**Paper 2: Core Pure Mathematics 2 Mark Scheme**

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **1(i)** |  | B1 | 3.1a |
|  | M1 | 1.1b |
|  | A1ft | 1.1b |
|  | **(3)** |  |
| **(ii)** |  | M1 | 1.1b |
|  | A1 | 1.1b |
|  | A1 | 1.1b |
|  | **(3)** |  |
| **Alternative:** |  |  |
|  | M1 | 1.1b |
|  | A1 | 1.1b |
|  | A1 | 1.1b |
|  | **(3)** |  |
| **(iii)** |  | M1 | 3.1a |
|  | A1ft | 1.1b |
|  | **(2)** |  |
| **(8 marks)** | | | |
| **Notes:** | | | |
| **(i)**  **B1:** Identifies the correct values for all 3 expressions (can score anywhere)  **M1:** Uses a correct identity  **A1ft:** Correct value (follow through their 8, 28 and 32) | | | |
| **(ii)**  **M1:** Attempts to expand  **A1:** Correct expansion  **A1:** Correct value | | | |
| **Alternative:**  **M1:** Substitutes *x* – 2 for *x* in the given cubic  **A1:** Calculates the correct constant term  **A1:** Changes sign and so obtains the correct value | | | |
| **(iii)**  **M1:** Establishes the correct identity  **A1ft:** Correct value (follow through their 8, 28 and 32) | | | |

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| **Question** | **Scheme** | | | **Marks** | | **AOs** | |
| **2(a)** |  | | | M1 | | 3.1a | |
|  | | | M1 | | 1.1b | |
|  | | | A1 | | 1.1b | |
|  | | | **(3)** | |  | |
| **(b)** |  | | | M1 | | 2.1 | |
| is perpendicular to *Π*2 | | | A1 | | 2.2a | |
|  | | | **(2)** | |  | |
| **(c)** |  | | | M1 | | 1.1b | |
|  | | | M1 | | 2.1 | |
| So angle between planes \* | | | A1\* | | 2.4 | |
|  | | | **(3)** | |  | |
| **(8 marks)** | | | | | | | |
| **Notes:** | | | | | | | |
| **(a)**  **M1:** Realises the need to and so attempts the scalar product between the normal and the  position vector  **M1:** Correct method for the perpendicular distance  **A1:** Correct distance | | | | | | | |
| **(b)**  **M1:** Recognises the need to calculate the scalar product between the given vector and both  direction vectors  **A1:** Obtains zero both times and makes a conclusion | | | | | | | |
| **(c)**  **M1:** Calculates the scalar product between the two normal vectors  **M1:** Applies the scalar product formula with their 11 to find a value for cos *θ*  **A1\*:** Identifies the correct angle by linking the angle between the normal and the angle between  the planes | | | | | | | |
| **Question** | | **Scheme** | | | **Marks** | | **AOs** | |
| **3(i)(a)** | |  | | | M1 | | 2.3 | |
| The matrix **M** has an inverse when | | | A1 | | 1.1b | |
|  | | | **(2)** | |  | |
| **(b)** | | or | | | B1 | | 1.1b | |
|  | | | M1 | | 1.1b | |
|  | 2 correct rows or columns. Follow through their det**M** | | A1ft | | 1.1b | |
| All correct. Follow through their det**M** | | A1ft | | 1.1b | |
|  | | | **(4)** | |  | |
| **(ii)** | | When *n* = 1, lhs = , rhs =  So the statement is true for *n* = 1 | | | B1 | | 2.2a | |
| Assume true for *n* = *k* so | | | M1 | | 2.4 | |
|  | | | M1 | | 2.1 | |
|  | | | A1 | | 1.1b | |
|  | | | A1 | | 1.1b | |
| If the statement is true for *n* = *k* then it has been shown true for  *n* = *k* + 1 and as it is true for *n* = 1, the statement is true for all positive integers *n* | | | A1 | | 2.4 | |
|  | | | **(6)** | |  | |
| **(12 marks)** | | | | | | | | |

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| **Question 3 notes:** |
| **(i)(a)**  **M1:** Attempts determinant, equates to zero and attempts to solve for *a* in order to establish the  restriction for *a*  **A1:** Provides the correct condition for *a* if **M** has an inverse |
| **(i)(b)**  **B1:** A correct matrix of minors or cofactors  **M1:** For a complete method for the inverse  **A1ft:** Two correct rows following through their determinant  **A1ft:** Fully correct inverse following through their determinant |
| **(ii)**  **B1:** Shows the statement is true for *n* = 1  **M1:** Assumes the statement is true for *n* = *k*  **M1:** Attempts to multiply the correct matrices  **A1:** Correct matrix in terms of *k*  **A1:** Correct matrix in terms of *k* + 1  **A1:** Correct complete conclusion |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **4(a)** |  | M1 | 2.1 |
|  | A1\* | 1.1b |
|  | **(2)** |  |
| **(b)** |  | B1 | 2.1 |
|  | M1 | 2.1 |
|  | A1 | 1.1b |
|  | M1 | 2.1 |
| cos4 *θ* = (cos 4*θ* + 4cos 2*θ* + 3)\* | A1\* | 1.1b |
|  | **(5)** |  |
| **(7 marks)** | | | |
| **Notes:** | | | |
| **(a)**  **M1:** Identifies the correct form for *zn* and *z*-*n* and adds to progress to the printed answer  **A1\*:** Achieves printed answer with no errors | | | |
| **(b)**  **B1:** Begins the argument by using the correct index with the result from part (a)  **M1:** Realises the need to find the expansion of  **A1:** Terms correctly combined  **M1:** Links the expansion with the result in part (a)  **A1\*:** Achieves printed answer with no errors | | | |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **5(a)** |  | M1 | 1.1a |
|  | M1 | 1.1b |
|  | M1 | 1.1b |
|  | A1\* | 2.1 |
|  | **(4)** |  |
| **(b)** |  | B1 | 3.1a |
| Uses with their values | M1 | 1.1b |
|  | A1 | 1.1b |
|  | A1 | 1.1b |
|  | **(4)** |  |
| **(c)** |  | M1 A1 | 3.1a  2.2a |
|  | **(2)** |  |
| **(10 marks)** | | | |
| **Notes:** | | | |
| **(a)**  **M1:** Realises the need to use the product rule and attempts first derivative  **M1:** Realises the need to use a second application of the product rule and attempts the second  derivative  **M1:** Correct method for the third derivative  **A1\*:** Obtains the correct 4th derivative and links this back to *y* | | | |
| **(b)**  **B1:** Makes the connection with part **(a)** to establish the general pattern of derivatives and  finds the correct non-zero values  **M1:** Correct attempt at Maclaurin series with their values  **A1:** Correct expression un-simplified  **A1:** Correct expression and simplified | | | |
| **(c)**  **M1:** Generalising, dealing with signs, powers and factorials  **A1:** Correct expression | | | |

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| **Question** | **Scheme** | | | **Marks** | **AOs** |
| **6(a)(i)** | Im  Re | | | M1 | 1.1b |
| A1 | 1.1b |
| **(a)(ii)** |  | | | M1 | 2.1 |
|  | | | A1 | 1.1b |
|  | | | M1 | 2.1 |
|  | | | A1\* | 2.2a |
|  | | | **(6)** |  |
| **(b)(i)** | Im  Re | | | B1 | 1.1b |
| B1ft | 1.1b |
| **(b)(ii)** |  | | | M1 | 3.1a |
|  | | | M1 | 1.1b |
|  | | | A1 | 1.1b |
|  | | | M1 | 2.1 |
|  | | | A1 | 1.1b |
|  | | | **(7)** |  |
| **Question** | **Scheme** | | | **Marks** | **AOs** |
|  | **(b)(ii) Alternative:** | | |  |  |
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| Candidates may take a geometric approach e.g. by finding sector + 2 triangles | | |
| Angle *ACB* = so area sector *ACB* =  Area of triangle *OCB* | | | M1 | 3.1a |
| Sector area *ACB* + triangle area *OCB* = + 12 | | | A1 | 1.1b |
| Area of triangle *OAC*:  Angle *ACO* =  so area *OAC* = | | | M1 | 1.1b |
| Total area = | | | M1 | 2.1 |
|  | | | A1 | 1.1b |
| **(13 marks)** | | | | | |

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| **Question 6 notes:** |
| **(a)(i)**  **M1:** Draws a circle which passes through the origin  **A1:** Fully correct diagram |
| **(a)(ii)**  **M1:** Uses *z* = *x* + i*y* in the given equation and uses modulus to find equation in *x* and *y* only  **A1:** Correct equation in terms of *x* and *y* in any form – may be in terms of *r* and *θ*  **M1:** Introduces polar form, expands and uses leading to a polar equation  **A1\*:** Deduces the given equation (ignore any reference to *r* = 0 which gives a point on the curve) |
| **(b)(i)**  **B1:** Correct pair of rays added to their diagram  **B1ft:** Area between their pair of rays and inside their circle from (a) shaded, as long as there is an  intersection |
| **(b)(ii)**  **M1:** Selects an appropriate method by linking the diagram to the polar curve in (a), evidenced by  use of the polar area formula  **M1:** Uses double angle identities  **A1:** Correct integral  **M1:** Integrates and applies limits  **A1:** Correct area |
| **(b)(ii) Alternative:**  **M1:** Selects an appropriate method by finding angle *ACB* and area of sector *ACB* and finds area  of triangle *OCB* to make progress towards finding the required area  **A1:** Correct combined area of sector *ACB* + triangle *OCB*  **M1:** Starts the process of finding the area of triangle *OAC* by calculating angle *ACO* and attempts  area of triangle *OAC*  **M1:** Uses the addition formula to find the exact area of triangle *OAC* and employs a full correct  method to find the area of the shaded region  **A1:** Correct area |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **7(a)** |  | M1 | 2.1 |
|  | M1 | 2.1 |
|  | A1\* | 1.1b |
|  | **(3)** |  |
| **(b)** |  | M1 | 3.4 |
|  | A1 | 1.1b |
|  | M1 | 3.4 |
|  | A1 | 1.1b |
|  | **(4)** |  |
| **(c)** |  | M1 | 3.4 |
|  | M1 | 3.4 |
|  | A1 | 1.1b |
|  | **(3)** |  |
| **(d)(i)** |  | M1 | 3.1b |
|  | M1 | 3.3 |
|  | M1 | 3.1b |
|  | A1 | 1.1b |
| 2019 | A1 | 3.2a |
| **(d)(ii)** | 3750 foxes | B1 | 3.4 |
| **(d)(iii)** | e.g. the model predicts a large number of foxes are on the island when the rabbits have died out and this may not be sensible | B1 | 3.5a |
|  | **(7)** |  |
| **(17 marks)** | | | |

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| **Question 7 notes:** |
| **(a)**  **M1:** Attempts to differentiate the first equation with respect to *t*  **M1:** Proceeds to the printed answer by substituting into the second equation  **A1\*:** Achieves the printed answer with no errors |
| **(b)**  **M1:** Uses the model to form and solve the auxiliary equation  **A1:** Correct values for *m*  **M1:** Uses the model to form the CF  **A1:** Correct CF |
| **(c)**  **M1:** Differentiates the expression for the number of foxes  **M1:** Uses this result to find an expression for the number of rabbits  **A1:** Correct equation |
| **(d)(i)**  **M1:** Realises the need to use the initial conditions in the model for the number of foxes  **M1:** Realises the need to use the initial conditions in the model for the number of rabbits to find  both unknown constants  **M1:** Obtains an expression for *r* in terms of *t* and sets = 0  **A1:** Rearranges and obtains a correct value for tan  **A1:** Identifies the correct year |
| **(d)(ii)**  **B1:** Correct number of foxes |
| **(d)(iii)**  **B1:** Makes a suitable comment on the outcome of the model |