Consumption-Based Asset Pricing, Part 2: Habit Formation, Conditional Risks, Long-Run Risks, and Rare Disasters

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Abstract

Following Part 1 of this article, which reviews late-1970s to 1990s classic derivations and tests of the consumption capital asset pricing model, here in Part 2 we review more recent developments, some of which are based on utility functions with non-time-separable preferences. Important second-generation consumption-based asset pricing advances are also reviewed, including models with habit formation and long-run risk. These models give large cyclical changes in relative risk aversion and risk premiums as well as lagged impacts of aggregate consumption changes on risk premiums. We review asset pricing with rare disasters and models focused on consumer spending on durables and real estate, as well as the fraction of spending financed by labor income. The second-generation models discussed have more free parameters and fit the empirical data better than did the first-generation consumption-based asset pricing models.

1. INTRODUCTION

Part 1 of this review showed the very strong theory in support of the consumption capital asset pricing model (CCAPM) contrasted with weak empirical support in early tests (pre 1999). This motivated researchers to improve both their theoretical and empirical modeling. The equity premium puzzle article of Mehra & Prescott (1985) stimulated a huge amount of additional research. One first response was the now-classic disaster risk analysis of Rietz (1988), which stimulated excellent research two decades later by Barro (2006), Barro & Ursua (2008), and Wachter (2013).

Also in the late 1980s, theorists began intensive modeling of preferences that were not based on time-additive utility functions, but instead had a representative utility function displaying decreasing relative risk aversion (RRA), time-complementarity, and habit formation or were of a recursive, forward-looking form. On the theoretical side, Pye (1972, 1973) and Greenig (1986) developed time-multiplicative utility functions. Then, Sundaresan (1989) and Constantinides (1990) modeled internal habit formation, and Abel (1990) modeled "keeping up with the Joneses." Epstein & Zin (1989) and Weil (1989) (often jointly referred to as EZ-W) developed forward-looking preference structures that displayed time complementarity in utility for consumption streams, allowing researchers to separate effects of different levels of intratemporal RRA from levels of the elasticity of intertemporal substitution (EIS). Constantinides (1990) found promising results from a model where individuals had internal habits, in that their preferences for consumption risk were dependent on their own past levels of consumption. Ferson & Constantinides (1991) found sensible evidence of habit persistence in durables expenditures.

In the 1990s, important articles were published on changes in conditional risks and conditional risk premiums. Ferson & Harvey (1991) demonstrated that changes in conditional risk premiums (per unit of risk) had larger impacts on predicted monthly return variations for many industries and size-ranked portfolios than did changes in conditional risks. Jagannathan & Wang (1996) modeled changes in conditional betas as being related to the movements of the credit yield spread on corporate bonds, which is quite sensitive to moves in the overall economy. They also estimated values for human capital to get a broader estimate of the market portfolio, which, when combined with changing conditional betas, helped them explain the size effect found by Fama & French (1993).

Building on the theoretical developments of utility functions with habit formation, Campbell & Cochrane (1999) developed an insightful model with an external habit, using a habit estimated from past movements in aggregate consumption per capita. With a subsistence level of consumption for a representative individual, their model allows for dramatic rises in RRA as surplus consumption (consumption above the habit level) goes toward zero in severe recessions. With the flexibility afforded by this model, they fit many aspects of empirical data on stock and bond returns as related to real consumption growth. Their model explained the level of the risk premium on the stock market, which Mehra & Prescott (1985) had found was substantially too high (the equity premium puzzle), given the low volatility of real consumption growth.

Another insightful contribution is Lettau-Ludvigson's (2001a,b) use of deviations of consumption from total wealth (which includes a human capital estimate in addition to stock market wealth) as a conditioning or scaling variable for changing mean returns. They found results quite compatible with Merton's (1973) and Breeden's (1984) intertemporal theories, in that high consumption versus wealth is an indicator of a good investment or income opportunity set, as most consumers optimally smooth forward those changes in expected returns or income fluctuations. Lettau & Ludvigson (2001a,b) found significant differences in the movements of consumption betas of value versus growth stocks during recessions, i.e., changing conditional risks. They found that value stocks tend to have much larger increases in consumption betas during recessions, when risks and risk premiums are high, which helps to explain the findings by Fama & French (1993) of higher returns on value stocks than those predicted by the unconditional CCAPM betas. As shown in Part 1, Jagannathan & Wang (2007) used recession and expansion periods identified by the National Bureau of Economic Research (NBER) as a conditioning variable and found that conditional consumption betas are excellent in describing conditional mean returns on the Fama-French portfolios.

Bansal & Yaron (2004) have had major impact by modeling the long-run risks caused by small, persistent shocks in the drift and volatility of real consumption growth. They showed that variance of real consumption growth grows more than proportionally with time, which is consistent with the persistence of growth shocks. Additionally, they provided evidence that shows that the conditional volatility of consumption is time varying, which leads naturally to time-varying risk premiums. Much subsequent research has been done on this long-run risks model, most notably in papers by Bansal, Dittmar & Lundblad (2005) and by Bansal, Dittmar & Kiku (2009). Bansal, Dittmar & Kiku (2009) showed that aggregate consumption and aggregate dividends have a cointegrating relation. They observed, "the deviation of the level of dividends from consumption (the error correction variable) is important for predicting dividend growth rates and returns at all horizons" (1, 5, and 10 years) (Bansal, Dittmar & Kiku 2009, p. 1344). Imposing cointegration allows them to predict 11.5% of the variation in 1-year returns, whereas only 7.5% of the variation is predicted without cointegration. Their conditional consumption betas account for approximately 75% of the cross-sectional variation in risk premiums for the 1-year horizon and 85% for long horizons.

Santos & Veronesi (2006) presented a very interesting model on time variation in the equity risk premium and changes in conditional risks of assets. They modeled consumer spending as being funded partly by labor income and partly by asset returns. They argued that when labor income provides more of the funding for consumption, stock returns will be less correlated with consumption and the risk premium on stocks will be smaller. Their data confirm this theory. In another strand of consumption-based research, spending on durable goods is used as a sharp indicator of changes in marginal utility. Spending on durables usually is more costly (cars, homes, and furniture), so it is quite plausible that consumers would be thoughtful and calculating as they make those purchases. Yogo (2006) and Gomes, Kogan & Yogo (2007) found some excellent results on the usefulness of considering durables spending when estimating consumption risks and risk changes. Yogo (2006) found this useful in explaining both size and value effects found by Fama and French as well as in modeling time variation in the equity risk premium.

Consumer spending on real estate was studied by Piazzesi, Schneider & Tuzel (2007). They argued that consumers really do not like to change their spending on housing and do so only in large amounts when they are in dire straits, which makes drops in real estate spending shares good indicators of sharp increases in marginal utility. In an application of consumption-based asset pricing to foreign exchange, Lustig & Verdelhan (2007) showed that there are substantial consumption risks in foreign exchange portfolios. Conditional consumption risks change depending on movements in the spread of the yield on a riskier currency relative to the yields of safer currencies. When yield spreads widen, consumption risks increase. They found the modeling of these conditional risk changes to be quite helpful in Fama-MacBeth (1973) regressions of returns on consumption risks.

The flow of this article is as follows: Section 2 presents the advances in modeling utility maximization with non-time-separable utility functions and empirical tests with a model of internal habit. Section 3 presents the 1990s advances in modeling changes in conditional risks and conditional risk premiums. Section 4 discusses some empirical applications of models with external habit formation, led by the Campbell-Cochrane model. Section 5 presents some of our simple graphs and statistics showing movements in conditional estimates of equity risk premiums as the unemployment rate changes and employment growth changes. Section 6 presents the

2001 results on modeling changes in conditional consumption risk and changes in the investment and job opportunity sets, led by Lettau & Ludvigson (2001a,b). Section 7 presents the long-run risks model and tests and developments based on the original work by Bansal & Yaron (2004). Section 8 discusses the work on risks of rare disasters. Section 9 presents the research on return predictability from the labor income share. Section 10 discusses works on using durable goods spending as an indicator of changes in marginal utility, with consequences for risk measurement. Section 11 has the real estate spending analysis, whereas Section 12 has a sample of foreign exchange work. Section 13 makes some concluding remarks.

2. NON-TIME-SEPARABLE UTILITY FUNCTIONS: HABIT FORMATION AND RECURSIVE UTILITY

A weak link in the theory of the 1970s and 1980s used to derive the CCAPM was the assumption that the utility of a lifetime consumption plan is additive over time. Thus, the utility of consumption expenditure at time *t* depended only on the real amount consumed at that time. The utility function could be quite general and nonlinear, but utility was not allowed to depend on the prior history or the expected future path of consumption for the individual. This assumption was made primarily for mathematical tractability, as behavioral researchers have known for some time (and most people know introspectively) that people really do not like to reduce consumption very significantly, once a standard of living is established. Researchers now say that an individual typically establishes a habit of consuming a certain set of goods costing a certain amount and is very averse to falling much below that level. When individuals are fortunate and consume significantly above a well-established habit level, perhaps they are not too risk averse to falling back in consumption somewhat, as long as they do not fall below a baseline habit level. Habit levels presumably evolve over time and are established gradually as a household's standard of living improves and is maintained at a higher level. If one moves from spending \$50,000 per year for 10 years up to spending \$100,000 per year for several years, the habit or subsistence level of consumption will have moved up toward the new level of \$100,000. Consumption flows from purchases of durables such as houses and autos are costly to reverse, which tends to reinforce this effect.

In attempts to solve Mehra & Prescott's (1985) equity premium puzzle, researchers developed utility functions with time complementarity, wherein utility for consumption at one point in time is affected by consumption levels at other points in time. Leaders in deriving and using utility functions with time complementarity included very early articles by Pye (1972, 1973), Kreps & Porteus (1978), and then Bergman (1985), Greenig (1986), Sundaresan (1989), Epstein & Zin (1989), Weil (1989), Abel (1990), and Constantinides (1990). Pye (1972, 1973) and Greenig (1986) modeled maximization of the expected utility of lifetime consumption with a multiplicative function of consumption at different dates, raised to various powers. Pye's time-multiplicative lifetime utility function is

$$U = \delta \prod\nolimits_0^T C_i^{\gamma\alpha^i}, \quad \delta = \begin{cases} +1 & \text{if } \gamma > 0, \\ -1 & \text{if } \gamma < 0, \end{cases} \quad \gamma \neq 0. \tag{1}$$

This equation allowed Pye to model RRA that is age dependent as RRA $= q$, where q is

$$q_t = 1 - \lambda_{t+1} = 1 - \gamma \sum_{t+1}^{T} \alpha^i. \tag{2}$$

Thus, Pye found that RRA increases with age for those more tolerant of risk than with log utility, and decreases with age for those less tolerant than log utility.

In 1989–1990, a flurry of five significant papers were published with non-time-separable utility functions: Sundaresan (1989) and Constantinides (1990) modeled internal habit formation, whereby the utility that consumers get from a certain consumption level today depends on how that level compares with their own past levels of consumption, which forms their habit. Abel (1990) proposed "catching up with the Joneses," a model of external habit formation. It is external because the habit is not a choice variable for the individual: Utility from consumption is modeled as a function of the person's consumption relative to that of lagged aggregate per capita consumption. Epstein & Zin (1989) and Weil (1989), following Bergman (1985), developed recursive utility models that consider anticipated future consumption levels in determining the utility of alternative consumption levels today. These utility functions have been used in several empirical tests.

With preferences that display time complementarity, both Bergman (1985), using recursive preferences, and Sundaresan (1989), using habit formation, found that "Merton's multi-beta Intertemporal CAPM is still valid, but it can no longer be collapsed to Breeden's (1979) single consumption beta model" (Bergman 1985, abstr.). Sundaresan (1989, p. 74) added, "Nor are these models based on time-separable utility able to explain the remarkably stable behavior of the per capita consumption series, despite the tremendous volatility of the wealth series." Sundaresan showed that the utility increment is diminished owing to the negative utility effect of having a higher consumption standard (internal habit) going forward, even though higher consumption increases current utility. Knowing this causes consumers with nonseparable utility to optimally dampen their consumption responses to wealth shocks (both up and down). Thus, with nonseparable preferences, any given shock in the system must cause greater wealth fluctuations to have a given impact on consumption. On the portfolio policy side, Sundaresan (1989, p. 85) showed that, with his nonseparable preferences, "the optimal investment policy is to invest (in the risky asset) a constant proportion of wealth in excess of the capitalized value of the consumption standard." This justifies a portfolio insurance creation strategy, as in Black & Perold (1987).

Constantinides's (1990) work was especially influential. Similar to Sundaresan (1989), he had a model of consumers maximizing expected utility with an internal habit, meaning one that is established by the consumer's own history of past consumption. This is more intuitive than an external habit but is mathematically more complex. In contrast, with an external habit, consumers gauge their satisfaction by comparison with consumption of others or comparison with average consumption per capita (see Abel 1990, Campbell & Cochrane 1999). In the latter case, a consumer's current decisions do not affect the habit that is developed, so the mathematical solutions are simplified. Constantinides (1990) assumed consumers maximize the expected value of the following utility function:

$$E_0 \int_0^\infty e^{-\rho t} \gamma^{-1} [c(t) - x(t)]^\gamma \, \mathrm{d}t, \tag{3}$$

where

$$x(t) = e^{-at} x_0 + b \int_0^t e^{a(s-t)} c(s) \mathrm{d}s. \tag{4}$$

Thus, Constantinides modeled habit as an exponentially decaying weighted average of past consumption rates, quite a sensible mathematical model for an internal habit. As consumption drops down toward the habit level, it is as if consumption approaches zero in prior consumption RRA models and marginal utilities approach infinity, which makes it optimal never to go to zero. Intuitively, habit formation could be interpreted as a kinked utility function with the marginal utility of consumption having a large upward jump as consumption falls below the habit. In contrast, the above formulation is an extreme version of habit formation that implies a Duesenberry-type

Table 1 **Mean and variance of the consumption growth rate generated by the model with habit persistence**

	Decay rate for past consumption in habit					
Parameter *a*, per year	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**
Parameter b	0.093	0.172	0.250	0.328	0.405	0.492
Mode ($\hat{z}$) of the state variable z	0.86	0.82	0.81	0.80	0.79	0.81
Mean annual growth rate in consumption						
Unconditional mean	0.018	0.019	0.018	0.018	0.018	0.018
At $z = \hat{z}$	0.011	0.013	0.014	0.014	0.014	0.014
Standard deviation of the annual growth rate in consumption						
Unconditional mean	0.036	0.036	0.036	0.036	0.036	0.034
At $z = \hat{z}$	0.023	0.029	0.032	0.033	0.034	0.032
RRA coefficient						
Unconditional mean	8.67	4.37	3.47	3.09	2.88	2.81
At $z = \hat{z}$	7.03	4.09	3.36	3.03	2.84	2.78
Elasticity of substitution (s) at $z = \hat{z}$	0.06	0.08	0.09	0.09	0.09	0.09
$s \cdot$ RRA at $z = \hat{z}$	0.42	0.33	0.30	0.27	0.26	0.25

Here a is the exponential decay rate for weighting past consumption levels in the habit; b is the multiplier for past consumption in the utility function. Assumed parameter values are $r = 0.01$, the annual rate of return of the riskless technology; $\mu - r = 0.06$, the difference between the mean annual rate of return of the risky technology and the annual rate of return of the riskless technology; $\sigma = 0.165$, the standard deviation of the annual rate of return of the risky technology; $\gamma = -1.2$, the power in the utility function; and $\rho = 0.037$, the rate of time preference in units (year)$^{-1}$. Abbreviation: RRA, relative risk aversion. Table reproduced from Constantinides (1990).

ratcheting of consumption demand that prevents consumption from falling below the exponentially weighted average of past consumption.

With this model of time complementarity, Constantinides (1990) showed that a wedge is driven between the EIS and the RRA for an individual, as later emphasized by Vissing-Jørgensen (2002). Constantinides demonstrated that habit persistence can generate the sample mean and variance of the historic consumption growth rate with a low exponent on the excess consumption term $[c(t) - x(t)]$. **Table 1** describes economies that can be generated with his model of habit persistence, where a is the exponential decay rate in weighting past consumption levels in the habit and b is the multiplier for the past consumption in the utility function.

The recursive preferences developed by Epstein & Zin (1989) and Weil (1989), who built upon fundamental preference modeling by Kreps & Porteus (1978), are frequently used in modern financial models, such as the long-run risks model of Bansal & Yaron (2004). The recursive preferences of Epstein & Zin (1989) allow the EIS to be disentangled from the coefficient of RRA. In the notation of Boguth & Kuehn (2013), the agent with EZ-W preferences maximizes recursive utility over consumption, using the following formula:

$$U_t = \{(1-\beta)C_t^{\rho} + \beta(\mathbb{E}_t[U_{t+1}^{1-\gamma}])^{\rho/(1-\gamma)}\}^{1/\rho}, \qquad (5)$$

where C_t denotes consumption, $\beta \in (0, 1)$ the rate of time preference, $\rho = 1 - 1/\psi$ and ψ the EIS, and γ the RRA. In a representative agent model, Epstein & Zin (1989, p. 958) found, "Thus, both consumption and the market return enter into the covariance that defines systematic risk . . . [and an asset's] price equals the discounted value of future dividends, where the discount factors involve both consumption and market returns."

In a follow-up article to his 1987 paper that derived recursive preferences similar to Epstein & Zin (1989), Weil (1989) focused on what he saw as the risk-free rate puzzle. Actual riskless rates observed are lower than model results. However, as Weil (1989, p. 416) stated,

> [I]ntroducing heterogeneity between agents in the form of undiversifiable individual consumption risk goes a long way towards explaining both the equity premium and risk-free rate puzzles. If individual consumption is more risky than aggregate consumption, one can explain why the risk premium is large, even though agents are only moderately risk-averse in the aggregate. At the same time, the price a consumer will be willing to pay for a safe unit of consumption tomorrow will rise—i.e., the risk-free rate will decrease. Therefore, the existence of heterogeneity and of market imperfections is likely to hold center stage in the explanation of the equity premium and risk-free rate puzzles.

This new approach led nicely into research on limited participation, incomplete markets, and much larger individual consumption risks, rather than on aggregate per capita consumption risks, which are discussed in Section 7 of Part 1 of this review. Section 3 below examines research on changes in conditional risks and risk premiums, which advanced significantly in the 1990s.

3. CHANGING CONDITIONAL RISKS AND CONDITIONAL RISK PREMIUMS

In addition to more general preferences with time complementarity and habits, important empirical research was done on changing conditional risks and changing risk premiums through time. Several articles were produced that appeared to demonstrate predictability in mean returns, a result that researchers had doubted, based upon earlier research on market efficiency. However, researchers began to realize that if risks change through time and in different economic conditions (e.g., in risky recessions versus stable growth periods), then it is economically sensible that mean returns should also vary with economic conditions to reward investors more when risk is higher. Keim & Stambaugh (1986) found that the credit yield spread of Baa-rated bonds over Aaa-rated bonds had some ability to predict future bond and stock returns. Fama & French (1988) and Campbell & Shiller (1988) found that trailing dividend yield, an easily measured variable, could predict returns, especially over the longer term, as much as 7 years out. Kandel & Stambaugh (1991) used dividend yield, a credit risk yield spread, and the slope of the term structure to model time-varying risk premiums.

In a particularly insightful article, Ferson & Harvey (1991) built on this prior work to model both changing conditional betas and changing conditional risk premiums. They found the changing risk premium for beta was a much larger explanatory variable in returns than were changing betas for 12 major industries, 10 deciles of size-ranked portfolios, and government and corporate bonds and Treasury bills (these results are shown in **Table 2**). Ferson & Harvey (1991) showed that the estimated risk premium for equities varies with economic conditions, generally increasing in recessions (as risk and premiums per unit of risk increase) and decreasing during growth periods, when risk and premiums per unit of risk appear to subside (see **Figure 1**).

Jagannathan & Wang (1996) significantly advanced the case for modeling conditional variation in betas and risk premiums. They modeled changes in betas as being related to the credit yield spread between low- and high-grade bonds, which is sensitive to perceived risks of default and is quite related to the state of the economy (as shown in Part 1 of this review). Additionally, they used a proxy for human capital to get a better estimate of returns on the true, but unobservable market portfolio. With the broader market portfolio, combined with changing conditional risks and conditional risk premiums, they explained much of the size effect identified by Fama & French (1993). Duffee (2005) also found significant variation in consumption risk intertemporally.

Table 2 Decomposing the predicted variation of monthly portfolio returns 1964:5–1986:12 (272 observations)

	Decomposition by economic risk variables							Decomposition by betas versus price of beta		
Portfolio	XVW	Prem	ΔSLOPE	UI	CGNON	REALTB	Interaction effects	Changing beta	Changing price of beta	Interaction effects
Size decile										
1	45.2	6.09	4.74	9.16	0.31	23.22	11.3	1.41	67.8	30.8
2	51.4	3.90	3.00	5.59	0.25	17.50	18.3	1.20	68.6	30.2
3	58.6	3.13	2.53	3.39	0.15	14.38	17.8	1.31	75.3	23.4
4	64.5	1.45	2.01	2.57	0.36	12.06	17.1	0.85	78.7	20.4
5	69.7	1.81	1.43	1.99	0.16	8.47	16.5	0.46	80.0	19.5
6	80.1	0.87	1.28	2.00	0.19	8.36	7.3	0.25	85.7	14.1
7	89.4	0.38	0.61	1.34	0.12	3.62	4.5	0.16	91.6	8.2
8	86.0	0.38	0.67	1.50	0.14	5.32	6.0	0.25	89.6	10.2
9	97.1	0.20	0.39	2.65	0.14	1.78	−2.3	0.04	90.9	9.1
10	105.5	0.10	0.26	0.47	0.06	1.36	−7.7	0.04	111.4	−11.5
Industry										
Petroleum	150.6	1.91	1.80	67.18	6.80	149.50	−277.8	0.83	146.7	−47.5
Finance/real estate	112.2	1.32	0.20	7.18	1.80	6.36	−29.1	0.42	101.5	−1.9
Consumer durables	88.0	0.89	0.87	4.12	0.73	15.90	−10.4	0.04	93.9	6.1
Basic industries	89.1	0.10	0.22	2.01	1.91	4.09	2.6	0.05	95.8	4.2
Food/tobacco	82.1	0.96	0.52	5.81	1.50	15.90	−6.8	0.37	89.2	10.4
Construction	90.0	1.43	0.66	4.80	0.52	8.44	−5.9	0.33	83.4	16.3
Capital goods	75.3	0.61	1.11	15.26	3.20	13.07	−8.6	3.14	97.2	−0.4
Transportation	75.5	0.67	4.28	6.49	1.56	6.25	5.3	0.66	84.0	15.3
Utilities	87.6	5.56	4.28	9.41	13.60	8.27	−28.7	1.73	87.7	10.6
Textiles/trade	65.8	0.27	0.37	7.73	6.37	35.00	−15.5	0.15	80.4	19.5
Services	74.9	0.44	0.37	7.19	0.32	20.71	−3.9	0.75	68.7	30.6
Leisure	73.1	0.10	1.31	17.07	1.13	43.64	−36.4	0.80	84.0	15.2
Government bonds	7.2	132.27	3.24	1.96	6.56	9.23	−60.5	1.68	91.9	6.4
Corporate bonds	18.3	92.70	6.73	13.21	21.77	20.07	−72.8	3.92	59.8	36.3
6-month Treasury bill	0.5	39.74	60.74	12.04	5.40	18.76	−37.2	1.19	46.2	52.7

Abbreviations and definitions: XVW, value-weighted NYSE stock excess return; Prem, Baa bond return in excess of long-term US government bond return; ΔSLOPE, change in the 10 year–3 month slope of the term structure of interest rates; UI, unexpected inflation (CPI); REALTB: 1-month Treasury bill return less monthly CPI inflation. Table reproduced from Ferson & Harvey (1991, table 8), with numbers rounded here.

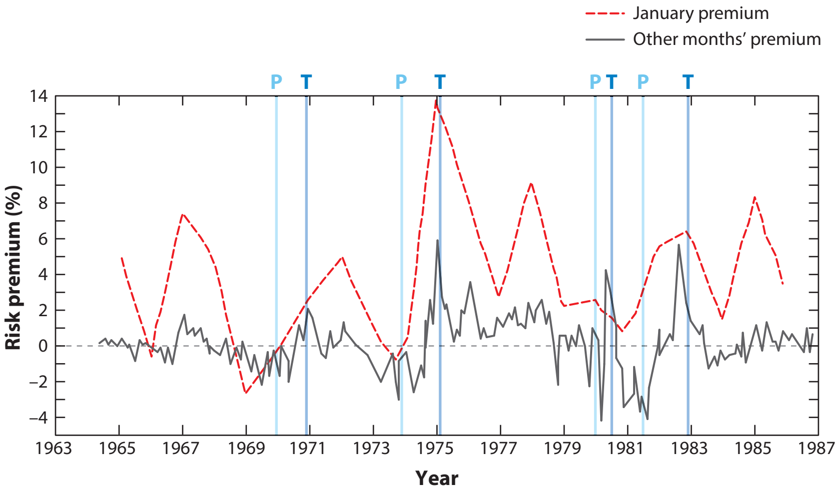

Figure 1

Changing estimated risk premium for equities. Vertical lines indicate recession periods, P's indicate peaks, and T's indicate troughs. Figure adapted with permission from Ferson & Harvey (1991, figure 1).

4. AN EXTERNAL HABIT-FORMATION MODEL

In a major second-generation consumption-based asset pricing model, 20 years after the original CCAPM, Campbell & Cochrane (1999) developed a model of asset pricing using a utility function of a representative individual with an external habit. Under an external habit, individuals do not consider the impact of their current consumption decision on their habit in future periods, which simplifies the optimization problem. Campbell & Cochrane (1999) successfully employed the utility function to fit a countercyclical equity risk premium. Three features of their model are worthy of note: (*a*) a slow-moving external habit based on per capita consumption, (*b*) independent and identically distributed (i.i.d.) per capita consumption growth, and (*c*) highly nonlinear utility and RRA that approaches infinity near the external habit level. By assuming a representative individual with an external habit, they sidestepped the aggregation of heterogeneous individuals, limited participation issues, and the impact of current consumption decisions on the current or future habit. Their model generates countercyclical fluctuations and long-term predictability of equity risk premium by having RRA become arbitrarily large as current consumption approaches the external habit.

Individuals are assumed to maximize an expected utility function of the following form:

$$E\sum_{t=0}^{\infty}\delta^{t}\frac{(C_t - X_t)^{1-\gamma}-1}{1-\gamma}. \tag{6}$$

Note that this preference function is similar to an extended power utility function with an intercept equal to minus the external habit. This utility function displays decreasing RRA, and RRA approaches infinity as the representative individual's consumption declines toward the external habit. Thus, the habit intuitively seems more like a subsistence level of consumption, rather than a habit motivated by consumption envy that the "keep up with the Joneses" motive seems to suggest. Because an individual views a habit as exogenous, the representative individual's consumption decision does not consider the impact on the habit. Under the assumption of identical powers, γ, this preference function could be aggregated from individual extended power utility with diverse external habit levels. The modeling of the aggregated external habit as a lagged function of past per capita consumption is intuitive. The preference function is not defined for a negative habit level, which requires consumption to be strictly above the habit. This preference function could be used with a stochastic process on per capita consumption that was consistent with this constraint. For example, if C_t followed a shifted lognormal process with a shift parameter of X_t, realizations of excess consumption would be positive without making X_t a function of C_t, which would not be intuitive for subsistence-level consumption. However, Campbell & Cochrane (1999) assumed that per capita consumption is lognormally distributed and made the external habit an implicit function of current consumption, such that the external habit's downward moves assure that excess consumption is positive for all realizations of per capita consumption.

Campbell & Cochrane (1999) used a variable called the surplus consumption ratio as the difference between per capita consumption in the economy and the representative individual's external habit level, X_t, expressed as a fraction of per capita consumption:

$$S_t^a = \frac{C_t^a - X_t}{C_t^a}. \tag{7}$$

The log surplus consumption function s_t^a is modeled as an AR(1) process with a speed depending on parameter ϕ and a monotonically decreasing sensitivity function $\lambda(s_t^a)$, where lowercase letters are logs of the uppercase variables:

$$s_{t+1}^a = (1-\phi)\bar{s} + \phi s_t^a + \lambda(s_t^a)(c_{t+1}^a - c_t^a - g). \tag{8}$$

Substituting the surplus consumption ratio into the AR(1) process demonstrates that the external habit X_t adjusts to C_t as well as to the history of average per capita consumption. The external habit adjusts slowly and geometrically to past consumption with coefficient ϕ. The log transformation constrains the surplus consumption to be non-negative. Campbell & Cochrane (1999) imposed several restrictions on the parameters to produce a constant risk-free rate and a predetermined habit level around the steady state so as to make sure the excess comoves with consumption but is always positive. Under their specification, as C_t approaches zero, changes in X_t offset the impact of changes in C_t on the excess consumption ratio. The justification for this specification for a learned habit, which intuitively should be slowly moving in response to past levels of consumption of others, is not provided. The implications of their parameter specifications are shown in **Figures 2** and **3**.

Campbell & Cochrane (1999) then priced bonds and stocks using classic Euler equations and chose the free parameters in the model to fit the moments of postwar data. Empirical calibration shows their model can fulfill its goals and generate a nonlinear countercyclical risk premium and cyclical equity volatility. When surplus consumption drops to near zero during recessions, both the equity risk premium and volatility of stock returns increase at an increasing pace, as shown in the **Figures 4** and **5**.

As **Table 3** also shows, there are enough parameters that their simulated data can fit the four moments of the postwar data quite well. The equity risk premium and its Sharpe ratio fit almost perfectly. In addition, their data provide good fits for consumption growth's mean and volatility as well as equity market volatility.

As one of the early papers modeling the effects of habit-formation utility functions on asset pricing, Campbell & Cochrane (1999) plays an important role in modeling the time-varying and countercyclical risk premium by a having a slowly adjusting habit level and highly nonlinear utility responses. Effectively, by making RRA become very large as excess consumption approaches zero, large variations in risk premiums can be explained. Campbell & Cochrane (1999, p. 244) observed, "Risk aversion is about 80 at the steady state . . . rises to values in the hundreds for low surplus consumption ratios and is still as high as 60 at the maximum surplus consumption ratio." Their

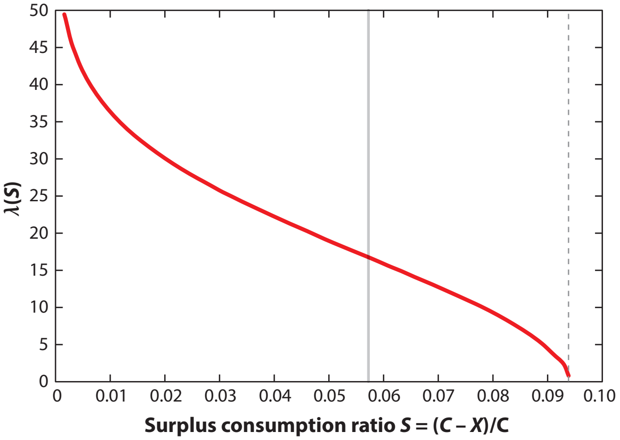

Figure 2

Sensitivity function $\lambda(s_t)$. Declining sensitivity function is employed to hold riskless rate constant while giving countercyclical variation in the price of risk. Figure adapted with permission from Campbell & Cochrane (1999).

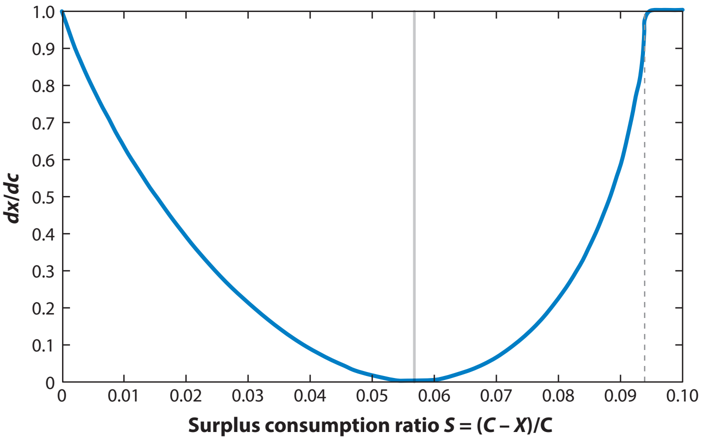

Figure 3

Implied sensitivity of habit x to contemporaneous consumption. Vertical line shows the steady-state surplus consumption ratio s. Dashed vertical line shows the maximum surplus consumption ratio. Figure adapted with permission from Campbell & Cochrane (1999).

empirical results do not seem to depend on time complementarity per se. In retrospect, this helps us see that the whole literature on excess volatility seems to have implicitly focused on constant RRA (CRRA).[1] Of course, a potential drawback is that having many parameters and imposing some delicate restrictions gives considerable flexibility to overfit the data. Out-of-sample testing using the in-sample parameter would be informative. Many more interesting articles have been written using the habit-formation model; to name only three, these include Boldrin, Christiano & Fisher (2001); Santos & Veronesi (2010); and Verdelhan (2010).

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[1] An exception to this is the model by Brunnermeier & Nagel (2006), who model risk aversion as time-varying owing to wealth fluctuations.

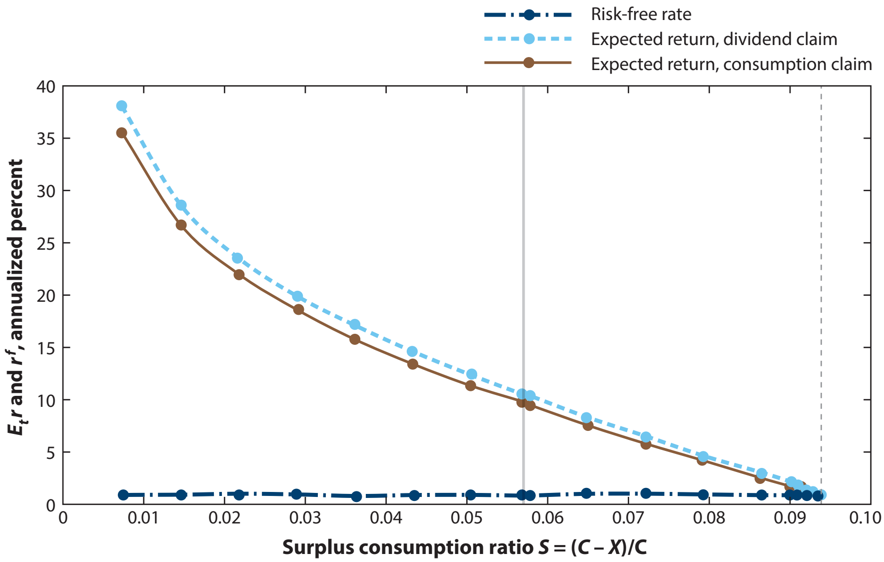

Figure 4

Risk-free rate and expected returns as functions of the surplus consumption ratio. Figure adapted with permission from Campbell & Cochrane (1999).

5. CHANGES IN RISK AND THE RISK PREMIUM

Campbell & Cochrane (1999) provided a very important and plausible prediction that RRA and risk premiums increase quite nonlinearly as surplus consumption goes toward zero, as occurs in major recessions. To amplify on this important aspect of changes in risk premiums, we have examined data for real consumption growth and the level and changes in the unemployment rate over long time

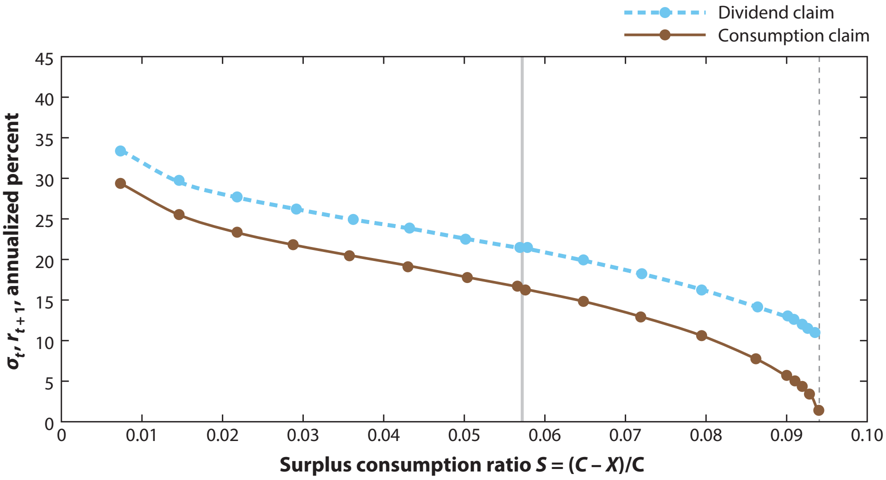

Figure 5

Conditional standard deviations of returns as functions of the surplus consumption ratio. Figure adapted with permission from Campbell & Cochrane (1999).

Table 3 Fit of Campbell-Cochrane habit formation model; means and standard deviations of simulated and historical data

	Model values		Historic values	
Statistic	**Consumption claim**	**Dividend claim**	**Postwar sample**	**Long sample**
$E\,(\Delta c)$	1.89[a]		1.89	1.72
$\sigma(\Delta c)$	1.22[a]		1.22	3.32
$E(r^f)$	0.094[a]		0.094	2.92
$E(r - r^f)/$ $\sigma\,(r - r^f)$	0.43[a]	0.33	0.43	0.22
$E(R - R^f)/$ $\sigma\,(R - R^f)$	0.50		0.50	
$E(r - r^f)$	6.64	6.52	6.69	3.90
$\sigma(r - r^f)$	15.2	20.0	15.7	18.0
$\exp[E(p - d)]$	18.3	18.7	24.7	21.1
$\sigma(p - d)$	0.27	0.29	0.26	0.27

The model is simulated at a monthly frequency; statistics are calculated from artificial time-averaged data at an annual frequency. All returns are annual percentages. Table reproduced from Campbell & Cochrane (1999).

[a] Statistics that model parameters were chosen to replicate.

horizons. We started with Shiller's (2015) long-term database for stock prices and consumption growth and added long-term data on the unemployment rate from NBER and the St. Louis Federal Reserve historical database, FRED. The NBER has a monthly series of unemployment rates that goes back to April 1929, and we used monthly data on employment changes from the 1932 Supplement to the Survey of Current Business to estimate monthly unemployment rates from January 1923 to April 1929. Given this, our first data point for the 12-month change in the unemployment rate is for January 1924, so we have slightly more than 90 years of monthly data to July 2014.

We inverted Shiller's long-term estimates of his cyclically adjusted price earnings ratio to get an earnings yield number (biased low) from which we subtracted the long-term (10-year) US Treasury interest rate to get a long data series of estimated risk premiums for US equities, $E/P - R_f$. These are surely biased low because earnings grow over time, so forward forecasts will be higher than these backward-looking earnings numbers. To test the stochastic properties of this backward-looking earnings yield with a forward one, we obtained monthly observations of the 12-month forward S&P 500 earnings estimates from 1986 to August 2014 from Edward Yardeni's (2014) website and computed a forward-looking earnings yield based on that series. **Figure 6** shows that the Shiller-type backward-looking long-term earnings yield gap is highly correlated ($\rho = 0.95$) from 1986 to 2014 with the forward-looking earnings yields: As expected, earnings yields based on the next 12 months' earnings estimates are persistently higher than the 10-year historic earnings divided by current price. The bias averages approximately 3% over the long-term. Furthermore, corporate investment at rates of return in excess of capital cost would result in a forward-looking earnings yield that is a downward-biased estimate of the expected rate of return. However, this bias should be greater in prosperous periods with higher returns on real investment than in recessionary periods with lower returns on real investment.

Given this high correlation of 10-year historical and 1-year forward yield gaps, we feel it is not unreasonable to look at a Shiller-type earnings yield gap time series as an estimate of what

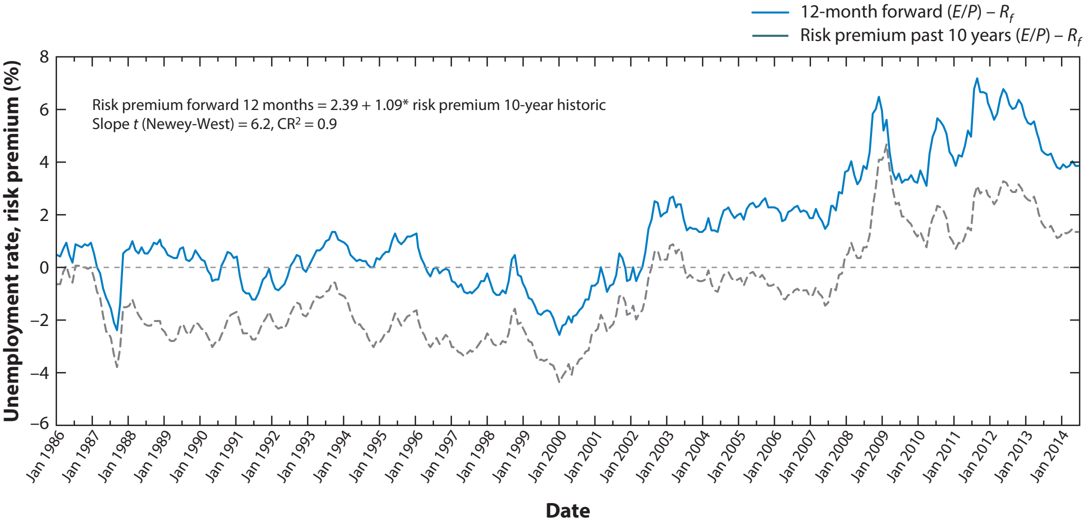

Figure 6

Risk premium ($E/P - R_f$) with 12-month forward S&P 500 earnings estimate versus past 10-year earnings yield gap, monthly from January 1986 to July 2014.

the stochastic properties of the time series of investors' forward earnings yield spreads to risk-free rates were in past years. Given the cyclical bias of forward earnings yields as estimates of expected returns, higher earnings yield spreads in recessionary periods are consistent with higher risk premiums in recessionary periods.

Figures 7 and **8** show the relationship of the 12-month changes in the estimated equity risk premium to 12-month changes in the unemployment rate. As the figures show, the series are highly correlated. When the economy falls into recession and the unemployment rate jumps, the estimated risk premium also tends to jump. We find a very strong and nonlinear relationship of the estimated risk premium to the unemployment rate and to real consumption growth. This helps us to understand why the model of Campbell & Cochrane (1999) is very helpful in modeling movements in the real economy. The correlation of the moves in estimated risk premium and changes in the unemployment rate is 0.40 over the entire 1924–2014 sample and is 0.53 if the World War II (WWII) years of 1939–1947 are excluded because unemployment fell sharply in WWII and risk and risk premiums increased, resulting in an abnormal economy.

The picture is much the same if one uses real growth of consumption of nondurables and services in modeling changes in the equity risk premium. When real consumption growth is high and the economy is good, the surplus consumption of Campbell & Cochrane (1999) increases and risk aversion along with risk premiums likely drop. The picture is very similar if we look at forecasted forward earnings yields, less the 10-year Treasury yield, and compare that yield gap's moves to moves in real consumption growth and to changes in the unemployment rate. Once again, the relationships are all strong and in the right direction. Thus, we believe that, as predicted by the Campbell-Cochrane model, it is quite plausible that RRA and risk premiums are significantly countercyclical (see **Figure 9**).

In conclusion, we believe that habit-formation models and the large cyclical swings in RRA and risk premiums are of great economic and statistical importance and have much to offer to finance researchers and practitioners.

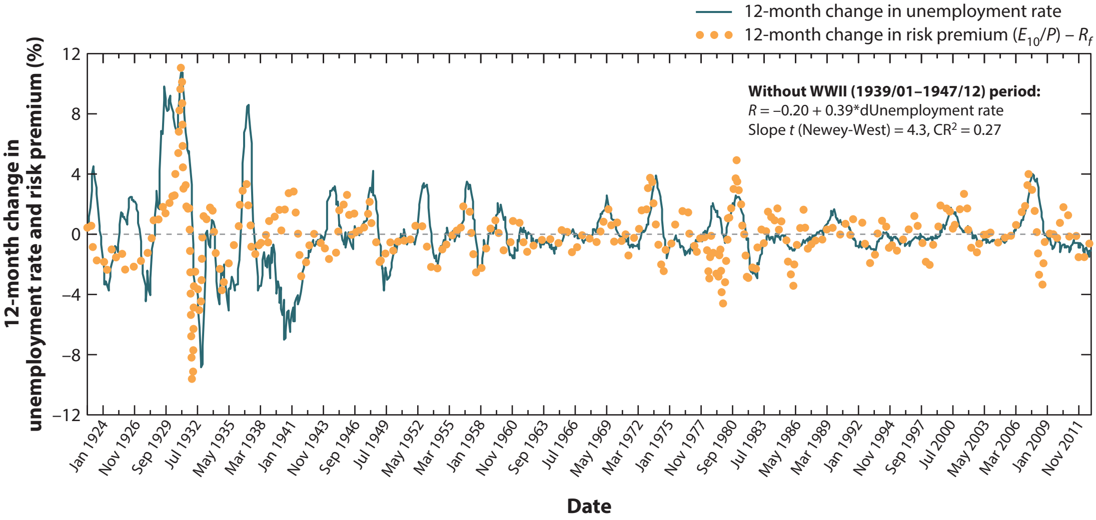

Figure 7

Change in unemployment rate versus change in risk premium $(E_{10}/P) - R_f$, showing 12-month changes, monthly from January 1924 to August 2014.

6. RESURRECTING THE CCAPM: CONDITIONAL CONSUMPTION RISKS

In a pair of innovative and impactful articles, Lettau & Ludvigson (2001a,b) built an econometric model where consumption, wealth, and labor income are cointegrated and consumption's deviations from the shared trend summarize agents' expectations of future returns on the market

Figure 8

Change in risk premium $(E_{10}/P) - R_f$ versus change in unemployment rate, monthly from January 1924 to August 2014, excluding World War II. Changes in the unemployment rate explain changes in the estimated risk premium, a statistically significant relation ($R^2 = 0.27$ and a Newey-West t-statistic of 4.3).

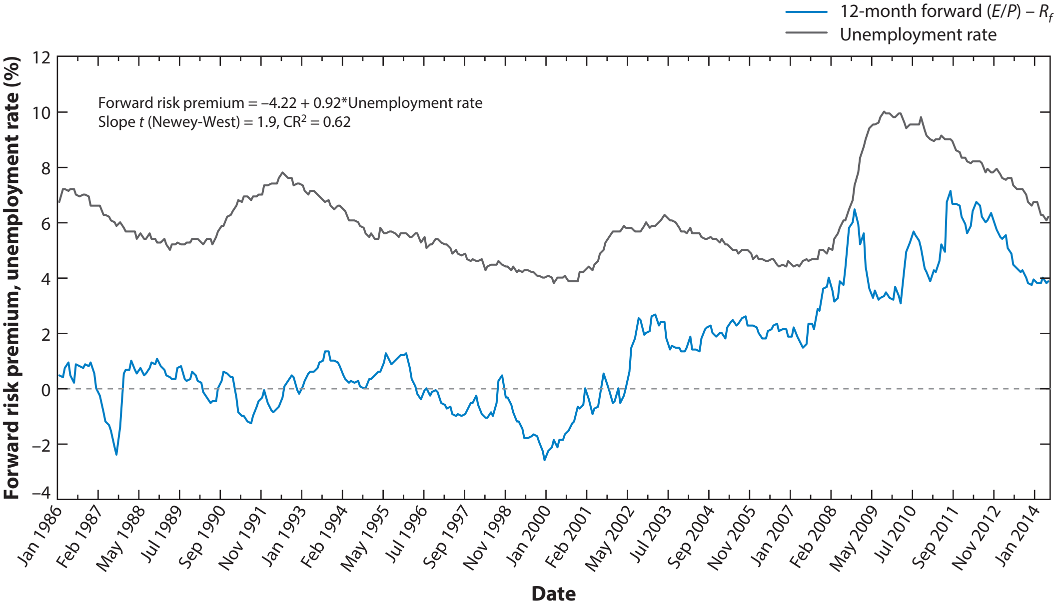

Figure 9

Risk premium $(E/P) - R_f$ with 12-month forward S&P 500 earnings estimates versus US unemployment rate, monthly from January 1986 to July 2014.

portfolio. This ties in very nicely with the continuous-time intertemporal portfolio theory of Merton (1971, 1973) and Breeden (1979, 1984). It also is consistent with prior analyses of consumption responses to shocks to permanent and transitory income. In Breeden's (1984) theoretical analysis (see equation 17 in Part 1 of this review), optimal consumption's sensitivities to the state variables that describe the investment and income opportunity set are proportional and opposite to the compensating variations in wealth for changes in those state variables. If the opportunity set improves, compensating variations in wealth are negative and we expect most individuals to respond positively with increased consumer spending. In contrast, if the opportunity set is believed to deteriorate, individuals optimally reduce consumption spending today to smooth their forward-looking lifetime consumption paths.

Going backward from consumption's moves, a high level of consumption relative to wealth and income indicates a good investment and income opportunity set, whereas a low consumption/wealth ratio is indicative of views of a poor investment and income opportunity set (perhaps a poor job market?). Lettau & Ludvigson (2001a) estimated the cointegrating relationship among consumption, wealth, and income and used positive deviations of consumption from the shared trend as a predictor of better than normal investment returns and negative deviations as predictors of poor future returns.[2] Using a dynamic least squares technique that accounts for both leading

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[2] However, both in theory and in practice, movements in volatility could well possibly be offsetting to the impact of movements in mean returns and thus cause the relation not to be as sought by Lettau & Ludvigson (2001a). If, for example, volatility dropped sharply at the same time that mean returns dropped modestly, the Sharpe ratio (slope of the capital market line) could improve, indicating a better investment opportunity set and causing optimal consumption to increase. Similarly, the mean return on the market could increase modestly at a time that volatility increased sharply, and optimal consumption would decline. These results are found in Breeden (1989), wherein high consumption growth was more likely a reflection of low future investment risk than of high future investment returns.

and lagging relationships among the cointegrated variables, they generated the following point estimates for the parameters of shared consumption, labor income, and wealth (with lowercase letters indicating natural logarithms of real nondurables and services consumption, asset wealth, and labor income per capita, respectively), using data from 1952Q4 to 1998Q3. Note that t-statistics are in parentheses:

$$c_{n.t} = \underset{(7.96)}{0.61} + \underset{(11.70)}{0.31}\, a_t + \underset{(23.92)}{0.59}\, y_t. \tag{9}$$

The residual term, which they denoted as *cay*, measures the difference between log consumption and its conditional expectation based of household net worth and labor income.

From the website of Lettau & Ludvigson (2014), given another 15 years of data, some substantial data revisions by the government in 2003, and some changes to their structural model, their current estimated log consumption trend deviation, using data from 1952Q1 to 2013Q3, is of the form

$$cay_t = c_t - 0.87 - 0.12a_t - 0.78y_t. \tag{10}$$

The change in coefficients from 1998 to 2013 indicates that, in describing consumption moves, labor income has become relatively more important than wealth, which is measured as household net worth reported quarterly by the US Federal Reserve and includes real estate, bond values, and stocks.

The results of Lettau & Ludvigson (2001a) during the time period studied (1952–1998) are quite strong, finding that "a one-standard-deviation increase in *cay* leads to a 220 basis points rise in the expected real return in the next quarter on the S&P Index and about the same rise in the excess return, roughly a nine percent increase at an annual rate" (p. 829). The Newey-West t-statistic, corrected for generalized autocorrelation, is above 3.0, which is statistically significant. Longer horizon forecasts are also impressive. As shown in **Table 4**, the *cay* variable has significant explanatory power at all intervals from 1 quarter to 6 years, with robust t-statistics of 3.0 or more.

Table 4 Long-horizon regressions: forecastability of consumption growth and the equity risk premium

	Forecast Horizon H (in quarters)							
Regressors	**1**	**2**	**3**	**4**	**8**	**12**	**16**	**24**
Consumption growth								
$\widehat{cay}_t$	0.11 (0.33) [0.00]	0.62 (0.87) [0.01]	1.23 (1.09) [0.02]	1.98 (1.33) [0.03]	2.29 (1.13) [0.02]	0.33 (0.14) [0.01]	−1.17 (−0.41) [0.00]	0.21 (0.05) [0.01]
Excess stock returns								
$\widehat{cay}_t$	**2.16** (3.44) [0.09]	**3.80** (3.34) [0.12]	**5.43** (3.37) [0.16]	**6.72** (3.70) [0.18]	**8.35** (3.73) [0.16]	**8.57** (3.24) [0.15]	**7.86** (2.99) [0.11]	**12.44** (3.41) [0.16]
$d_t - p_t$	0.03 (1.40) [0.01]	0.06 (1.23) [0.02]	0.10 (1.16) [0.03]	0.13 (1.22) [0.04]	0.24 (1.18) [0.06]	0.27 0.27 [0.07]	0.30 0.30 [0.07]	**0.76** (3.12) [0.25]

Shown are results from long-horizon regressions of excess returns on lagged variables. H denotes the return horizon in quarters. The dependent variable in the top half of the table is H-period consumption growth $\Delta c_{t+1} + \ldots + \Delta c_{t+H}$. In the bottom half of the table, the dependent variable is the sum of H log excess returns on the S&P Composite Index, $\mathrm{r}_{t+1} - r_{f,t+1} + \ldots + r_{t+H} - r_{f,t+H}$. The regressors are one-period lagged values of the deviations from trend $\widehat{cay}_t = c_t - \hat{\beta}_a a_t - \hat{\beta}_y y_t$, the log dividend yield $d_t - p_t$, the dividend earnings ratio $d_t - e_t$, the detrended short-term interest rate RREL_t, and combinations thereof. Reported for each regression are the ordinary least squares estimates of the regressors, with the Newey-West-corrected t-statistics in parentheses, and adjusted R^2 statistics in square brackets. Significant coefficients at the 5% level are highlighted in bold. The sample period is the fourth quarter of 1952 to the third quarter of 1998. Table reproduced from Lettau & Ludvigson (2001a, table 6).

During the 1952–1998 period, dividend yield (in log terms, log *D* minus log *P*) was less statistically significant as a forecaster of future real stock returns than in prior studies, but it still had strong significance forecasting 6 years out returns. The *cay* variable of Lettau & Ludvigson (2001a) had even stronger explanatory power for future stock returns for both shorter time horizons (1–4 years) and the longer horizon (6 years). **Table 4** shows that consumption deviations were not successful in forecasting future real consumption growth, which is consistent with Hall's (1988) prior results. Updating the statistics with data from 1998 to 2013 from the website data of Lettau & Ludvigson (2014), we find that a 1 standard deviation move in their *cay* variable is associated with a move that is approximately 65 basis points (bp) less per quarter than in the original study, perhaps 155 bp/quarter, giving a still-large increment of returns of approximately 6–6.5% annualized, rather than the original finding of 9.0%.

In their companion article, Lettau & Ludvigson (2001b) used their new consumption trend deviation, cay_t, as a scaling variable for measuring conditional expected returns of assets. Lettau & Ludvigson (2001b) first illustrated the poor results of using unconditional beta estimates in cross-sectional fits of mean returns with market-based CAPM betas (**Figure 10*a***) and with CCAPM betas (**Figure 10*c***) for the 25 Fama-French portfolios sorted by size and book/market. They showed the much better fit of mean returns from the three-factor statistical model of Fama & French (1993) (**Figure 10*b***), for which the underlying risk factors are unknown. Finally, using their *cay* variable for conditioning, they found that their conditional version of the CCAPM fits nearly as well as the Fama-French three-factor statistical model (**Figure 10*d***).

Lettau & Ludvigson (2001b, p. 1241) noted, "Intuitively, conditioning improves the fit of the CCAPM because some stocks are more highly correlated with consumption growth in bad times, when risk or risk aversion is high, than they are in good times, when risk or risk aversion is low. This conditionality on risk premia is missed by unconditional models because they assume that those risk premia are constant over time." This logic is consistent with Campbell & Cochrane (2000), who argued that conditional models will perform far better than unconditional models based on the presence of an external habit.

To see the changes in conditional consumption betas between "good states" and "bad states," Lettau & Ludvigson (2001b) denoted the good and bad states as those where *cay* was 1 standard deviation above and below, respectively, the unconditional mean. The estimated conditional consumption betas for the 25 Fama-French size and book/market-sorted portfolios are in **Table 5**.

Note the differences in the systematic changes in consumption betas between good and bad states and how they are related to whether the stocks are growth stocks (B1/B2) or value stocks (B4/B5). Consumption betas of value stocks increase in bad times, which is when risks are highest, so their equilibrium returns on average need to be higher, as the data has shown they are. Thus, Lettau & Ludvigson (2001b) explained the value effect with the conditional changes in consumption betas. In contrast, betas for growth stocks seem to fall in bad times, giving them lower consumption risk then. As previously discussed, this pattern of conditional consumption beta indicated that the growth stock portfolio has an unconditional convex relation to consumption growth and, conversely, the value stock portfolio has an unconditional concave relation to consumption growth. Under decreasing absolute risk aversion, there is a preference for positive skewness and the unconditional risk premium for the growth stock portfolio would, ceteris paribus, be less than that for the value stock portfolio.

The model of Campbell & Cochrane (1999) of external habit formation (and conditionally changing and nonlinear risks and RRA) as well as Lettau & Ludvigson (2001a,b) were important in reestablishing the CCAPM and consumption-based asset pricing as a leading model of asset pricing. These authors demonstrated that we need more advanced econometric techniques to

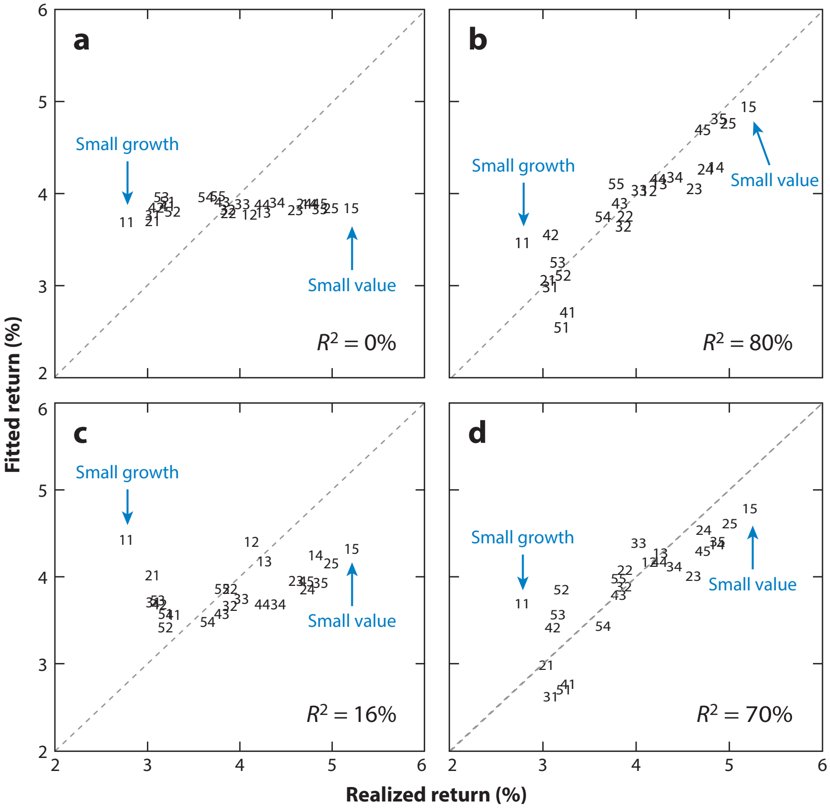

Figure 10

Realized versus fitted returns showing the pricing errors for each of the 25 Fama-French portfolios: (*a*) CAPM, (*b*) Fama-French, (*c*) consumption CAPM, and (*d*) consumption CAPM scaled. Conditional consumption capital asset pricing model (CAPM) fits nearly as well as the Fama-French three-factor model. Each two-digit number represents one portfolio. The first digit refers to book-to-market quintiles (with 1 indicating the portfolio with the lowest book-to-market ratio and 5 with the highest). The pricing errors are generated using the Fama-MacBeth regressions. The scaling variable is $\widehat{cay}$. Figure adapted with permission from Lettau & Ludvigson (2001b, figure 1).

properly model changing consumption risks and changing risk aversion and risk premiums through time and economic scenarios.

7. LONG-RUN RISKS

In another major second-generation consumption-based asset pricing model, 25 years after the original CCAPM derivations, Bansal & Yaron (2004) developed a model of long-run risks in consumption growth. This has been an influential version of consumption-based asset pricing for the past decade, spawning a substantial amount of additional research. Their key innovations were to model (*a*) expected consumption and dividend growth rates as containing a small, persistent long-run predictable component and (*b*) changing volatility of consumption growth rates. They use the forward-looking preferences of Epstein & Zin (1999), which are recursive and exhibit time complementarity for consumption.

Table 5 Conditional betas with *cay* in the consumption capital asset pricing model for Fama-French size and book/market portfolios

Portfolio		All states	Good states[a]	Bad states[b]	Change in beta bad-good
S1B1	Growth	6.4	7.3	5.7	−1.6
S1B2		6.3	5.8	6.6	0.8
S1B3	**Small**	5.1	4.4	5.6	1.1
S1B4		5.5	4.4	6.2	1.8
S1B5	Value	5.8	3.4	7.4	4.0
S2B1	Growth	4.4	7.7	2.1	−5.6
S2B2		3.6	3.6	3.6	0.0
S2B3		3.9	4.1	3.8	−0.4
S2B4		3.5	2.1	4.5	2.4
S2B5	Value	4.8	3.1	6.0	3.0
S3B1	Growth	2.7	7.4	−0.5	−7.8
S3B2		2.8	3.5	2.3	−1.2
S3B3	**Midsize**	2.9	2.0	3.5	1.6
S3B4		2.6	2.6	2.6	0.0
S3B5	Value	3.7	2.7	4.4	1.7
S4B1	Growth	2.0	6.2	−0.9	−7.1
S4B2		2.6	4.8	1.2	−3.6
S4B3		1.9	3.0	1.2	−1.8
S4B4		2.5	2.4	2.6	0.2
S4B5	Value	3.8	3.1	4.3	1.2
S5B1	Growth	1.6	6.1	−1.4	−7.5
S5B2		1.2	2.0	0.6	−1.5
S5B3	**Large**	2.3	4.1	1.2	−2.9
S5B4		1.2	3.4	−0.3	−3.7
S5B5	Value	3.1	3.3	2.9	−0.5

Table reproduced from Lettau & Ludvigson (2001b), with numbers rounded here.

[a] Good states are states with *cay* more than 1 standard deviation above the mean.

[b] Bad states are states with *cay* more than 1 standard deviation below the mean.

With regard to the modeling of a small, persistent long-run fluctuation in consumption growth rates, Bansal & Yaron (2004) noted the great difficulty of distinguishing in finite samples between a purely i.i.d. process and one that incorporates a small persistent component. Despite the difficulty of distinguishing econometrically between the two alternative processes, the asset pricing implications across them are very different. Bansal & Yaron (2004, pp. 1502) observed, “If, indeed, news about consumption has a nontrivial impact on long-term expected growth rates or economic uncertainty, then asset prices will be fairly sensitive to small growth rate and consumption volatility news,” and (p. 1482), “For these channels to have a significant quantitative impact on the risk premium and volatility of asset prices, the persistence in expected growth rate has to be quite large, close to 0.98.” They showed that their combination of assumptions for consumption and dividend growth rates, which incorporate the fluctuating persistent component, are consistent with the historic data and help them explain several puzzling aspects of asset price levels and fluctuations.

The Epstein-Zin preferences assumption drives a wedge between the EIS and RRA, which allows separate modeling of each. Flexibility in fitting EIS to a high level (EIS $=$ 1.5) allows them to match the low level of real short-term interest rates. Flexibility in fitting RRA to a relatively high level (RRA $= 10$) allows Bansal & Yaron (2004) to fit the relatively large risk premium on equities. Their modeling of changing conditional volatility of the growth rate of consumption across time allows them to model time-varying risk and risk premiums. As shown by Bansal, Khatchatrian & Yaron (2002) and here confirmed, there is a significant negative correlation between price-dividend ratios and consumption volatility. When consumption volatility is high, stock prices are low in relation to dividends. Bansal, Khatchatrian & Yaron (2002, p. 3) noted, "about half of the volatility of the price-dividend ratios in the model can be attributed to variation in expected growth rates, and the remaining can be attributed to variation in economic uncertainty." Bansal et al. (2013) more recently focused on relating volatility to macroeconomic performance, showing the impact on asset prices.

Some of the specifics of the long-run risks model of Bansal & Yaron (2004) are as follows. Defining consumption growth as g_t and dividend growth as $g_{d,t}$ and letting x_t be the small, predictable component of consumption growth,

$$\begin{aligned} x_{t+1} &= \rho x_t + \varphi_e \sigma e_{t+1}, \\ g_{t+1} &= \mu + x_t + \sigma \eta_{t+1}, \qquad\qquad e_{t+1}, u_{t+1}, \eta_{t+1} \sim \text{N.i.i.d.}(0, 1), \\ g_{d,t+1} &= \mu_d + \phi x_t + \varphi_d \sigma u_{t+1}, \end{aligned} \tag{11}$$

with the three shocks e_{t+1}, u_{t+1}, and η_{t+1} being mutually independent (Bansal & Yaron 2004, equation 4, p. 1485). Two additional parameters, $\phi > 1$ and $\phi_d > 1$, allow us to calibrate the overall volatility of dividends (which in the data are significantly larger than that of consumption) and its correlation with consumption. The parameter ϕ, as in Abel (1992), can be interpreted as the leverage ratio on expected consumption growth.

Table 6 shows the fit of the Bansal-Yaron model with the statistical properties of historic real consumption growth. From the table, comparing the historic statistics with the means from simulations of the long-run risks model, we see that the fit of consumption volatility and autocorrelation is quite good, on average. The variance ratio statistics for both the data and the model are all above 1.0 for the data shown (up to 10 years), which is consistent with positive autocorrelation in consumption growth and persistent shocks, as the ratio would be 1.0 and variance would grow proportionally through time, absent those effects. However, note that the 10-year variance ratio in the historic data is less than the 5-year variance ratio (as in Cochrane 2008), whereas the Bansal-Yaron long-run risks model has the variance ratio continuing to increase from 5 to 10 years out. Perhaps at some point, mean reversion of real consumption growth sets in (which seems quite plausible) and offsets the effects of the persistent shocks. This long-run risks model would likely miss that effect.

The Bansal-Yaron model for the time-varying variance of real consumption growth has both a purely random component and a mean-reverting component, with ν_1 describing the speed of mean reversion:

$$\sigma_{t+1}^2 = \sigma^2 + \nu_1(\sigma_t^2 - \sigma^2) + \sigma_w w_{t+1,} \qquad e_{t+1}, u_{t+1}, \eta_{t+1}, w_{t+1} \sim \text{N.i.i.d.}(0, 1). \tag{12}$$

The risk premium comes as a compensation for three consumption risks: short run, long run, and volatility. Time variation in the risk premium is governed by the conditional variance of consumption growth (i.e., it is high when current volatility is high). With fluctuating economic uncertainty, as well as the persistent growth shocks, **Table 7** shows that the Bansal-Yaron long-run risks model can replicate historic data quite well for many key asset market returns when CRRA is between 7.5 and 10.

Another nice feature of the Bansal-Yaron long-run risks model is its ability to mimic broadly the predictability of returns, growth rates, and price-dividend ratios (see **Table 8**). Bansal & Yaron (2004, p. 1481) observed, "The model can justify the equity premium, the risk-free rate, the

Table 6 Real consumption growth: historic statistical properties versus the Bansal-Yaron model fit annual data, 1929–1998, real consumption of nondurables and services

	Historic data		Model				
Variable	**Estimate**	**Standard error**	**Mean**	**95%**	**5%**	***p*-value**	**Population values**
$\sigma(g)$	2.93	(0.69)	2.72	3.80	2.01	0.37	2.88
AC(1)	0.49	(0.14)	0.48	0.65	0.21	0.53	0.53
AC(2)	0.15	(0.22)	0.23	0.50	−0.17	0.70	0.27
AC(5)	−0.08	(0.10)	0.13	0.46	−0.13	0.93	0.09
AC(10)	0.05	(0.09)	0.01	0.32	−0.24	0.80	0.01
VR(2)	1.61	(0.34)	1.47	1.69	1.22	0.17	1.53
VR(5)	2.01	(1.23)	2.26	3.78	0.79	0.63	2.36
VR(10)	1.57	(2.07)	3.00	6.51	0.76	0.77	2.96
$\sigma(g_d)$	11.49	(1.98)	10.96	15.47	7.79	0.43	11.27
AC(1)	0.21	(0.13)	0.33	0.57	0.09	0.53	0.39
$\text{corr}(g,g_d)$	0.55	(0.34)	0.31	0.60	−0.03	0.07	0.35

AC(t) is the autocorrelation for real consumption growth over t periods, VR(t) is the variance ratio for t periods, and g_d is the dividend growth rate. Table reproduced from Bansal & Yaron (2004, table 1).

volatility of the market return and the price-dividend ratio. As in the data, dividend yields predict returns and the volatility of returns is time-varying." Three critical observations are (*a*) the number of tuning parameters is so large that they have considerable flexibility to overfit the data; (*b*) their model requires CRRA near 10 to duplicate the data, which is relatively high; and (*c*) EIS needs to be large (1.5) to replicate the negative correlation between consumption volatility and price-dividend ratio present in the data, which is well above the EIS indicated by Vissing-Jørgensen's (2002) research. Nonetheless, their results provide an economic rationale for the equity premium puzzle and duplicate a 6% risk premium, while having a low risk-free rate and reasonable volatility. To get these results with simply a small persistent growth term and with time-varying volatility is surprising. The following sections include articles on the long-run risks model of Bansal & Yaron

Table 7 Equity risk premium, real riskless rate, volatilities, and price-dividend ratio historic statistics versus model fit with relative risk aversion of 7.5 and 10.0

	Historic data		Bansal-Yaron model	
Variable	**Estimate**	**Standard error**	$\gamma = 7.5$	$\gamma = 10$
Returns				
$E(r_m - r_f)$	6.33	(2.15)	4.01	6.84
$E(r_f)$	0.86	(0.42)	1.44	0.93
$\sigma(r_m)$	19.42	(3.07)	17.81	18.65
$\sigma(r_f)$	0.97	(0.28)	0.44	0.57
Price-dividend ratio				
$E(\exp(p-d))$	26.56	(2.53)	25.02	19.98
$\sigma(p-d)$	0.29	(0.04)	0.18	0.21
$AC1(p-d)$	0.81	(0.09)	0.80	0.82
$AC2(p-d)$	0.64	(0.15)	0.65	0.67

Table reproduced from Bansal & Yaron (2004, table 4).

Table 8 Predictability of returns, growth rates, and price-dividend ratios in the Bansal-Yarom long-term risks model versus historical predictability

	Excess returns			Growth rates			Volatility		
Variable	Data	SE	Model	Data	SE	Model	Data	SE	Model
$B(1)$	−0.08	(0.07)	−0.18	0.04	(0.03)	0.06	−8.78	(3.58)	−3.74
$B(3)$	−0.37	(0.16)	−0.47	0.03	(0.05)	0.12	−8.32	(2.81)	−2.54
$B(5)$	−0.66	(0.21)	−0.66	0.02	(0.04)	0.15	−8.65	(2.67)	−1.56
$R^2(1)$	0.02	(0.04)	0.05	0.13	(0.09)	0.10	0.12	(0.05)	0.14
$R^2(3)$	0.19	(0.13)	0.10	0.02	(0.05)	0.12	0.11	(0.04)	0.08
$R^2(5)$	0.37	(0.15)	0.16	0.01	(0.02)	0.11	0.12	(0.04)	0.05

Evidence is provided on predictability of future excess returns and growth rates by price-dividend ratios, and the predictability of price-dividend ratios by consumption volatility. Entries in the Excess returns columns correspond to regressing $r^e_{t+1} + r^e_{t+2} + \ldots + r^e_{t+j} = \alpha(j) + B(j)\log(P_t/D_t) + v_{t+j}$, where r^e_{t+1} is the excess return, and j denotes the forecast horizon in years. Entries in the Growth rates columns correspond to regressing $g^a_{t+1} + g^a_{t+2} + \ldots + g^a_{t+j} = \alpha(j) + B(j)\log(P_t/D_t) + v_{t+j}$, where g^a is annualized consumption growth. Entries in the Volatility columns correspond to $\log(P_{t+j}/D_{t+j}) = \alpha(j) + B(j)|\epsilon_{g^a,t}| + v_{t+j}$, where $|\epsilon_{g^a,t}|$ is the volatility of consumption defined as the absolute value of the residual from regressing. $g^a_t = \sum_{j=1}^{5} A_j g^a_{t-j} + \epsilon_{g^a,t}$. Model is based on the process in Equation 8, with the parameter configuration given in **Table 4** and $\gamma = 10$. Entries for the model are based on 1,000 simulations each with 840 monthly observations that are time aggregated to an annual frequency. Standard errors (SEs) are corrected for Newey & West (1987) using 10 lags. Table reproduced from Bansal & Yaron (2004, table 6).

(2004) because it continues to be one of the most impactful articles in consumption-based asset pricing in the past decade.

In a test of whether consumption risks of cash flows are helpful measures of risk, as posited by Breeden & Litzenberger (1978), Bansal, Dittmar & Lundblad (2005) examined dividends and share repurchases versus real nondurables and services consumption growth for 30 portfolios: 10 decile portfolios ranked by size, book/market, and momentum, respectively. Bansal, Dittmar & Lundblad (2005, p. 1640) showed "that the cross-sectional dispersion in the measured cash flow beta explains approximately 62% of the cross-sectional variation in observed risk premia. Further, the estimated market price of consumption risk is sizable, statistically significant, and positive in all cases." They duplicated much of the spread in mean returns of the extreme momentum portfolio (winner minus loser) and similarly defined size and value portfolios, using quarterly data from 1967 to 2001. The estimated cash flow (dividend) growth rates for the 30 portfolios are interesting (see **Table 9**).

Small firms having the highest cash flow growth is as expected. However, high book/market firms, value stocks, display higher growth rates than did the growth stocks, which helps Bansal, Dittmar & Lundblad (2005) duplicate the value effect. Key to their being able to duplicate momentum effects is the result that the high positive momentum stocks display faster cash flow growth, whereas the negative momentum stocks have negative growth, perhaps not surprising, but interesting.

To estimate consumption risks, Bansal, Dittmar & Lundblad (2005) used two alternative methods. First, they estimated γ_i to be the "projection of portfolio-specific dividend growth on the moving average of consumption growth," using an 8-quarter moving average of prior consumption growth with a lag length of 4 quarters, which is akin to Parker & Julliard's (2005) 11-quarter consumption calculations for ultimate consumption betas. Second, they estimated the sensitivity of the innovation in dividend growth rates to the estimated innovation in consumption growth, which gives their β_{ig} estimates. There is some instability in these cash flow consumption risk estimates, and there is no clear relation of beta estimates to size. However, for book/market portfolios, the value stocks do have noticeably higher cash flow consumption betas than do the growth

Table 9 Summary statistics: portfolio real cash flow (dividend) growth for portfolios sorted by size, book/market, and momentum

Size	Mean	Standard deviation	Book/market	Mean	Standard deviation	Momentum	Mean	Standard deviation
S1 (small)	0.011	0.055	B1 (low)	−0.001	0.040	M1 (losers)	−0.039	0.228
S2	0.010	0.039	B2	0.002	0.051	M2	−0.019	0.130
S3	0.008	0.038	B3	0.003	0.072	M3	−0.009	0.112
S4	0.007	0.039	B4	0.005	0.070	M4	−0.002	0.080
S5	0.007	0.040	B5	0.003	0.047	M5	−0.003	0.090
S6	0.003	0.030	B6	0.006	0.032	M6	0.002	0.075
S7	0.005	0.037	B7	0.005	0.034	M7	0.004	0.104
S8	0.004	0.065	B8	0.009	0.040	M8	0.012	0.092
S9	0.002	0.042	B9	0.008	0.046	M9	0.021	0.122
S10 (large)	0.000	0.018	B10 (high)	0.011	0.089	M10 (winners)	0.028	0.178

Table reproduced from Bansal, Dittmar & Lundblad (2005, table 2).

stocks, and the high momentum stocks also have higher cash flow consumption betas than do the negative momentum stocks. The standard errors are high relative to risk estimates, and R^2 are very low. So it is surprising that the estimated cross-sectional relations for returns versus these cash flow consumption risks show such a strong relationship. The cross-sectional evidence using generalized method of moments (GMM) estimates is shown in **Table 10**, with cash flows and risks each measured in two ways.

Table 10 shows a significant cross-sectional relationship of risk premiums with cash flow consumption betas from dividends, which explains 62% to 66% of return premiums. With repurchases added, the relationship is a bit weaker, but still strong, with R^2 ranging from 46% to 61%. All in all,

Table 10 Cross-sectional evidence: excess returns of size, book/market, and momentum portfolios versus cash flow consumption betas

	λ_0	Standard error	*t*-Statistic	λ_c	Standard error	*t*-Statistic	R^2
Dividends							
Independent variable is γ_i							
Coefficient	1.754	(0.815)	2.15	0.177	(0.072)	2.46	0.663
Independent variable is $\beta_{i,g}$							
Coefficient	1.658	(0.837)	1.98	0.118	(0.027)	4.37	0.620
Dividends plus repurchases							
Independent variable is γ_i							
Coefficient	1.741	(0.851)	2.05	0.166	(0.057)	2.91	0.607
Independent variable is $\beta_{i,g}$							
Coefficient	1.697	(0.859)	1.98	0.105	(0.030)	3.50	0.456

Shown are generalized method of moments estimates. Here, γ_i is the projection of portfolio-specific dividend growth on the 8-quarter moving average of prior consumption growth; $\beta_{i,g}$ is the sensitivity of the innovation in dividend growth to the innovation in consumption growth. Regressions are: excess return $= \lambda_0 + \lambda_1$ (consumption risk measure). Table reproduced from Bansal, Dittmar & Lundblad (2005, table 4).

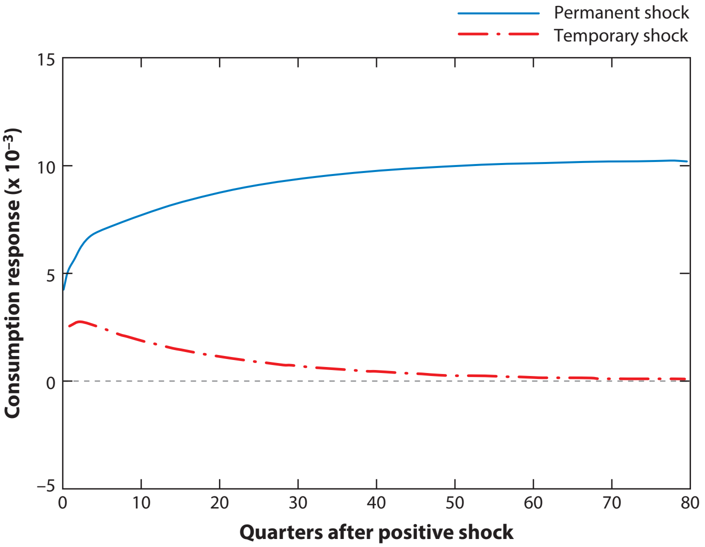

Figure 11

Impulse responses of consumption to permanent and temporary shocks. Figure adapted with permission from Hansen, Heaton & Li (2008, figure 3).

this is quite a good showing for cash flow consumption betas. Note that Bansal, Dittmar & Lundblad (2005) confirmed the result of Lettau & Ludvigson (2001b) that unconditional consumption betas fit returns very poorly.

Hansen, Heaton & Li (2008) carefully developed an interesting model that contained both short-term transitory shocks and long-term permanent shocks in an economy of consumers with EZ-W preferences. Impulse response functions for both shocks are shown in **Figure 11**.

Hansen, Heaton & Li (2008) modeled corporate profits as being cointegrated with consumption and studied the valuation of growth and value stocks as an application of their model. **Figure 12** shows how differently cash flows of value and growth stocks move, relative to consumption.

Whereas the return of value stocks over growth is well known, the much greater long-run volatility of value stocks' performance in consumption units is quite noticeable. By contrast, movements in growth stocks and the market portfolio are tame. As Hansen, Heaton & Li (2008, p. 261) observed, "We find that the cash flows of value portfolios exhibit positive comovement in the long run with macroeconomic shocks, whereas the growth portfolios show little covariation with these shocks. Equilibrium pricing reflects this heterogeneity in risk exposure: risk-averse investors must be compensated to hold value portfolios." Using data for the high book/market portfolio 5 versus that for the growth portfolio 1 of Fama & French (1993), they graphed the impulse responses of cash flows of both to a permanent shock in consumption (see **Figure 13**). Their results show that consumption shocks have transitory impacts on growth stock performance but permanent impacts on value stocks, thus leading to the value premium.

In another significant article with conditional consumption betas, Bansal, Dittmar & Kiku (2009) found that betas generated by an error-correction vector-autoregressive (EC-VAR) model with cointegration restrictions between dividends and consumption can explain cross-sectional differences at many horizons, outperforming the model without the cointegration restriction. They utilized classic techniques of Granger & Engle (1987). As they noted,

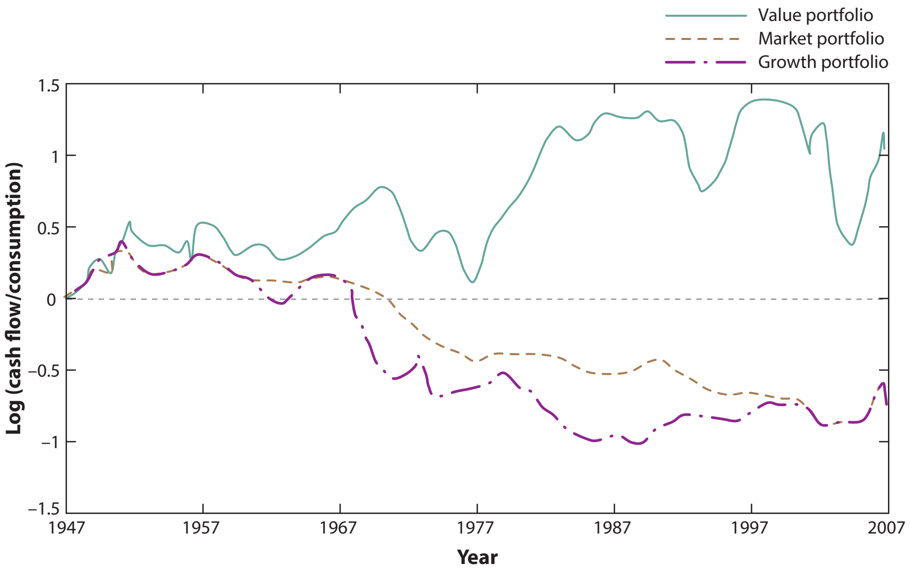

Figure 12

Cash flows relative to consumption for value, market, and growth portfolios using the model of Hansen, Heaton & Li (2008). Figure adapted with permission from Hansen, Heaton & Li (2008, figure 1).

> The deviation of the level of dividends from consumption (the error correction variable) is important for predicting dividend growth rates and returns at all horizons. . . . Imposing cointegration, we are able to predict on average 11.5% of the variation in one-year returns, compared to 7.5% when we do not impose cointegration. This difference is even starker at longer horizons: at the 10-year horizon, the EC-VAR specification results in an average 44.0% adjusted R^2, compared to 9.9% for the standard growth-rate VAR specification. (Bansal, Dittmar & Kiku 2009, p. 1344)

This predictability evidence, they claimed, has important implications for measuring return innovations and, consequently, conditional consumption betas. They argued that the EC term of the cointegrated VAR contains important information and can predict future dividend growth by conditioning, which translates into longer-term return predictability. Their empirical results suggest that return predictability increases more than dividend predictability does when adding the EC term into the VAR model.

Please note that Bansal, Dittmar & Kiku (2009) assumed all individuals are identical and have a standard, time-additive power utility function with everyone having the same CRRA. This is a special case of the original derivations of the CCAPM in the late 1970s (see Section 2), some of which were derived for heterogeneous individuals and more general time-additive utility and do not assume that anyone has CRRA utility. So all the results and insights of those original models must apply to this one and the contributions here are primarily empirical. Bansal, Dittmar & Kiku (2009) used annual data from 1929–2002, quite a different sample than for the 1963–2001 quarterly sample of Bansal, Dittmar & Lundblad (2005). The longer-term calculations of Bansal, Dittmar & Kiku (2009) used more stable annual data and appear to have more continuous estimated movements in returns and risk measures for size and book/market portfolios than in Bansal, Dittmar & Lundblad (2005).

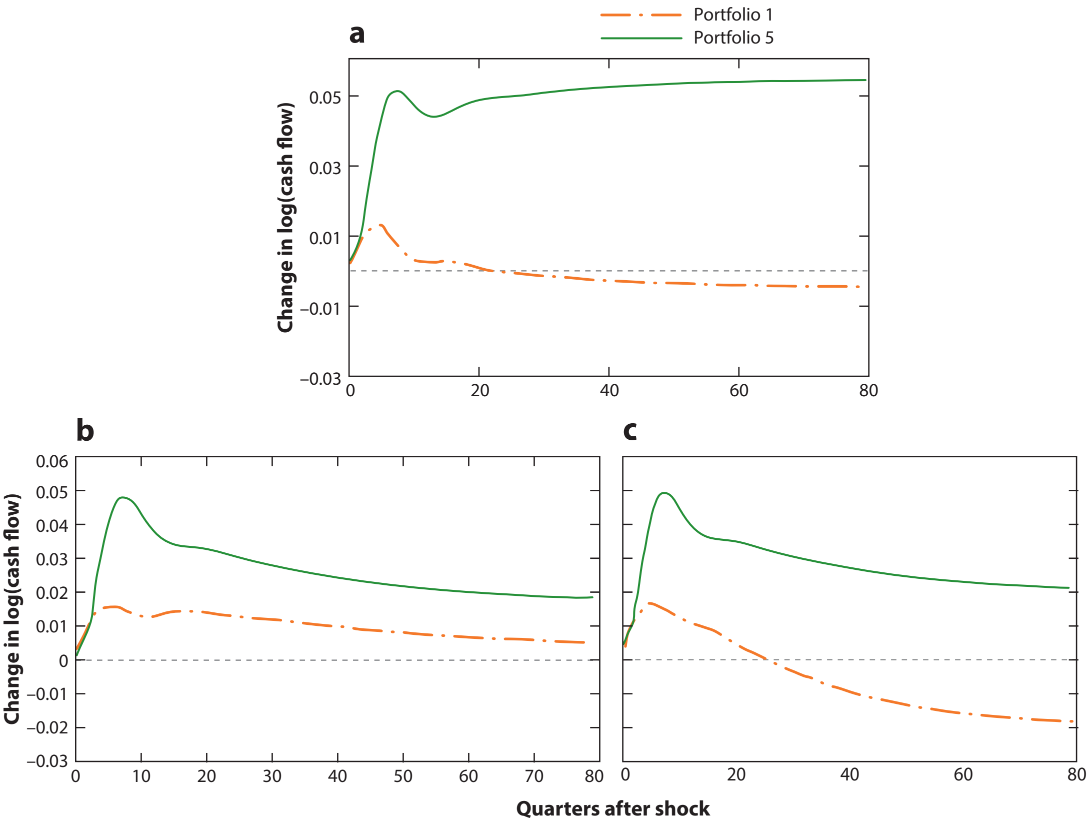

Figure 13

Impulse responses to a permanent shock to consumption of the cash flows to portfolios 1 (growth) and 5 (value). Panel (*a*) uses a first difference specification, (*b*) a level specification without time trends, and (*c*) a level specification with time trends. Figure adapted with permission from Hansen, Heaton & Li (2008, figure 8).

As Bansal, Dittmar & Kiku (2009, pp. 1356–7) observed, "Cointegration implies that the dividend growth rates are predicted by the cointegrating residuals. That is, the current deviations of an asset's cash flows from their long-run relation with consumption should forecast the dynamics of dividend growth rates while dividends are moving back toward equilibrium." As dividends are a key element in investment returns, especially in the longer term, this may also translate into return predictability.

Examining **Table 11**, we see the following: The cointegrated model, EC-VAR, is better than the standard VAR in the short- and medium-term, which makes sense as the EC model picks up transitory variation in dividend growth rates. With regard to return predictability, the cointegrated model, EC-VAR, has much better predictive accuracy. Thus, Bansal, Dittmar & Kiku (2009, p. 1358) observed, "the cointegrating residual, included in the error-correction specification, contains distinct information about future returns beyond that in the growth-rate-based model." As return innovations differ across the two models, consumption betas will also differ between the models. **Table 11** gives the estimated consumption betas in the cointegrated model. Notice the large differences in EC-VAR versus the VAR estimates.

A close examination of the results in **Tables 11** and **12** confirms, "Neither the unconditional betas nor those based on the VAR reflect the cross-sectional differences in mean returns on size

Table 11 Consumption betas by horizon: cointegration model

			Horizon					
			1 year		5 years		10 years	
	Portfolio	Unconditional	EC-VAR	VAR	EC-VAR	VAR	EC-VAR	VAR
Size deciles	S1	0.71 (1.5)	4.12 (2.4)	1.77 (2.1)	4.51 (4.7)	−1.46 (3.9)	6.54 (4.3)	−1.55 (4.0)
	S2	0.80 (1.4)	2.09 (1.1)	0.89 (1.1)	1.82 (2.7)	−1.24 (2.4)	4.21 (3.0)	0.13 (2.4)
	S3	0.52 (1.4)	4.14 (1.3)	2.95 (2.0)	2.14 (2.4)	0.04 (2.4)	3.38 (2.3)	0.35 (2.9)
	S4	0.77 (1.1)	3.52 (1.2)	2.66 (1.5)	2.09 (2.0)	0.79 (1.9)	3.56 (2.4)	1.59 (1.8)
	S5	0.36 (1.2)	2.76 (1.0)	2.36 (1.2)	0.99 (1.7)	0.39 (1.5)	2.05 (2.1)	1.14 (1.4)
Size deciles	S6	0.64 (1.1)	3.07 (0.8)	2.58 (1.1)	1.42 (1.5)	0.71 (1.9)	2.50 (1.8)	1.41 (1.6)
	S7	0.33 (1.1)	2.37 (0.9)	2.20 (0.8)	0.46 (1.4)	0.14 (1.3)	1.00 (1.7)	0.61 (1.2)
	S8	−0.31 (1.1)	1.62 (0.7)	1.43 (0.8)	−0.12 (1.4)	−0.44 (1.6)	0.35 (1.7)	−0.13 (1.6)
	S9	0.13 (1.1)	1.58 (0.8)	1.38 (0.8)	0.60 (1.4)	0.27 (1.6)	1.02 (1.6)	0.62 (1.5)
	S10	0.69 (0.8)	1.54 (0.6)	1.64 (0.5)	0.31 (1.1)	0.51 (1.1)	0.34 (1.1)	0.67 (1.0)
Book/market deciles	B1	0.82 (1.0)	1.81 (0.5)	2.16 (0.3)	−0.58 (1.4)	0.31 (1.5)	−0.83 (1.3)	0.14 (1.6)
	B2	−0.18 (0.8)	0.16 (0.6)	0.65 (0.4)	−1.69 (0.9)	−0.76 (1.0)	−2.05 (0.9)	−0.86 (1.1)
	B3	−0.33 (0.8)	−0.09 (0.4)	0.32 (0.4)	−1.79 (0.8)	−1.34 (0.9)	−1.70 (0.8)	−1.33 (1.0)
	B4	0.29 (1.1)	1.48 (1.3)	1.94 (1.5)	−0.67 (2.0)	−0.07 (2.1)	−0.59 (2.1)	−0.28 (2.2)
	B5	0.27 (1.1)	1.94 (0.9)	1.67 (1.0)	1.18 (1.6)	0.81 (1.8)	1.60 (1.7)	1.10 (1.6)
Book/market deciles	B6	2.24 (1.0)	3.18 (1.5)	2.64 (2.0)	2.75 (2.4)	1.91 (2.0)	3.27 (2.4)	2.17 (1.8)
	B7	0.21 (1.2)	2.74 (1.0)	1.46 (1.7)	2.67 (1.5)	1.01 (1.6)	4.22 (1.6)	1.70 (1.7)
	B8	0.84 (1.2)	4.34 (1.8)	2.18 (2.1)	4.39 (2.7)	0.37 (2.2)	6.36 (2.7)	0.98 (2.3)
	B9	−0.39 (1.5)	5.47 (2.3)	2.16 (2.9)	6.11 (3.8)	−1.55 (2.7)	8.32 (3.4)	−2.03 (2.8)
	B10	0.14 (1.6)	3.89 (1.1)	2.87 (1.0)	2.14 (2.6)	0.54 (3.2)	4.33 (3.4)	1.49 (3.5)

Presented are the consumption betas for investment horizons of 1, 5, and 10 years for each of the 20 portfolios sorted by market capitalization (S1–S10) and book-to-market ratio (B1–B10). In columns labeled EC-VAR (error-correction vector autoregression), betas are measured using the error correction specification for consumption and asset returns. Columns labeled VAR (vector autoregression) present betas measured using a growth-rate VAR omitting the error correction information. These consumption betas are estimated as in Equation 17, using the covariance matrices implied by the relevant time-series model. The column labeled Unconditional represents the standard consumption beta. Robust standard errors are reported in parentheses. The number of lags used in the covariance estimator of Newey & West (1987) is 8. Table reproduced from Bansal, Dittmar & Kiku (2009, table 6), with standard errors rounded for table readability.

and book-to-market-sorted portfolios" (Bansal, Dittmar & Kiku, p. 1362). **Table 12** shows the one-step GMM estimates of the market prices of risk that are jointly estimated with the time-series parameters. The R^2 values for the EC model are very high, at 73% to 84%, in contrast to the standard VAR results. Also, the estimated prices of risk are all estimated as two to three times their standard errors (also see **Figure 14**).

The long-run risk model of Bansal & Yaron (2004) has stimulated a large body of research, with additional results by (to name a few) Koijen et al. (2010); Constantinides & Ghosh (2011); Drechsler & Yaron (2011); and Ferson, Nallareddy & Xie (2013).

Strong results with consumption betas require modeling of conditional consumption betas. From the works of Lettau & Ludvigson (2001b), Bansal, Dittmar & Lundblad (2005), Jagannathan & Wang (2007), and Bansal, Dittmar & Kiku (2009), we learned that it is very important to have a conditioning variable for consumption betas because they change over time and economic states. Lettau & Ludvigson (2001b) conditioned on *cay*, their variable that represents consumption's deviations from a broad wealth variable (stocks, bonds, and real estate), which also includes capitalized wage income. In contrast, Jagannathan & Wang (2007) quite reasonably conditioned on NBER-designated recession and expansion periods. Bansal, Dittmar & Kiku (2009) conditioned on the deviations of dividends from their long-term trend with consumption.

Table 12 Cross-sectional regressions of portfolio returns on consumption rise by horizon: EC model with cointegration versus VAR

	Unconditional	Horizon					
		1 year		5 years		10 years	
		EC-VAR	VAR	EC-VAR	VAR	EC-VAR	VAR
$\lambda_{1,s}$	0.51	1.19	1.28	0.73	−0.31	0.65	−0.07
SE	(2.24)	(0.41)	(1.57)	(0.32)	(0.40)	(0.24)	(0.44)
$\bar{R}^2$	−0.04	0.75	0.22	0.73	−0.03	0.84	−0.05

This table presents results for cross-sectional regressions for a set of 10 portfolios sorted by market capitalization and 10 portfolios sorted by book-to-market ratio. The row labeled $\lambda_{1,s}$ reports the estimated prices of risk. Consumption risk for different investment horizons is measured by the corresponding consumption beta. In columns labeled EC-VAR, betas are measured using the error correction specification for consumption and asset returns. Columns labeled VAR present betas measured using a growth-rate VAR omitting the error correction information. Consumption betas are estimated as in Equation (17), using the covariance matrices implied by the relevant time series model. The column labeled Unconditional represents the standard consumption beta. All risk prices are expressed in annual percentage terms. Robust standard errors, reported in parentheses, are computed by estimating time-series and cross-sectional parameters in one step via GMM. The number of lags used in the Newey & West (1987) covariance estimator is 8. Abbreviations: EC-VAR, error-correction vector autoregression; SE, standard error; VAR, vector autoregression. Table reproduced from Bansal, Dittmar & Kiku (2009, table 7).

8. RISKS OF RARE DISASTERS

An influential early article by Rietz (1988) proposed a model of rare disasters, such as stock market crashes, to explain the high level of the equity premium and low riskless returns. This model is quite compatible with the model of Kraus & Litzenberger (1983) that implies a sufficiently concave characteristic line of equities with consumption growth could help explain the high risk premium on equities because of asymmetrically large exposure to contractions versus expansions. Rietz (1988) extended the Mehra-Prescott model to include a third state representing a very low probability of a major depression or crash. Using a simple power utility function with CRRA,

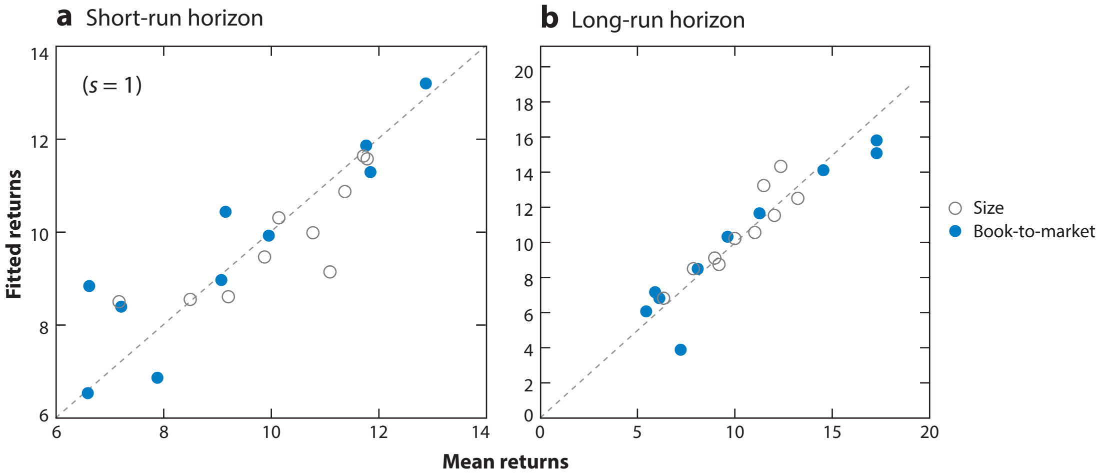

Figure 14

Fitted versus mean returns of very strong (*a*) 1-year and (*b*) long-term fits of the error-correction vector-autoregression model. Figure adapted with permission from Bansal, Dittmar & Kiku (2009, figure 2).

Table 13 Rare disasters; output falls to one-half its normal value during a crash; combinations of crash probabilities and risk aversion that give risk-free and market returns that match history

Crash probability (η)	Risk-aversion parameter (α)	Time preference parameter (β)	Corresponding risk-free return (annual %)	Corresponding risk premium (annual %)
0.0008	7.05	0.997	0.77	6.36
0.0008	7.00	0.999	0.83	6.18
0.0009	6.90	0.994	0.87	6.38
0.0009	6.90	0.995	0.77	6.38
0.0009	6.85	0.997	0.83	6.19
0.0009	6.85	0.998	0.73	6.19
0.0010	6.75	0.993	0.88	6.34
0.0010	6.75	0.994	0.78	6.33
0.0010	6.70	0.996	0.84	6.15
0.0010	6.70	0.997	0.74	6.14
0.0010	6.65	0.999	0.79	5.96
0.0020	5.75	0.989	0.83	5.92
0.0020	5.75	0.990	0.73	5.92
0.0030	5.30	0.980	0.89	6.15

Shown are parameter configurations that give risk-free returns and risk premia very near the economy's sample values. Table reproduced from Rietz (1988, table 3).

Rietz matched both a low real riskless rate (less than 1.0%) and the historic equity market risk premium of 6% to 7% with a relatively moderate level of CRRA (5 to 7) (see **Table 13**).

Almost two decades after Rietz's (1988) classic work providing a disaster risk explanation of the equity premium puzzle of Mehra & Prescott (1985), Barro (2006), Barro & Ursua (2008), and then Wachter (2013) gathered new data and built new models of the risks of rare disasters. As Barro (2006, p. 823) said, "I think that Rietz's basic reasoning is correct, but the profession seems to think differently [and] the major reason for skepticism about Rietz's argument is the belief that it depends on counterfactually high probabilities and sizes of economic disasters." Using data from Maddison (2003) (with some corrections), Barro (2006) found 60 instances among 20 OECD (Organisation for Economic Cooperation and Development) countries of peak-to-trough declines in real per capita GDP of 15% or more in the twentieth century. The average decline was 29% of GDP. Using the data available for the 20 OECD countries, Barro computed the probability of such a −15% or more disaster to be 1.7% per year, on average. The frequency distribution of contractions is shown in **Figure 15**.

Barro (2006) also computed the combination of disaster probability and RRA that would explain the observed equity risk premium. The result is shown in **Figure 16**, where b is the loss that occurs in a typical disaster, which is simulated to be a 25% to 50% decline in real GDP, peak to trough. Note that CRRA = 4.3, a not implausible level of risk aversion, combined with the historic probability of 1.7% per year, fits the historic equity risk premium.

More recently, Wachter (2013) pointed out that the models of Rietz (1988) and Barro (2006) would have stock market volatility equal to the volatility of dividends, which is so low as not to be realistic. Wachter showed that modeling the probability of a consumption disaster as time varying can solve this problem. As she noted, "The possibility of this poor outcome substantially increases the equity premium, while time variation in the probability of this outcome drives high

Figure 15

Frequency distribution for sizes of contractions of real GDP more than 15% for 20 countries in the Organisation for Economic Co-operation and Development (OECD). Figure adapted with permission from Barro (2006, figure I).

stock market volatility and excess return predictability" (Wachter 2013, p. 987). Wachter's specific model for the stochastic process for aggregate consumption is as follows:

$$dC_t = \mu C_{t^-} dt + \sigma C_{t^-} dB_t + (e^{Z_t} - 1) C_{t^-} dN_t, \tag{13}$$

where B_t is a standard Brownian motion and N_t is a Poisson process time-varying intensity λ_t. This intensity follows the process

$$d\lambda_t = \kappa(\bar{\lambda} - \lambda_t)dt + \sigma_\lambda \sqrt{\lambda_t}\, dB_{\lambda,t}. \tag{14}$$

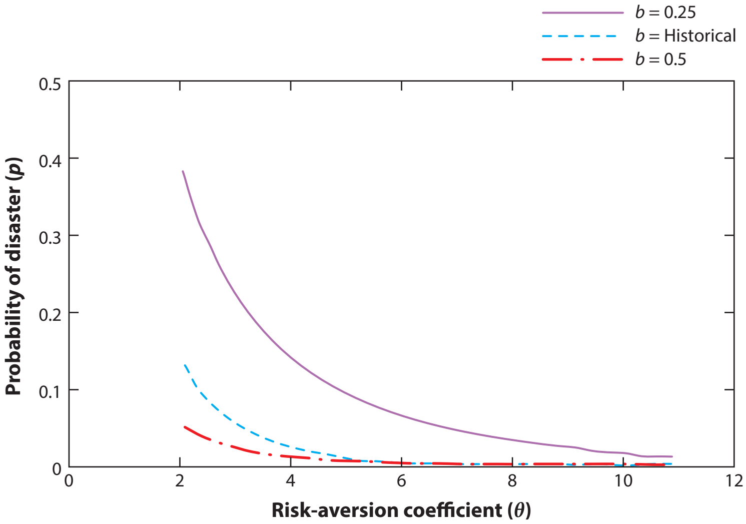

Figure 16

Combinations of disaster probability and relative risk aversion that would explain historic equity risk premium for contraction sizes of 25%, 50%, and historical. Figure adapted with permission from Barro (2006, figure II).

Thus, Wachter (2013) has a mixed jump-diffusion process. In normal times, when no disaster takes place, consumption follows a continuous diffusion process. Disasters are captured by the Poisson jumps downward in consumption: "Roughly speaking, λ_t can be thought of as the disaster probability over the course of the next year" (Wachter 2013, p. 991).

The equity premium in Wachter's model is different from previous models, reflecting the presence of disaster risk as well as the time variation in disaster risk (from Wachter 2013, equation 28):

$$r_t^e - r_t = \underbrace{\phi\gamma\sigma^2}_{\text{standard model}} \underbrace{- \lambda_t \frac{G'}{G} b\sigma_\lambda^2 + \underbrace{\lambda_t E_\nu[(e^{-\gamma Z} - 1)(1 - e^{\phi Z})]}_{\text{static disaster risk}}}_{\text{time—varying disaster risk}}. \qquad (15)$$

Here, the second term produces time variation in the equity premium that reflects changes in the disaster jump intensity, λ_t, whereas the third term gives the impact of a static amount of rare disaster risk.

For her calibration and simulation, Wachter (2013) used data from Barro & Ursua (2008) on consumption declines, wherein a 10% or more decline is termed a disaster [rather than a 15% decline as in Barro (2006)]. With this definition, the disaster probability, λ, equals 3.55%, using data from 22 countries from 1870 to 2006. A developed-country subset (termed OECD countries) has a slightly lower disaster probability of 2.86%. The frequency distributions of consumption declines for the two data sets are shown in **Figure 17**.

Wachter (2013) assumed RRA of 3.0 and a rate of time preference equal to 1.2%, which allowed her to match the average real return on the 3-month Treasury bill in postwar US data. Her simulation results using the data for all 22 countries are shown in **Table 14**.

Wachter's (2013) conditional results, i.e., when there was no disaster, match the US postwar data quite well. However, the population results, which include a normal fraction of disasters, have consumption volatility over 6%, in contrast to the 2% volatility with no disasters. Volatility of dividends also depends greatly on whether there was a disaster in the sample period. In summary, Wachter's mixed jump-diffusion model mimics well the historic data when there are no disasters

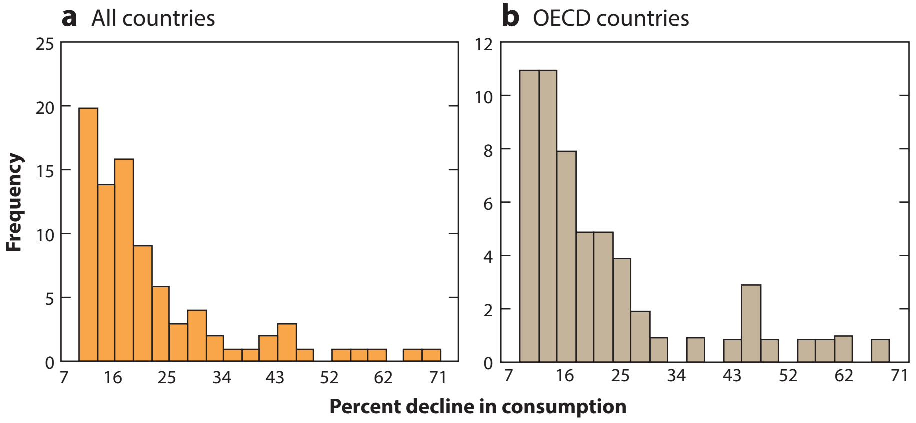

Figure 17

Frequency distribution of real consumption decline >10% for (*a*) all countries and (*b*) developed countries in the Organisation for Economic Co-operation and Development. Figure adapted with permission from Wachter (2013, figure 7).

Table 14 Population moments from simulated data. Sample moments from historical time-series. Wachter's (2013) rare disasters model fit

	Model		
	Population	Conditional	US data
$E[R^b]$	0.99	1.36	1.34
$\sigma(R^b)$	3.79	2.00	2.66
$E[R^e - R^b]$	7.61	8.85	7.06
$\sigma(R^e)$	19.89	17.66	17.72
Sharpe ratio	0.39	0.49	0.40
$\sigma(\Delta c)$	6.36	1.99	1.34
$\sigma(\Delta d)$	16.53	5.16	6.59

Model is simulated at a monthly frequency, and simulated data are aggregated to an annual frequency. Data moments are calculated using overlapping annual observations constructed from quarterly US data, from 1947 through the first quarter of 2010. With the exception of the Sharpe ratio, moments are in percentage terms. The second column reports population moments from simulated data. The third column reports moments from simulated data that are calculated over years in which a disaster did not occur. The last column reports annual sample moments. R^b denotes the gross return on the government bond, R^e the gross equity return, Δc growth in log consumption, and Δd growth in log dividends. Table reproduced from Wachter (2013, table 2).

and shows what data should look like over the very, very long-term when we have a normal (small) proportion of disasters in the sample.

9. PREDICTABILITY OF THE EQUITY RISK PREMIUM

Much as in their 2001 article that developed *cay*, Lettau & Ludvigson (2005) modeled consumption, stock dividends, and labor income (dividends from human capital) as having a three-way cointegrated relationship, but in their new model, asset wealth was replaced by dividend growth via the derivation of Campbell & Mankiw (1989). In the post-WWII era, dividend yield has been a less useful predictor of stock returns. Lettau & Ludvigson (2005) argued that this was in part the result of the offsetting effects of dividend growth and equity risk premiums. In a recession, risks likely increase, dividend yields increase as stock prices fall more than dividends, and projected returns on stocks increase to compensate for higher risk. However, if dividends have fallen, expected dividend growth may increase, partially offsetting the effects of the higher risk premium. Per Lettau & Ludvigson (2005), the positive covariation of dividend growth with market risk premiums masks greater volatility in both during the post-WWII period. For the prewar era, they found no evidence of this cointegrating relation and little predictive power in the pre-WWII data.

Santos & Veronesi (2006) have a straightforward, but powerful model of time variation in the equity risk premium and changes in conditional risks of assets. They model consumer spending as being composed of two parts—one part funded by labor income and the other by financial assets, such as stock returns. According to their hypothesis, when labor income provides a larger fraction of financing for consumption (and stock returns provide a smaller portion), stock returns will be less correlated with optimal consumption and will earn smaller risk premiums owing to their smaller consumption betas. In contrast, when stocks provide a large fraction of funds for consumption, covariance of consumption with stock returns increases and risk premiums on stocks should also increase. Thus, as the ratio of labor income (compensation of employees data) to consumption (shown in **Figure 18**) increases, the equity risk premium should fall, yielding a negative relationship.

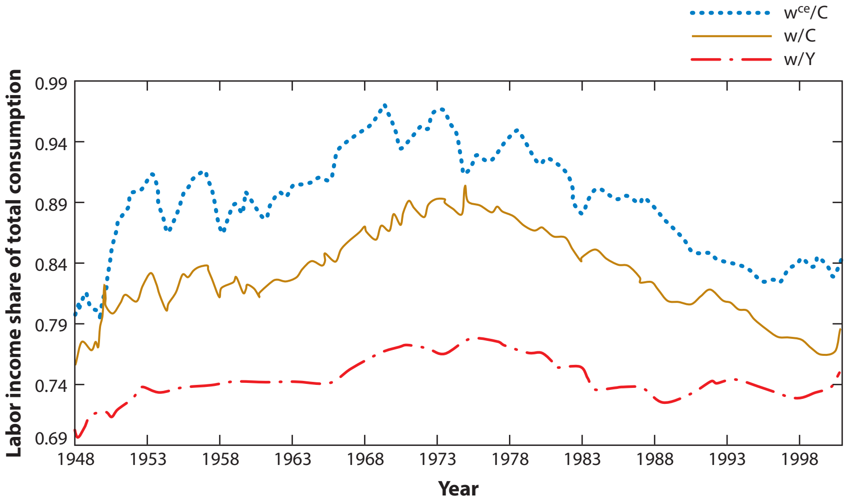

Figure 18

Shares of labor income to consumption. Compensation of employees (*blue dotted line*) is the broader measure of labor income, including bonuses as well as wages and salaries. Note that in the sharp recessions in 1974/1975 and 1981/1982, the share of consumption paid for by labor income (which was quite high) dropped sharply, so that income paid by financial assets provided a larger fraction while the conditional consumption beta of stocks and the time-varying risk premium should have increased. Plots are: labor income/consumption (*blue dotted line*), compensation of employees/consumption (*orange solid line*), labor income/disposable income (*red broken line*). Figure adapted with permission from Santos & Veronesi (2006, figure 1).

Figure 19 compares the ability of the labor income share to (negatively) predict subsequent 4-year returns on equities against the ability of dividend yields to do so. The labor income/consumption ratio has greater ability to predict returns than does dividend yield during the post-WWII period. Long-term (4-year) predictability is quite high, with $R^2 = 0.42$ for the labor income/consumption ratio, versus 0.14 for dividend yield. Combining both predictors is even more statistically significant, with $R^2 = 0.57$ over the 1948–2001 sample period. Using the labor income/consumption ratio as a conditioning variable, much as Lettau & Ludvigson (2001b) did with *cay*, Santos & Veronesi (2006) also obtained very positive results for the conditional CAPM in cross-sectional fits of returns of the 25 Fama-French portfolios sorted by size and book/market.

10. DURABLE GOODS CONSUMPTION, SYSTEMATIC RISK, AND ASSET PRICING

Researching durables consumption flows, Yogo (2006) found strong results on asset pricing, risk measurement, and risk premium. These findings are especially interesting, as so many researchers use just the nondurables and services part of consumption and exclude durables because only a portion of the durable is consumed annually (6%, as estimated by the US Bureau of Economic Analysis). Yogo found that the ratio of durables consumption flows to nondurables and services is highly procyclic: Compared with nondurables and services consumption, durables move more sharply up and down with the economy. This seems plausible introspectively: Often, when individuals have moved up significantly in wealth or income, they buy nice durable goods and luxury items, such as cars, jewelry, or vacation homes, and in so doing, they control (reduce) marginal

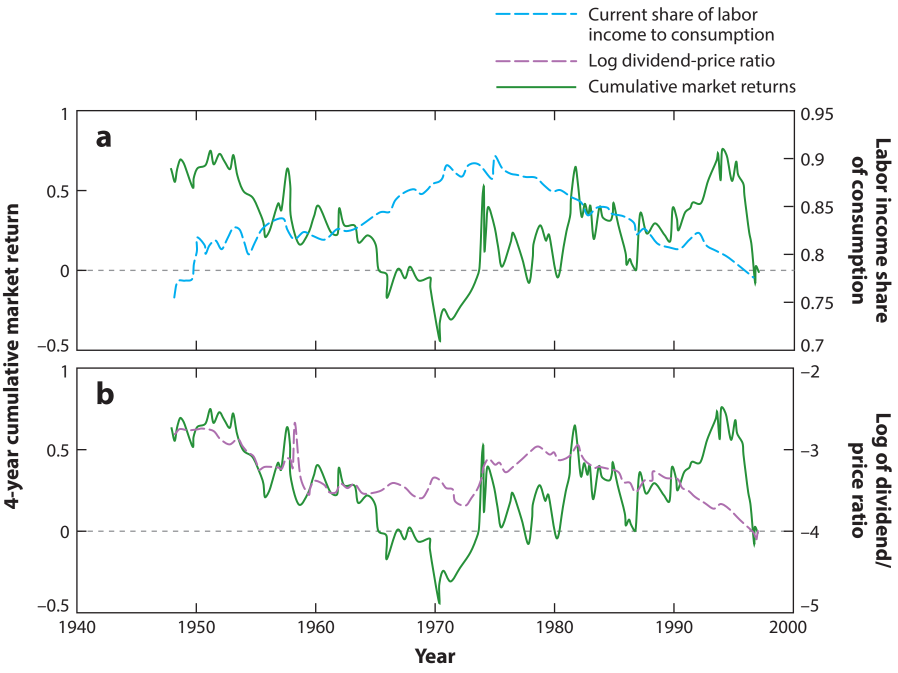

Figure 19

Long-term returns and predictive variables. The next 4 years of equity returns (*solid line*) are plotted versus (*a*) share of labor income to consumption (*dashed line*) and (*b*) log dividend/price ratio (*dashed line*). Figure adapted with permission from Santos & Veronesi (2006, figure 2).

utility optimally. In recessions, however, durables purchases are sharply curtailed, and households live off their old durables stocks. Thus, durables spending could be an excellent signal of changes in marginal utility, which is indeed what Yogo (2006) found. **Figure 20** shows the ratio of durables stocks to nondurables consumption.

Shown in **Table 15** are the correlations of the three Fama-French factors with nondurables and durables consumption with quarterly data from 1951 to 2001, along with mean, volatility, and autocorrelation statistics. Note that durables consumption growth is highly autocorrelated, much more so than is nondurables (0.88 versus 0.28). Correlations of durables consumption growth with the stock market and small-minus-big as well as high-minus-low factors are all low, and the correlation of these quarterly changes with nondurables spending is only 0.19.

Yogo (2006) provided first-stage GMM estimates of consumption betas of the 25 (5 × 5) Fama-French portfolios sorted by size and book/market (shown in **Table 16**). Note the much sharper correspondence of stocks' average excess returns with stocks' betas using durables consumption than when using betas with nondurables (also see **Figure 21**).

Interestingly, Yogo (2006) also provided the consumption betas for various industries relative to nondurables and durables, estimated by first-stage GMM and shown in **Table 17**. Note that value stocks (high book/market) have much higher durables betas than do growth stocks, whereas the nondurables betas are not much different. This helps explain the ability of the durables betas to do so well in explaining the cross section of average returns.

Finally, Yogo (2006) also demonstrated that durables can model time variation in the estimated equity premium, as durables move sharply down in recessions at a time when risk and risk aversion

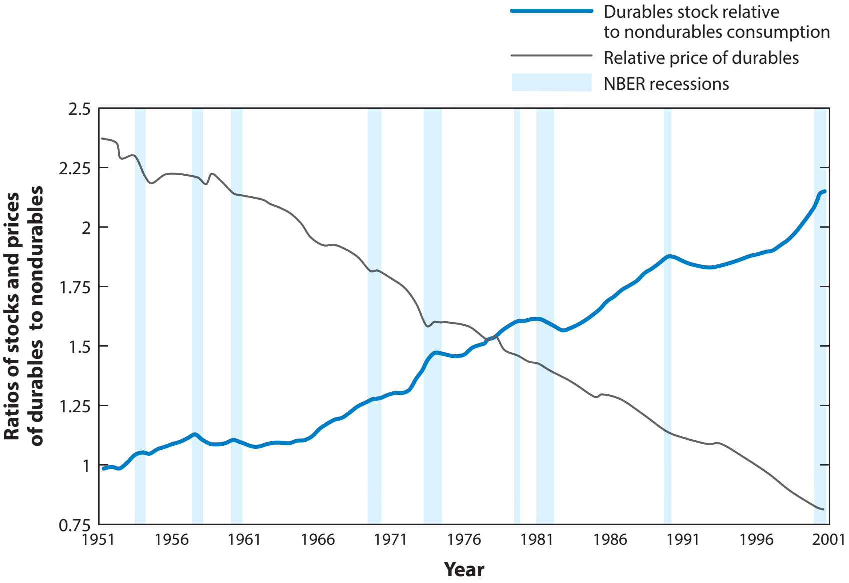

Figure 20

Ratio of durables stocks to nondurables consumption, showing a very procyclic nature as well as the increasing share of real durables consumption in total consumption, in part responding to the relative price decline of durables. Data are quarterly 1951–2001. Shaded regions are NBER recessions. Figure adapted with permission from Yogo (2006, figure 1).

cause the equity premium to surge (see **Figure 22**). A weakness in Yogo's model is that the implied level of RRA is extremely high at CRRA $=$ 174. This high estimate is due to the significant risk premiums, despite the very low volatility of durables consumption stocks and durables consumption flows. The sales of high-end durable goods such as BMWs and Mercedes are considerably more volatile and more cyclical than the aggregate consumption of durables. Assuming no complementarity across consumption goods and that the marginal investors are the wealthy, Aït-Sahalia, Parker & Yogo (2004) find implied risk aversions for luxury goods that are more than an order of magnitude less than for aggregate durables; e.g., for BMW and Mercedes sales over the 1970–1999 period, their estimate of relative risk aversion (corrected for time aggregation bias) is 10.0.

Table 15 Descriptive statistics: durables and nondurables real consumption growth and Fama-French three-factor model correlations

				Correlation			
Variable	Mean (%)	SD (%)	Autocorrelation	Market	SMB	HML	Nondurables
Market	1.88	8.82	0.05				
SMB	0.51	5.58	−0.03	0.42			
HML	1.09	5.54	0.15	−0.39	−0.14		
Nondurables	0.51	0.54	0.28	0.28	0.13	0.00	
Durables	0.92	0.54	0.88	−0.11	−0.04	0.04	0.19

Shown are the mean, standard deviation (SD), and first-order autocorrelation of excess market return, small-minus-big (SMB) return, high-minus-low (HML) return, and nondurables and durables consumption growth. Also shown are the correlations among these variables. Table reproduced from Yogo (2006, table 1), with numbers rounded here.

Table 16 Average returns and consumption betas for the Fama-French portfolios. Durables and nondurables consumption betas

	Book-to-market equity					
Size	**Low**	**2**	**3**	**4**	**High**	**High to low**
Average excess return (%)						
Small	1.12	2.45	2.53	3.16	3.46	2.34
2	1.46	2.23	2.72	2.93	3.15	1.69
3	1.71	2.35	2.31	2.76	2.94	1.23
4	1.90	1.80	2.42	2.57	2.73	0.83
Big	1.69	1.65	2.02	1.99	2.14	0.45
Small minus big	−0.57	0.80	0.52	1.17	1.32	
Nondurables consumption beta						
Small	6.5	6.1	5.8	5.4	6.2	−0.3
2	6.1	5.1	5.2	5.4	5.9	−0.2
3	5.5	5.1	5.1	5.2	5.9	0.5
4	4.9	4.3	4.5	5.2	5.1	0.1
Big	4.8	3.5	3.0	4.2	4.0	−0.8
Small minus big	1.8	2.6	2.8	1.2	2.2	
Durables consumption beta						
Small	0.3	1.2	1.6	2.3	2.5	2.2
2	0.1	1.1	1.8	1.8	2.0	1.8
3	0.5	1.2	1.4	1.9	2.0	1.5
4	0.9	0.7	1.3	1.8	1.8	0.9
Big	1.0	0.8	1.3	1.4	1.3	0.4
Small minus big	−0.6	0.5	0.4	0.9	1.2	

The top third of the table shows the average excess returns (per quarter) on the 25 Fama-French portfolios sorted by size and book-to-market equity. The middle and bottom thirds present nondurable and durable consumption betas, implied by the first-stage generalized method of moments estimate of the durable consumption model, respectively. The last row in each section reports the difference between small and big stocks, and the last column reports the difference between high and low book-to-market stocks. Table reproduced from Yogo (2006, table 4), with numbers rounded here.

Following up on Yogo's (2006) analysis of the consumption of durable goods, Gomes, Kogan & Yogo (2007) provided an innovation with their construction of better industry classifications than those provided by the US government. These authors used the input/output tables of the US Bureau of Economic Analysis for cash flows in the economy. Gomes, Kogan & Yogo (2007, p. 943) then documented four new facts in the cross section of cash flows and stock returns: First, cash flows of durable goods producers are more volatile and are more correlated with aggregate consumption than are other industries. Second, returns on the durable goods portfolio are higher and more volatile on average. Third, cash flows of durable-goods producers are conditionally more volatile when the durable expenditure/stock ratio is low, which generally coincides with recessions. Fourth, returns on durable goods portfolios are more predictable. Supporting their second point over the 1927–2007 period, a portfolio that is long durable goods and short the services portfolio earned an average annual return over 4.0%. Supporting their fourth point, a portfolio that is long durables and short the market portfolio has countercyclical expected returns, reliably predicted by the durables expenditure/stock ratio.

A key mechanism in the model of Gomes, Kogan & Yogo (2009, p. 944) is that "a proportional change in the service flow (or the stock) of durable goods requires a much larger proportional change in the expenditure on durable goods." They argued, "the difference in the conditional

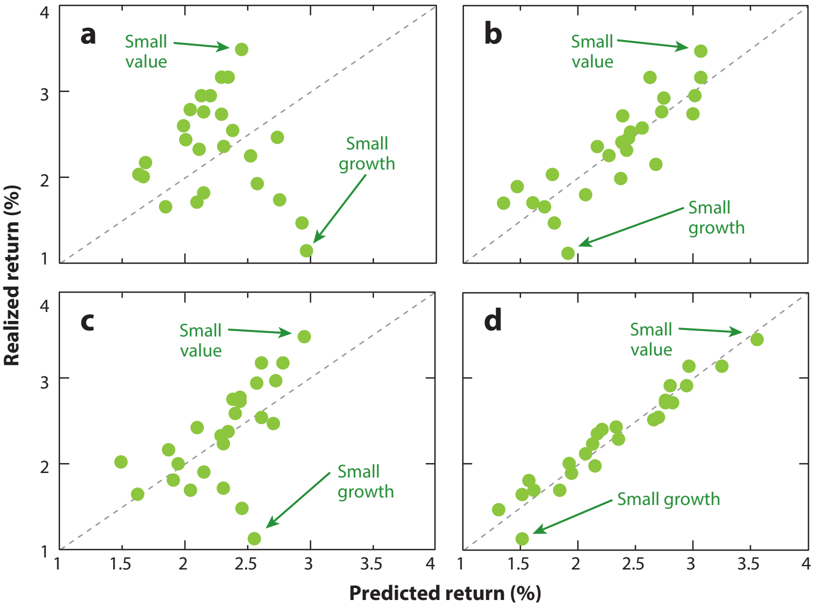

Figure 21

Realized versus predicted returns for Fama-French portfolios, sorted by size and back-to-market equity (value or growth). Risks are measured by (*a*) market betas, (*b*) the Fama-French three-factor model, (*c*) nondurables consumption betas, and (*d*) durables betas. Figure adapted with permission from Yogo (2006, figure 4).

cash flow risk between durable-goods producers and nondurable-good producers is relatively high when the existing stock of durables is high relative to current demand. This mechanism leads to a testable implication that the durable expenditure-stock ratio predicts cross-sectional differences in the conditional moments of cash flows and stock returns" (Gomes, Kogan & Yogo 2009, p. 944). **Figure 23** shows the cyclical movements of the durables expenditure-stock ratio. Note the sharp drops during the Great Depression as well as during the sharp recessions in 1974/1975 and 1981/1982.

Gomes, Kogan & Yogo (2007) used the durables expenditure-stock ratio as a predictor variable for excess returns and showed that its performance is comparable to that of aggregate dividend yield, especially in the postwar period. Consistent with the conditionally higher expected returns in recessions, additional results confirm that cash flow risk and 5-year dividend growth risk are higher when the durables expenditure-stock ratio is low, as occurs during recessions. Thus, in summary, Gomes, Kogan & Yogo (2007) demonstrated the greater cyclicality of durables and the potential use of the durables expenditure-stock ratio as a conditioning variable for modeling risk changes in the economy.

11. REAL ESTATE

Piazzesi, Schneider & Tuzel (2007) examined housing and consumption and the composition risk between housing and nonhousing consumption and its impact on asset pricing. They argued several theoretical and empirical points for the importance of this split. Perhaps the most persuasive result they found is that individuals really dislike reducing housing consumption (habit formation). Piazzesi, Schneider & Tuzel (2007, p. 532) observed, "In our model, investors' concern with

Table 17 Average returns and consumption betas for portfolios sorted by book-to-market (B/M) equity within industry

Industry	Average return (%)			Nondurables beta			Durables beta		
	Low B/M	Med B/M	High B/M	Low B/M	Med B/M	High B/M	Low B/M	Med B/M	High B/M
Manufacturing, nondurables	1.90	2.27	2.82	4.2	4.3	5.1	1.20	1.27	1.55
Manufacturing, durables	1.73	2.40	3.74	6.1	5.9	8.4	−0.05	0.77	2.80
Manufacturing, other	1.52	1.89	2.66	5.5	3.7	4.8	0.57	1.17	2.24
Retail, nondurables	1.96	2.63	2.52	6.0	4.5	4.9	0.40	0.89	1.04
Retail, durables	2.26	2.05	3.48	5.7	6.3	5.6	0.38	−0.06	0.90
Services	1.67	1.30	2.18	4.4	3.4	5.8	−0.21	0.11	1.51
Finance	1.54	2.58	3.10	5.0	5.0	3.8	0.29	1.34	1.65
Natural resource	0.28	1.63	2.93	2.0	3.5	4.3	−0.33	1.26	2.78

Average excess returns (per quarter) are reported on 24 portfolios sorted by B/M equity within industry, along with nondurable and durable consumption betas, implied by the first-stage generalized method of moments estimate of the durable consumption model. See notes to Yogo (2006, table 6) for details on portfolio formation. Table reproduced from Yogo (2006, table 7), with numbers rounded here.

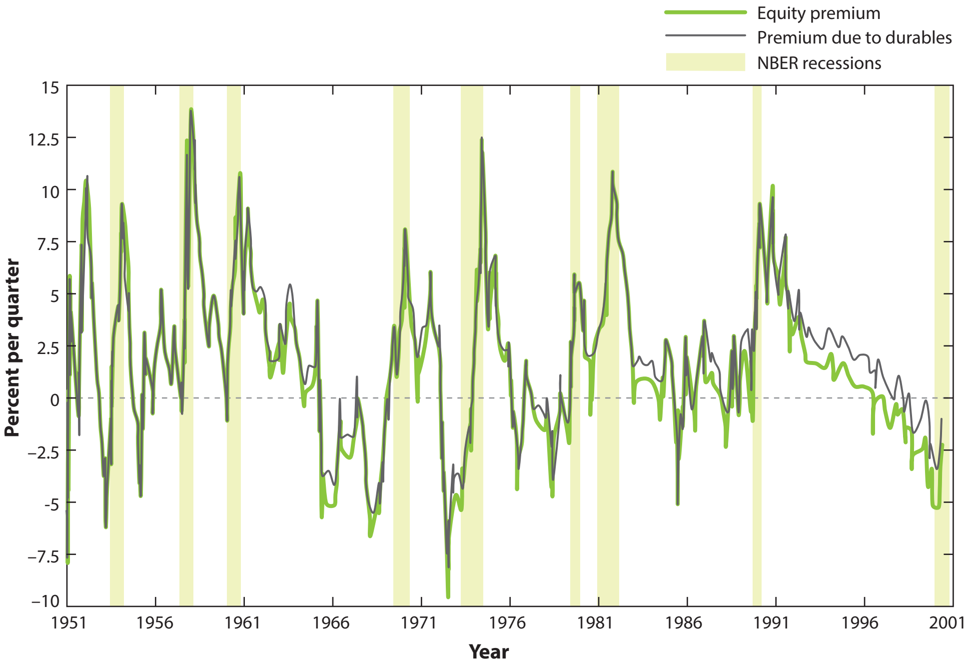

Figure 22

Time variation in the equity premium. The figure is a time series plot of expected excess returns on the market portfolios. Sample period is 1951–2001 quarterly. Shaded regions are NBER recession. Figure adapted with permission from Yogo (2006, figure 5).

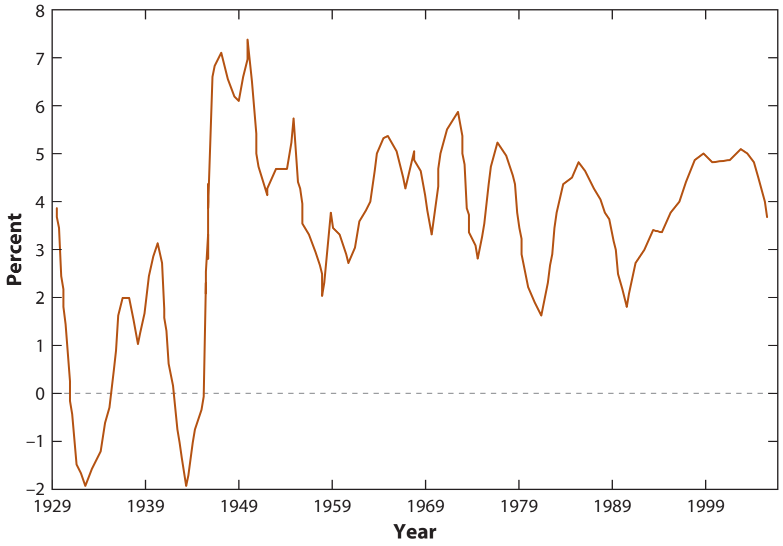

Figure 23

Cyclical movements of the durables expenditure-stock ratio. The stock of durables is the sum of the stock of consumer durable goods and the the stock of private residential fixed assets. Sample period is 1929–2007. Figure adapted with permission from Gomes, Kogan & Yogo (2007, fig. 2).

composition risk implies that recessions are perceived as particularly severe when the share of housing consumption is low." Housing is a necessity, and people do not reduce it unless circumstances are very bad and marginal utility is very high. Furthermore, "stocks have . . . especially low payoffs in severe recessions, when housing consumption is relatively low (and α is high). This generates higher equity premia than under the standard model" (Piazzesi, Schneider & Tuzel 2007, p. 540).

Piazzesi, Schneider & Tuzel (2007, p. 548) made another significant point, noting that "times characterized by relatively little housing correspond to times when the volatility of shocks is higher. In other words, times with little housing are times of high uncertainty." As they further observed, "Interestingly, the model implies that a macroeconomic variable, the (nonhousing) expenditure share, α_t, should be a good forecasting variable. Intuitively, the model implies that α_t is high in severe recessions, when expected excess returns are high" (Piazzesi, Schneider & Tuzel 2007, p. 560). **Table 18** shows their results predicting excess stock returns with the housing expenditure share.

12. FOREIGN EXCHANGE

Foreign exchange rates are quite important in the global economy. Their movements make goods, services, and labor cheaper or more expensive country by country (or for the eurozone) and, thereby, direct the flow of international investment and employment around the world. When a country's economy is strong, its unemployment rate drops, there is less slack in the economy, and its foreign exchange rate strengthens, which directs global investment to other, less strong areas in the world. So there is and should be a positive correlation of foreign exchange rates in a country with the country's economic strength relative to other countries.

Lustig & Verdelhan (2007) examined data for eight different currency portfolios, with portfolio 1 in low interest rate currencies and portfolio 8 in high rate currencies. They examined the interest rate spreads to the US dollar, the average rate of depreciation, and average inflation rate in the

Table 18 Predicting excess stock returns with the housing expenditure share

	High-perceived-risk model[a]		High-risk-aversion model[b]		Long sample			Postwar sample		
Horizon (year)	Slope	R^2	Slope	R^2	Slope	*t*-Statistic	R^2	Slope	*t*-Statistic	R^2
Regressions on expenditure share										
1	2.00	0.05	2.40	0.06	1.36	1.47	0.02	1.42	1.68	0.03
2	3.80	0.09	4.55	0.10	3.30	2.03	0.07	3.68	2.24	0.08
3	5.42	0.13	6.50	0.14	5.01	2.40	0.14	6.25	3.21	0.20
4	6.88	0.16	8.25	0.18	6.58	2.84	0.18	8.63	3.95	0.28
5	8.19	0.19	9.83	0.21	8.44	3.65	0.22	10.73	4.92	0.30

	Long sample: 1936–2001 historical data					Postwar sample: 1947–2001 historical data				
	$\ln 1/v_t^s$		$\ln\alpha_t$			$\ln 1/v_t^s$		$\ln\alpha_t$		
Horizon (year)	Slope	*t*-Statistic	Slope	*t*-Statistic	R^2	Slope	*t*-Statistic	Slope	*t*-Statistic	R^2
Regression on expenditure share and dividend yield										
1	0.10	2.04	0.43	0.44	0.07	0.10	1.84	0.50	0.13	0.08
2	0.17	1.60	1.75	1.11	0.14	0.16	1.39	2.14	0.75	0.14
3	0.15	1.01	3.65	1.77	0.18	0.10	0.66	5.30	2.11	0.21
4	0.16	0.86	5.08	2.29	0.20	0.06	0.30	8.09	3.04	0.28
5	0.28	1.49	5.87	2.64	0.26	0.15	0.81	9.24	3.43	0.31

The top panel reports regression results of log excess stock returns $\sum_{j=1}^{n} r_{t+j}^{s} - r_{t+j}^{f}$ on a constant and the log expenditure share $\ln\alpha_t$, for $n = 1, \ldots, 5$ years. High-perceived risk and high-risk-aversion models contain the average slope and R^2 over 50,000 simulated samples with 65 observations. The model with high-perceived risk is the boldface parameterization in Piazzesi, Schneider & Tuzel (2007, table 3) with $\varepsilon = 1.05$, $\beta = 0.99$, and $1/\sigma = 5$. The model with high-risk aversion is the boldface parameterization in Piazzesi, Schneider & Tuzel (2007, table 3) with $\varepsilon = 1.25$, $\beta = 1.24$, and $1/\sigma = 16$. The Long sample columns run regressions with 1936–2001 historical data, and the Postwar sample columns use 1947–2001 data. *t*-Statistics are based on the standard errors of Newey & West (1987) to correct for overlapping observations. The bottom panel reports regression results of $\sum_{j=1}^{n} r_{t+j}^{s} - r_{t+j}^{f}$ on a constant, and $\ln\alpha_t$ and the log dividend yield $\ln 1/v_t^s$. Table reproduced from Piazzesi, Schneider & Tuzel (2007, table 5).

Table 19 US investor's excess returns: foreign exchange portfolios formed on yield spreads

Portfolio	1	2	3	4	5	6	7	8
1953–2002								
Mean yield spread	−2.34	−0.87	−0.75	0.33	−0.15	−0.21	2.99	2.03
Sharpe ratio	−0.36	−0.13	−0.11	0.04	−0.02	−0.03	0.37	0.16
1971–2002								
Mean yield spread	−2.99	−0.01	−0.83	1.14	−0.69	−0.00	3.94	1.48
Sharpe ratio	−0.38	−0.00	−0.10	0.11	−0.07	−0.00	0.39	0.10

Shown are the mean of the real excess returns (in percentage points) and the Sharpe ratio for a US investor, reporting annual returns for annually rebalanced portfolios. Portfolios are constructed by sorting currencies into eight groups at time t based on the nominal interest rate differential at the end of period $t-1$. Portfolio 1 contains currencies with the lowest interest rates. Portfolio 8 contains currencies with the highest interest rates. Table reproduced from Lustig & Verdelhan (2007, table 1).

countries over the full 1953–2002 sample as well as the post–Bretton Woods period of 1971–2002. Excess returns to the eight portfolios show that the high-rate currencies provided positive excess returns, whereas the low-rate currencies provided negative excess returns over both periods (shown in **Table 19**).

If one considers investing in a currency, or playing a carry trade that is long a high interest currency and short a low interest currency, one likely has a position that has nontrivial relative consumption risk, in that returns from the trade will depend on the relative performances of the economies, which most likely are reflected in their relative growths in real consumption. The estimates of unconditional consumption betas of Lustig & Verdelhan (2007) (**Table 20**) showed that currencies with higher interest rates on average have higher consumption betas, whether betas are measured relative to nondurables or relative to durables (as suggested in Yogo 2006).

Lustig & Verdelhan (2007) also performed conditional estimates of betas, using the interest rate differential as the sole conditioning variable. Thus, when risks get larger and a portfolio's yield spreads widen, estimated consumption betas increase, and when spreads shrink, risks decrease. As the authors observed, "For every 4-percentage point reduction in the interest rate gap, the

Table 20 Estimation of factor betas for eight currency portfolios sorted on interest rates

	Low rates							**High rates**
Portfolios	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1953–2002								
Nondurables	0.11	0.76	0.26	0.18	0.63	0.26	1.10	0.09
Durables	0.24	0.49	0.64	0.89	0.55	0.70	1.30*	0.68
Market	−0.07*	−0.03	−0.01	−0.12*	−0.00	−0.01	−0.06	0.03
1971–2002								
Nondurables	0.01	0.90	0.36	0.67	0.70	0.32	1.55	−0.46
Durables	0.54	0.79	1.29*	2.03*	1.23*	1.36	2.18*	0.85
Market	−0.11*	−0.10*	−0.03	−0.17*	−0.02	−0.01	−0.08	0.05

Each column reports ordinary least squares estimates of the following time-series regression of excess returns on the factor for each portfolio j: $R^{j,e}_{t+1} = \beta^j_0 + \beta^j_1 f_t + \varepsilon^j_{t+1}$. Estimates are based on annual data: The top panel presents results for 1953–2002, and the bottom panel shows results for 1971–2002. We use eight annually rebalanced currency portfolios sorted on interest rates as test assets. The asterisk indicates significance at 5% level. We use Newey-West heteroskedasticity-consistent standard errors with an optimal number of lags. Table reproduced from Lustig & Verdelhan (2007, table 6), with numbers rounded here.

Table 21 Estimation of linear factor models: eight currency portfolios sorted on interest rates

Consumption models				
	CCAPM	**DCAPM**	**EZ-CCAPM**	**EZ-DCAPM**
Nondurables	1.71 [1.09]	1.62 [1.10]	2.50 [0.91]	2.42 [0.91]
Durables		2.56 [0.96]		2.92 [0.91]
Market			15.26 [7.80]	8.48 [7.26]
MAE	2.65	1.66	2.28	1.28
R^2	0.26	0.54	0.36	0.64
p-Value	[0.312]	[0.535]	[0.222]	[0.479]
Factor models				
	CAPM	**FF equity**	**FF bonds**	
Market	1.94 [8.44]	5.17 [8.68]		
Small minus big		9.53 [5.19]		
High minus low		−6.53 [5.97]		
Slope			3.97 [9.63]	
Default			0.66 [2.39]	
Statistics				
MAE	3.55	2.91	3.46	
R^2	0.01	0.19	0.03	
p-Value	[0.001]	[0.001]	[0.001]	

Abbreviations: CAPM, capital asset pricing model; CCAPM, consumption CAPM; DCAPM, EZ-CCAPM, Epstein-Zin CCAPM; FF, Fama-French. Shown are the Fama-MacBeth estimates of the factor prices (in percentage points) using eight annually rebalanced currency portfolios as test assets. The sample is 1971–2002 (annual data). The standard errors are reported in brackets. The factors are demeaned. The last three rows report the mean absolute pricing error (in percentage points), the R^2, and the *p*-value for an x^2 test. Table adapted from Lustig & Verdelhan (2007 table 11), with numbers rounded here.

nondurable consumption betas decrease by about 100 basis points" (Lustig & Verdelhan 2007, p. 102). **Table 21** shows the results for the estimated prices of consumption and market risks from Fama-MacBeth regressions of cross-sectional returns on conditional betas for four consumption models: what they describe as the original CCAPM (using nondurables), the durables CCAPM of Yogo (2006), and Epstein-Zin preference versions of both. Also shown are the preference versions for the Sharpe-Lintner CAPM and the Fama-French three-factor and bond factor models

All four consumption-based models pass the chi-squared test and are not rejected, but "only the models with durable consumption growth as a factor explain a large fraction of the cross-sectional variation in returns" (Verdelhan 2007, p. 108). Their benchmark model, using durables consumption measures explains 54% to 64% of the variation, depending on whether CRRA or the Epstein-Zin preferences are used. "In this subsample, the CAPM explains none of the variation, and the Fama-French factor models explain less than 18%" (Verdelhan 2007, p. 108). Thus, as Lustig & Verdelhan (2007, p. 89) stated, "Because high interest rate currencies depreciate on average when domestic consumption growth is low and low interest rate currencies appreciate under the same conditions, low interest rate currencies provide domestic investors with a hedge against domestic aggregate consumption growth risk." To provide that consumption hedge, lower returns are earned on average on the low interest rate currencies, even with negative excess returns. Also interesting would be for researchers to study the nonlinear risks, given the likelihood of substantial nonlinear risks in currency movements.

13. CONCLUSION

Another research dimension established in the late 1980s and early 1990s was based on models that exhibited time complementarity in utility for consumption, as in habit-formation models (which are backward looking, in that individuals' utilities for current consumption depends on habits formed from prior consumption levels). Yet another popular approach involves forward-looking recursive models of utility maximization (think of an anticipated future inheritance, for example). The asset pricing implications of these more general models allowed more flexibility in replicating financial and economic data and historic means, volatilities, and correlations. Perhaps not surprisingly, as they have more degrees of freedom, these models fit the data considerably better than did prior models.

The 1990s and 2000s were periods of great progress in understanding and modeling changing risks and changing risk premiums (conditional risks) in asset returns. In recessions, consumer spending was reduced toward habit levels and consumers became very risk averse (as they did not want to consume below their habit levels) just at a time when risk was often very high. Thus, risk premiums skyrocketed in big recessions because both risk aversion and risk were very high and risk premiums were the product of the two. Studying this, researchers found the clue to the value/growth puzzle: Value stocks have higher consumption risks when risk premiums are high, whereas growth stocks have higher risks when risk is low. Thus, the unconditional returns should be higher on value stocks than on growth stocks, ceteris paribus. Accordingly, value and growth stocks have different concavity or convexity, which relates closely to equilibrium returns on these investments. With this new understanding and risk estimates, several researchers explained the value and size premiums.

Along with continuing development of the implications of habit formation on utility and asset pricing, a new model of long-run risks in consumption was developed in 2004. In this model, small movements in the changes in real consumption growth are so persistent that they have really large implications for long-term consumer spending and for current asset prices, which anticipate long-term effects. This long-run risks model has spawned a great amount of research and has had a lot of empirical success.

At the present time, the long-run risks model and the habit-formation model are the two main empirical models for consumption-based asset pricing. Each model has its strengths, its weaknesses, and its truths. Most theoretical insights still fit relatively comfortably within the most general original models of asset pricing of the late 1970s. In this article, with few exceptions, we have presented the outstanding research in this area in the past four decades much as did the authors cited herein, often using their tables, graphs, and descriptions for authenticity and correctness. As there should be, substantial academic debate concerns the merits and challenges of the various models. We have not taken the time and space to review the various claims and counterclaims by the authors of those competing models, but instead refer readers to the following articles: Campbell & Cochrane (2000); Campbell (2003); Beeler & Campbell (2012); Bansal, Kiku & Yaron (2012); Mehra (2012); Ludvigson (2013); and The Economic Sciences Committee (2013) for the Nobel Prize. In addition, Lewellen & Nagel (2006) and Nagel & Singleton (2010) raised important concerns and objections to significant amounts of the important research reviewed here and should also be examined carefully, so readers more fully understand the larger issues of econometric testing of consumption-based asset pricing models.

Consumption-based asset pricing has yielded many theoretical and empirical insights in the past four decades. The financial economics literature has produced a vast body of competing economic rationales for what earlier authors characterized as puzzles and paradoxes. Again, we borrow the econometric description: We have overidentified these puzzles and paradoxes. Further research should attempt to integrate models and sort through them to determine the most important results and develop tests based on the models' different predictions.

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