Calculus BC - 2021 AP Live Review Session 5
Examining Differential Equations and Logistics

1. After completing the 2020 AP Exams, students uploaded their responses to be scored. Let E represent the total number of AP Calculus responses, in thousands, that have been uploaded t minutes after testing is complete. At time $t = 10$ minutes, 75 thousand responses have been uploaded and $E'(10) = 18$.

(**a**) Write an equation of the line tangent to the graph of E at $t = 10$. Use the tangent line to approximate $E(13)$.

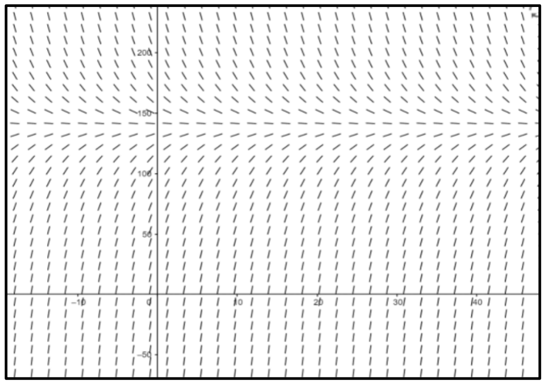
(**b**) Yoangel believes that E can be modeled by the logistic equation $y = R(t)$ where $\frac{dR}{dt} = \frac{8}{25}R\left(1 - \frac{R}{300}\right)$ and $R(10) = 75$. According to this model, what is the rate, in thousands of responses per minute, at the moment when responses are being uploaded fastest?

(**c**) According to the model R in part B, is the rate that responses are uploaded increasing or decreasing at time $t = 10$? Give a reason for your answer.

(**d**) For $0 \le t \le 30$ minutes, Emma believes that E can be modeled by the function $y = U(t)$, where $\frac{dU}{dt} = \frac{2}{25}U$ and $U(10) = 75$ (U is measured in thousands of responses) Find the particular solution $y = U(t)$. According to this model, how many responses have been uploaded at time $t = 13$?

2. For $0 \leq t \leq 30$ minutes, the number of cars that have exited a parking garage can be modeled by the function $y = L(t)$ which satisfies the differential equation $\frac{dL}{dt} = \frac{2}{15}(280 - 2L)$. At time $t = 0$, there are 435 cars in the garage and no cars have exited the garage.

(**a**) Sketch the solution curve to the differential equation $\frac{dL}{dt} = \frac{2}{15}(280 - 2L)$ through the point $(0, 0)$.

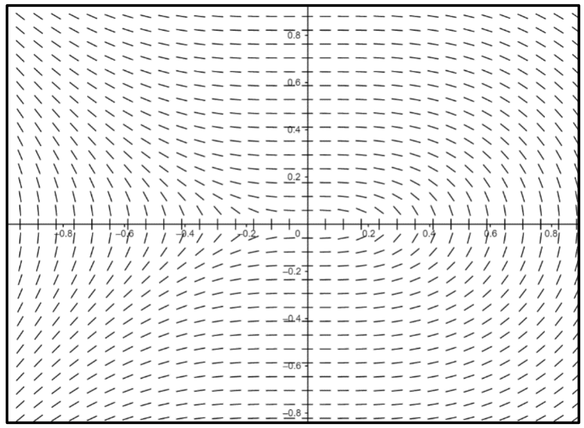


(**b**) Use separation of variables to find the particular solution $y = L(t)$ to the differential equation above with condition $L(0) = 0$.

(**c**) For $0 \leq t \leq 30$ minutes, cars enter the parking garage at a rate modeled by $E(t) = 6.7 - 1.2\cos\left(\frac{\pi t}{15}\right)$. Using your solution from part (b), find the number of cars in the parking garage at time $t = 15$.

(**d**) For $0 \leq t \leq 30$ minutes, find the minimum number of cars in the parking garage. Justify your answer.

5 for 5: MC Practice for Differential Equations and Logistics



1. Which of the following could be the equation of the differential equation that satisfies the slope field given in the figure above?

(A) $\frac{dy}{dx} = -\frac{x^2}{y}$ (B) $\frac{dy}{dx} = -\frac{x}{y}$ (C) $\frac{dy}{dx} = x - y$ (D) $\frac{dy}{dx} = \frac{y}{x}$

2. Which of the following functions satisfies the differential equation $y'' - 2y' + 10 = 0$?

(A) $y = e^{2x} + 10$

(B) $y = e^{2x} + 10x$

(C) $y = 2e^{2x} + 5x$

(D) $y = 5e^{2x}$

3. A rumor is spreading that Trevor Packer is going to join Bryan and Tony for a guest appearance on AP Live. The total number of people that have heard the rumor R, in thousands, at time t hours follows a logistic model. Which of the following could be the logistic differential equation for the rate that the rumor is spreading at time t?

(A) $\frac{dR}{dt} = 10t\left(1 - \frac{t}{4000}\right)$

(B) $\frac{dR}{dt} = 10R\left(1 - \frac{t}{4000}\right)$

(C) $\frac{dR}{dt} = 4000t - 10t^2$

(D) $\frac{dR}{dt} = 4000R - 10R^2$

4. The function $y = f(x)$ satisfies the logistic differential equation $\frac{dy}{dx} = 6y\left(1 - \frac{y}{20}\right)$. It is known that $f(4) = 10$.

$\lim_{x \to 4} \frac{f(x) - 4x + 6}{f'(x) - 2x^2 + 2}$ is

(A) $-\frac{13}{8}$ (B) 0 (C) $\frac{96}{5}$ (D) nonexistent

5. Let $y = H(t)$ be the particular solution to the differential equation $\frac{dH}{dt} = \frac{1}{10}(7 - 5H)$. Which of the following gives the value of $\frac{d^2H}{dt^2}$ at the point $H(2) = 3$?

(A) $-\frac{1}{2}$ (B) $\frac{2}{5}$ (C) $\frac{3}{2}$ (D) 4