

Section 1: Surds

Solutions to Exercise level 3 (Extension)

1. (i) $h^2 = 3^2 + x^2$

and $L = 20 - h - x$

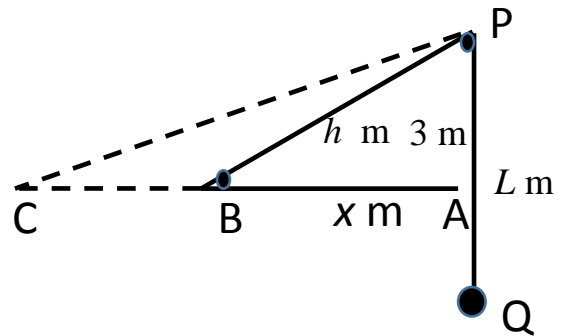
$$= 20 - x - \sqrt{x^2 + 9}$$

(ii) $T = 2\pi\sqrt{\frac{L}{g}}$, with $g = 10$, $x = 4$

$$\Rightarrow T = 2\pi\sqrt{\frac{20 - 4 - \sqrt{4^2 + 9}}{10}}$$

$$= 2\pi\sqrt{\frac{11}{10}}$$

$$= \frac{\pi\sqrt{2}\sqrt{11}}{\sqrt{5}}$$



(iii) In general, $T = 2\pi\sqrt{\frac{20 - x - \sqrt{x^2 + 9}}{10}}$

$$= \frac{\pi\sqrt{2}\sqrt{20 - x - \sqrt{x^2 + 9}}}{\sqrt{5}}$$

(iv) $x = 8 \Rightarrow T = \frac{\pi\sqrt{2}\sqrt{12 - \sqrt{73}}}{\sqrt{5}}$

(v) $T_1 = 2\pi\sqrt{\frac{L_1}{g}}$ and $T_2 = 2\pi\sqrt{\frac{L_2}{g}} \Rightarrow \frac{T_2}{T_1} = \sqrt{\frac{L_2}{L_1}}$

$$x_1 = 4 \Rightarrow T_1 = \frac{\pi\sqrt{2}\sqrt{11}}{\sqrt{5}} \approx 6.5898\dots$$

and $L_1 = 11$

$$x_2 = 8 \Rightarrow T_2 = \frac{\pi\sqrt{2}\sqrt{12 - \sqrt{73}}}{\sqrt{5}} \approx 3.6937\dots$$

and $L_2 = 20 - 8 - \sqrt{73} = 3.4559\dots$

Check: $\frac{T_2}{T_1} \approx 0.560\dots$ and $\sqrt{\frac{L_2}{L_1}} \approx \sqrt{0.314\dots} \approx 0.560\dots$

as expected.