Falling Objects – Empire State Building

Working with Quadratics

"When am I ever going to use this?"

Using the concepts in this worksheet, you will be able to estimate the speed and height of any falling object.

The Empire State Building, at a height of 373 meters (excluding its antenna), is one of the tallest buildings in the United States. The building's 86th Floor Observatory is at a height of 320 meters (Source: www.esbnyc.com.) Dropping any object from this height can be extremely dangerous to people below.

The height of a falling object above the ground (in meters) may be modeled by $s(t) = \frac{1}{2}gt^2 + v_0t + s_0$ where $g = -9.8$ m/sec^2 is acceleration due to gravity, v_0 is the initial velocity, and s_0 is the initial height of the object.

1. Explain the practical meaning of $g = -9.8$ m/sec^2 as related to the velocity of the object.

2. If an object is dropped from a height of 320 meters, how fast is it falling after 3 seconds?

3. To understand why it is dangerous to throw anything from an observatory, we investigate the height and velocity of a penny thrown from 2 meters above the observatory floor into the air at a velocity of 8 meters per second.

   a. Write an equation for the velocity of the penny at time t seconds.

b. Write an equation for the height of the penny at time t seconds.

c. When with the penny hit the ground? (Round to the nearest tenth of a second.)

d. How fast will the penny be moving when it hits the ground? Give your answer in meters per second.

e. Convert the velocity in (d) to kilometers per hour. (Hint: There are 1000 meters in 1 kilometer.)

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The height of a falling object above the ground (in meters) may be modeled by $s(t) = \frac{1}{2}gt^2 + v_0t + s_0$ where $g = -9.8$ m/sec^2 is acceleration due to gravity, v_0 is the initial velocity, and s_0 is the initial height of the object.

1. Explain the practical meaning of $g = -9.8$ m/sec^2 as related to the velocity of the object.

Another way to write $g = -9.8$ m/sec^2 *is* $\dfrac{-9.8\,meters\,per\,second}{1\,second}$. *This means that the velocity of the object is decreasing by 9.8 meters per second each second.*

2. If an object is dropped from a height of 320 meters, how fast is it falling after 3 seconds?

Since the object is dropped, its initial velocity is 0 meters per second. After 1 second, its velocity is −9.8 meters per second. After 2 seconds, its velocity is −19.6 meters per second. After 3 seconds, its velocity is −29.4 meters per second. With each additional second, the velocity changes by −9.8 meters per second.

3. To understand why it is dangerous to throw anything from an observatory, we investigate the height and velocity of a penny thrown from 2 meters above the observatory floor into the air at a velocity of 8 meters per second.

a. Write an equation for the velocity of the penny at time t seconds.

The initial velocity is 8 meters per second and it is changing by −9.8 meters per second each second. Therefore, $v(t) = -9.8t + 8$ *is the velocity function.*

b. Write an equation for the height of the penny at time t seconds.

The observatory is at 320 meters so the initial height of the penny is 322 meters. The initial velocity is 8 meters per second and $g = -9.8$ m/sec^2. *Substituting these values into* $s(t) = \frac{1}{2}gt^2 + v_0t + s_0$, *yields*

$$s(t) = -4.9t^2 + 8t + 322.$$

c. When with the penny hit the ground? (Round to the nearest tenth of a second.)

We need to determine when the height of the penny will be 0 meters.

$$s(t) = -4.9t^2 + 8t + 322$$

We use the quadratic formula $t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ *with* $a = -4.9$, $b = 8$, *and* $c = 322$.

$$t = \frac{-8 \pm \sqrt{(8)^2 - 4(-4.9)(322)}}{2(-4.9)}$$

$$\approx -7.3,\ 9.0$$

It doesn't make sense to talk about negative time, so we throw out the negative value. The penny will hit the ground in about 9.0 seconds.

d. How fast will the penny be moving when it hits the ground? Give your answer in meters per second.

We know from (a) that $v(t) = -9.8t + 8$. *We need to find* $v(9.0)$.

$$v(9.0) = -9.8t + 8$$

$$= -80.2 \text{ meters per second}$$

The penny will be moving downward at a speed of 80.2 meters per second.

e. Convert the velocity in (d) to kilometers per hour. (Hint: There are 1000 meters in 1 kilometer.)

$$-80.2\frac{\cancel{\text{meters}}}{\cancel{\text{second}}}\left(\frac{60\ \cancel{\text{second}}}{1\ \cancel{\text{minute}}}\right)\left(\frac{60\ \cancel{\text{minutes}}}{1\,\text{hour}}\right)\left(\frac{1\,\text{kilometer}}{1000\ \cancel{\text{meters}}}\right) \approx -288.7\,\text{km per hour}$$

−80.2 meters per second is the same as −288.7 kilometers per hour.