	**4.82**	**1.34**	2.72	2.65	2.09	1.20	0.59	**0.41**
12/31/2007	22.2	**1468**	**3.36**	**3.05**	**4.36**	2.24	1.87	1.25	0.80	0.38	**0.42**
6/30/2008	22.3	**1280**	**1.90**	**2.63**	**4.61**	2.31	1.74	1.31	0.76	0.36	**0.40**
10/31/2008	39.4	**969**	**0.46**	**1.56**	**7.02**	1.09	0.92	0.60	0.46	0.19	**0.43**
12/31/2008	36.3	**903**	**0.11**	**0.76**	**6.60**	1.17	0.97	0.69	0.48	0.22	**0.41**
2/27/2009	36.9	**735**	**0.26**	**1.00**	**6.30**	1.19	0.96	0.71	0.48	0.22	**0.42**
6/30/2009	26.8	**919**	**0.19**	**1.11**	**4.89**	1.78	1.37	1.04	0.62	0.29	**0.39**
12/31/2009	22.8	**1115**	**0.06**	**1.14**	**3.99**	2.17	1.62	1.27	0.71	0.33	**0.38**
Europe's Sovereign Debt Crisis and USA Budget Impasse (Summer 2011)											
6/30/2011	19.4	**1321**	**0.03**	**0.45**	**3.36**	2.80	2.17	1.85	0.80	0.32	**0.31**
7/29/2011	20.7	**1292**	**0.10**	**0.36**	**3.70**	2.57	2.00	1.63	0.77	0.30	**0.33**
8/31/2011	25.3	**1219**	**0.02**	**0.20**	**4.49**	2.10	1.62	1.36	0.66	0.27	**0.34**
9/30/2011	30.8	**1131**	**0.02**	**0.25**	**5.21**	1.56	1.28	0.99	0.57	0.23	**0.37**
10/31/2011	24.5	**1253**	**0.01**	**0.25**	**4.45**	2.15	1.67	1.29	0.67	0.27	**0.33**
12/30/2011	24.1	**1258**	**0.02**	**0.25**	**4.39**	2.19	1.70	1.29	0.68	0.26	**0.34**
1/31/2012	21.2	**1312**	**0.06**	**0.22**	**3.86**	2.62	1.99	1.47	0.74	0.27	**0.31**
Global Coronavirus Pandemic of 2020-2021											
12/31/2019	15.6	**3231**	**1.55**	**1.58**	**1.81**	5.87	3.64	1.78	0.91	0.22	**0.29**
2/28/2020	20.7	**2954**	**1.27**	**0.86**	**3.77**	4.67	2.47	0.73	0.75	0.25	**0.34**
3/16/2020	38.7	**2386**	**0.24**	**0.36**	***7.02***	*0.98*	*1.00*	*0.39*	*0.46*	*0.08*	***0.36***
3/31/2020	29.7	**2585**	**0.11**	**0.23**	**4.63**	4.16	1.83	0.87	0.55	0.19	**0.35**
4/30/2020	26.6	**2912**	**0.09**	**0.20**	**5.26**	3.67	1.83	0.79	0.59	0.06	**0.32**
6/30/2020	24.4	**3100**	**0.16**	**0.16**	**4.82**	4.65	2.16	0.71	0.63	0.10	**0.30**
12/31/2020	20.5	**3756**	**0.09**	**0.13**	**3.66**	6.31	2.75	1.51	0.70	0.12	**0.28**
Global Inflation and Interest Rate Surge 2022-2024											
12/31/2021	19.3	**4766**	**0.06**	**0.73**	**3.00**	7.23	3.17	1.40	0.76	0.04	0.30
9/30/2022	26.3	**3586**	**3.33**	**4.22**	**4.33**	3.29	1.85	1.13	0.73	0.30	0.42
12/30/2022	23.5	**3840**	**4.42**	**4.41**	**3.59**	3.44	2.09	1.16	0.81	0.31	0.46
12/29/2023	15.8	**4770**	**5.40**	**4.23**	**1.42**	4.61	3.29	2.57	1.13	0.37	0.37
6/28/2024	15.0	**5460**	**5.48**	**4.71**	**1.28**	4.49	3.32	2.83	1.21	0.43	0.37
Averages	24.6	**2257**	**1.25**	**1.48**	**4.19**	3.11	2.00	1.29	0.73	0.26	0.36
	ATM	S&P 500	Treasury Yields		**Left Tail**	90	95	100	105	110	**Right Tai**
Date	mplied σ	pot Inde	3 mnth	2 Year	**Spread**	Butterfly	Butterfly	Butterfly	Butterfly	Butterfly	**Spread**

Figure 19

Price/Frequency Multiples: Conditional Marginal Utilities in Calm. "Risk On" Markets										
Low	Average	Average	-9999	-12.5	-7.5	-2.5	2.5	7.5	12.5	B-L Skew
High	VIX	S&P500	-12.51	-7.51	-2.51	2.49	7.49	12.49	9999	
1. The Calm Before the Storm: Prior to the Financial Panic of 2008/2009										
12/30/2005	14.3	1231.4	1.87	2.35	2.22	2.07	1.20	0.61	0.44	-8.0%
6/30/2006	13.9	1284.4	1.46	2.61	2.54	2.05	1.21	0.62	0.42	-9.7%
12/29/2006	13.9	1352.2	1.33	2.70	2.66	2.11	1.19	0.59	0.41	-9.9%
6/29/2007	14.1	1463.7	1.37	2.71	2.67	2.09	1.23	0.44	0.45	-11.9%
12/31/2007	20.3	1492.5	3.49	2.56	2.03	1.46	0.89	0.31	0.45	3.7%
2. 2017 Low Volatility Generated the Fed Conference on "Global Risk, Uncertainty and Volatility.										
12/30/2016	16.3	2179.4	2.50	4.70	2.93	1.61	0.83	0.27	0.27	9.3%
6/30/2017	14.2	2370.8	1.82	5.30	3.36	2.11	0.93	0.32	0.25	3.7%
12/29/2017	13.4	2559.6	1.33	5.52	3.80	2.32	1.01	0.30	0.24	-0.4%
6/29/2018	16.0	2713.3	1.96	5.09	3.20	1.86	0.99	0.33	0.30	2.6%
12/31/2018	16.2	2768.4	1.93	4.76	3.13	2.01	1.04	0.34	0.31	1.4%
6/28/2019	15.5	2827.1	1.75	5.16	3.24	1.86	1.01	0.32	0.31	0.1%
12/31/2019	16.2	3048.8	1.99	5.56	3.35	1.69	0.90	0.22	0.30	2.5%
6/30/2020	23.4	2970.2	4.17	4.65	2.34	0.91	0.68	0.17	0.32	20.6%
3. 2023/2024. Fed Engineers A Soft Landing, From High Inflation, 500 Basis Points of Rate Increases										
12/30/2022	24.1	3910.4	3.82	3.78	2.10	1.21	0.78	0.28	0.42	10.4%
6/30/2023	18.8	4159.3	2.12	4.79	2.98	1.62	0.99	0.28	0.40	-2.0%
12/29/2023	16.8	4486.0	1.54	4.58	3.17	2.20	1.10	0.39	0.39	-7.0%
6/28/2024	15.5	5161.7	1.38	4.43	3.21	2.66	1.18	0.47	0.37	-8.7%

VI. Decomposing the Risk Premium: Expected Returns on Arrow Securities

If one thinks about estimating the expected returns on "Arrow securities" for the various states, the calculation is straightforward. Arrow securities pay $1.00 if the state occurs, and 0 if it does not, so the expected payoff on each Arrow security is simply the probability of the state in which it pays off. All other states have zero payoffs, so they add nothing to the expected cash flow from the Arrow security. The price paid for the Arrow security, φ_{ts}, is given in equation 1 and should be equal to the probability of the time-state times the marginal utility in that time-state, normalized by the current marginal utility. So state prices are proportional to probabilities and conditional marginal utilities.

If we use historical state frequencies of various stock market moves as estimates of probabilities of those moves going forward, then from eq. 1, the price/frequency ratios given in Figures 17-19 should equal marginal utilities in the various states, normalized by the current marginal utility of consumption, as discussed in the prior section.

If we invert the price/frequency ratios and if state probabilities equal frequencies, we get probability to price ratios. As the probability equals the expected payoff on the Arrow security, and the state price is what you pay for that, the ratio, minus 1, is the expected return on the Arrow security. Figure 20 shows the calculations of expected returns on 1-year Arrow securities for our 7 states:

Figure 20

		Expected Returns on Arrow Securities						
State	Low	-9999	-12.5	-7.5	-2.5	2.5	7.5	12.5
Range	High	-12.51	-7.51	-2.51	2.49	7.49	12.49	9999
Average 2005-2014		-63%	-59%	-49%	-35%	23%	193%	187%
Average 2015 2024 June		-57%	-79%	-65%	-36%	17%	296%	230%
Average 2005-2024		-60%	-69%	-57%	-35%	20%	243%	208%

From Figure 20, we see that the Arrow securities (butterfly spreads and tail spreads) are highly levered securities, in that you pay small prices, like probabilities, which might be, say 20 cents, and receive a $1.00 payoff if your state occurs, a 400% return! And, like homeowners insurance, with the hedge securities on the left, you often buy insurance that ends up worthless. Expected returns on those can be very negative, like -60%, but they do provide insurance against stock market falls and the associate economic problems. On the upside, if you buy an out of the money call option or Arrow security for the right tail, you can have huge returns and huge positive betas.

We have tentatively estimated the betas for various Arrow securities. It is surprisingly tedious to do, as we purchased a certain set of butterfly strike options at the beginning of the year and had daily data that gave us implied volatilities every day of a different set of strikes, varying with the moves of the S&P 500. For example, if the S&P 500 is at 5,000 one day, the 10% OTM call option strikes at 5500. If the stock market goes up 2% the next day, the S&P 500 is then at 5100, and the 10% OTM quote from Bloomberg reflects then reflects a strike of 5610. So we must interpolate the volatilities daily and estimate the prices for the same option purchased at the beginning of the year. I thank my Research Assistant, Jack Yan, for carefully doing this tedious work year by year from 2005 to 2022, examining a new set of options at the beginning of each year. The betas estimated are in Figure 21. The betas are lower in absolute value than expected, though all are well above 1.0. Reviewing and replicating these estimates must be left for future research.

Figure 21

	Arrow Security Betas (Daily Data, Average of 2005-2022 Annual Data)								
		111%	105%	100%	95%	91%	87%	83%	
	Left Tail Spr	$90.00	$95.00	$100.00	$105.00	$110.00	$115.00	$120.00	Right Tail Spread
Average Beta	*-20.1*	*-6.6*	*-0.9*	*3.0*	*5.1*	*6.8*	*4.7*	*4.0*	*3.8*

VII. Conclusion.

In this article, we have presented our proposed "Breeden-Litzenberger Skew" and shown how it moves sensibly as a measure of risk aversion in times of economic stress and in risk-on scenarios. We have shown that it forecasts expected real returns from 3 to 10 years out and does quite well compared to other forecasts reviewed. We then used historic frequency data as estimates of probabilities to allow us to estimate conditional marginal utilities and examine them through time and across states of the world/economic scenarios, finding large differences in marginal utility of $1.00 in good states (right tail) and in bad states (left tail).

We computed returns on Arrow securities for the various states and found them to be quite large, both in positive and negative directions, reflecting the inherent leverage in these relatively pinpoint bets. We provided tentative estimates of the betas for Arrow securities, which reflect the positive and negative betas as expected.

Derivatives traders likely will find in these calculations (especially from the Price/Frequency multiples) trades that appear quite attractive, and trades that appear quite unattractive, given their own probabilistic views. As always, much future research can be done in this area.

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