Set 2: Multiple-Choice Questions on Limits and Continuity

1. $\lim\limits_{x\to 2}\dfrac{x^2-4}{x^2+4}$ is

(A) 1 (B) 0 (C) $-\dfrac{1}{2}$ (D) −1 (E) ∞

2. $\lim\limits_{x\to \infty}\dfrac{4-x^2}{x^2-1}$ is

(A) 1 (B) 0 (C) −4 (D) −1 (E) ∞

3. $\lim\limits_{x\to 3}\dfrac{x-3}{x^2-2x-3}$ is

(A) 0 (B) 1 (C) $\dfrac{1}{4}$ (D) ∞ (E) none of these

4. $\lim\limits_{x\to 0}\frac{x}{x}$ is

(A) 1 (B) 0 (C) ∞ (D) −1 (E) nonexistent

5. $\lim\limits_{x\to 2}\dfrac{x^3-8}{x^2-4}$ is

(A) 4 (B) 0 (C) 1 (D) 3 (E) ∞

6. $\lim\limits_{x\to \infty}\dfrac{4-x^2}{4x^2-x-2}$ is

(A) −2 (B) $-\dfrac{1}{4}$ (C) 1 (D) 2 (E) nonexistent

7. $\lim\limits_{x\to -\infty}\dfrac{5x^3+27}{20x^2+10x+9}$ is

(A) −∞ (B) −1 (C) 0 (D) 3 (E) ∞

8. $\lim\limits_{x\to \infty}\dfrac{3x^2+27}{x^3-27}$ is

(A) 3 (B) ∞ (C) 1 (D) −1 (E) 0

9. $\lim\limits_{x\to \infty}\dfrac{2^{-x}}{2^x}$ is

(A) −1 (B) 1 (C) 0 (D) ∞ (E) none of these

10. $\lim_{x\to-\infty} \frac{2^{-x}}{2^x}$ is

(A) -1 (B) 1 (C) 0 (D) ∞ (E) none of these

11. If $[x]$ is the greatest integer not greater than x, then $\lim_{x\to 1/2}[x]$. is

(A) $\frac{1}{2}$ (B) 1 (C) nonexistent (D) 0 (E) none of these

12. (With the same notation) $\lim_{x\to-2}[x]$ is

(A) -3 (B) -2 (C) -1 (D) 0 (E) none of these

13. The graph of $y = \arctan x$ has

(A) vertical asymptotes at $x = 0$ and $x = \pi$
(B) horizontal asymptotes at $y = \pm\frac{\pi}{2}$
(C) horizontal asymptotes at $y = 0$ and $y = \pi$
(D) vertical asymptotes at $x = \pm\frac{\pi}{2}$
(E) none of these

14. $\lim_{x\to\infty} \sin x$

(A) is -1 (B) is infinity (C) oscillates between -1 and 1
(D) is zero (E) is 1

15. The graph of $y = \frac{x^2-9}{3x-9}$ has

(A) a vertical asymptote at $x = 3$ (B) a horizontal asymptote at $y = \frac{1}{3}$
(C) a removable discontinuity at $x = 3$ (D) an infinite discontinuity at $x = 3$
(E) none of these

16. The function $f(x) = \begin{cases} x^2/x & (x \neq 0) \\ 0 & (x = 0) \end{cases}$

(A) is continuous everywhere
(B) is continuous except at $x = 0$
(C) has a removable discontinuity at $x = 0$
(D) has an infinite discontinuity at $x = 0$
(E) has $x = 0$ as a vertical asymptote

17. $\lim_{x\to 0} \frac{\sin x}{x^2+3x}$ is

(A) 1 (B) $\frac{1}{3}$ (C) 3 (D) ∞ (E) $\frac{1}{4}$

18. $\lim_{x \to 0} \sin \frac{1}{x}$ is

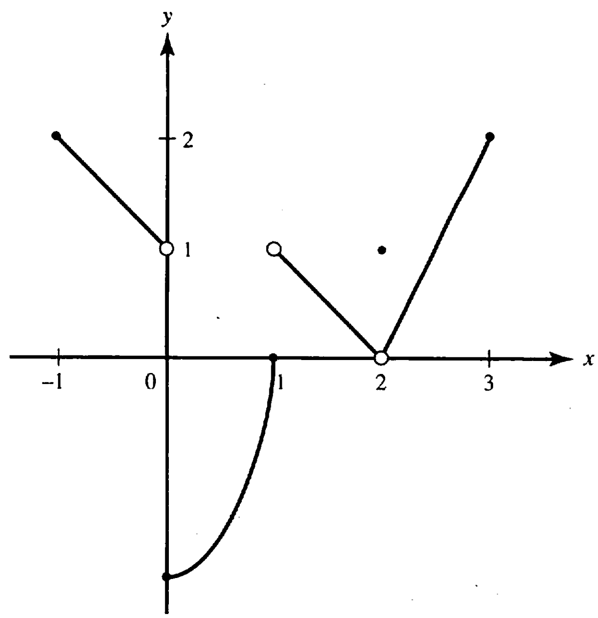
(A) ∞ **(B)** 1 **(C)** nonexistent **(D)** -1 **(E)** none of these

19. Which statement is true about the curve $y = \frac{2x^2 + 4}{2 + 7x - 4x^2}$?

(A) The line $x = -\frac{1}{4}$ is a vertical asymptote.
(B) The line $x = 1$ is a vertical asymptote.
(C) The line $y = -\frac{1}{4}$ is a horizontal asymptote.
(D) The graph has no vertical or horizontal asymptotes.
(E) The line $y = 2$ is a horizontal asymptote.

Questions 20 through 24 are based on the function f shown in the graph and defined below:

$$f(x) = \begin{cases} 1 - x & (-1 \leq x < 0) \\ 2x^2 - 2 & (0 \leq x \leq 1) \\ -x + 2 & (1 < x < 2) \\ 1 & (x = 2) \\ 2x - 4 & (2 < x \leq 3) \end{cases}$$



20. $\lim_{x \to 2} f(x)$

(A) equals 0 **(B)** equals 1 **(C)** equals 2
(D) does not exist **(E)** none of these

21. The function f is defined on $[-1,3]$

(A) if $x \neq 0$ **(B)** if $x \neq 1$ **(C)** if $x \neq 2$
(D) if $x \neq 3$ **(E)** at each x in $[-1,3]$

22. The function f has a removable discontinuity at

(A) $x = 0$ **(B)** $x = 1$ **(C)** $x = 2$ **(D)** $x = 3$ **(E)** none of these

23. On which of the following intervals is f continuous?

(A) $-1 \leq x \leq 0$ **(B)** $0 < x < 1$ **(C)** $1 \leq x \leq 2$
(D) $2 \leq x \leq 3$ **(E)** none of these

24. The function f has a jump discontinuity at

(A) $x=-1$ **(B)** $x=1$ **(C)** $x=2$
(D) $x=3$ **(E)** none of these

25. $\lim\limits_{x\to\infty} \dfrac{2x^2+1}{(2-x)(2+x)}$ is

(A) -4 **(B)** -2 **(C)** 1 **(D)** 2 **(E)** nonexistent

26. $\lim\limits_{x\to 0} \dfrac{|x|}{x}$ is

(A) 0 **(B)** nonexistent **(C)** 1 **(D)** -1 **(E)** none of these

27. $\lim\limits_{x\to\infty} x\sin\dfrac{1}{x}$ is

(A) 0 **(B)** ∞ **(C)** nonexistent **(D)** -1 **(E)** 1

28. $\lim\limits_{x\to\pi} \dfrac{\sin(\pi-x)}{\pi-x}$ is

(A) 1 **(B)** 0 **(C)** ∞ **(D)** nonexistent **(E)** none of these

29. Let $f(x)=\begin{cases} \dfrac{x^2-1}{x-1} & \text{if } x\neq 1 \\ 4 & \text{if } x=1 \end{cases}$

Which of the following statements is (are) true?

I. $\lim\limits_{x\to 1} f(x)$ exists. II. $f(1)$ exists. III. f is continuous at $x=1$.

(A) I only **(B)** II only **(C)** I and II
(D) none of them **(E)** all of them

30. If $\begin{cases} f(x)=\dfrac{x^2-x}{2x} \text{ for } x\neq 0, \\ f(0)=k, \end{cases}$

and if f is continuous at $x=0$, then $k=$

(A) -1 **(B)** $-\dfrac{1}{2}$ **(C)** 0 **(D)** $\dfrac{1}{2}$ **(E)** 1

31. Suppose $\begin{cases} f(x)=\dfrac{3x(x-1)}{x^2-3x+2} \text{ for } x\neq 1,2, \\ f(1)=-3, \\ f(2)=4. \end{cases}$

Then $f(x)$ is continuous

(A) except at $x=1$ **(B)** except at $x=2$ **(C)** except at $x=1$ or 2
(D) except at $x=0$, 1, or 2 **(E)** at each real number

32. The graph of $f(x)=\frac{4}{x^2-1}$ has

(A) one vertical asymptote, at $x = 1$
(B) the y-axis as vertical asymptote
(C) the x-axis as horizontal asymptote and $x = \pm 1$ as vertical asymptotes
(D) two vertical asymptotes, at $x = \pm 1$, but no horizontal asymptote
(E) no asymptote

33. Suppose $\lim_{x\to -3^-} f(x) = -1$, $\lim_{x\to -3^+} f(x) = -1$, and $f(-3)$ is not defined. Which of the following statements is (are) true?

I. $\lim_{x\to -3} f(x) = -1$.
II. f is continuous everywhere except at $x = -3$.
III. f has a removable discontinuity at $x = -3$.

(A) None of them **(B)** I only **(C)** III only
(D) I and III only **(E)** All of them

34. The graph of $y=\frac{2x^2+2x+3}{4x^2-4x}$ has

(A) a horizontal asymptote at $y = +\frac{1}{2}$ but no vertical asymptotes
(B) no horizontal asymptotes but two vertical asymptotes, at $x = 0$ and $x = 1$
(C) a horizontal asymptote at $y = \frac{1}{2}$ and two vertical asymptotes, at $x = 0$ and $x = 1$
(D) a horizontal asymptote at $x = 2$ but no vertical asymptotes
(E) a horizontal asymptote at $y = \frac{1}{2}$ and two vertical asymptotes, at $x = \pm 1$

35. Let $f(x)=\begin{cases}\frac{x^2+x}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0\end{cases}$.

Which of the following statements is (are) true?

I. $f(0)$ exists. II. $\lim_{x\to 0} f(x)$ exists. III. f is continuous at $x = 0$.

(A) I only **(B)** II only **(C)** I and II only
(D) all of them **(E)** none of them

36. If $y=\frac{1}{2+10^{\frac{1}{x}}}$, then $\lim_{x\to 0} y$ is

(A) 0 **(B)** $\frac{1}{12}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{1}{3}$ **(E)** nonexistent

37. $\lim_{x\to 0} \sqrt{3+\arctan\frac{1}{x}}$ is

(A) $-\infty$ **(B)** $\sqrt{3-\frac{\pi}{2}}$ **(C)** $\sqrt{3+\frac{\pi}{2}}$
(D) ∞ **(E)** none of these

Answers for Set 2: Limits and Continuity

1.	B	9.	C	17.	B	24.	B	31.	B
2.	D	10.	D	18.	C	25.	B	32.	C
3.	C	11.	D	19.	A	26.	B	33.	D
4.	A	12.	E	20.	A	27.	E	34.	C
5.	D	13.	B	21.	E	28.	A	35.	D
6.	B	14.	C	22.	C	29.	C	36.	E
7.	A	15.	C	23.	B	30.	B	37.	E
8.	E	16.	A						

1. B. The limit as $x \to 2$ is $0 \div 8$.

2. D. Use the Rational Function Theorem (pages 30 and 31). The degrees of $P(x)$ and $Q(x)$ are the same.

3. C. Remove the common factor $x - 3$ from numerator and denominator.

4. A. The fraction equals 1 for all nonzero x.

5. D. Note that $\frac{x^3-8}{x^2-4} = \frac{(x-2)(x^2+2x+4)}{(x-2)(x+2)}$.

6. B. Use the Rational Function Theorem.

7. A. Use the Rational Function Theorem.

8. E. Use the Rational Function Theorem.

9. C. The fraction is equivalent to $\frac{1}{2^{2x}}$; the denominator approaches ∞.

10. D. Since $\frac{2^{-x}}{2^x} = 2^{-2x}$, therefore, as $x \to -\infty$, the fraction $\to +\infty$.

11. D. See Figure N2–1 on page 23.

12. E. Note, from Figure N2–1, that $\lim_{x\to -2^-}[x] = -3$ but $\lim_{x \to -2^+}[x] = -2$.

13. B. Graph $\tan^{-1} x$ on your calculator in $[-5, 5] \times [-\frac{\pi}{2}, \frac{\pi}{2}]$.

14. C. As $x \to \infty$, the function $\sin x$ does, indeed, oscillate between -1 and 1.

15. C. Since $\frac{x^2-9}{3x-9} = \frac{(x-3)(x+3)}{3(x-3)} = \frac{x+3}{3}$ (provided $x \neq 3$), y can be defined to be equal to 2 at $x = 3$, removing the discontinuity at that point.

16. A. Note that $\frac{x^2}{x} = x$ if $x \neq 0$ and that $\lim_{x\to 0} f = 0$.

17. B. Note that $\frac{\sin x}{x^2+3x} = \frac{\sin x}{x(x+3)} = \frac{\sin x}{x} \cdot \frac{1}{x+3} \to 1 \cdot \frac{1}{3}$.

18. C. As $x \to 0$, $\frac{1}{x}$ takes on varying finite values as it increases. Since the sine function repeats, $\sin \frac{1}{x}$ oscillates, taking on, infinitely many times, each value between -1 and 1. The calculator graph of `Y₁ = sin(1/X)` exhibits this oscillating discontinuity at $x = 0$.

19. A. Note that, since $y = \frac{2x^2+4}{(2-x)(1+4x)}$, both $x = 2$ and $x = -\frac{1}{4}$ are vertical asymptotes. Also, $y = -\frac{1}{2}$ is a horizontal asymptote.

20. A. $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = 0.$

21. E. Verify that f is defined at $x = 0$, 1, 2, and 3 (as well as at all other points in $[-1,3]$).

22. C. Note that $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = 0.$ However, $f(2) = 1$. Redefining $f(2)$ as 0 removes the discontinuity.

23. B. The function is not continuous at $x = 0$, 1, or 2.

24. B. $\lim_{x\to 1^-} f(x) = 0 \neq \lim_{x\to 1^+} f(x) = 1.$

25. B. $\frac{2x^2+1}{(2-x)(2+x)} = \frac{2x^2+1}{4-x^2}$. Use the Rational Function Theorem (pages 30 and 31).

26. B. Since $|x| = x$ if $x > 0$ but equals $-x$ if $x < 0$, $\lim_{x\to 0^-} \frac{|x|}{x} = \lim_{x\to 0^+} \frac{x}{x} = 1$ while $\lim_{x\to 0^-} \frac{|x|}{x} = \lim_{x\to 0^-} \frac{-x}{x} = -1.$

27. E. Note that $x \sin \frac{1}{x}$ can be rewritten as $\frac{\sin\frac{1}{x}}{\frac{1}{x}}$ and that, as $x \to \infty$, $\frac{1}{x} \to 0$.

28. A. As $x \to \pi$, $(\pi - x) \to 0$.

29. C. Since $f(x) = x + 1$ if $x \neq 1$, $\lim_{x\to 1} f(x)$ exists (and is equal to 2).

30. B. $f(x) = \frac{x(x-1)}{2x} = \frac{x-1}{2}$, for all $x \neq 0$. For f to be continuous at $x = 0$, $\lim_{x\to 0} f(x)$ must equal $f(0)$. $\lim_{x\to 0} f(x) = -\frac{1}{2}$.

31. B. Only $x = 1$ and $x = 2$ need be checked. Since $f(x) = \frac{3x}{x-2}$ for $x \neq 1, 2$, and $\lim_{x\to 1} f(x) = -3 = f(1)$, f is continuous at $x = 1$. Since $\lim_{x\to 2} f(x)$ does not exist, f is not continuous at $x = 2$.

32. C. As $x \to \pm\infty$, $y = f(x) \to 0$, so the x-axis is a horizontal asymptote. Also, as $x \to \pm 1$, $y \to \infty$, so $x = \pm 1$ are vertical asymptotes.

33. D. No information is given about the domain of f except in the neighborhood of $x = -3$.

34. C. As $x \to \infty$, $y \to \frac{1}{2}$; the denominator (but not the numerator) of y equals 0 at $x = 0$ and at $x = 1$.

35. D. The function is defined at 0 to be 1, which is also $\lim_{x\to 0} \frac{x^2+x}{x} = \lim_{x\to 0}(x+1)$.

36. E. As $x \to 0^+$, $10^{\frac{1}{x}} \to \infty$ and therefore $y \to 0$. As $x \to 0^-$, $\frac{1}{x} \to -\infty$, so $10^{\frac{1}{x}} \to 0$ and therefore $\to \frac{1}{2}$. Because the two one-sided limits are not equal, the limit does not exist.

37. E. As $x \to 0^-$, arctan $\frac{1}{x} \to -\frac{\pi}{2}$, so $y \to \sqrt{3 - \frac{\pi}{2}}$. As $x \to 0^+$, $y \to \sqrt{3 + \frac{\pi}{2}}$. The graph has a jump discontinuity at $x = 0$. (Verify with a calculator.)