Consumption-Based Asset Pricing, Part 1: Classic Theory and Tests, Measurement Issues, and Limited Participation

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Abstract

This article, Part 1 of 2, reviews the classical origins, development, and tests of consumption-based asset pricing theory, focusing mainly on the first two decades from 1976 to 1998. Starting with the original consumption capital asset pricing model (CCAPM) derivations, we review both theory and subsequent tests and provide some new applications. The consumption aggregation theorem and CCAPM are derived, and optimal consumption and portfolio strategies are discussed. The term structure of interest rates is derived from the term structures for expected growth, volatility, and inflation. Time aggregation biases in consumption betas as well as the usefulness of the "consumption-mimicking portfolio" are also derived. In addition to various empirical tests, models and tests of limited participation in asset markets as well as models of incomplete markets are presented. When certain measurement issues are taken into account, the CCAPM performs better than the original CAPM and nearly as well as the Fama-French three-factor model.

1. INTRODUCTION

Consumption-based asset pricing models have been among the leading multiperiod general equilibrium asset pricing models in financial economics research for the past 35 years. The consumption capital asset pricing model (CCAPM) was first derived in the late 1970s in successively more general models by Rubinstein (1976), Breeden & Litzenberger (1978), and Breeden (1979).[1] Although Lucas (1978) did not derive the CCAPM formula, his work on Euler equations was also helpful to many researchers in subsequent consumption-based asset pricing theory and tests. The CCAPM built on the classic single-period, market-based CAPM of Sharpe (1964) and Lintner (1965) as well as on the subsequent major work on the intertemporal CAPM (ICAPM) by Merton (1973). The CCAPM links asset pricing with macroeconomic risk.

The CCAPM states that the conditional expected excess return on any risky asset should be proportional to its conditional consumption beta. Rubinstein (1976), Breeden & Litzenberger (1978), and Breeden (1979) showed that securities with higher sensitivities of returns to movements in real consumption spending have more systematic risk and should have proportionately higher excess returns. They pay off more when consumption is high and marginal utility is low, and pay less when consumption is low and marginal utility is high, so they are worth less in price and have higher equilibrium returns. This is different from the original market-oriented CAPM insights of Sharpe (1964) and Lintner (1965), as real consumption growth is not perfectly correlated with market returns. In a multiperiod model, market wealth can be high and still have high marginal utility if the investment or income opportunity set is good, as shown by Merton (1973) and Breeden (1984).

The first two decades of CCAPM tests produced mixed results. Tests of the special case of the CCAPM under constant relative risk aversion (CRRA) by Hansen (1982), Hansen & Singleton (1983), Mehra & Prescott (1985), and Hansen & Jagannathan (1991) rejected the model. Chen, Roll & Ross (1986) found no significant consumption factor priced when in the presence of other factors, including industrial production, junk bond returns, and inflation hedges. Grossman, Melino & Shiller (1987), Wheatley (1988), and Breeden, Gibbons & Litzenberger (1989) examined measurement issues in consumption (such as time aggregation) and their biases of measures of volatility and consumption betas. Breeden, Gibbons & Litzenberger found a significant positive coefficient on consumption betas and separately found a significant positive coefficient on market betas. However, both the CCAPM and the CAPM were rejected on the basis of stronger form tests of their respective implied first-order conditions. They derived two useful results used by several subsequent authors: (*a*) biases in consumption betas due to time aggregation and how those biases are reduced with increased differencing intervals for consumption growth and (*b*) estimation of consumption betas relative to returns on a consumption mimicking portfolio (CMP). The latter accounts for measurement error in consumption expenditure data by using security returns as instrumental variables. This allows a greater number and frequency of observations, as well as a more synchronous timing of CMP observations with stock returns, than with direct use of consumption data. Thus, the use of the CMP gives more precise estimates of consumption betas.

The very strong theory in support of the CCAPM contrasted with weak early empirical support, which motivated researchers to improve both their theoretical and empirical modeling. On the

[1]The first CCAPM equations are provided by Rubinstein (1976, p. 412), Breeden & Litzenberger (1978, equations 26 and 27), and Breeden (1979, equations 19, 19′, and 35). Lucas's (1978) important paper has also been credited with CCAPM development, but the paper has no aggregation, no equations with consumption covariances or consumption betas, and no CCAPM asset pricing formula. Lucas's development of the relation of asset prices to marginal utility is similar in economic intuition to the time-state preference literature of Hirshleifer (1970, chapter 8), Rubinstein (1974, 1976, 1981), and Nielsen (1974). We view Lucas's most important contributions to be on the existence and stability of equilibrium and of an equilibrium pricing density with intuitive economic properties.

theoretical side, several authors examined utility functions with time complementarity, including habit formation, which are examined in Part 2 of this review. On the empirical side, a great deal of progress was made, many key articles of which are reviewed here.

In the early 1990s, Mankiw & Zeldes (1991) considered that many households owned no stock or none in significant amounts, a situation called limited participation. They pointed out that there is no reason that the Euler equation should hold for households who are not investing. They found that the implied estimates of relative risk aversion (RRA) were much more reasonable for households who actually owned stocks than for households who did not own stocks. Heaton & Lucas (1992, 1996) examined incomplete markets that did not permit full hedging of labor income, thus causing consumers to have more volatile consumption streams. Brav, Constantinides & Geczy (2002) studied consumer-spending data and found generally plausible estimates of RRA, given the high volatilities of individuals' consumption streams. Vissing-Jørgensen (2002) focused on estimating the elasticity of intertemporal substitution (EIS), which determines how much consumers change their expected consumption growth rate when interest rates or expected returns on assets change. She found the EIS to be quite different for stockholders than for nonstockholders, generally getting plausible estimates for those who chose to invest in stocks and bonds, trading off current consumption versus future consumption. Although Breeden's (1979) classic derivation of the CCAPM allowed for nonmarketable assets such as human capital, individuals hedged these exposures with corrective portfolios of tradable assets. Thus, the nonparticipation observations imply a type of extreme market segmentation that has strong empirical implications.

Also on the empirical side, advances were made in examining changes in conditional means, variances, and covariances and in testing conditional versions of the CAPM and CCAPM, as in Bollerslev, Engle & Woolridge (1988); Ferson & Harvey (1991); Harvey (1989b; 1991a,b); Jagannathan & Wang (1996); and Ferson & Harvey (1999). Particularly insightful papers testing the conditional CCAPM include Lettau & Ludvigson (2001a,b) (discussed in Part 2 of this review) and Jagannathan & Wang (2007). Jagannathan & Wang (2007) used recession and expansion periods identified by the National Bureau of Economic Research (NBER) as a conditioning variable and found that conditional consumption betas are excellent in describing conditional mean returns on the Fama-French portfolios.

After Grossman, Melino & Shiller (1987), Wheatley (1988), and Breeden, Gibbons & Litzenberger (1989) raised concerns about measuring consumption, Parker & Julliard (2005) showed that it is important to measure ultimate consumption betas because consumption changes are slow moving and could take 2–3 years for the full effects to be observed. Using measures of these ultimate consumption betas, they were able to explain much of the Fama & French (1992) effects for size-related portfolios and value versus growth (book/market) portfolios.

Jagannathan & Wang (2007) also showed that when consumption betas of stocks are computed using year-over-year consumption growth based on the fourth quarter, the consumption-based CAPM explains the cross section of stock returns as well as the Fama & French (1992) three-factor model. Jagannathan & Wang (2007) argued that major investment and job decisions are most often made in the fourth quarter, as investors and firms plan for the coming year, so this is when the Euler equations should fit best. For estimation of consumption betas, they followed Breeden, Gibbons & Litzenberger (1989) in using a CMP formed from the six Fama-French benchmark portfolios, using weights from an ordinary least squares (OLS) regression of consumption growth on the benchmark portfolios. This allowed them to substitute synchronous portfolio returns for (time-aggregated) real consumption growth in the empirical tests, giving more precisely estimated consumption betas.

The plan is as follows: Section 2 derives the main theoretical results for aggregation of consumption in a discrete time-state preference model, and then the CCAPM in Merton's continuous-time model. Section 3 derives the term structure of interest rates from the term structure of expected growth of consumption, the term structure of volatility, and the term structure of inflation and shows tests of this theory. Section 4 discusses early 1980s tests of the CCAPM and focuses on the equity premium puzzle of Mehra and Prescott, given its large impact on subsequent research. In response to the early empirical setbacks, Section 5 presents the Breeden, Gibbons & Litzenberger derivations for time aggregation and the CMP. Section 6 presents early estimates of cash flow betas versus consumption growth and market returns. Section 7 discusses significant research on limited participation and incomplete markets. Section 8 presents the 1990s' empirical modeling of changing conditional risks and changing risk premiums. Section 9 makes some concluding remarks.

2. REVIEW OF CCAPM THEORY

This section reviews and derives the principal theory for the CCAPM. Additional insights and derivations can be found in advanced textbooks and readings by Huang & Litzenberger (1988), Merton (1992), and Lo (2007).

2.1. Aggregation of Consumption with Heterogeneous Agents

In reality, investment counselors know that individuals are often quite different in their preferences and behavior, having different levels of risk aversion, different tax brackets, and different preferences with regard to nonlinear risks, such as those causing positive and negative skewness (i.e., tail risks). Some prefer to lever up to get high returns and are willing to accept high risks, whereas others choose to hold Treasury securities and low return/low risk combinations. Some write insurance by taking credit risks or mortgage prepayment risks, whereas others wish to purchase portfolio insurance, paying a price to limit downside risk while retaining much of the upside potential. Reactions to alternate possible consumption paths can also vary considerably: Some individuals may be comfortable taking the risk of having to reduce consumption significantly if markets fall sharply, whereas others may go to extreme lengths to smooth consumption or to protect their consumption from going below a certain subsistence threshold.

Dealing analytically with heterogeneous preferences such as these is a challenge to asset pricing theorists. Papers have been written on the so-called aggregation problem and on how to allow individuals to have heterogeneous preferences and still derive asset pricing results in terms of aggregate wealth or aggregate consumption. Yet it is surprising that many of the most well-known articles in consumption-based asset pricing simply assume either identical individuals or the existence of a representative individual (e.g., Lucas 1978, Campbell & Cochrane 1999, Bansal & Yaron 2004). This may lead students to believe that the aggregation problem is unsolvable and that we have to just assume it away. That is not true in some important cases, as shown by Merton (1973), Breeden & Litzenberger (1978), and Breeden (1979), who derive the ICAPM and the CCAPM and aggregate fully diverse preferences in the class of time-additive utility functions. It is insightful to show how market price signals coordinate optimal consumption plans such that legitimate aggregation results can be obtained.

Breeden & Litzenberger (1978, theorem 1) provide the most general aggregation theorem that we are aware of to date. We use their basic time-state preference model to derive the aggregation result. Each individual, k, chooses planned consumption, c^k_{ts}, for each time-state ts, which maximizes the expected value of a time-additive utility function, $u^k(c^k_{ts}, t)$, subject to the usual budget constraint for initial wealth, W^k_0. Individuals are assumed to agree on the subjective probabilities for states $\{\pi_{ts}\}$, which sum to 1.0 for each date. For this derivation, markets are assumed to be complete, and the Arrow (1964) security price of insurance that pays \$1.00 if and only if state s occurs at time t is ϕ_{ts}. To find the optimal contingent consumption plan, individuals maximize the Lagrangian

$$\max_{\{c_{ts}^k\}} L = u_0^k(c_0^k) + \sum_t \sum_{s \in s_t} \pi_{ts} u^k(c_{ts}^k, t) + \lambda^k \left[W_0^k - c_0^k - \sum_t \sum_s \varphi_{ts} c_{ts}^k \right]. \tag{1}$$

First-order conditions for a maximum give

$$\frac{\partial L}{\partial c_0^k} = u_0'^k - \lambda^k = 0 \Rightarrow \lambda^k = u_0'^k, \tag{2}$$

$$\frac{\partial L}{\partial c_{ts}^k} = \pi_{ts}^k u_{ts}'^k - \lambda^k \phi_{ts} = 0 \Rightarrow \phi_{ts} = \frac{\pi_{ts} u_{ts}'^k}{u_0'^k} = \text{price of \$1 in time-state } ts \tag{3}$$

$$\Rightarrow \frac{u_{ts}'^k}{u_0'^k} = \frac{\varphi_{ts}}{\pi_{ts}^k} \Rightarrow \frac{1}{u_0'^k} \begin{pmatrix} u_{t1s1}'^k \\ \vdots \\ u_{TS}'^k \end{pmatrix} = \begin{pmatrix} \frac{\phi_{t_1 s_1}}{\pi_{t_1 s_1}^k} \\ \vdots \\ \frac{\phi_{TS}}{\pi_{TS}^k} \end{pmatrix}. \tag{4}$$

High marginal utility means low consumption (for all individuals k), and low marginal utility means high consumption.

So if ordered from high to low, state price/probability ratios at the optimum are positively and monotonically related to marginal utilities in different states and negatively related to consumption across states. With homogeneous probability beliefs, the price/probability ratios computed for all are the same for each individual, and the ordering of states from high to low by price/probability ratios will also order states by optimal consumption levels from low to high. And if every individual's optimal consumption level is ordered the same across states, then we can clearly see that the total of everyone's consumption, aggregate consumption, C, is also ordered in the same way. Given that, there exists a positive, monotonic functional relationship of each individual's consumption to aggregate consumption, $c_{ts}^k = f^k(C_{ts}, t)$, where $f^{k\prime} > 0$.

Optimal responses of individuals' consumption plans to price/probability ratios across states coordinate everyone's consumption risks. Because every individual's marginal utility of consumption is the same for a given level of aggregate consumption and is the same monotonically decreasing function of aggregate consumption, an aggregate utility function that is monotonically increasing and strictly concave follows. Furthermore, if each individual's utility function has a positive third derivative (implied by decreasing absolute risk aversion), Kraus & Litzenberger (1983) have shown that the aggregate utility function would also have a positive third derivative and decreasing absolute risk aversion. The positive third derivative implies a preference for skewness, which implies, ceteris paribus, that assets having a convex relation to consumption would be preferred to those having a concave relation with consumption (for asset pricing implications for skewness, see Kraus & Litzenberger (1976), Harvey & Siddique (2000).

Understanding this consumption aggregation result in 1978 was key to Breeden's (1979) well-known derivation of the CCAPM in the very general framework of the continuous-time model of Merton (1969, 1971, 1973). Then at the University of Chicago, he brought together insights from the time-state preference models (West Coast models) developed at UC Berkeley, Stanford, and UCLA by Arrow, Debreu, Hirshleifer, Rubinstein, Litzenberger, Leland, Hakansson, Kraus, and Sharpe and the continuous-time models (East Coast models) developed at MIT, Yale, and Penn by Merton, Samuelson, Cox, Ingersoll, and Ross. Breeden reasoned that, if every individual's consumption was optimally a monotonically increasing function of aggregate consumption in a complete market, it must be the case that, even in an incomplete market, movements in aggregate consumption would locally determine movements in marginal utilities for everyone, to the extent permitted by existing securities. He showed that randomness in individuals' constrained optimal consumption in an incomplete market would be uncorrelated with all assets' returns, because if any of the fluctuations were correlated with some asset's return, individuals' consumption changes would then not be maximally correlated with aggregate consumption and the consumption plan was not optimal.

Substituting the fact that each individual's real consumption is a monotonic function of aggregate real consumption, we see that state prices depend only on their probabilities, the level of real aggregate consumption in the state, and the level of aggregate consumption today:

$$\Rightarrow \phi_{ts} = \frac{\pi_{ts} u'_t(C_{ts})}{u'_0(C_0)} \quad \text{and} \quad \frac{\phi_{ts}}{\pi_{ts}} = \frac{u'_t(C_{ts})}{u'_0(C_0)}. \tag{5}$$

These state prices for aggregate consumption claims can be used to value any security's time-state contingent payoffs at time t in terms of its joint probability distribution with aggregate real consumption at that date, which gives consumption-based asset pricing for all assets.

If the cash flows to a security at different future dates and states are the set $\{X_{ts}\}$, then those cash flows can be replicated by purchasing Arrow securities and, to avoid arbitrage, must have a present value, V_0, that is the cost of the replicating portfolio, which is (substituting Equation 5)

$$V_0\{X_{ts}\} = \sum_t \sum_{s \in S(t)} \phi_{ts} X_{ts} = \sum_t \frac{E_0\left[\widetilde{X}_t u'(\widetilde{C}_t, t)\right]}{u'(C_0, t_0)}. \tag{6}$$

Dividing by the initial price to put the payoffs in return form, we get the Euler equation forms

$$1 = \sum_t \frac{E_0\left[\widetilde{R}_t u'(\widetilde{C}_t, t)\right]}{u'(C_0, t_0)} \quad \text{for every asset} \tag{7}$$

$$\text{and} \quad 0 = \sum_t \frac{E_0\left[(\widetilde{R_{i,t}} - \widetilde{R_{j,t}}) u'(\widetilde{C}_t, t)\right]}{u'(C_0, t_0)} \quad \text{for any two assets } i \text{ and } j. \tag{8}$$

These Euler equations are often tested by econometricians, following Hansen & Singleton (1983). While the Euler conditions have frequently been tested assuming a representative investor with CRRA, the above analysis is consistent with any monotonically increasing, strictly concave aggregate utility function that is based on diverse individual preferences and endowments. For example, an aggregate utility function displaying decreasing RRA is consistent with the above equations and would imply risk premiums that increase in economic contractions and decrease in economic expansions. The parameters of the aggregate utility function should be estimated by the econometrician, rather than restricted a priori without any theoretical justification.

2.2. Intertemporal and CCAPM in Continuous Time with Incomplete Markets

After Sharpe (1964) and Lintner's (1965) development of the single-period CAPM, Merton (1969), Samuelson (1969), Fama (1970b), and Hakansson (1970) recognized that multiperiod optimal portfolios are different from prescriptions of single-period models because individuals' indirect utility functions for wealth depend on the investment and income opportunity sets (except for the special case of log utility). In his continuous-time model, Merton (1969, 1971, 1973) developed the most elegant solution to this problem. He first derived optimal consumption and portfolio rules, finding additional risk elements that make individuals' optimal portfolios differ from just holding the market portfolio, because individuals desire to hedge or reverse hedge against changes in the investment and/or job market opportunity set. These new risks are priced in a multibeta ICAPM, extending the Sharpe-Lintner model quite significantly. Merton's model was a groundbreaking contribution because of three key elements of generality that researchers in financial economics view as quite important and attractive:

1. Individuals were allowed to be fully diverse in preferences, within the class of time-additive utility functions, which were the common assumption at the time. So, 1 billion individuals could have 1 billion different preference functions (and quite general and changeable risk aversions) for consumption. Note that nothing prevented RRAs of individuals and the market risk premium to get very large as consumption fell to low levels, for example, as in more recent models with external habit formation.
2. Asset prices and consumption levels were allowed to follow very general diffusion processes, with conditionally changing drifts, volatilities, and correlations, which can generate tremendously different probability distributions over discrete intervals (displaying nonnormality, nonlognormality, and optionlike features, for example). Thus, Merton's (1973, p. 872) ICAPM and Breeden's (1979, p. 268) subsequent CCAPM were done in terms of conditionally expected returns and conditional consumption betas (a point that does not appear to be appreciated by some subsequent authors).
3. Markets were not assumed to be complete, in the sense that Merton did not assume that there were assets or portfolios that would replicate the behavior of any or all economically important state variables. Thus, the allocation that was achieved was not necessarily Pareto-optimal, or efficient, but was merely the best that could be attained with existing markets. Of course, the complete-markets case is a special case of his model, so Merton's (1973) and Breeden's (1979) continuous-time models apply to both complete and incomplete markets. Models by Breeden (1984), Duffie & Huang (1985), and Cox & Huang (1989) show how dynamic portfolio policies help investors achieve more complete market allocations from a smaller number of securities.

Merton's (1973) ICAPM showed that, in equilibrium, the vector of instantaneous expected excess returns on risky assets, $\mu - \mathbf{r}$, is equal to the matrix of betas relative to the market portfolio and relative to the S state variables for the investment opportunity set, multiplied by the vector of risk premia for each of the $S + 1$ risks:

$$\boldsymbol{\mu}_{\mathbf{a}} - \mathbf{r} = \boldsymbol{\beta}_{\mathbf{a},\mathbf{Ms}} \begin{pmatrix} \mu_M - r \\ \boldsymbol{\mu}_{\mathbf{s}}^{*} - \mathbf{r} \end{pmatrix} \quad \text{(ICAPM)}. \tag{9}$$

Merton's ICAPM also has empirical implications that are similar to those of Ross's (1976) arbitrage pricing theory (APT). However, Ross uses statistical assumptions and merely the absence of arbitrage to derive his APT, whereas Merton uses arguably weaker statistical assumptions and stronger economic assumptions to derive the ICAPM. Testing of Merton's ICAPM and Ross's APT was inhibited by the fact that there were an unspecified number of opportunity set state variables (or factors) and that it was unclear whether their corresponding risk premiums would be positive or negative for different state variables. Many empirical articles focused on the weak prediction of these theories, i.e., the existence of several price factors, rather than predictions of which factors should be priced on the basis of the a priori theory. Indeed, these models seemed to give those who tried to apply the model too much freedom to data mine in choosing state variables that had "statistically significant" risk premiums.

Breeden's (1979) article derived the CCAPM in the exact same, very general intertemporal model that Merton used, showing that Merton's $S + 1$ betas and risk premia could be replaced with a single beta relative to consumption and that risk premia for the state variables should all be proportional to their consumption betas. An outline of his derivation follows: Per Merton (1973), at each instant, each individual k chooses dynamically an optimal consumption rate, c^k, and an optimal $A \times 1$ vector of risky asset portfolio weights, $\mathbf{w}^k$, where the residual is invested in the riskless asset, $w_0 = 1 - \sum_j w_j^k$. An optimal policy at every instant maximizes the sum of instantaneous utility of current consumption plus the expected change in remaining expected utility of lifetime consumption, $J^k(W^k, \mathbf{s}, t)$, where $\mathbf{s}$ is an $S \times 1$ vector of state variables that describe the investment and income opportunity sets. The Bellman equation says that the following maximum must be 0 or else the dynamic plan is not optimal:

$$0 = \max_{\{c^k, \mathbf{w}^k\}} u^k(c^k, t) + E_t \text{“}\left[\frac{dJ^k}{dt}\right]\text{”}. \tag{10}$$

Merton then invokes Ito's Lemma to compute the expected change in $J(W, \mathbf{s}, t)$:

$$0 = \max_{\{c^k, \mathbf{w}^k\}} \left\{ \begin{array}{l} u^k(c^k, t) + \begin{pmatrix} J_W^k & \mathbf{J}_\mathbf{s}^k & J_t^k \end{pmatrix} \begin{pmatrix} W^k[\mathbf{w}^{k'}(\boldsymbol{\mu}_a - \mathbf{r}) + r] + y^k - c^k \\ \boldsymbol{\mu}_\mathbf{s} \\ 1 \end{pmatrix} \\ + \dfrac{1}{2}\left[\begin{pmatrix} J_{WW}^k & \mathbf{J}_{W\mathbf{s}}^k \\ \mathbf{J}_{\mathbf{s}W}^k & \mathbf{J}_{\mathbf{ss}}^k \end{pmatrix} \square \begin{pmatrix} (W^k)^2 \mathbf{w}^{k'} \mathbf{V}_{aa} \mathbf{w}^k & W^k \mathbf{w}^{k'} \mathbf{V}_{a\mathbf{s}} \\ \mathbf{V}_{\mathbf{s}a} \mathbf{w}^k W^k & \mathbf{V}_{\mathbf{ss}} \end{pmatrix} \right] \end{array} \right\}, \tag{11}$$

where $\mathbf{V}_{aa} = A \times A$ covariance matrix of risky asset returns, $\mathbf{V}_{a\mathbf{s}} = A \times S$ covariance matrix with state variables, and $\mathbf{V}_{\mathbf{ss}} = S \times S$ covariance matrix for state variables. Mean vectors are $\boldsymbol{\mu}_a$ and $\boldsymbol{\mu}_s$, respectively. Subscripts on the J function indicate first and second partial derivatives. Setting derivatives of control variables equal to 0 and solving for the optimal portfolio and consumption gives

$$\mathbf{w}^k W^k = T^k \mathbf{V}_{aa}^{-1}(\boldsymbol{\mu} - \mathbf{r}_f) + \mathbf{V}_{aa}^{-1} \mathbf{V}_{a\mathbf{s}} \mathbf{H}_\mathbf{s}^k \tag{12}$$

and

$$u_c^k(c^k, t) = J_W^k(W^k, \mathbf{s}, t) \quad \text{(envelope condition)}, \tag{13}$$

where $T^k = -J_W^k / J_{WW}^k$ is k's risk tolerance for wealth and $\mathbf{H}_\mathbf{s}^k = -\mathbf{J}_{\mathbf{s}W}^k / J_{WW}^k$. $\mathbf{H}_\mathbf{s}^k$ is equal to individual K's $s \times 1$ vector of hedging or reverse hedging demands for the s portfolios that best hedge against changes in the investment and income opportunity set vector, $\mathbf{s}$.

The envelope condition shows that the marginal utility of \$1 consumed must be equal to the marginal utility of \$1 invested. This means that in individual portfolio equilibrium an individual's indirect marginal utility of wealth, which depends on the investment opportunity set as well as the individual's wealth, would equal the marginal utility of her consumption, which depends only on her consumption. Differentiating the envelope condition with respect to wealth W^k and state variables and then substituting into $\mathbf{H}_s^k$ gives

$$u_{cc}^k c_W^k = J_{WW}^k \quad \text{and} \quad u_{cc}^k \mathbf{c}_\mathbf{s}^k = \mathbf{J}_{\mathbf{s}W}^k. \tag{14}$$

So

$$T^k = -\frac{J_W^k}{J_{WW}^k} = -\frac{u_c^k}{u_{cc}^k c_W^k} \quad \text{and} \quad \mathbf{H}_\mathbf{s}^k = -\frac{\mathbf{J}_{\mathbf{s}W}^k}{J_{WW}^k} = -\frac{\mathbf{c}_\mathbf{s}^\mathrm{k}}{c_W^k}. \tag{15}$$

To gain insight into the optimal dynamic portfolios and consumption plans, assume the special case where individual investors have CRRA, which differs across individuals. Let $\boldsymbol{\gamma}_\mathbf{s}^k$ be the vector of percentage compensating variations in wealth that would hold lifetime utility constant for state variables' changes. Breeden (1984) showed that

$$\mathbf{H}_\mathbf{s}^k = W^k(1 - T^{*k})\boldsymbol{\gamma}_\mathbf{s}^k \tag{16}$$

and that optimal consumption sensitivities to state variables are

$$\mathbf{c}_{\mathbf{s}}^{k} = -c_{W}^{k} W^{k}(1 - T^{*k})\boldsymbol{\gamma}_{\mathbf{s}}^{k}. \tag{17}$$

Many authors have estimated typical RRA to be much greater than 1.0, so that the inverse of RRA, relative risk tolerance T^{*k}, is much smaller than 1.0. Therefore, for most people we assume that $(1 - T^{*k})$ is positive.

If a state variable j is a good one (in that higher s_j means higher expected lifetime utility), its compensating variation in wealth, γ_{s_j}, will be negative. Equation 16 shows that with normal RRA, H_s^k is then negative, and the individual holds assets that hedge against adverse changes in the opportunity set by giving higher wealth when the good state variables decline. We see from Equation 17 that current consumption will increase when s_j increases, smoothing lifetime consumption, given normal RRA, as shown in Grauer & Litzenberger (1979) and Breeden (2004).

In contrast, denote a speculator as an individual who has a much higher tolerance for risk, with $T^{*k} > 1$. It follows from Equation 17 that such a speculator would reduce current consumption to invest more when investment opportunities improve. The speculator would also invest in assets that give higher wealth when opportunities are good and lower wealth when opportunities are poor. Such consumption and portfolio strategies give the speculator a higher lifetime mean consumption stream, but with much higher volatility (for empirical analyses of consumption's relation to permanent income, see Hall 1979, Flavin 1981, Shiller 1982).

Substituting Equations 14 and 15 into Equation 12 and premultiplying by $c_W^k \mathbf{V}_{aa}$ and rearranging gives

$$\mathbf{V}_{aW^k} c_W^k + \mathbf{V}_{a\mathbf{s}} \mathbf{c}_{\mathbf{s}}^k = T_c^k(\boldsymbol{\mu}_a - \mathbf{r}). \tag{18}$$

Using Ito's Lemma for $c^k(W^k, \ \mathbf{s}, \ t)$, the stochastic part of c^k is $\widetilde{dc^k} = c_W^k \widetilde{dW^k} + \mathbf{c}_{\mathbf{s}}^{k\prime} \widetilde{d}\mathbf{s}$. Thus,

$$\Rightarrow \text{Vector of covariances:} \quad \mathbf{V}_{\mathbf{a}c^k} = \mathbf{V}_{\mathbf{a}W^k} c_W^k + \mathbf{V}_{\mathbf{as}} \mathbf{c}_{\mathbf{s}}^k. \tag{19}$$

Substituting Equation 19 into Equation 18 gives

$$\mathbf{V}_{ac^k} = T_c^k(\boldsymbol{\mu}_a - \mathbf{r}). \tag{20}$$

Accordingly, for each individual, assets are held in the portfolio in proportions that result in an optimal consumption rate that covaries with each asset in proportion to its expected excess return. Individuals do not influence $\boldsymbol{\mu}_a - \mathbf{r}$. They control W^k and, thus, $c^k(W^k, \ \mathbf{s}, \ t)$:

$$\text{The aggregate consumption rate} \quad \tilde{C} = \sum_k \widetilde{c^k} \Rightarrow \sum_k \mathbf{V}_{ac^k} = \mathbf{V}_{aC} = \left(\sum_k T_c^k\right)(\boldsymbol{\mu}_a - \mathbf{r}), \tag{21}$$

$$\text{Dividing by } C: \quad \boldsymbol{\mu}_a - \mathbf{r} = (T_c^{*M})^{-1}\mathbf{V}_{a,\ln C}, \quad \text{where} \quad T_c^{*M} = \frac{\sum_k T_c^k}{C} = \frac{T_c^M}{C}, \tag{22}$$

$$\text{For any portfolio } M: \quad \boldsymbol{\mu}_M - r = (T_c^{*M})^{-1}\sigma_{\mathbf{M,lnC}} \Rightarrow (T_c^{*M})^{-1} = \frac{\mu_M - r}{\sigma_{\mathbf{M,lnC}}}, \quad \forall M. \tag{23}$$

Substituting Equation 23 into Equation 22 gives the CCAPM:

$$\boldsymbol{\mu}_a - \mathbf{r} = \frac{\boldsymbol{\beta}_{\mathbf{a,lnC}}}{\beta_{\mathbf{M,lnC}}}[\mu_M - r] \, (\text{CCAPM}). \tag{24}$$

This shows that the $A \times 1$ vector of (conditional) expected excess returns on risky assets in equilibrium is proportional to the $A \times 1$ vector of the betas with respect to percentage changes in real aggregate consumption. Merton's $S + 1$ betas have been summarized in one consumption beta. Note that the CCAPM of Equation 22 is identical in form to the original Sharpe-Lintner CAPM, but with their vector of market betas replaced by a vector of relative consumption betas, which for asset j is its consumption beta divided by the market portfolio's consumption beta.

An interesting issue is how the CCAPM replicates the risk premia from Merton's ICAPM, both for World H, where there is a predominance of hedgers, and World RH, where there is a predominance of speculators (or reverse hedgers). For World H, the world is dominated by people who want to short sell assets positively related to a good state variable, driving down their prices and giving a positive risk premium. In that world of hedgers, consumption increases with increases in a good state variable, so the consumption beta for the state variable is positive, causing the CCAPM to also give a positive risk premium.

In contrast, if the world is RH and is dominated by speculators (reverse hedgers), investors will want to go long assets correlated with a good state variable, pushing up their prices and giving a negative risk premium in the ICAPM. However, in this world, aggregate consumption is reduced when the good state variable is high (so as to invest more with good opportunities), which gives a negative consumption beta for the good asset and a negative risk premium according to the CCAPM, duplicating that of the ICAPM.

Thus, in worlds dominated both by hedgers and by speculators (reverse hedgers), the CCAPM properly identifies the same risk premium as Merton's ICAPM. Ultimately, to sign and estimate the risk for a given state variable requires only an estimate of its consumption beta.

The CCAPM was extended to the global economy by Stulz (1981), who proved that the real expected excess return on a risky asset is proportional to the covariance of its return with changes in the world consumption rate. Additionally, as Backus & Smith (1993) proved, if there are no nontraded goods and markets are effectively complete, consumption in every country is optimally monotonically related to consumption in every other country, an extension of the aggregation result of Breeden & Litzenberger (1978). But as Stulz (1981) proved, if there are nontraded goods and consumption opportunity sets differ across countries, changes in real consumption rates will not generally be perfectly correlated across countries. Brandt, Cochrane & Santa-Clara (2006) showed that real consumption growth correlations are statistically significant among the major economies of the United States, the United Kingdom, Germany, and Japan, but far from 1.00. With quarterly data, US consumption growth correlations are 0.31, 0.17, and 0.27 versus the United Kingdom, Germany, and Japan, respectively, and with annual changes, correlations are higher at 0.42, 0.24, and 0.35.

3. TERM STRUCTURE OF INTEREST RATES, CONSUMPTION GROWTH, VOLATILITY, AND INFLATION

Section 2 focuses on the derivation of the CCAPM, which provides equilibrium expected returns for risky assets in terms of their return sensitivities to aggregate real consumption. In this section, we examine the pricing of riskless bonds and the term structure of interest rates and the relation of the term structure of rates to the term structure of expected consumption growth and the term structure of volatility for consumption growth. Although the general term structure results for the economies presented in the prior section are in Breeden (1986), those results end up being Taylor series approximations to the following simple model with CRRA (power utility) and lognormally distributed consumption, the CRRA-lognormal model (see Breeden 1977, chapter 7). To use less space, while gaining greater understanding, riskless bond prices and implicit annualized interest rates are derived in the simple CRRA-lognormal model, with identical powers for all individuals. This combination of identical CRRA and lognormality allows the computation of closed-form solutions for bond prices and interest rates. Garman (1977); Sundaresan (1984); Cox, Ingersoll

& Ross (1985a); Dunn & Singleton (1986); Campbell (1987); Campbell & Mankiw (1989); and Wachter (2006) present more complex models of the term structure.

We assume that the representative individual has the following power utility function, where γ is the CRRA for the individual. γ is also the inverse of the EIS:

$$\text{Let} \quad u^k(c^k, t) = \frac{e^{-\rho t}(c_t^k)^{1-\gamma}}{1-\gamma}, \tag{25}$$

$$\Rightarrow \text{RRA} = -\frac{u''}{u'}c_t = -\frac{-\gamma e^{-\rho t} c_t^{-\gamma-1}}{e^{-\rho t} c_t^{-\gamma}} c_t = \gamma. \tag{26}$$

From the time-state preference valuation model, Equation 6, where the cash flows are \$1.00 received for sure at T, we have the riskless zero coupon bond price at time t equal to expected marginal utility at time T divided by marginal utility at the present time, t. Given our aggregation result, this depends generally only on the utility function and the distribution of aggregate consumption at T and t. With the power utility function, this simplifies to a dependence only on the probability distribution of the percentage growth rate of aggregate consumption from t to T.

$$B_{t,T} = \frac{E_t[u'_T(c_T, T)]}{u'_t(c_t, T)} \quad \text{in general.} \tag{27}$$

$$B_{t,T} = \frac{E_t[e^{-\rho T} c_T^{-\gamma}]}{e^{-\rho t} c_t^{-\gamma}} = E_t\left[e^{-\rho(T-t)}\left(\frac{\widetilde{c_T}}{c_t}\right)^{-\gamma}\right] \quad \text{CRRA.} \tag{28}$$

Next we assume that aggregate per capita consumption is lognormally distributed, where the log's mean grows at a continuously compounded annual rate of $\mu^*_{t,T}$ and variance grows proportionally to time with annualized volatility of $\sigma_{t,T}$, i.e.,

$$c_t \sim \text{lognormal}[\ln c_0 + \mu^*_{t,T}(T-t), \sigma^{*2}_{t,T}(T-t)]. \tag{29}$$

Then note that for lognormals if $y \equiv e^x$, and x is normal with $E(x) = \mu, \text{var}(x) = \sigma^2$, then $E(y) = e^{\mu + \frac{1}{2}\sigma^2}$. For our lognormal consumption growth, we have

$$\frac{\widetilde{c_T}}{c_t} \equiv e^{\tilde{g}_{t,T}} \quad \text{has} \quad g_{t,T} \sim N(\mu_{t,T}, \sigma^2_{t,T}), \tag{30}$$

$$\Rightarrow \left(\frac{\widetilde{c_T}}{c_t}\right)^{-\gamma} = e^{-\gamma \tilde{g}_{t,T}} \quad \text{has} \quad -\gamma g_{t,T} \sim N(-\gamma\mu_{t,T}, \gamma^2\sigma^2_{t,T}),$$

$$\Rightarrow E_t\left[\left(\frac{\widetilde{c_T}}{c_t}\right)^{-\gamma}\right] = e^{-\gamma\mu_{t,T} + \frac{1}{2}\gamma^2\sigma^2_{t,T}}, \text{ which gives } \quad B_{t,T} = e^{-\rho(T-t) - \gamma\mu_{t,T} + \frac{1}{2}\gamma^2\sigma^2_{t,T}}, \tag{31}$$

$$\Rightarrow \begin{pmatrix} r_{t,1} \\ r_{t,2} \\ \vdots \\ r_{t,T} \end{pmatrix} = \rho \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} + \gamma \begin{pmatrix} \mu^*_{t,1} \\ \mu^*_{t,2} \\ \vdots \\ \mu^*_{t,T} \end{pmatrix} - \frac{1}{2}\gamma^2 \begin{pmatrix} \sigma^{*2}_{t,1} \\ \sigma^{*2}_{t,2} \\ \vdots \\ \sigma^{*2}_{t,T} \end{pmatrix}. \tag{32}$$

Applying Equation 32 for different dates to plot the term structure of interest rates, we see that the term structure of interest rates reflects the term structure of expected growth rates for consumption over different time horizons and the term structure of volatility of consumption growth over those same horizons.

Breeden (1986) examined the term structure in an economy (with individuals who have time-additive utility functions) with a multigood model and derived corresponding term structure results in a world with inflation and deflation. He derived the nominal term structure of interest rates to have real terms as above, but now the term structure of inflation is added, along with a risk premium or risk deduction for the consumption risk of inflation that is imbedded in returns of nominally riskless bonds. The equation derived is

$$r - \frac{\mu_I}{I} + \sigma^2_{\ln I} = \rho + \gamma \mu_{\ln e^*} - \frac{\gamma^2}{2} \sigma^2_{\ln e^*} + \gamma \sigma_{-\ln I, \ln e^*}. \tag{33}$$

Note that in a model with external habit formation presented in Part 2 of this review, Campbell & Cochrane (1999, equation 8) found a corresponding equation for riskless rates to be

$$r_t^f = -\ln(\delta) + \gamma g - \gamma(1-\phi)(s_t - \bar{s}) - \frac{\gamma^2 \sigma^2}{2} [1 + \lambda(s_t)]^2. \tag{34}$$

In both models, we see the classic positive relation of the real rate to the expected real consumption growth, g, as well as the negative relation to volatility of consumption, thus reflecting rational "flight to quality" responses of consumers. Additionally, in the external habit-formation model, the riskless rate is affected by where surplus consumption is relative to its long-term mean and the speed of adjustment parameter ϕ.

Harvey (1988, 1989a, 1991) empirically tested whether the slope of the term structure of interest rates actually forecasted expected real growth of the whole economy (given that consumption is 70% of GDP and is highly correlated). Both Breeden and Harvey observed that late in the economic cycle near an economic peak, when growth is expected to slow considerably and possibly enter a recession, the term structure should be negatively sloped, with lower real rates on longer maturities reflecting slower longer-term growth. Correspondingly, they argued that near the bottom of a recession, when consumers and investors usually expect that "things will likely get better over the longer term," longer-term growth forecasts should be much higher than shorter-term growth and the term structure should be strongly upward sloping (see **Figures 1** and **2**).

Harvey demonstrated that the slope of the yield curve (either the 5-year or 10-year Treasury minus the 3-month yield) had significant predictive ability for the subsequent four quarters of GNP growth, in both in-sample and out-of-sample forecasts. Graphically, Harvey shows that the term structure is quite helpful in explaining GNP growth (see **Figure 3**).

Harvey (1989a) also showed that this simple one-variable predictor had root-mean-squared forecast errors that were as low as those of most of the top professional forecasters over the periods examined, as shown in **Table 1**. Harvey (1991a,b) also demonstrated that the relationship of the slope of the term structure to subsequent economic growth is true both for the United States and for several other G-7 countries (**Table 2**).

Estrella & Hardouvelis (1991) published results quite similar to Harvey's, but with some additional tests. As shown in **Figure 4**, they estimated recession probabilities based on the slope of the term structure four quarters earlier. The correlation is quite striking.

Estrella & Hardouvelis (1991) also looked more broadly at the ability of the term structure slope to forecast the components of GDP—consumption, investment, and government spending. They found statistically significant predictability for approximately 2 years forward for consumption, both total and durables, and also for investment. However, the term structure slope does not have any explanatory power for government spending, which makes sense given periodic countercyclical fiscal policy.

In 1996, after the empirical work of Harvey (1991) and Estrella & Hardouvelis (1991), the slope of the term structure was added as a predictor variable in the Conference Board's Leading Economic Indicators series. In a subsequent study, Dotsey (1998) of the Federal Reserve Bank of Richmond found that a negative term structure slope gave 18 correct signals and 2 incorrect

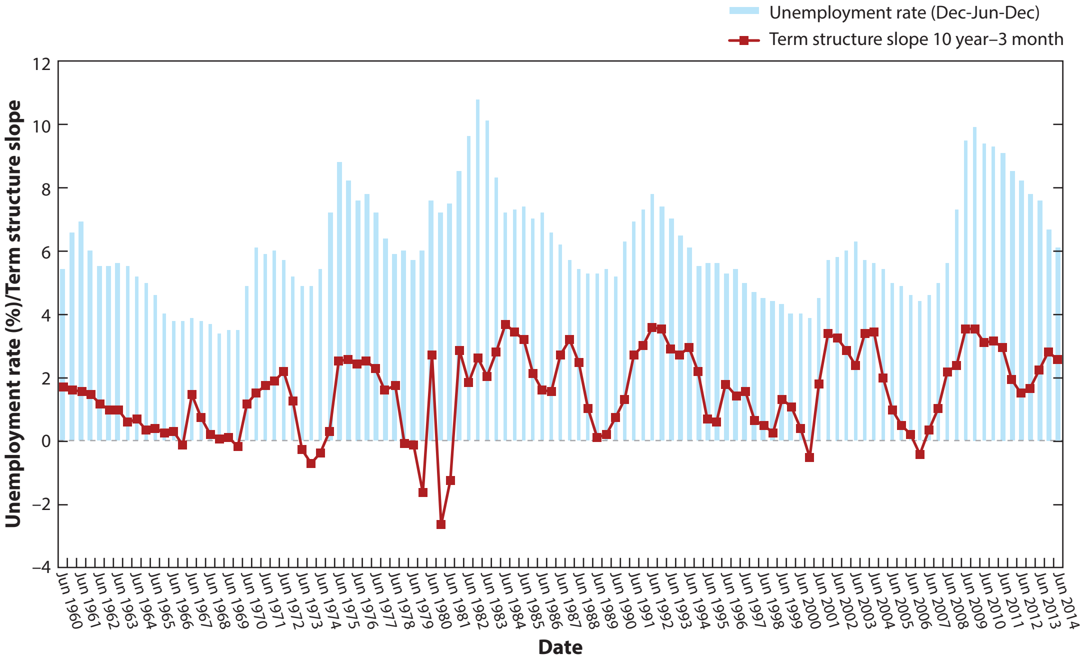

Figure 1

Term structure slope (10 year–3 month) and unemployment rate with semiannual data from 1960 to 2014Q2. Data indicate that upward-sloping term structures are the norm, as the spread between 10-year yields and 3-month Treasury yields is normally positive. With a negative slope, recession usually follows. The yield curve slope was near zero or negative in 1970, 1974, 1980, 1981, 1989, 2000–2001, and 2006–2007. As also shown for each of these periods, the unemployment rate subsequently surged.

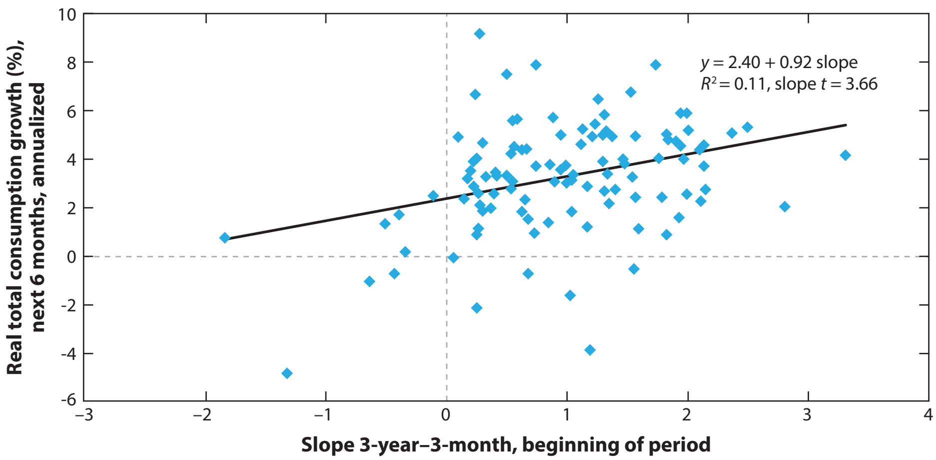

Figure 2

Scatter plot and regression results showing that the 3 year–3 month Treasury term structure slope was positively related to subsequent (next 6 months, annualized) real consumption growth in the 1959–2013 period, with a t-statistic of 3.6, indicating a significant relationship. A high slope leads to high growth, and a negative slope leads to recessions. This was also true in subperiods. Although a straight line fit is shown, the relationship has intriguing nonlinearity, worthy of further study.

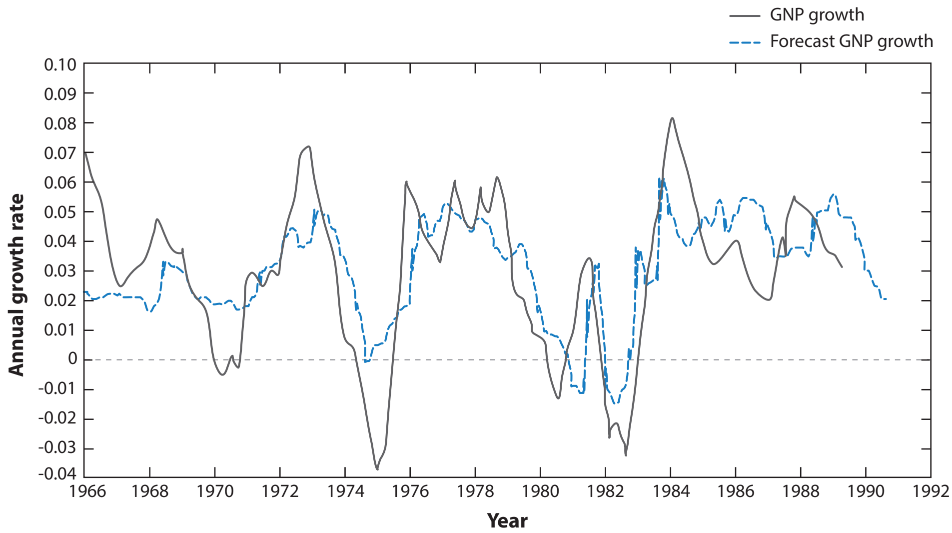

Figure 3

Annual GNP growth and forecast GNP growth. Figure adapted from Harvey (1989a).

signals of recession in the 1955–1995 time period. Thus, the slope of the term structure of interest rates is a closely watched barometer of the likely future growth of the economy.

Harvey's tests focused on the theoretically positive relation between real rates and expected future growth; however, the term structure of volatility is also a factor in the term structure equation (Equation 32). An upward-sloping term structure of volatility would help explain a downward-sloping term structure (not necessarily related to a decline in growth), a condition that occurred in 2005–2006. A downward-sloping term structure of volatility (as in late 2008 and early 2009,

Table 1 Yield spread model forecasting performance versus other econometric models from 1976Q1 to 1985Q1

Model	Mean absolute error (%)	Root-mean-squared error (%)
5-year spread	1.7	2.1
10-year spread	1.7	2.1
One-quarter stock return	2.4	3.0
Four-quarter stock return	2.5	3.0
BEA	1.7	2.4
BMARK	2.1	2.5
Chase	2.0	2.4
DRI	1.6	2.1
EFP	1.7	2.1
RSQE	1.7	2.1
WEFA	1.5	2.1

Abbreviations: BEA, Bureau of Economic Analysis; BMARK, benchmark forecast from Charles R. Nelson Associates; Chase, Chase Econometric Associates; DRI, Data Resources; EFP, Econometric Forecasting Project at Georgia State University; RSQE, Research Seminar on Quantitative Economics at the University of Michigan; WEFA, Wharton Econometric Forecasting Associates. Data reproduced from Harvey (1989b, table 3).

Table 2 Predicting global economic growth with the local term structures from 1970Q1 to 1989Q4, with 76 observations

Country	*a*	*b*	R^2
Canada	0.03 [6.22]	1.11 [4.53]	0.48
France	0.03 [6.11]	0.52 [2.14]	0.13
Germany	0.01 [2.60]	0.75 [4.50]	0.29
Italy	0.04 [7.19]	0.71 [5.15]	0.26
Japan	0.04 [10.58]	0.23 [1.38]	0.01
United Kingdom	0.02 [4.20]	0.42 [1.71]	0.08
United States	0.02 [3.20]	1.27 [5.71]	0.47
World	0.02 [5.66]	1.42 [6.8]	0.54

Regression is real GDP growth$_{t+1,t+5} = a + b$ [Slope$_t$], and t-statistics are in brackets. Data reproduced from Harvey (1991).

at the depths of the Great Recession) can explain a positive term structure slope at a time when growth may not have been projected to significantly increase quickly. **Figure 5** displays the slope of the term structure of volatility from 2005 to 2013, including during the Great Recession, using Black-Scholes implied volatilities from option price data for 3-month and 2-year at-the-money options on the S&P 500 stock price index. The slope of the term structure of volatility swung from being sharply positive in 2005–2006 (when volatilities were historically low) to being sharply negative in 2008–2009, when stock market volatility was very high. Accounting for these moves in the term structure of volatility would reduce the forecast errors made by the slope of the term structure during those periods. Combining analysis of changes in the term structure of volatility and the term structure of inflation with changes in the term structure of expected real growth is a promising area for future research on the term structure of nominal interest rates.

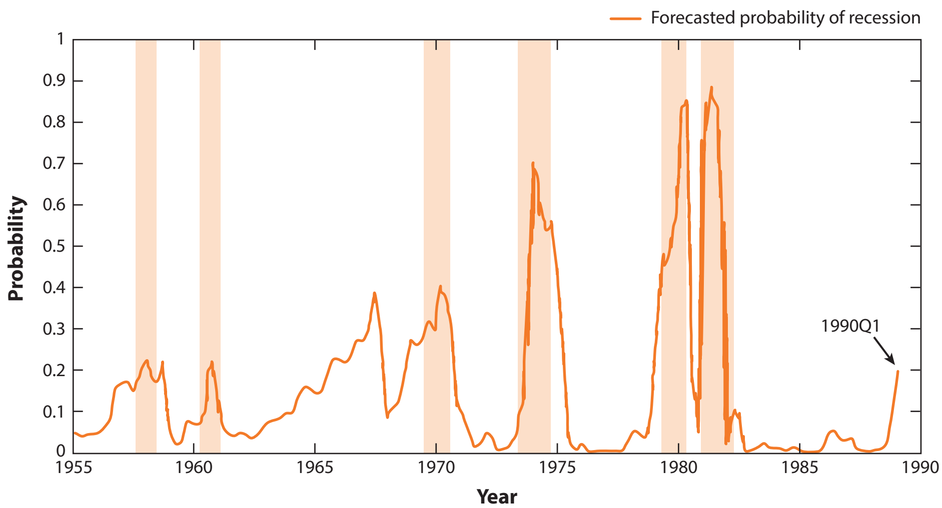

Figure 4

Forecasted probability of recession for current quarter based on the slope of the yield curve four quarters earlier. Shaded areas indicate recessions. Figure adapted from Estrella & Hardouvelis (1991, figure 2).

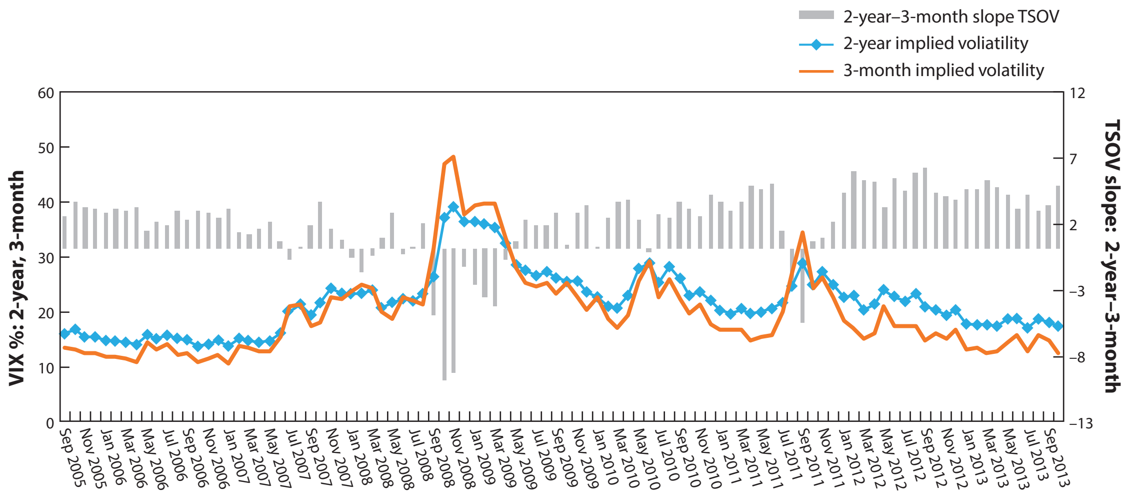

Figure 5

Term structure of volatility (TSOV) from S&P 500 options: 2-year and 3-month implied volatility and slope. Abbreviation: VIX, Volatility Index.

4. EARLY 1980s TESTS OF CCAPM: THE EQUITY PREMIUM PUZZLE

In this section, we review some of the early tests of consumption-based asset pricing and the CCAPM. The first two decades of CCAPM tests produced mixed relative results (versus the original market-oriented CAPM) and mainly negative absolute results, with initial strong model rejections in tests of the CCAPM Euler conditions based on a representative individual with CRRA. Most prominent were Hansen & Singleton (1983) for multiple assets and Mehra & Prescott (1985) for the S&P 500. The examination of prices of economic factors by Chen, Roll & Ross (1986) was also interpreted as rejecting the CCAPM. Grossman, Melino & Shiller (1987), Breeden, Gibbons & Litzenberger (1989), and Wheatley (1988) examined measurement issues in consumption (such as time aggregation) and their biases on measures of volatility and consumption betas. Breeden, Gibbons & Litzenberger found that the risk premium on consumption betas (based on the CMP) was significantly positive, as was that for the risk premium on market index betas, but mean-variance portfolio efficiencies of both the CMP and the stock market index portfolio were rejected. In subsequent years, empirical tests have become more sophisticated and have yielded much more positive results than did these earliest tests.

4.1. Commodity Futures: Consumption Betas Versus Market Betas

In one of the first applications of CCAPM insights, Breeden (1980) compared unconditional consumption and market beta estimates for commodity futures returns. Dusak's (1973) well-known study found S&P 500 betas of commodities to be near zero, which, on the basis of the CAPM, predicted that risk premiums of commodity futures should be near zero and futures prices should be unbiased estimates of future spot prices. Owing to different income elasticities of demand and relatively fixed supplies in the short run for commodities such as beef, pork, and chicken, Breeden argued that these commodities should have positive consumption betas. In his model, positive elasticities of supply with respect to price dampen the consumption betas for more distant maturity commodity futures contracts. His statistical estimates support these predictions. The near contracts of meats had an average consumption beta of 10.9 (average t-statistic of 3.0), indicating that their prices rose almost 11% for every 1% that real consumption growth was above forecast. This is almost twice the consumption beta for the S&P 500, indicating significant systematic risk in near futures contracts for livestock and meat. Consistent with supplies adapting over time, the third and fifth contracts to maturity had consumption betas of 7.3 and 4.3, respectively, dampening down the risk of the nearest contract. This pattern was also demonstrated in both consumption and market betas by the industrial metals and wood (copper, platinum, and plywood), because demand for these commodities is quite sensitive to the economic cycle and there is ability to adapt supplies over time. With these commodities, there is relatively little supply uncertainty in the short run, so demand risks dominate.

In contrast, grains and their derivatives have substantial supply uncertainty that is sometimes big enough to affect economic growth. A major drought in 1974–1975 drove grain prices substantially higher at the same time oil prices jumped and economies fell into a serious recession. Thus, the supply effect is offsetting to demand risks and resulted in consumption betas for grains that were near zero or even slightly negative. For these commodities, the CCAPM predicts near zero risk premiums and that futures prices would be unbiased estimates of future spot prices. Meats should have futures prices that are downward-biased estimates of future spot prices to provide rewards to investors for their positive systematic risks. Other foods with major supply uncertainties that are not normally large enough to affect economic growth (e.g., sugar, orange juice) have betas that can be large and positive or negative, owing to the coincidence of their crops with consumption growth during the sample period.

4.2. Euler Equation Tests and the Mehra-Prescott Equity Premium Puzzle

Grossman & Shiller (1981) showed that the CCAPM could be tested and should hold even when there are heterogenous probability beliefs, if empiricists conditioned only on information that was common knowledge in investors' information sets. This should include tests based on past prices, earnings, and dividends, for example, so it is a nontrivial extension. They also proved that the CCAPM holds with heterogeneous beliefs for nontradable risky assets, as long as consumption and the prices of tradable assets form an Ito process.

Hansen & Singleton (1982) provided a now widely used method for testing nonlinear stochastic Euler equation restrictions on the joint movements of asset returns and consumption (see Equations 7 and 8). Using a narrow power utility function for a representative investor, they obtained plausible estimates of CRRA but were able to reject the CRRA model when examining multiple asset returns simultaneously. The next year, Hansen & Singleton (1983) used maximum likelihood estimation to test Euler equation restrictions with monthly consumption growth data and concluded that the CRRA-lognormal model is unable to fit the data and that more general preference specifications or probability distributions are required. Kraus & Litzenberger (1983) followed up their well-known 1976 work on skewness preference and demonstrated that, with the plausible assumption of decreasing absolute risk aversion (less risk averse to dollar gambles as wealth increases), a three-moment CCAPM could be derived. Assets with a convex relation with real consumption growth have lower exposures to economic contractions and higher exposure to economic expansion, ceteris paribus, have lower expected returns in equilibrium. Conversely, assets with a concave relation with consumption have higher exposure to economic contractions than to economic expansions and, ceteris paribus, need to have higher expected returns to make them attractive to hold.

In a groundbreaking, controversial article, Mehra & Prescott (1985) examined 89 years of annual data from 1889 to 1978 on real consumption growth (nondurables and services), the real riskless rate estimated from Treasury bills, and the real return on the S&P 500 stock price index.

They tested whether the average level of the estimated real riskless rate and the average level of the equity risk premium could be explained by a consumption-based asset pricing model based on a representative individual with CRRA, using what were believed to be plausible levels of CRRA (1 to 10). They found a mean real stock return of 7.0% over the entire period, whereas the real riskless rate averaged 0.8%, giving an equity risk premium of 6.2%. The standard deviation of the annual real stock return was 16.5%, whereas the volatility of real consumption growth was 3.6%. Whereas the volatility of stocks showed no obvious trend over the decades, the volatility of annual consumption growth declined quite dramatically from approximately 5.0% in the first two decades (1889–1908) to approximately 1.1% over the most recent three decades from 1949 to 1978, following the end of World War II. This very low level of volatility of consumption growth and the moderate (0.40) correlation of stocks with consumption growth leads to CCAPM predicted risk premiums for stocks that are much lower than actually experienced by investors, which Mehra & Prescott termed the equity premium puzzle. They also found it difficult to fit what they viewed as the low level of the real riskless rate, 0.8%.

Mehra & Prescott (1985) simulated economies that mimicked historic volatility properties of stocks and consumption growth, as well as the −0.14 autocorrelation in real consumption growth in that time period, and enforced the following constraints: (*a*) CRRA could not exceed 10, and (*b*) the real riskless rate had to be between 0% and 4.0%. With these constraints, they estimated the admissible region for the equity risk premium to be related to the real riskless rate estimate as shown in **Figure 6**.

Quoting Mehra & Prescott (1985, p. 156), "The largest premium obtainable with the model is 0.35%, which is not close to the observed value" (which was 6.18%). Given that they found the volatility of the risk premium is 1,667 basis points annually, Mehra & Prescott (1985) estimated that a risk premium of 35 basis points would properly reward investors for taking equity risk. This is highly implausible unless equity risk is uncorrelated with marginal utilities of investors (i.e., there is little consumption risk). With that risk premium and volatility, equity investors would have barely more than a 50/50 chance of having returns that exceeded Treasury bills!

Let us confirm whether equities had much systematic consumption and marginal utility related risk over the sample period. We begin by looking at the correlations from 1889 to 2013 (124 years) of their annual returns with real consumption growth and with changes in the unemployment rate,

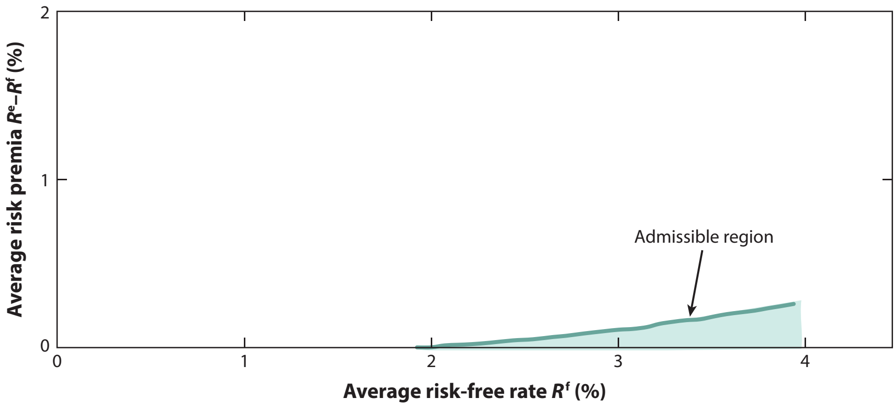

Figure 6

Set of admissible average equity risk premia and real returns. Figure adapted from Mehra & Prescott (1985, p. 155).

using data from the NBER MacroHistory database and early data from *Historical Statistics of the United States: Colonial Times to 1970* (US Dep. Commerce 1975). First, we compute the correlations of annual real stock returns with annual real consumption growth and annual changes in the unemployment rate to be 0.40 and −0.40, respectively, a moderate, but significant correlation, as expected. Simple OLS regressions of contemporaneous stock returns on contemporaneous values of those variables have slopes with the predicted signs and t-statistics of 4.4 and −3.9, respectively. Thus, the stock market clearly has significant consumption risk and likely is highly related to changes in marginal utility of consumption and wealth.

Going further, because stocks strongly lead changes in the unemployment rate and real growth (e.g., Breeden 2013), we examine "bad economy" years when the unemployment rate increased by 1.25% (0.5 sigma) or more in year t and another 1.25% in year $t + 1$. We find that the real stock return in period t averaged −15.3% in those years, approximately 22% below the unconditional mean real return on stocks of 7.0%. On the upside (in a good economy), when the unemployment rate dropped by 1.25% in year t and again in year $t + 1$, the real stock return in year t averages a gain of +13.3%, or 6% above the unconditional mean return. These statistics are little different if we look at economies with back-to-back 0.5 standard deviation moves in real consumption growth, as shown in **Table 3**.

Note the strong nonlinearity of the conditional means of stock returns to the economy: Good economies have real stock returns that are 6% to 7% above the unconditional mean, whereas bad economies have mean real stock returns that are approximately −22% below the unconditional mean. This is true whether one looks at real consumption growth or unemployment rate changes as the economic barometer for changes in individuals' marginal utilities. As the statistics cited make clear, the stock market has large, highly concave exposure to consumption growth and investors require risk premiums substantially in excess of the 35 basis points that Mehra & Prescott (1985) supports. Furthermore, their assumption of CRRA does not allow cyclical variations in risk premiums per unit of risk.

The equity premium puzzle introduced by Mehra & Prescott (1985) stimulated a huge amount of research, including the now-classic disaster risk analysis of Rietz (1988), which is discussed in Part 2 of this review, along with research that it stimulated two decades later. Also in the late 1980s, theorists began intensive modeling of preferences that were not based on time-additive narrow power utility functions, but instead had a representative utility function displaying decreasing

Table 3 Real stock returns in good and bad economies

		1890–1978		1890–2013	
		Bad economy	Good economy	Bad economy	Good economy
Real nondurables and services consumption growth	Number of observations	9	4	10	7
	Mean return	−15.5%	+14.0%	−15.7%	+15.0%
	Abnormal return	−22.5%	+7.0%	−22.7%	+7.0%
Unemployment rate change	Number of observations	4	10	4	10
	Mean return	−15.3%	+13.3%	−15.3%	+13.3%
	Abnormal return	−22.3%	+6.3%	−22.3%	+6.3%

Economic state based on real consumption growth and unemployment rate changes, showing years when economy is 0.5 standard deviations from the unconditional mean for 2 consecutive years.

RRA, time-complementarity, and habit formation, or were of a recursive forward-looking form. Several of the major articles in these areas are discussed in Part 2 of this review.

There is another strand of research spawned by this work. As Mehra & Prescott (1985, p. 159) quite presciently said in the concluding remarks to their article,

> In the absence of such markets, there can be variability in individual consumptions, yet little variability in aggregate consumption.... The fact that certain types of contracts may be nonenforceable is one reason for the nonexistence of markets that would otherwise arise to share risk.... To test such theories, it would probably be necessary to have consumption data by income or age groups.

Subsequently, the US government collected and disseminated such data. One of the most important strands of research that has helped us figure out the equity premium puzzle is a group of papers on incomplete markets and limited participation of investors in the stock market. In Section 7, we review some of the works in these areas and show some calculations of our own from the Consumer Expenditure Survey (CEX), which some authors have used.

5. MEASURING CONSUMPTION RISKS

In his book entitled *Neoclassical Finance*, Ross (2005, p. 37), the distinguished financial economist and developer of the APT, said, "The consumption beta model is marvelous theory but it surprises me that people take it as seriously as they do for empirical work." That is quite a challenging statement for CCAPM proponents. To understand the basis for his remark, let us examine the test of the APT and Merton's ICAPM done by Chen, Roll & Ross (1986) and the results they found for consumption risk. They did an interesting analysis of exposures of stock returns to economic state variables, such as (*a*) fluctuations in monthly industrial production (a good monthly measure of economic activity), (*b*) Consumer Price Index inflation, (*c*) changes in expected inflation, (*d*) credit risk as measured by the junk bond premium, and (*e*) the unanticipated change in the term structure as measured by Ibbotson's long-term government bond return less the return on 1-month Treasury bills. The junk bond premium was computed as the return on bonds rated Baa or less (junk rated below Baa), less Ibbotson's long-term US Government bond return.

Chen, Roll & Ross (1986) estimated the risk premiums for exposures to the various fundamental risk factors (excluding consumption). Plausibly, they found that, standing alone, exposure to equity market beta carries a positive risk premium, where betas are estimated with 60-month rolling regressions with prior data. They presented results for their five economic factors, which are also plausible. In a multiple regression, the coefficient of the market return variable is negative, and their betas "do not explain cross-sectional differences in average returns after the betas of the economic state variables have been included" (Chen, Roll & Ross 1986, p. 399). The key results (which are quite sensible) are that exposure to general economic activity (industrial production) and that exposure to credit risk, which is also very procyclic, are rewarded with a significantly positive risk premium. Exposure to both expected and unexpected inflation gives negative risk premiums, as inflation hedging assets appear to be priced higher and give lower average returns, which are also plausible results. Stocks with returns inversely related to interest rates received negative premiums for that exposure, perhaps as falling interest rates indicate weak economies, so such stocks have less real consumption risk. Thus, all the risk premiums, except for the market risk premium, are plausible. Chen, Roll & Ross (1986) then added a factor for 1-month percentage changes in real per capita consumption growth (similar to Hansen & Singleton 1983), with results shown in **Table 4**.

Table 4 Factor pricing with consumption

Years	CG	MP	DEI	UI	UPR	UTS	Constant
1964–1984	0.68 (0.11)	14.96(3.8)	−0.17 (−1.74)	−0.85 (−2.25)	8.81 (2.58)	−6.92 (−1.79)	2.29 (0.63)
1964–1977	−0.49 (−0.66)	18.15 (3.54)	0.17 (−2.42)	−0.95 (−2.49)	11.44 (3.29)	−9.19 (−2.41)	−1.91 (−0.44)
1978–1984	1.17 (1.00)	8.59 (1.48)	−0.17 (−0.66)	−0.65 (−0.77)	3.56 (0.47)	−2.38 (−0.27)	10.69 (1.61)

In parentheses are t-statistics. Abbreviations: CG, consumption growth; DEI, change in expected inflation; MP, growth of monthly industrial production; UI, unanticipated inflation; UPR, junk-bond return premium; UTS, long-term minus short-term government bond returns, reflecting term structure. Data reproduced from Chen, Roll & Ross (1986, table 6).

Monthly consumption data in the United States starts in 1959, so using 60 prior months of data to estimate rolling consumption betas means that the first data point is in 1964. Over the entire sample period, the point estimate of the risk premium is slightly positive, but statistically insignificant. For the 1964–1977 subperiod, the consumption risk premium was actually negative! Signs and significance of the other economic state variables were essentially unchanged.

Why did Chen, Roll & Ross (1986) and Hansen & Singleton (1983) get such poor results for consumption risk premiums? Breeden, Gibbons & Litzenberger (1989) give some clues. First, they note that consumption is measured with a great deal of error and is affected by considerable noise in the monthly data that is related to weather effects on spending and to major strikes and significant tax changes that affect income and spending. In many cases, these are short-term effects that are followed by catch-up gains or sharp declines in spending in subsequent months. Monthly consumption expenditures include many types of goods with different degrees of durability that provide consumption flows in subsequent months. Breeden, Gibbons & Litzenberger (1989) found that monthly data have significantly negative autocorrelation (−0.28), whereas quarterly consumption data have positive first-order autocorrelation (+0.29), as shown in **Table 5**.

Thus, monthly growth rates of consumption are likely being dominated by the noise of weather, strikes, and tax changes, whereas the quarterly data (and semiannual and annual percentage changes) are much more likely to pick up real changes in consumption flows. Subsequent empirical tests by Parker & Julliard (2005) and Jagannathan & Wang (2007) appear to support this explanation: Parker & Julliard (2005) examined consumption betas measured by cumulating consumption growth over 11 quarters, and Jagannathan & Wang (2007) examined four-quarter changes. To have more nonoverlapping data, we prefer the use of 6-month or two-quarter percentage changes, as in Vissing-Jørgensen (2002) and Breeden (2013). Tests based on monthly

Table 5 Time-series properties of percentage changes in real, per capita consumption of nondurable goods and services

Time period	T	$\hat{c}$	$\widehat{SD}(c)$	$\hat{\rho}_1$	$\hat{\rho}_2$	$\hat{\rho}_3$	$\hat{\rho}_4$	$\hat{\rho}_8$	$(SD^*(\hat{\rho}_k)$	$\widehat{SD}(\hat{\rho}_1)$	$\widehat{SD}(\hat{\rho}_K)$	Q_{12}	p-value
Quarterly consumption data													
1939Q2–1982Q4	175	0.00543	0.00951	0.29	0.03	−0.00	0.07	0.02	0.08	0.07	0.08	23.93	0.02
Monthly consumption data													
1959–1982	287	0.00178	0.00447	−0.28	−0.02	−0.14	−0.12	−0.19	0.06	0.05	0.06	43.09	0.00
Quarterly sampling of monthly consumption data													
1959Q2–1982Q4	95	0.00521	0.00568	0.13	−0.13	0.20	0.04	−0.17	0.10	0.09	0.11	13.42	0.34

Data are seasonally adjusted. T denotes the number of observations, whereas $\hat{c}$ and $\widehat{\mathrm{SD}}$(c) are the sample mean and standard deviation, respectively. Under the hypothesis that the observations are serially uncorrelated, the asymptotic standard errors for the sample autocorrelations are $1/\sqrt{T}$, as given by SD*($\hat{\rho}_k$). Under the hypothesis that $\rho_1 = 0.25$ and $\rho_k = 0\,\forall|k| > 1$, SD($\rho_1$) and SD($\rho_k$) report the asymptotic standard errors. The test statistic for the joint hypothesis that all autocorrelations are zero for lags 1 through 12 is given by Q_{12}; the modified Box-Pierce Q-statistic. Q_{12} is asymptotically distributed as chi-squared is well known with 12 degrees of freedom. The p-value is the probability of drawing a Q_{12} statistic larger than the current value under the null hypothesis. Data taken from from Breeden, Gibbons & Litzenberger (1989, table 1).

Table 6 Consumption and marginal utility: changes in the unemployment rate, employment growth, and loan charge-offs related to contemporaneous real stock returns and real consumption growth

Dependent variable/independent variable	6-month change in unemployment rate			6-month employment growth annualized percentage			Total loan charge-offs percent of loans		
	Slope	*t*-Statistic	CRSQ	Slope	*t*-Statistic	CRSQ	Slope	*t*-Statistic	CRSQ
S&P 500 (6-month real return)	−0.01	−1.5	0.01	0.00	0.0	−0.01	−0.007	−1.0	−0.00
Total consumption (6-month real growth)	−0.14	−6.4	0.27	0.34	5.6	0.22	−0.12	−3.3	0.14
Nondurables and services consumption (6-month real growth)	−0.16	−5.2	0.20	0.48	5.8	0.23	−0.15	−3.4	0.15

Shown are univariate regressions with nonoverlapping data on 6-month growth rates, June 1960–June 2014. Abbreviation: CRSQ, corrected R-squared. Data updated from Breeden (2013).

percentage changes in real consumption are simply not reliable, given the low signal-to-noise ratio and the presence of large reversals, which can statistically dominate the results.

To illustrate that real consumption growth measured with 6-month or two-quarter differencing is picking up real economic risks and fluctuations in marginal utilities, consider the relation of contemporaneous changes in the unemployment rate versus real consumption growth and real stock market returns found by Breeden (2013). The 6-month changes in unemployment rate are much more contemporaneously correlated with real consumption growth than with real returns on the stock market, with a slope *t*-statistic of −6.7 versus −1.3, respectively, and corrected R^2 values of 0.30 versus 0.02, using nonoverlapping data from 1960 to 2013. Thus, even given the difficulties of measuring consumption growth, it appears to be strongly correlated with likely measures of marginal utility. Correlations of real consumption growth with total employment growth and loan charge-offs show similar results, as shown in **Table 6**.

Chen, Roll & Ross (1986) found the credit risk variable (junk bond premium) to be highly significant in pricing risks. **Table 6** shows that loan charge-offs are closely correlated with real consumption growth (t = 4.0 to 4.5 for total real consumption and nondurables and services growth, respectively). Thus, Chen, Roll & Ross (1986) likely picked up part of the consumption risk of the assets but with more precisely measured variables. **Figure 7** shows the relation of the credit spread (Baa-rated bond yield minus 10-year Treasury yield) and the unemployment rate. It is quite easy to see the close relation of credit spreads to the unemployment rate and economic recessions and growth periods.

Because credit yield spreads are forward-looking, much as the stock market is quite forward-looking, they are contemporaneously more highly correlated with stock returns than are loan charge-offs, as **Table 7** demonstrates. The results found suggest that the economic factors of Chen, Roll & Ross (1986) be viewed as instrumental variables for the unobservable true rate of growth in per capita consumption. In an early article, Mankin & Shapiro (1986) also studied consumption and market risks and found stock returns more closely related to stock market risks. Breeden, Gibbons & Litzenberger (1989),[2] Grossman, Melino & Shiller (1987), and Wheatley (1988) examined measurement issues in consumption (such as time aggregation) and their biases on measures of volatility and consumption betas. Breeden, Gibbons & Litzenberger (1989) derived two useful results used by many subsequent authors: (*a*) biases in consumption betas due to time aggregation,

[2] Note that Breeden, Gibbons & Litzenberger's (1989) results in this section were complete by 1984 and were contemporaneous with or prior to Grossman, Melino & Shiller (1987).

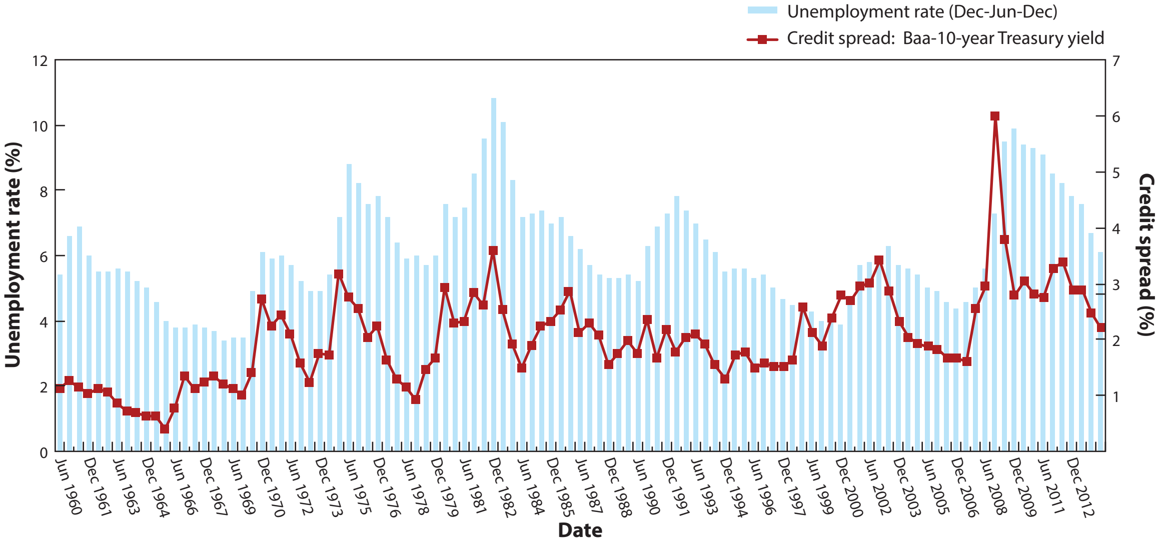

Figure 7

Credit spread (Baa-10-year Treasury yield) versus unemployment rate using semiannual data from 1960 to 2013Q2. During recessions, there is high unemployment and high credit risk.

Table 7 Consumption and marginal utility: loan credit risk

Independent variable	Coefficient	*t*-Statistic	CRSQ
S&P 500 (6-month real % change)	−1.98	−4.2	0.13
Total real consumption (6-month real % change)	−8.41	−4.1	0.13
Nondurables and services consumption (6-month real % change)	−7.82	−2.6	0.05

Basis points change in yield spread of Baa bonds—10-year Treasury related to real stock returns and real consumption growth, showing univariate regressions, June 1960–June 2014 (nonoverlapping 6-month changes). Abbreviation, CRSQ, corrected R-squared.

and how those biases are reduced with increased differencing intervals for consumption growth, and (*b*) estimation of consumption betas relative to returns on a CMP, which allows for a greater number and frequency of observations and more precise estimates of consumption betas. Breeden, Gibbons & Litzenberger's (1989) main results are derived next.

5.1. Time Aggregation Biases in Consumption Growth[3]

Ignoring other measurement problems, the reported (interval) consumption rate for a quarter is the integral of the instantaneous (spot) consumption rates during the quarter. The CCAPM relates expected quarterly returns on assets (e.g., from January 1 to March 31) and the covariances of those returns with the change in the spot consumption rate from the beginning of the quarter to the end of the quarter. This section derives the relation between the desired population covariances (and betas) of assets' returns relative to changes and the population covariances (and betas) of assets' returns relative to change in interval consumption. The variance of interval consumption changes

[3]This section is taken from Breeden, Gibbons & Litzenberger (1989, section II.B).

has only two-thirds the variance of spot consumption changes, whereas the autocorrelation of interval consumption is 0.25 owing to the integration of spot rates.

Without loss of generality, consider a two-quarter period with $t = 0$ as the beginning of the first quarter and $t = T$ as the end of the first quarter. All discussion analyzes annualized consumption rates, so $T = 0.25$ for a quarter. Initially, let the change in the spot consumption rate over a quarter be the cumulative of n discrete changes, $\{\tilde{\Delta}_1^C, \tilde{\Delta}_2^C, \ldots, \tilde{\Delta}_n^C\}$ for the first quarter and $\{\tilde{\Delta}_{n+1}^C, \tilde{\Delta}_{n+2}^C, \ldots, \tilde{\Delta}_{2n}^C\}$ for the second quarter. That is, $\widetilde{C_T} = C_0 + \sum_1^n \tilde{\Delta}_i^C$. Similarly, let the wealth at time T from buying one share of an asset at time 0 (and reinvesting any dividends) equal its initial price plus n random increments $\{\tilde{\Delta}_i^a\}$: $P_T = P_0 + \sum_1^n \tilde{\Delta}_i^a$.

Changes in consumption, $\tilde{\Delta}_i^C$, the asset's return, $\tilde{\Delta}_i^a$, are assumed to be homoscedastic and serially uncorrelated. The contemporaneous covariation of an asset's return with consumption changes is σ_{ac}. The variance of the change in the spot consumption from the beginning of a quarter to the end of the quarter is

$$\mathrm{var}(\widetilde{C_T} - \widetilde{C_0}) = \mathrm{var}\left(\sum_1^n \tilde{\Delta}_i^C\right) = \sigma_C^2 \mathrm{T}. \tag{35}$$

The first quarter's reported annualized consumption, C_{Q1}, is a summation of the consumption during the quarter, annualized by multiplying by 3 (or $1/T$):

$$C_{\mathrm{Q1}} = \frac{1}{T}\sum_{j=1}^{n} C_j \Delta t = \frac{1}{T}\sum_{j=1}^{n}\left(C_0 + \sum_{i=1}^{j}\Delta_i^C\right)\Delta t. \tag{36}$$

The annualized consumption rate for the second quarter, C_{Q2}, is the same as Equation 36, but with the first summation for j being $n+1$ to $2n$.

Continuous movements in consumption and asset prices can be approximated by letting the number of discrete movements per quarter, n, go to infinity ($\Delta t \to 0$). Doing this, the change in reported consumption becomes

$$C_{\mathrm{Q2}} - C_{\mathrm{Q1}} = \int_0^T \frac{t}{T}\Delta_t^C dt + \int_T^{2T} \frac{2T - t}{T}\Delta_t^C dt. \tag{37}$$

Given the independence of spot consumption changes over time, Equation 37 implies that the variance of reported annualized consumption changes is

$$\mathrm{var}(\tilde{C}_{\mathrm{Q2}} - \tilde{C}_{\mathrm{Q1}}) = \int_0^T \left(\left(\frac{t}{T}\right)^2 \sigma_C^2\right) dt + \int_T^{2T}\left(\left(\frac{2T - t}{T}\right)^2 \sigma_C^2\right) dt = \frac{2}{3}\sigma_C^2 T. \tag{38}$$

Thus, the population variance of reported (interval) consumption changes for a quarter is two-thirds of the population variance for changes in the spot consumption from the beginning of a quarter to the end of the quarter. The averaging caused by the integration led to the lower variance for reported consumption.

Next, consider the covariance of an asset's quarterly return with quarterly changes in the consumption. The covariance of the change in spot consumption from the beginning of a quarter to the end of the quarter with an asset's return over the same period is $\sigma_{ac} T$, given the independent and identically distributed assumption. With reported, interval consumption data, the covariance can be computed from Equation 38:

$$\mathrm{cov}(\tilde{C}_{\mathrm{Q2}} - \tilde{C}_{\mathrm{Q1}}, \tilde{P}_{2T} - \tilde{P}_T) = T^{-1}\int_T^{2T} (2T - t)\sigma_{aC} dt = \frac{1}{2} T \sigma_{aC}. \tag{39}$$

Thus, from Equation 39 the population covariance of an asset's quarterly return with reported (interval) consumption is one-half the population covariance of the asset's return with spot consumption changes.

Given Equations 38 and 39, betas measured relative to reported quarterly consumption changes are three-fourths times the corresponding betas with spot consumption:

$$\beta_{ac}^{\text{sum}} = \frac{\frac{1}{2}T\,\sigma_{aC}}{\frac{2}{3}\sigma_C^2 T} = \frac{3}{4}\beta_{ac}^{\text{spot}}. \tag{40}$$

Because the CCAPM relates quarterly returns to spot betas, the subsequent empirical tests multiply the mean-adjusted consumption growth rates by three-fourths to obtain unbiased spot betas. The three-fourths relation of interval betas to spot betas in Equation 40 is a special case of the multiperiod differentiating relation: $\beta_{ac}^{\text{sum}} = \beta_{ac}^{\text{spot}}\frac{K-(1/2)}{K-(1/3)}$, where K is the differencing interval. Thus, monthly data sampled quarterly (i.e., $K = 3$) should give interval betas that are $\frac{5/2}{8/3} = 0.9375$ times the spot betas. When quarterly consumption growth rates are calculated from monthly data, the quarterly numbers are mean adjusted and multiplied by 0.9375.

Although changes in spot consumption are uncorrelated, changes in reported, interval consumption rates have positive autocorrelation. To see this, use Equation 37 to compute the covariance of the reported consumption change from Q1 to Q2 with the reported change from Q2 to Q3, noting that all covariance arises from the time overlap from T to $2T$:

$$\text{cov}(\tilde{C}_{\text{Q3}} - \tilde{C}_{\text{Q2}}, \tilde{C}_{\text{Q2}} - \tilde{C}_{\text{Q1}}) = \int_T^{2T} \frac{(t-T)(2T-t)}{T^2}\sigma_C^2 dt = \frac{1}{6}\sigma_C^2 T. \tag{41}$$

The first-order autocorrelation in reported consumption is 0.25 because

$$\rho_1 = \frac{\text{cov}(\tilde{C}_{\text{Q3}} - \tilde{C}_{\text{Q2}}, \tilde{C}_{\text{Q2}} - \tilde{C}_{\text{Q1}})}{\text{var}(\tilde{C}_{\text{Q2}} - \tilde{C}_{\text{Q1}})} = \frac{(1/6)\sigma_C^2 T}{(2/3)\sigma_C^2 T} = 0.25. \tag{42}$$

By similar calculations, higher-order autocorrelations are zero. Breeden, Gibbons & Litzenberger (1989, table 1) presented the time-series properties of reported unspliced quarterly consumption growth rates. First-order autocorrelation of quarterly real consumption growth for the entire 1939–1982 period is estimated to be 0.29, which is insignificantly different from the theoretical value of 0.25 at usual levels of significance. Higher-order autocorrelations are not significantly different from zero. Thus, the model for reported consumption is not rejected by the sample autocorrelations.

Monthly growth rates of real consumption from 1959 and 1982 exhibit negative autocorrelation of −0.28, which is significantly different from zero and from the hypothesized 0.25. This may be caused by vagaries such as bad weather and strikes in major industries, which cut current consumption temporarily but are followed by catch-up purchases. Quarterly growth rates in consumption computed from the monthly series again have positive autocorrelation of 0.13, more closely in line with the value 0.0625 (or one-sixteenth) predicted by the summation bias. The longer the differencing interval, the less affected the data are by temporary fluctuations and measurement errors in consumption.

Chen, Roll & Ross (1986) and Hansen & Singleton (1983) used monthly data on unadjusted consumption growth. Because that data's autocorrelation statistics suggest significant departures from the random-walk assumption, the statistics they presented warrant reexamination. The use of larger differencing intervals should be fruitful and have been found in subsequent research (see Vissing-Jørgensen 2002, Parker & Julliard 2005, Jagannathan & Wang 2007).

Because even consumption goods classified as nondurables (such as clothing) often have consumption flows in future periods, the marginal utility of consumption depends in part on past consumption expenditure, so current consumption expenditures impact the future marginal utility of consumption. Therefore, even under time-additive utility, past consumption expenditures should have an impact of current risk premiums. This suggests longer consumption horizons or distributed lags in empirical test of the time additive CAPM, e.g., the longer horizon used by Parker & Julliard (2005), which is discussed in Section 8.

5.2. Maximum Correlation Portfolio or Consumption Mimicking Portfolio[4]

Testing of the CCAPM is hampered by the infrequent measurement of consumption relative to the frequency of measuring stock returns, as aggregate consumption in the United States has been measured only with annual totals back into the 1800s, quarterly only since 1939, and monthly only since 1959. Many other countries have even less frequent measurements of consumption than does the United States. In contrast, we have monthly indexes for the S&P 500 back to the 1800s and daily data from the University of Chicago's Center for Research in Security Prices (CRSP) return series that begins in 1926. The data are available on Dartmouth Professor Kenneth French's website. Thus, for example, we have daily data on stock returns during the very important economic time period covering the Great Depression of 1929–1938, but only annual data on aggregate consumption. The United States is one of the few countries with monthly consumption data for the past 55 years. Most other countries in the world have only quarterly data on aggregate consumption, which contrasts with daily data on stock returns, giving stock measurements approximately 66 times the frequency of consumption measurements for most countries at present and for the past 50 years or more.

Fortunately for econometric testing, Breeden, Gibbons & Litzenberger (1989), proved that if one would first find the portfolio that has maximum correlation of its return with real consumption growth, then the CCAPM should hold where betas are measured against the returns of that portfolio. Articles by Breeden, Gibbons & Litzenberger (1989), Jagannathan & Wang (2007), and Malloy, Moskowitz & Vissing-Jørgensen (2009) used this technique to get more powerful tests of the CCAPM, especially as conditional consumption betas can be estimated more precisely using this much higher frequency data. This portfolio was called the maximum correlation portfolio by Breeden, Gibbons & Litzenberger (1989), the CMP by Jagannathan & Wang (2007), and the consumption growth factor–mimicking portfolio by Malloy, Moskowitz & Vissing-Jørgensen (2009). As it seems more descriptive, we use the CMP nomenclature of Jagannathan & Wang (2007) in this review. A simplified version (where a riskless asset exists) of the Breeden, Gibbons & Litzenberger (1989) derivation of the CCAPM in terms of the CMP's return is as follows.

First, given a riskless asset, let us choose the minimum variance $A \times 1$ portfolio of risky assets, **w**, that is levered or unlevered to have a consumption beta of 1.00:

$$\beta_{pC} = \rho_{pC}\frac{\sigma_p}{\sigma_C} = 1.00 \quad \text{implies that} \quad \sigma_p = \frac{\sigma_C}{\rho_{pC}}, \quad \text{so minimizing } \sigma_p \text{ maximizes } \rho_{pC},$$

$$\min_{\{w\}}: w'Vw + \lambda(1 - w'\beta_C), \tag{43}$$

$$\text{at the optimum:} \quad w_{cmp} = \left(\frac{1}{2}\right)\lambda V^{-1}\beta_C, \tag{44}$$

$$\beta_{cmp} = \frac{Vw_{cmp}}{\sigma^2_{cmp}} = \frac{\lambda}{2\sigma^2_{cmp}}\beta_C. \tag{45}$$

[4]This section is from Breeden, Gibbons & Litzenberger (1989, section II.C), following Breeden (1979, footnote 8).

Substituting Equation 45 into Equation 24 gives

$$\boldsymbol{\mu} - \mathbf{r}_f = \frac{2\sigma^2_{cmp}}{\lambda}\left(\frac{\mu_M - r_f}{\beta_{MC}}\right)\beta_{cmp}. \tag{46}$$

Premultiplying Equation 46 by the row vector of market portfolio weights gives

$$\mathbf{w}'_M(\boldsymbol{\mu} - \mathbf{r}_f) = \mu_M - r_f = \frac{2\sigma^2_{cmp}}{\lambda}\left(\frac{\mu_M - r_f}{\beta_{MC}}\right)\mathbf{w}'_M\beta_{cmp}, \tag{47}$$

$$\Rightarrow \frac{\lambda}{2\sigma^2_{cmp}} = \frac{\beta_{M,cmp}}{\beta_{MC}}. \tag{48}$$

Substituting Equation 48 into Equation 46 gives

$$\boldsymbol{\mu} - \mathbf{r}_f = \beta_{cmp}\frac{\mu_M - r_f}{\beta_{M,cmp}} \quad (\text{CCAPM} - \text{CMP}), \tag{49}$$

which is the CCAPM, where the $A \times 1$ vector of consumption betas is measured relative to the CMP, and the market price of risk is the risk premium of the market portfolio, divided by its beta relative to the CMP.

Breeden, Gibbons & Litzenberger (1989) used their quarterly data on industry stock returns and bond returns to estimate a CMP for the 1929–1982 period (using spliced consumption growth estimates for 1929–1939). Note that the junk bond premium is the strongest variable by t-statistic, confirming that the statistics of Chen, Roll & Ross (1986) for that economic state variable may be attributed to its real consumption risk. Breeden, Gibbons & Litzenberger (1989) then estimated consumption betas for the various industry stock portfolios and bonds using the data on real consumption growth, the returns on the CMP, and the value-weighted return on stocks from the CRSP (see **Table 8** for the results). Consumption betas estimated from 1929 to 1982 from quarterly consumption data have t-statistics that for stocks are primarily in the 6.0 to 7.5 range. By contrast, the estimates relative to the CMP have t-statistics that are approximately 19.0 to 23.5, and those for stock market betas relative to the stock market index are typically 45.0 to 60.0 or more. Thus, using the CMP appears to improve the estimation of consumption risk quite significantly. Subsequent articles by Jagannathan & Wang (2007) and Malloy, Moskowitz & Vissing-Jørgensen (2009) used the Breeden, Gibbons & Litzenberger (1989) technique in fitting CMPs employing data for the Fama-French portfolios stratified by size and book to market.

Breeden (2005) illustrated a simple, three-variable CMP, which likely has more robust coefficients than does the original Breeden, Gibbons & Litzenberger (1989) portfolio, which estimated weights for 17 portfolios. With so many portfolios, there are legitimate concerns about estimation risk and possible data snooping biases, as studied by Lo & MacKinlay (1990). **Table 9** shows that a portfolio of just the S&P 500 stock index, the credit spread of Baa versus 10-year bonds, and the 3-year versus 3-month Treasury slope had a corrected $R^2 = 0.24$ for real nondurables and services consumption growth and 0.29 for real total consumption growth, both statistics quite comparable with the Breeden, Gibbons & Litzenberger (1989) fit with an $R^2 = 0.25$. All these variables can be measured daily and intraday, so they hold promise for better estimates of a good CMP that can be used to estimate consumption betas more precisely.

6. CASH FLOW BETAS: CONSUMPTION RISK VERSUS MARKET RISK

In their original derivation of the CCAPM, Breeden & Litzenberger (1978, p. 646) realized that measuring consumption betas from cash flows such as dividends or profits, rather than from stock market prices and returns, could be useful in practice:

Table 8 Estimated betas relative to growth in real, per capita consumption,[a] maximum-correlation portfolio for consumption, and CRSP value-weighted index

Asset (SIC codes)	Spliced consumption: quarterly 1929–1982 ($T = 215$)			Maximum correlation consumption portfolio: monthly 1926–1982 ($T = 684$)			CRSP value-weighted index: monthly 1926–1982 ($T = 684$)		
	$\hat{\beta}_c$	$t(\hat{\beta})$	R^2	$\hat{\beta}_{MCP}$	$t(\hat{\beta})$	R^2	$\hat{\beta}_{CRSP}$	$t(\hat{\beta})$	R^2
US Treasury bills	−0.1	−1.3	0.01	0.03	3.9	0.02	0.01	2.0	0.01
Long-term government bonds	0.0	0.0	0.00	0.07	2.5	0.01	0.07	4.9	0.03
Long-term corporate bonds	0.2	0.9	0.00	0.07	2.5	0.01	0.08	6.6	0.06
Junk bond premium	2.5	6.9	0.18	0.63	18.5	0.33	0.33	20.5	0.38
Petroleum (13, 29)	4.3	6.4	0.16	1.41	20.6	0.38	0.92	38.6	0.69
Finance and real estate (60–69)	5.9	6.3	0.16	1.50	18.8	0.34	1.19	76.0	0.89
Consumer durables (25, 30, 36, 37, 50, 55, 57)	6.9	6.8	0.18	1.79	22.0	0.42	1.29	80.8	0.91
Basic industries (10, 12, 14, 24, 26, 28, 33)	5.5	7.0	0.18	1.48	22.0	0.41	1.09	100.8	0.94
Food and tobacco (1, 20, 21, 54)	3.3	5.7	0.13	0.99	18.6	0.34	0.76	58.2	0.83
Construction (15–17, 32, 52)	7.4	7.0	0.19	1.57	19.2	0.35	1.20	61.2	0.85
Capital goods (34, 35, 38)	5.3	6.7	0.18	1.45	21.1	0.39	1.08	85.9	0.92
Transportation (40–42, 44, 45, 47)	5.2	5.0	0.10	1.27	13.5	0.21	1.19	49.0	0.78
Utilities (46, 48, 49)	3.7	6.1	0.15	1.04	19.4	0.35	0.75	46.3	0.76
Textiles and trade (22, 23, 31, 51, 53, 56, 59)	5.6	7.8	0.22	1.66	30.5	0.58	0.95	48.7	0.78
Services (72, 73, 75, 80, 82, 89)	4.2	4.2	0.08	1.65	13.0	0.20	0.80	12.8	0.19
Leisure (27, 58, 70, 78, 79)	7.4	7.0	0.18	1.85	23.0	0.44	1.22	49.8	0.78
CSRP value-weighted index	4.9	7.1	0.19	1.37	23.7	0.45	1.00	NA	NA

The MCP is constructed from the 17 assets given in table 3 of Breeden, Gibbons & Litzenberger (1989). The weights of the MCP are determined by maximizing the sample correlation between the return on the portfolio and the growth rate of real consumption (see Breeden, Gibbons & Litzenberger 1989, table 3, for more details. Abbreviations, CRSP, Center for Research in Security Prices; MCP, maximum correlation portfolio; NA, not available. Data reproduced from Breeden, Gibbons & Litzenberger (1989), with numbers rounded here for greater readability.

[a]The spliced consumption data are scaled to adjust for summation bias. Real growth in per capita consumption is multiplied by 0.75 for observations between 1939Q2 and 1959Q1 and by 0.9375 otherwise.

Table 9 Consumption mimicking portfolio multivariate regressions of real consumption growth (annual) on three key variables: real stock return, credit spread, and term structure slope

	PCE total coefficient	PCE total *t*-statistic	PCE NDS coefficient	PCE NDS *t*-statistic
Constant	2.32	7.43	2.34	9.90
S&P 500 real return	0.078	3.91	0.060	3.93
Δ(Baa-10-year Treasury) (credit spread %)	−0.87	−2.28	−0.24	−0.83
Lagged 3 year–3 month term structure slope	0.85	3.49 RSQ = 0.30	0.59	3.18 RSQ = 0.23

Shown are nonoverlapping semiannual data (December-June) from 1960 to 2014. Abbreviations: NDS, nondurables and services; PCE, personal consumption expenditure; RSQ, R-squared. Data updated from Breeden (2005).

Table 10 Estimates of cash-flow risks, 1930–1988 (excluding 1939–1948)

	X = market			X = total consumption			X = NDS consumption		
NIPA variable	***b***	***t (b)***	**RSQ**	***b***	***t (b)***	**RSQ**	***b***	***t (b)***	**RSQ**
AT profits	0.059	5.34	0.36	0.357	6.26	0.44	0.400	5.50	0.37
AT profits IVA CCA	0.047	4.63	0.30	0.354	8.09	0.57	0.406	7.21	0.51
BT profits IVA CCA	0.062	5.48	0.37	0.441	9.02	0.62	0.494	7.50	0.53
Net dividends	1.79	4.67	0.31	15.53	9.40	0.65	18.88	9.33	0.65
Net cash flow	3.56	4.09	0.25	15.79	2.77	0.12	15.16	2.11	0.07

Regression is in the form $Y = a + bX$, where X = (market, PCE total, PCE NDS). Abbreviations: AT, after tax; BT, before tax; CCA, capital consumption adjustment; IVA, inventory adjustment; NDS, nondurables and services; PCE, personal consumption expenditure; RSQ, R-squared. Data reproduced from Breeden (1989b, table 3).

> The correct beta to be used in finding the risk-adjusted discount rate is the cash flow's volatility with respect to aggregate consumption, not with respect to the market portfolio. For capital budgeting, these betas may be easier to estimate than "market" betas, since the cash flows of many projects may be more closely related to GNP or aggregate consumption than to the level of the market portfolio.

Breeden (1991) presented OLS regression results that confirmed this intuition by estimating (unconditional) cash flow betas, using annual data from national income and product accounts for real growth of profits, dividends, and cash flows versus real market returns and real consumption growth (separately for total and nondurables and services consumption) (**Table 10**).

In Breeden (2005), unconditional consumption and market betas for various industries were presented, on the basis of two-quarter changes in real profits over the 1948–2004 period.[5] The consumption betas from profits are quite different from betas estimated with stock market returns, though they are significantly correlated. Profits reflect leverage and are highly volatile. For the US total, profits move by 4.3% when real nondurables and services consumption growth moves by 1%, and motor vehicles, which are quite cyclic, have 1-year profits growth that moves by 15.1% then (or 3.51 times as much as aggregate profits move). Motor vehicles, durables, and wholesale trade all have stock market betas that are quite similar, between 1.08 and 1.16, but their relative profit betas are 3.51, 2.42, and 0.84, respectively. Utilities, oils, and banks had low consumption betas of profits, but higher stock market betas. Banks' profit betas should increase in more recent samples that include the recent Great Recession. Breeden (1991) also found that industry profits were often mean-reverting over 3–5 years, which makes sense given competition and freedom of entry. Very high profits entice additional supplies and competition. Stock market betas and dividends should pick up these longer-term effects (**Table 11**).

Updated data for 1930–2012 for the relation of real profits changes to real S&P 500 returns and to the growth rate of real nondurables and services consumption confirmed these results with 24 more years of data, as real profit growth had an R^2 of 0.26 with real stock returns, whereas its relationship to real nondurables and services and total consumption growth have R^2 of 0.42 and 0.51, respectively. Annual profit growth is more related to consumption changes than to stock returns. Thus, whereas betas of stock returns are estimated more precisely vis-à-vis the market portfolio return, cash flow betas may be more precisely estimated versus real consumption growth.

[5]To deal with negative profits or near-zero profits in certain industries, actual calculations were of changes in real profits, divided first by prior quarter employee compensation, and then divided by overall average profits/compensation.

Table 11 Cash-flow risks ranked by industry: consumption betas versus market betas

Industry	Profit beta, real 2Q, versus % change real NDS consumption	Relative consumption beta (industry divided by US total)	Industry stock beta versus real return on S&P 500
Motor vehicles	15.1	3.51	1.08
Durables	10.4	2.42	1.16
Retail trade	9.2	2.14	1.09
Construction	8.5	1.98	1.23
Wholesale trade	3.6	0.84	1.10
Food and beverage	2.9	0.67	0.84
Banks	1.7	0.40	1.03
Oil	1.6	0.37	0.76
Utilities	−1.0	−0.23	0.61
US total	4.3	1.00	1.00

Nonoverlapping two-quarter (2Q) real percentage changes in real profits, 1948–2004. S&P 500 betas are Kenneth French real industry returns versus S&P 500 real return. Abbreviation: NDS, nondurables and services. Data reproduced from Breeden (2005).

7. LIMITED PARTICIPATION AND INCOMPLETE MARKETS

Mankiw & Zeldes (1991) were first to consider the implications of limited participation in asset markets. Their article, using the Panel Study of Income Dynamics, separates and analyzes the consumption of stockholder and nonstockholder households to explain the equity risk premium, given that stockholders accounted for only 25% of all US households in 1984. The authors argued, "One group is involved in the stock market and is at an interior solution with respect to the holding of stocks, and the other group holds no stocks at all. The relationship between aggregate consumption and the stock market considered above is no longer valid, because aggregate consumption includes the consumption of both the individuals who satisfy the first-order conditions and those who do not" (Mankiw & Zeldes 1991, p. 185).

Mankiw & Zeldes (1991) found that stockholder consumption growth has higher volatility and correlation with the market risk premium than does nonstockholder consumption growth. As **Table 10** shows, stockholders have a correlation of consumption growth with stock returns equal to 0.49, almost five times as large as that of nonstockholders, who have a correlation of 0.10. Volatility of stockholders' consumption growth is 0.032, almost 60% higher than for nonstockholders at 0.020. Combining higher correlation and higher volatility gives stockholders a covariance of consumption with stock returns that is seven times higher. Mankiw & Zeldes (1991) then computed that the implied CRRA for a representative shareholder that is consistent with these statistics drops to approximately one-third the level implied by full participation, from 100 to 35 (**Table 12**).

The estimate of 35 for the CRRA of a representative shareholder is still implausibly large and might be caused by the extremely small sample (1970–1984 annual data) of only 14 annual food consumption growth observations and by the assumption that stockholding status remains unchanged for each household throughout the whole sample period. The size of equity risk premium is reduced, though not fully resolved, owing to data limitations or misspecification of the representative shareholder's utility function. Although the short sample period is a major limitation, the observed differences in estimates of consumption correlation, volatility, covariance, and the implied risk-aversion parameters of these two groups are dramatic enough to stimulate future research.

Table 12 Calibrating the equity premium: stockholders versus nonstockholders

	$\rho(GC, r^m - r^f)$	$\sigma\,(GC)$	$\sigma\,(r^m - r^f)$	$\text{cov}(GC,\ r^m - r^f)$	$E\,(r^m - r^f)$	Implied value of A
PSID all families	0.26	0.021	0.148	0.000796	0.080[b]	100.4
PSID[a] nonstockholders	0.10	0.020	0.148	0.000305	0.080[b]	261.9
PSID[a] stockholders	0.49	0.032	0.148	0.002270	0.080[b]	35.2

GC is the growth of consumption based on the Panel Study of Income Dynamics (PSID), and $r^m - r^f$ is the difference between the return on the S&P 500 and the return on 3-month Treasury bills. A is the coefficient of relative risk aversion implied by the corresponding estimates. Data are from 1970 to 1984. Data reproduced from Mankiw & Zeldes (1991).

[a]Based on split 3 (a household is a stockholder if it holds at least $10,000 of stock, else nonstockholder).

[b]Uses value from 1948 to 1988.

Heaton & Lucas (1992, 1996) provide two important articles on incomplete markets and the volatilities of individuals' consumption growths vis-à-vis aggregate consumption. Heaton & Lucas (1992) cleverly employed a three-period, two-person model to demonstrate how market incompleteness, combined with market frictions (such as borrowing, short-sale constraints, and transactions costs), can explain the equity risk premium. The market is incomplete in having uninsurable labor income shocks, either transitory or permanent. By setting up three periods, they modeled trade and differentiated between transitory and permanent shocks to income. With two heterogeneous agents in the model, they modeled unemployment risk in a possible recession. Heaton & Lucas (1992) found that the assumed market structure has a large, systematic impact on predicted asset prices. They noted that whether consumers use asset markets to smooth consumption depends critically on the persistence of idiosyncratic shocks: "The ability to self-insure diminishes as shocks become more persistent, because more persistent shocks have a larger impact on permanent income and hence on desired consumption" (Heaton & Lucas 1992, p. 607). This model is thought-provoking but problematic because it has so much model flexibility that almost any empirical data may be explained.

Heaton & Lucas (1996) decomposed the two effects of transactions costs on the equity premium. The direct effect is because individuals equate marginal benefits, net of transactions costs. The indirect effect occurs because transaction costs result in individual consumption that more closely tracks individual income. In the simulations, they found that the direct effect dominates and can produce a sizable equity premium only if transaction costs are large or the quantity of tradable assets is limited. They could not resolve whether there was a realistic assumption about transactions costs that can simultaneously explain the low volatility of short bond rates and the high volatility of stock returns. They seemed to recognize that taxation of capital gains based on realizations is a form of very large transaction costs on assets with high appreciation that could lead to concentrated portfolios in their model.

Brav, Constantinides & Geczy (2002) and Vissing-Jørgensen (2002) are two insightful articles documenting inefficient allocations of consumption and analyzing their implications. In both articles, the authors used the data set provided by the US Bureau of Labor Statistics in its CEX (**http://www.bls.gov/cex/**). Brav, Constantinides & Geczy (2002) provided an excellent detailed explanation of this quarterly series of cross sections of household-level consumption data. Each quarter approximately 5,000 households are surveyed about their spending on a list of consumption goods and services that account for approximately 95% of all household expenditures. Households are chosen randomly according to stratification criteria determined by the US Census and are surveyed for five quarters in a row, one training quarter and four regular ones. Every quarter,

Table 13 Summary statistics on per capita quarterly consumption

	Number of households			Household consumption level		Household consumption growth rate	
	Minimum	Median	Maximum	Mean	Standard deviation	Mean	Standard deviation
Total assets ≥ $0							
January	552	692	825	2,437	368	−0.01	0.06
February	569	682	761	2,466	370	−0.01	0.07
March	533	688	794	2,436	378	−0.01	0.06
Total assets ≥ $2,000							
January	31	80	108	3,351	528	0.00	0.08
February	30	81	104	3,426	554	−0.01	0.09
March	39	81	113	3,469	606	0.00	0.09
Total assets ≥ $10,000							
January	22	53	80	3,541	560	0.00	0.10
February	18	54	76	3,621	583	−0.02	0.10
March	23	56	83	3,665	631	−0.01	0.11
Total assets ≥ $20,000							
January	14	40	69	3,657	605	0.00	0.10
February	13	40	63	3,764	606	−0.02	0.10
March	16	40	71	3,773	681	−0.01	0.12

Data reproduced from Brav, Constantinides & Geczy (2002).

one-fifth of the sample is replaced by a newly selected household, so four-fifths of the sample is the same from one quarter to the next. The data started in 1980Q1; however, they omitted the first 2 years since Attanasio & Weber (1989) explained that the 1980 and 1981 data are of questionable quality. **Table 13** gives the summary statistics for per capita consumption of households that are sampled quarterly starting in January, February, and March tranches.

The estimated standard deviation of real consumption growth for these subsets of households in **Table 13** is particularly informative. Quarterly growth rates have volatilities from 6% to 12%, with wealthier households having higher volatilities. If consumption growth were independent from quarter to quarter, these would annualize proportionally to the square root of time, so the annualized volatilities of real consumption growth would range from 12% to 24%, which is quite a lot of volatility and is considerably above the approximately 1.0% annualized volatility of aggregate per capita consumption in the postwar period in the United States and 3.6% from 1889 to 1978, as given in table 2 of Mehra & Prescott (1985). This level of volatility in actual individual household level consumption and limited participation households is one potential rationale for the equity premium puzzle.

Brav, Constantinides & Geczy (2002) had a dual goal of investigating the pricing implications of the incompleteness of markets that insure against idiosyncratic income shocks and the limited participation of households in the capital markets. Brav, Constantinides & Geczy (2002) defined a stochastic discount factor (SDF) or pricing kernel, m_t, as a function with the following property for all assets j:

$$E[\widetilde{m}_t \widetilde{R}_{j,t} | F_{t-1}] = 1, \quad j = 1, \ldots, J. \tag{50}$$

And applying this to any two assets, such as the market portfolio M and risk-free rate F, we have

$$E[\widetilde{m}_t(\widetilde{R}_{M,t} - R_{F,t})] = 0. \tag{51}$$

They noted that, quite generally, owing to the Euler equations given in Equations 7 and 8, every individual's marginal rate of substitution of consumption $\left(u'_{ts}/u'_0\right)$ across dates and states should be a valid SDF. In addition, any weighted sum of the households' SDFs is also a valid SDF. This is true whether markets are complete or incomplete. Accordingly, they computed the unexplained mean premium statistic, u, using historic data for the equity risk premium from

$$u = T^{-1} \sum_{t=1}^{T} m_t (R_{M,t} - R_{F,t}). \tag{52}$$

They also examined the value premium by using returns for high book/market and low book/market portfolios and computing a "conditional Euler equation, where the attribute of book/market is the conditioning variable" (Brav, Constantinides & Geczy 2002, p. 797), resulting in their unexplained premium statistic:

$$u = T^{-1} \sum_{t=1}^{T} m_t (R_{H,t} - R_{L,t}). \tag{53}$$

Then, using an assumption of power utility/CRRA, Brav, Constantinides & Geczy (2002) estimated the individual household's marginal rate of substitution by raising the household's consumption growth to a power equal to the negative of the RRA coefficient. With a time discount factor of $\beta = e^{-\rho t}$ and $\alpha =$ RRA in Brav, Constantinides & Geczy's notation, the SDF for a group of households is

$$m_t = \beta \left(\frac{\sum_{i=1}^{I} c_{i,I}}{\sum_{i}^{I} c_{i,t-1}} \right)^{-\alpha}. \tag{54}$$

Brav, Constantinides & Geczy filtered the data so that extreme outliers do not dominate the results and then tested that the Euler equations hold and the mean values of the unexplained premium are individually zero.

Some of Brav, Constantinides & Geczy's (2002) test results are as follows: Using CRRA values from 0 to 9, they found that the unexplained equity risk premium test statistic is positive for low values of CRRA and negative for high CRRA, crossing zero at CRRA between 3 and 4. Many economists view this range as plausible. When skewness in consumption growth is considered, the unexplained equity premium remains positive for all levels of CRRA from 0 to 9 and even increases as CRRA increases, but the amount is statistically insignificant for CRRA greater than 2. Without considering skewness in consumption growth, the unexplained premium of value stocks over growth stocks crosses zero when CRRA is between 3 and 4. It also crosses with CRRA between 4 and 5 when skewness is considered. Thus, for very reasonable levels of a representative household's CRRA, Brav, Constantinides & Geczy appeared to provide an explanation for a higher-risk premium for the value stock portfolio compared with the growth stock portfolio based on limited participation of households in incomplete capital markets.

Vissing-Jørgensen (2002) focused on estimating the EIS with the Consumer Expenditure Survey data, using information on asset holdings to identify limited participation in stock and bond markets. As Vissing-Jørgensen (2002, p. 826) states, "The elasticity of intertemporal substitution determines how much consumers change their expected consumption growth rate in response to changes in the expected return to any such asset." Using the riskless rate, it would be the change in expected consumption growth for a 1% change in the interest rate. As the riskless rate increases, consumers reduce current consumption and have more to consume later, so the expected growth of consumption increases. The amount of that sensitivity is the EIS. A high EIS means that consumers are willing to vary their growth rate a lot in response to interest rate moves, giving a higher volatility of the lifetime consumption path. This would be consistent with a higher tolerance for lifetime consumption risk and a lower degree of risk aversion. Thus, as shown for the term structure theory (Equation 32), in the CRRA-lognormal model, the EIS would equal the inverse of RRA, i.e., EIS = 1/RRA. However, as Hall (1988) and Kocherlakota (1990) found, EIS can be quite different from relative risk tolerance (the inverse of RRA) when preferences have time complementarity. More precisely, risk aversion is perhaps best thought of with regard to intratemporal (timeless) gambles or choices among different risky assets at a point in time, whereas the EIS is more a descriptor of intertemporal responses to changes in the reward for deferring consumption.

Vissing-Jørgensen (2002) used micro data from the Consumer Expenditure Survey to argue that accounting for limited asset market participation is crucial for obtaining consistent estimates of the EIS. As she says, "The Euler equation should hold for a given household only if the household holds a nonzero position in the asset" (Vissing-Jørgensen 2002, p. 827). If households do not hold an asset, there is little reason why they would vary their consumption in response to changes in its expected return. Including them in estimates of EIS could lead to substantially downward-biased estimates. Using data for stockholders, Vissing-Jørgensen (2002) estimated EIS to be approximately 0.3 to 0.4, whereas for bondholders the EIS estimates are approximately 0.8 to 1.0. Inverting these numbers gives a CRRA estimate of 2.5 to 3.3 using the stockholder data and of 1.0 to 1.25 using the bondholder data. To estimate the EIS, she estimated the following equation:

$$\begin{aligned}\sigma \;=\; \frac{1}{H_t^s}\sum_{h=1}^{H_t^s}\Delta\ln C_{t+1}^{h,s} &= \sigma^s \ln(1+R_{s,t}) + \delta_1^s D_1 + \delta_2^s D_2 + \ldots + \delta_{12}^s D_{12}\\ &\quad + \alpha^s\frac{1}{H_t^s}\sum_{h=1}^{H_t^s}\Delta\ln(\text{family size})_{t+1}^{h,s} + u_{t+1}^s, \qquad (55)\end{aligned}$$

where σ denotes the EIS, $D_1, \ldots, D_{12}$ are seasonal dummies, R_s denotes the real net stock return, and H_t^s denotes the number of stockholders in the cross section at date t.

Vissing-Jørgensen (2002) performed the estimation using GMM with three different sets of instrumental variables: (*a*) the log dividend/price ratio; (*b*) the former and lagged real stock return; and (*c*) the dividend/price ratio, the bond horizon premium, and the bond default premium. She showed that the EIS for stockholders is approximately 0.30, which is much larger than the 0.06 for nonstockholders, a value that shows little sensitivity of nonstockholders to returns in the stock market. Stockholders show much more willingness to respond to better returns than do nonstockholders. Vissing-Jørgensen (2002) showed that this is even truer for wealthier households, as the lowest one-third of households by wealth have an estimated EIS of 0.05, the middle one-third have an EIS of 0.18, and the wealthiest one-third have an EIS of 0.49. Thus, Vissing-Jørgensen showed that limited asset market participation is very important for estimating the EIS. Differences across stockholders and nonstockholders, as well as between bondholders and nonbondholders, are large and statistically significant. This research should be quite useful to the Federal Reserve and other policy makers as they estimate the likely responses of consumers to changes in interest rates and risky investment returns.

Using updated household-level Consumer Expenditure Survey data (with help from Dana Kiku), we computed the volatility of real consumption growth for households with 1 person, then for those with two to three and four or more members. Volatility was also computed according to three levels of income: upper third, middle third, and lower third. Individual percentage

Table 14 Nondurables and services consumption growth volatility

	Persons in household	Income: Lower third	Middle third	Upper third	Row average
	1	11.7	12.4	12.5	9.4
	2–3	9.8	8.4	9.1	6.2
	≥4	13.5	10.3	7.6	6.7
Column average		8.6	8.0	7.3	6.4

consumption growth volatility in **Table 14** compares with volatility of approximately 1.0% for aggregate per capita consumption growth of nondurables and services. The calculations show that individuals experience real consumption volatility of more than 7%, which should reduce CRRA estimates by a factor of at least 7, bringing many risk aversion estimates into the 1 to 10 range that many believed was reasonable.

Malloy, Moskowitz & Vissing-Jørgensen (2009) found that, for example, at a horizon of 4 years, nonstockholders' real consumption growth moves approximately one-to-one with aggregate real growth, whereas the growth of all stockholders moves by 2.7 times as much and that of the wealthiest third, top stockholders moves by 3.9 times as much as the aggregate. There is no reason for the Euler equations for stocks to hold for nonstockholders, but they should hold for stockholders, especially for those with the most invested. Given the much higher sensitivities to aggregate consumption (picking up only systematic consumption fluctuations), Malloy, Moskowitz & Vissing-Jørgensen (2009) estimated RRA are much lower "at around 10" (p. 2427) and nearing a range that many economists find more plausible (**Table 15**).

Malloy, Moskowitz & Vissing-Jørgensen (2009) estimated consumption growth covariances of the 25 Fama-French portfolios with the different groups' consumption rates and found substantial differences, depending on whether consumption growth for top stockholders, stockholders, or nonstockholders was used (shown in **Figure 8**). Note the dispersion of consumption covariances (betas) between value and growth portfolios for stockholders versus for nonstockholders. Using stockholder consumption shows value stocks to have much higher consumption risk than for nonstockholders. This makes the cross-sectional relation of average stock returns (using 1926–2004 data) to consumption covariances (using the CEX data period 1982–2004) have a much better fit for stockholders (especially top stockholders), than for nonstockholders, as shown in **Figure 8**.

Table 15 Sensitivity of stockholder, top stockholder, and nonstockholder consumption growth to aggregate consumption growth across horizons

$S =$	1	2	4	8	12	16	20	24
Stockholder	0.68 (0.35)	0.93 (0.37)	1.21 (0.32)	1.57 (0.36)	2.12 (0.39)	2.68 (0.49)	2.68 (0.49)	2.42 (0.41)
Top stockholder	0.70 (0.90)	1.01 (0.77)	1.56 (0.62)	2.14 (0.49)	2.88 (0.53)	3.94 (0.67)	3.91 (0.73)	3.48 (0.63)
Nonstockholder	0.51 (0.23)	0.41 (0.27)	0.59 (0.26)	0.84 (0.27)	0.96 (0.29)	1.01 (0.26)	0.95 (0.24)	0.79 (0.27)

The sensitivity of stockholder, top stockholder, and nonstockholder consumption growth to aggregate consumption growth from national income and product accounts is reported over horizons of $S = 1, 2, 4, 8, 12, 16, 20$, and 24 quarters. The sensitivity of each group's consumption growth to aggregate consumption growth is computed as the regression coefficient from regressing a group's discounted consumption growth over horizon S on aggregate discounted consumption growth over the same horizon. Standard errors (in parentheses) on the regression sensitivity measure are computed using a Newey-West estimator that allows for autocorrelation of up to $S \times 3 - 1$ month lags. Group consumption growth rates are calculated using data from the Consumer Expenditure Survey over the period March 1982 to November 2004. Data excerpted from Malloy, Moskowitz & Vissing-Jorgensen (2009, table 1).

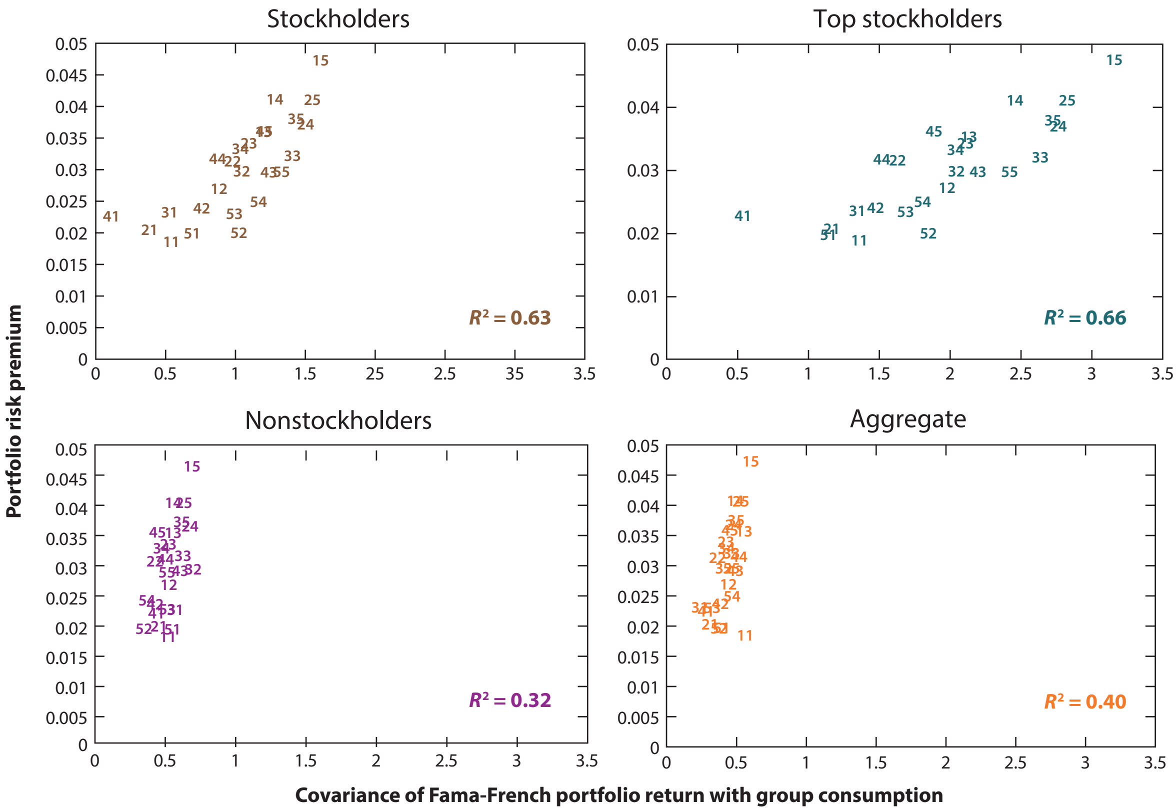

Figure 8

Mean returns versus consumption covariances for stockholders, top stockholders, and nonstockholders. Data from the Consumer Expenditure Survey of the US Bureau of Labor Statistics. Figure adapted from Malloy, Moskowitz & Vissing-Jørgensen (2009, figure 1).

Malloy, Moskowitz & Vissing-Jørgensen (2009) extended their computations by using the CMP technique of Breeden, Gibbons & Litzenberger (1989), as in Section 5. Their CMP is from regressions of 16 quarters of consumption growth on portfolios of stocks representing stocks with small growth, large growth, small value, and large value. With this CMP, they extended their data to the entire Center for Research in Security Prices period from 1926 to 2004. With this long data set, they computed time-varying factor loadings and conditional consumption risks by having cross products with cay, the deviation of consumption from wealth and income of Lettau & Ludvigson (2001a). They also showed that cay and the consumption share of stockholders are quite strongly negatively correlated (−0.44), as shown in **Figure 9**. Malloy, Moskowitz & Vissing-Jørgensen (2009, p. 2,459) found, "For each group, risk aversion estimates are similar in magnitude" to their prior estimates and indicate "that accounting for time variation in factor loadings from 1951 to 2004 does not seem to alter our estimates."

Constantinides & Duffie (1996) developed an elegant theoretical model of the impact of substantial heterogeneity of individuals' incomes on asset pricing. They modeled consumers who have power utility functions (CRRA) but have uninsurable, persistent, and heteroscedastic labor income shocks. This is quite a reasonable and important model, given the differences we have seen in households' consumption volatility versus aggregate consumption volatility (as shown in **Tables 9** and **10**). Constantinides & Duffie (1996, p. 223) observed, "the model predicts that a

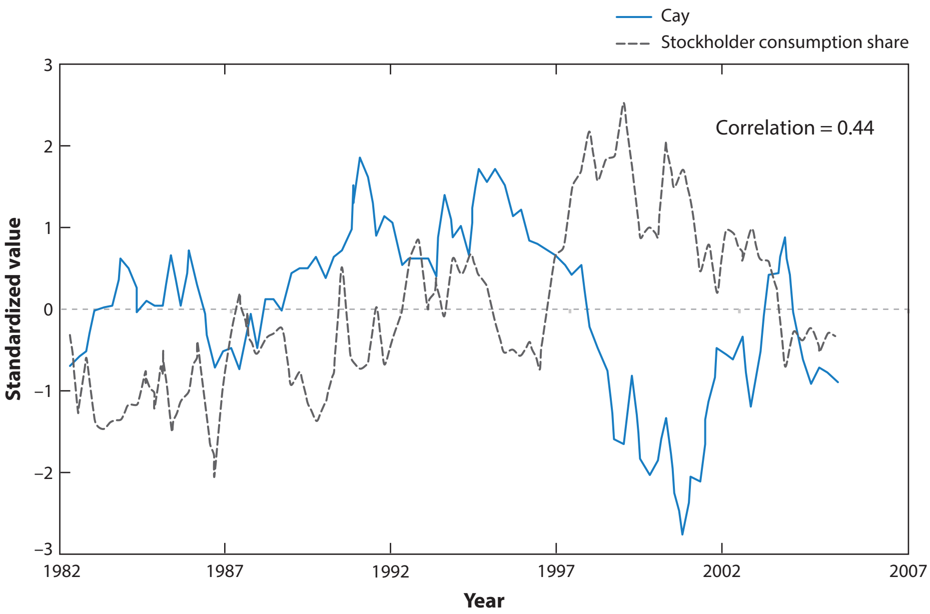

Figure 9

Consumption-to-wealth ratio (cay) (*blue line*) and stockholder consumption share (*black dashed line*). Plots show the consumption-to-wealth ratio of Lettau & Ludvigson (2001a) along with the ratio of the quarterly consumption of stockholders (using our baseline stockholder definition) to total quarterly Consumer Expenditure Survey (CEX) consumption. For readability, each time series is standardized by subtracting its mean and dividing by its standard deviation calculated over the CEX sample period. Figure adapted from Malloy, Moskowitz & Vissing-Jørgensen (2009, figure 2).

potential source of the equity premium is the covariance of the securities returns with the cross-sectional variance of individual consumers' consumption growth, a source that is (under typical conditions) irrelevant in an economy with full consumption insurance." With CRRA $= \alpha$ and their carefully built model of individual income shocks, the Euler equation is derived in terms of aggregate consumption, C_{t+1}, and a measure y^2_{t+1} that "is interpreted as the variance of the cross-sectional distribution of log $[(C_{i,t+1}/C_{t+1})/(C_{i,t}/C_t)]$" (Constantinides & Duffie 1996, p. 229), where $i = 1$ to N represents different individuals:

$$E\left[\widetilde{R}_{j,t+1}e^{-\rho}\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)^{-\alpha}\exp\left(\frac{\alpha(\alpha+1)}{2}y^2_{t+1}\right)|\phi_t\right] = 1. \tag{56}$$

If consumers were identical in preferences and incomes, then y^2 would be zero and the Euler equation reduces to the familiar one for CRRA.

Constantinides & Duffie (1996) also proved the equilibrium solution of their heterogeneous consumer model is isomorphic to that of the representative consumer model, by assuming specific forms of the y_t function, such as the following:

$$y^2_{t+1} = a + b\log\left(\frac{C_{t+1}}{C_t}\right). \tag{57}$$

Substituting this function into the general Euler equation gives a reduced form of the Euler equation corresponding to the classical one with representative consumers:

$$E\left[\widetilde{R}_{j,t+1}e^{-\hat{\rho}}\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)^{-\hat{\alpha}}|\Phi_t\right]=1, \quad (58)$$

where the modified risk aversion coefficient $\hat{\alpha}$ deviates from the one without heterogeneity, α:

$$\hat{\alpha}=\alpha-\frac{\alpha(\alpha+1)}{2}b. \quad (59)$$

Though this specific function of y_t is rejected when testing the Euler equation, their discussion is of general interest in explaining the size of the equity risk premium. If the cross-sectional variation is countercyclical, b is negative and the modified risk aversion $\hat{\alpha}$ is higher than the classical risk aversion parameter α. Therefore, an econometrician unaware of the heterogeneity would mistake $\hat{\alpha}$ as α and overestimate the risk aversion parameter under a homogenous model.

The Constantinides-Duffie derivation of the importance for asset pricing of the cross-sectional heterogeneity of individual consumers' actual incomes and consumption volatilities, rather than just the volatility of the aggregate, is a significant and economically intuitive result in asset pricing. Almost nobody has consumption volatility as low as the aggregate measures, so is it not intuitive that market pricing would reflect covariances with individuals' actual marginal utilities, which reflect their individual consumption volatilities? This is an area worthy of additional research. We need to understand the consistency of this result with Breeden's (1979) continuous-time CCAPM with heterogeneous consumers, wherein consumer spending can be aggregated in asset pricing results.

8. CONSUMPTION MEASUREMENT AND CHANGES IN CONDITIONAL RISKS AND RISK PREMIA

In their classic articles, Fama & French (1992, 1993) showed that average returns on stocks and bonds were related to five major factors (three from stocks, two from bonds):

1. a general stock market risk estimated by market betas,
2. a factor related to differential performance of small stocks versus large stocks (small minus big),
3. a factor related to differential performance of high versus low book/market stocks, which picks up a premium of value versus growth stocks (high minus low),
4. a term premium variable measured by the return on long-term government bonds, less that on 1-month Treasury bills, and
5. a default premium, measured by the return on corporate bonds, less that of government bonds.

As the following regression from Fama & French (1993, equation 1) shows, the market portfolio's excess return is a hodgepodge of the common risk factors, even showing statistically significant common variation with the two bond market–related factors (t-statistics in parentheses; $R^2 = 0.38$):

$$\mathrm{RM}-\mathrm{RF}=\underset{(2.55)}{0.50}+\underset{(6.48)}{0.44}\mathrm{SMB}-\underset{(-8.23)}{0.63}\mathrm{HML}+\underset{(9.09)}{0.81}\mathrm{TERM}+\underset{(4.62)}{0.79}\mathrm{DEF}+e. \quad (60)$$

Equation 60 shows that stocks tend to do better when small stocks outperform large stocks, when growth stocks beat value stocks, when long-term bonds beat Treasury bills, and when credit risk is rewarded.

Fama & French (1992, 1993) raised two new puzzles that stimulated a large literature in subsequent years. In 1992, they found that the "relation between market beta and average return is flat" (Fama & French 1992, p. 427). Contrary to the prediction of the CAPM, they found no significant reward for taking equity beta risk in their 1963–1991 sample. Their first puzzle showed that small stocks outperformed large stocks (small minus big effect > 0 on average), even after taking into account equity market beta differentials. Their second puzzle showed that value stocks outperformed growth stocks (high minus low effect > 0 on average), also after taking into account equity market beta differentials.

These surprising results have been attributed to the relatively small sample (1963–1991), and their estimates have high standard errors. Although the average returns versus betas relationship improves with longer data series and more asset classes, the size and book/market results have held over longer time periods. As shown in subsequent key articles by Lettau & Ludvigson (2001b) and Jagannathan & Wang (2007) on the CCAPM, perhaps the size and value/growth effects can be explained by their differential conditional consumption risks. For example, value stocks have relatively higher consumption betas in recessions, when risks and risk premiums per unit of risk are high, and they have lower conditional consumption betas in growth periods, when risks and risk premiums are low. This effect makes the unconditional risk premium for value stocks larger, owing to the positive correlation of their consumption betas with market risk premiums. Alternatively, the growth stock portfolio has an unconditional convex relation to consumption indicated by a higher CCAPM beta conditional on expansion periods and a lower CCAPM beta conditional on recessionary periods. Conversely, the value stock portfolio has an unconditional concave relation to consumption indicated by a lower CCAPM beta conditional on expansion periods and higher CCAPM beta conditional on recessionary periods. Consistent with Jagannathan & Wang (2007), the analysis of Kraus & Litzenberger (1983) suggests that, under decreasing absolute risk aversion, an individual would have a preference for positive skewness and the unconditional risk premium for the growth stock portfolio would, ceteris paribus, be less than that for the value stock portfolio.

In an important article, Parker & Julliard (2005) developed a model of ultimate consumption risk that captures the longer-run relationship of consumption with asset returns. They argued,

> [M]ost importantly, the ultimate risk may be a better measure of the true risk of an asset if consumption is slow to adjust to returns. If consumption responds with a lag to changes in wealth, then the contemporaneous covariance of consumption and wealth understates or mismeasures the true risk of a portfolio. Ultimate consumption risk, on the other hand, can provide the correct measure of risk under several extant explanations of slow consumption adjustment, such as some models of (*a*) measurement error in consumption; (*b*) costs of adjusting consumption; (*c*) nonseparability of marginal utility of consumption from factors such as labor supply or housing stock, which themselves are constrained to adjust slowly; or (*d*) constraints on information flow or calculation so that household behavior is 'near-rational.' (Parker & Julliard 2005, p. 186)

Their main finding is that ultimate consumption risk can largely explain the cross-sectional pattern of expected portfolio returns. Whereas the covariance of each portfolio and contemporaneous consumption growth explains little of the variation in expected returns across portfolios, at a horizon of 3 years the ultimate risk to consumption explains 44–73% of the variation in expected returns across portfolios, depending on the specification. The performance of ultimate consumption risk rivals that of the three-factor models of Fama & French (1992) and of Lettau & Ludvigson (2001b), two important linear factor models that have been used to price the expected returns in the Fama-French portfolios.

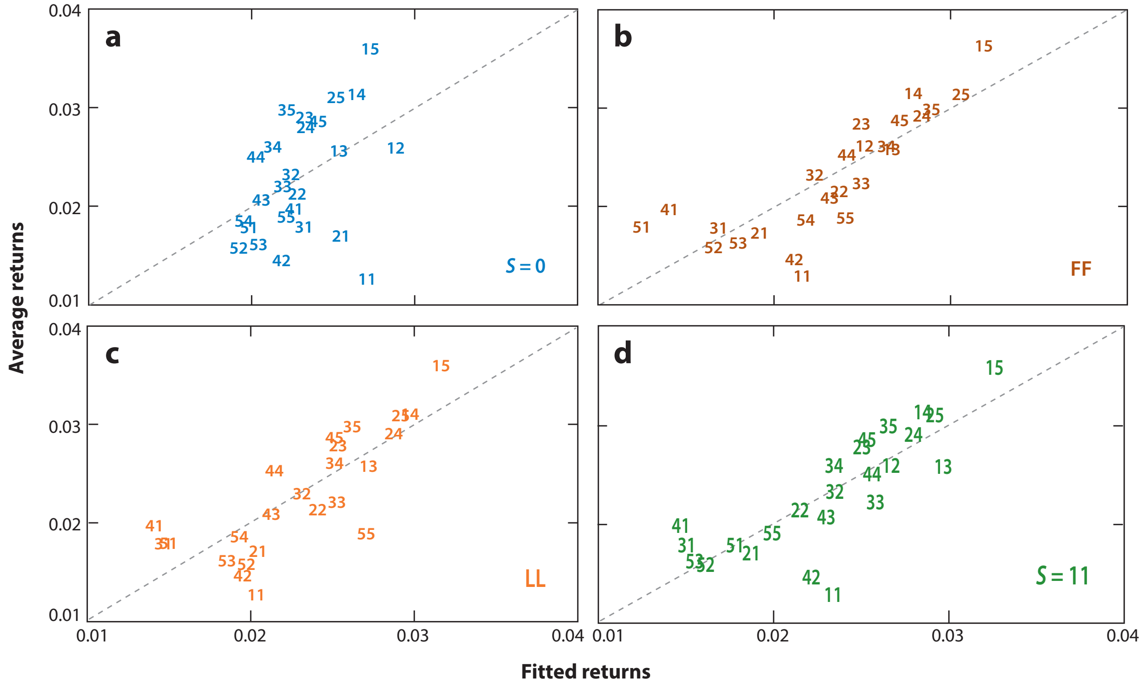

Figure 10

Comparison of fitted returns against average returns across four models: (*a*) consumption capital asset pricing model (CCAPM) with only contemporaneous consumption risk, (*b*) the Fama-French (FF) three-factor model, (*c*) the conditional test of the CCAPM of Lettau & Ludvigson (2001b) (LL) using their consumption/wealth ratio, cay, as a conditioning variable, and (*d*) ultimate consumption risk over 11 quarters. Figure adapted from Parker & Julliard (2005).

Parker & Julliard (2005) proposed ultimate consumption risk to explain the cross-sectional heterogeneity of the 25 Fama-French portfolios. By ultimate consumption, they mean that increasing the consumption growth horizon to longer terms can increase cross-sectional explanatory power of the canonical CCAPM model, and the optimal point is achieved at approximately 3 years (11 quarters) with a 44–73% R^2, depending on the specifications. They found that R^2 grows from near zero for contemporaneous consumption risk to 44% when consumption risk is measured by cumulative real nondurables consumption growth over the next 11 quarters. Note that, even though ultimate consumption risk helps explain the cross section of returns, the RRA estimate is quite high at approximately 25. Parker & Julliard (2005) also compared the empirical performance of four models (see **Figure 10**).

What drives the excellent results for the ultimate consumption risk measure of Parker & Julliard (2005) is the fact that the excess returns of the Fama-French portfolios predict future consumption growth. As they state, "both the excess return of small firms less large firms and the excess return of high-value stocks less low-value stocks predict consumption growth. The joint significance of these two excess returns in predicting consumption growth peaks at the horizon of three years" (Parker & Julliard 2005, p. 187). However, using the GMM estimator, the overidentification test still rejects the ultimate consumption model with 26 moments and only three parameters, so further analysis is required. Though their observations concerning longer-term consumption effects are intuitive, the specification based on declining weights over time would be more intuitive than equal weighting over a fixed horizon. **Figure 11** shows the difference in return to factor portfolios

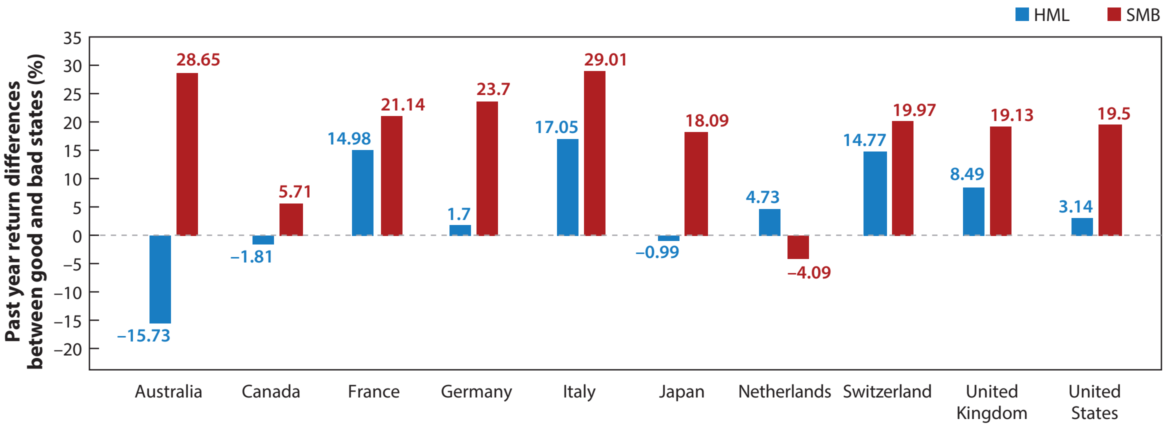

Figure 11

Difference in return to factor portfolios in the year prior to above-average versus below-average GDP growth for various countries. Abbreviations: HML, high minus low; SMB, small minus big. Figure adapted from Liew & Vassalou (2000).

in the year prior to above average versus below average GDP growth (see Liew & Vassalou 2000; Bodie, Kane & Marcus 2011, figure 13.1). Both small-minus-big and high-minus-low portfolios are typically higher prior to improving GDP growth.

Jagannathan & Wang (2007) also provided strong empirical support for consumption-based asset pricing. They proposed a theory of consumers who use discretion in periodically reviewing consumption and investment decisions, as opposed to continuously optimizing their consumption and investment rules. They argued that consumers are more likely to review their decision making at tax-year ends (in the fourth quarter) and during economic contractions. They demonstrated how this assumption helps explain the unsatisfactory previous testing results of the CCAPM. They computed the excess returns and consumption betas for size-ranked and book/market-ranked ($5 \times 5 = 25$) portfolios as shown in **Table 16**. A simple scatter plot of returns versus consumption betas for the 25 Fama-French portfolios sorted on size and value also shows a strong relationship (**Figure 12**).

Jagannathan & Wang (2007) further showed that the differencing interval and data series matters for relations of cross-sectional consumption betas to average returns. They found that four-quarter percentage changes in real consumption growth (Q4–Q4) and returns are most significant, as their model of normal decision making and planning suggests (see **Table 17**). Jagannathan & Wang (2007) modeled conditional consumption betas by using NBER business-cycle dates as a dummy variable for regressions, which is important in modeling risks because consumption betas increase during contractions, relative to expansions (see **Table 18**).

Petkova & Zhang (2005) provided another way of seeing the changing conditional risks in comparisons of betas for value and growth stocks (**Figure 13**). Jagannathan & Wang (2007) found that the CCAPM performs as well as, if not better than, the Fama-French three-factor model in explaining cross-sectional return differences among the 25 Fama-French portfolios sorted by size and book/market, using the year-over-year fourth quarter consumption growth rate. In addition, tests show the contraction beta is more significant than expansion beta. Both findings support their hypothesis (see **Figure 14**).

The Jagannathan-Wang discretionary consumption decision model was motivated by several important prior empirical tests, including conditioning, long-run risk, and ultimate consumption, as all these models take into consideration the large errors in estimating short-term consumption

Table 16 Annual excess returns and consumption betas

	Book to market (from low to high)				
Average annual excess returns (%)					
Size (small to large)	6.19	12.47	12.24	15.75	17.19
	5.99	9.76	12.62	13.65	15.07
	6.93	10.14	10.43	13.23	13.94
	7.65	7.91	11.18	12.00	12.35
	7.08	7.19	8.52	8.75	9.50
Consumption betas					
Size (small to large)	3.5	5.5	4.3	4.8	5.9
	2.9	3.0	4.8	4.3	5.2
	2.9	4.1	4.4	4.8	5.7
	2.6	3.4	3.9	4.8	5.6
	3.4	2.3	2.8	4.1	4.4
***t*-values**					
Size (small to large)	0.9	1.7	1.6	1.8	2.1
	1.0	1.3	2.0	1.8	2.1
	1.2	1.9	2.2	2.1	2.4
	1.1	1.8	1.9	2.3	2.4
	1.7	1.3	1.7	2.2	2.0

The top panel presents average annual excess returns on the 25 Fama-French portfolios from 1954 to 2003. Annual excess return is calculated from January to December in real terms. All returns are annual percentages. The middle panel presents these portfolios' consumption betas estimated by the time-series regression $R_{i,t} = \alpha_i + \beta_{i,c}\Delta c_t + \varepsilon_{i,t}$, where $R_{i,t}$ is the excess return over the risk-free rate and Δc_t is Q4–Q4 consumption growth calculated using fourth-quarter consumption data. The bottom panel presents *t*-values associated with consumption betas. Data reproduced from Jagannathan & Wang (2007), with numbers rounded here.

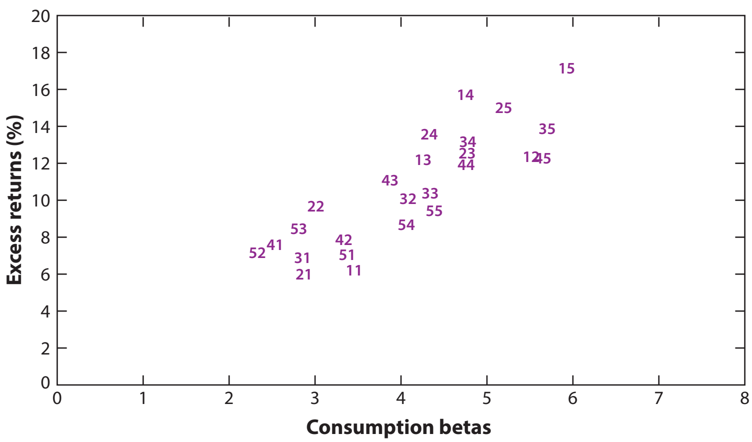

Figure 12

Annual excess returns and consumption betas using 25 Fama-French portfolios from 1954 to 2003. Figure adapted from Jagannathan & Wang (2007, figure 1).

Table 17 Consumption capital asset pricing model (CCAPM) with different frequency data

	Monthly consumption data			Quarterly consumption data			Annual consumption data		
Consumption growth									
Monthly growth	Month–month								
Quarterly growth	December–March, March–June			Quarter–quarter					
	June–September, September–December								
Annual growth	December–December			Q4–Q4			Annual–annual		
Cross-sectional regression results									
	Monthly consumption data			**Quarterly consumption data**			**Annual consumption data**		
	λ_0	λ_1	R^2	λ_0	λ_1	R^2	λ_0	λ_1	R^2
Monthly return	7.70	0.02	0.00						
***t*-value**	2.61	0.17	−0.04						
Quarterly return	8.34	0.03	0.00	4.52	0.33	0.22			
***t*-value**	2.80	0.15	−0.04	1.83	1.59	0.18			
Annual return	−1.83	2.01	0.41	−1.19	2.68	0.69	10.12	1.32	0.21
***t*-value**	−0.51	2.33	0.38	−0.37	3.49	0.68	3.70	1.61	0.18

Different frequency returns and consumption data are used to test the CCAPM. The top panel describesHow consumption growth is calculated. For example, with monthly consumption data, annual consumption growth is measured using December consumption of one year and December consumption of the following year. The bottom panel presents cross-sectional regression estimation results for the CCAPM: $E[R_{i,t}] = \lambda_0 + \lambda_1 \beta_{i,c}$. Test portfolio returns are annualized excess returns on the 25 portfolios of Fama & French (1993) from 1960 to 2003. (Monthly consumption data are available from 1959.) Data reproduced from Jagannathan & Wang (2007, table 7).

data and the discretionary psychology involved in consumption decision making. As Jagannathan & Wang (2007) pointed out, discretion is perhaps less consistent with the habit-formation models of Constantinides (1990) and Campbell & Cochrane (1999) because consumption is likely less dependent on the historical levels if more discretion is taken to review the decision. They reconciled the two effects and showed that the habit-formation effect is dominated by the classic consumption beta in asset pricing when the time between consumption decisions increases. However, this analysis is based on the assumption of simultaneous and infrequent decision making of all consumers, which is exogenous to the model and requires scrutiny. When a high implied risk aversion parameter is found, Jagannathan & Wang (2007) attributed it to the lack of realism of the assumption that consumers simultaneously make choices only once a year. Overall, Jagannathan & Wang (2007) added to the string of significant papers that found consumption-based asset pricing, using alternative specifications of individuals' consumption and portfolio choices, explains cross-sectional returns and the Fama-French size and book/market anomalies. However, the different specifications lead to different economic understandings of the causal behavior generating the observed returns.

9. CONCLUSION

In this article, we first review the late-1970s' major theoretical derivations that led to consumption-based asset pricing models, one of the leading theories of asset pricing in the past four decades. Next, we examine empirical testing and the evolution of both theory and empirical testing and of applications of consumption-based asset pricing. We begin with heterogeneous individuals

Table 18 Consumption beta in contractions and expansions

	Intercept	Contraction	Expansion	R^2 (adjusted R^2)
Estimate	0.86	0.98	0.23	0.65
t-value	0.50	6.11	0.67	0.62
Estimate	0.84	1.06		0.65
t-value	0.50	7.51		0.62
Estimate	6.10		1.40	0.33
t-value	4.71		4.78	0.26

Cross-sectional regression results of the consumption capital asset pricing model during different subperiods. First, we estimate the correlation consumption beta and the expansion consumption beta by the time-series regression $E_t[R_{i,t+4}] = \alpha_{i,\mathrm{const}}\mathrm{I}_t + \alpha_{i,\exp}(1 - I_t) + \beta_{i,\mathrm{cont}}\Delta \mathrm{c}_{t+4}\mathrm{I}_t + \beta_{i,\exp}\Delta \mathrm{c}_{t+4}(1 - \mathrm{I}_t)$, where $I_t = 1$ if the economy is contracting according to the National Bureau of Economic Research business cycle dating, otherwise $I_t = 0$; $\beta_{i,\mathrm{cont}}$ is the contraction consumption beta, and $\beta_{i,\exp}$ is the expansion consumption beta. Then we run the cross-sectional regression $E\left[R_{i,t+4}\right] = \lambda_0 + \lambda'\beta_i$. $R_{i,t+4}$ are annual excess returns of the 25 Fama-French portfolios from quarter t to quarter $t+4$ for all quarters from 1954 to 2003. The total number of observations is 200, including 43 quarters of contractions and 157 quarters of expansions. Within the 43 recession quarters, there are 11 first quarters, 9 second quarters, 11 third quarters, and 12 fourth quarters. Data reproduced from Jagannathan & Wang (2007, table XI).

with time-additive utility functions and derive that their optimal responses to market price signals should lead them to coordinate their consumption plans so that they all have high consumption when aggregate real consumption increases and have low consumption when aggregate real consumption decreases, leading to the major aggregation theorem. Even without effectively complete markets with a full set of hedges available, each individual optimally chooses a dynamic portfolio and optimal consumption plan to achieve the maximum correlation possible for the individual's

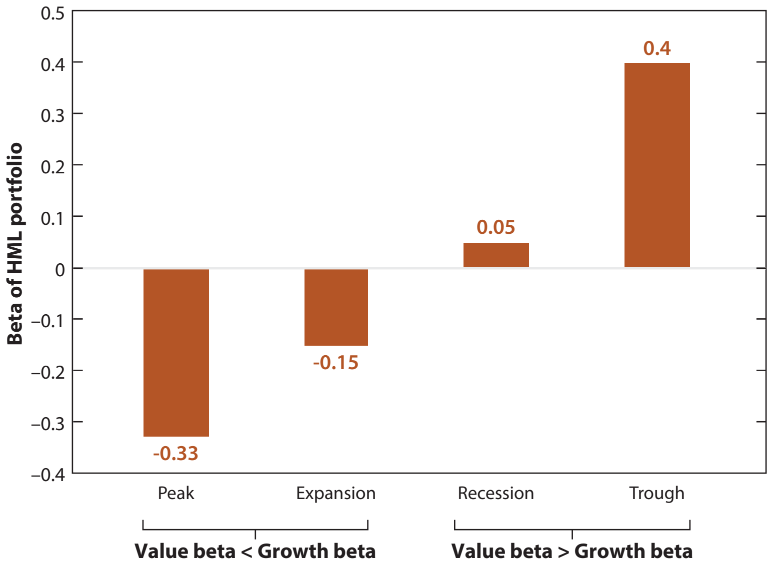

Figure 13

Conditional betas showing risks of value stocks increase in recessions. Abbreviation: HML, high minus low. Figure adapted from Petkova & Zhang (2005), previously printed in Bodie, Kane & Marcus (2011, figure 13.2).

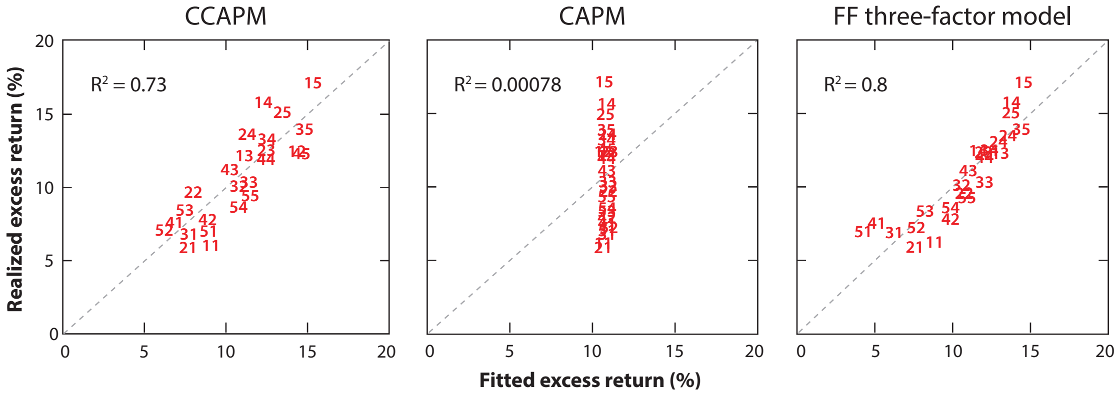

Figure 14

Realized excess returns mapped against fitted excess returns, showing (*left to right*) consumption capital asset pricing model (CCAPM), CAPM, and a Fama-French (FF) three-factor model. Figure adapted from Jagannathan & Wang (2007).

consumption with aggregate, per capita consumption. In that economy in a continuous-time model, the CCAPM is derived. Consumption betas capture all of the risks and risk premiums in Merton's continuous-time ICAPM, even with heterogeneous individuals and incomplete markets. Identical investors or a representative individual are not required.

The 1980s empirical tests showed a risk premium for consumption risk, much as there was for market risk, but the CCAPM was rejected, much like the original market-oriented CAPM was. Assets' conditional risk premiums are not higher in proportion to their conditional consumption betas, as predicted by the CCAPM and the market-oriented CAPM. An equity premium puzzle was identified in the 1980s, which identified a mismatch between a relatively high risk premium on equities and very low volatility of real aggregate consumption and relatively low correlation of equity returns with consumption fluctuations. Researchers also found that models often had real riskless interest rates that were higher than historical data, which gave rise to what some identified as a riskless rate puzzle.

Research proceeded along several dimensions, first by recognizing measurement of consumption as an integral of daily/instantaneous consumption, as opposed to stock returns as point-to-point returns. Next, researchers noted the considerable noise in consumption measurements versus consumers' fundamental economic desires, reflecting often large but transitory impacts of weather, tax changes, labor strikes, etc., on income and consumption. Furthermore, expenditures on a bundle of consumption goods with varying degrees of durability yield consumption flows in subsequent months. Monthly data for real consumption growth showed negative autocorrelation (presumably owing to weather, which can cause large monthly swings that dominate regression results), whereas quarterly, semiannual, and annual autocorrelations were positive, reflecting business-cycle impacts. Tests that relied on monthly consumption growth were misleading. Subsequent researchers found it more sensible to use 6-month, two-quarter, or four-quarter returns, or even ultimate consumption betas that measured covariances over almost 3 years (11 quarters).

From 1991 to 2002, researchers used survey data compiled by the US government on actual consumer spending by individual households to see how individual consumption volatility differed from aggregate consumption volatility. Differences were presumed to be due to incomplete markets for insuring major risks (such as for labor income) as well as to limited participation, in that some consumers simply did not invest in the stock market. If they did not invest, their first-order conditions for an optimum likely were not met. Thus, examining the consumption of stockholders versus that of nonstockholders yielded considerable insights. First, volatility for a typical household was 5–10 times as large as for aggregate real consumption, which helped to resolve the risk premium puzzle caused in part by the low volatility of aggregate consumption. Second, both risk aversion and EIS estimates (which are closely related to riskless interest rates) are much different between stockholders and nonstockholders and between bondholders and nonbondholders.

These findings on incomplete markets and limited participation helped researchers to resolve both the risk premium puzzle and the riskless rate puzzle. However, in the early 1990s, Fama & French (1992, 1993) found two new puzzles in cross-sectional stock returns over long periods of time: the size effect (small stocks earned higher returns than did large stocks, beta adjusted) and the value premium (where high book/market stocks, value stocks, earned more than did growth stocks, beta adjusted). Subsequent researchers tested their models on 25 (5×5) portfolios stratified by size and value to see if they could explain these important results. Lettau & Ludvigson (2001b) (see Part 2 of this review), Parker & Julliard (2005), and Jagannathan & Wang (2007) found that their better measurements of conditional consumption betas helped explain the size and book/market effects.

Consumption-based asset pricing yielded many theoretical and empirical insights in its first two-plus decades. Part 2 of this review presents more recent models of consumption-based asset pricing, including models with habit formation in utility functions as well as long-run risks in consumption growth. The financial economics literature has produced a vast body of competing economic rationales for what earlier authors characterized as puzzles and paradoxes. To borrow an econometric term, we have overidentified these puzzles and paradoxes. Further research should attempt to integrate models and sort through them for the most important results and develop tests based on different model predictions.

DISCLOSURE STATEMENT

The authors are not aware of any affiliations, memberships, funding, or financial holdings that might be perceived as affecting the objectivity of this review.

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