**Paper 1: Core Pure Mathematics 1 Mark Scheme**

|  |  |  |  |
| --- | --- | --- | --- |
| **Question** | **Scheme** | **Marks** | **AOs** |
| **1** |  | M1 | 3.1a |
|  | M1 | 2.1 |
|  | A1 | 2.2a |
|   | M1 | 1.1b |
|   | A1 | 1.1b |
|  | **(5)** |  |
|  | **Alternative by induction:**  | M1 | 3.1a |
| Assume true for *n* = *k*  so  |  |  |
|  | M1 | 2.1 |
|  | A1 | 2.2a |
|  | M1 | 1.1b |
|  So true for *n* = *k* + 1 So  | A1 | 1.1b |
|  | **(5)** |  |
| **(5 marks)** |

|  |
| --- |
| **Question 1 notes:** |
| **Main Scheme****M1:** Valid attempt at partial fractions**M1:** Starts the process of differences to identify the relevant fractions at the start and end**A1:** Correct fractions that do not cancel**M1:** Attempt common denominator**A1:** Correct answer |
| **Alternative by Induction:****M1:** Uses *n* = 1 and *n* = 2 to identify values for *a* and *b***M1:** Starts the induction process by adding the (*k* + 1)th term to the sum of *k* terms**A1:** Correct single fraction**M1:** Attempt to factorise the numerator**A1:** Correct answer and conclusion |

|  |  |  |  |
| --- | --- | --- | --- |
| **Question** | **Scheme** | **Marks** | **AOs** |
| **2** | When *n* = 1, 391 = 17 × 23 so the statement is true for *n* = 1 | B1 | 2.2a |
| Assume true for *n* = *k* so is divisible by 17 | M1 | 2.4 |
|  | M1 | 2.1 |
|  |  |  |
|   | A1 | 1.1b |
|   | A1 | 1.1b |
| If the statement is true for *n* = *k* then it has been shown true for *n* = *k* + 1 and as it is true for *n* = 1, the statement is true for all positive integers *n* | A1 | 2.4 |
|  | **(6)** |  |
| **(6 marks)** |
| **Notes:** |
| **B1:** Shows the statement is true for *n* = 1**M1:** Assumes the statement is true for *n* = *k***M1:** Attempts f(*k*+1) – f(*k*) **A1:** Correct expression in terms of f(*k*)**A1:** Correct expression in terms of f(*k*)**A1:** Obtains a correct expression for f(*k* + 1)**A1:** Correct complete conclusion |

|  |  |  |  |
| --- | --- | --- | --- |
| **Question** | **Scheme** | **Marks** | **AOs** |
| **3** | is also a root | B1 | 1.2 |
| **or**Sum of roots = 6, Product of roots = 13 | M1 | 3.1a |
|   | A1 | 1.1b |
|  | M1 | 3.1a |
|  | A1 | 1.1b |
|   | M1 | 1.1a |
|   | A1 | 1.1b |
| Im(-1, 2)(3, 2)Re(-1, -2)(3, -2) | B13 ± 2iPlottedcorrectly | 1.1b |
| B1ft̶ 1 ± 2iPlottedcorrectly | 1.1b |
|  **(9 marks)** |
| **Notes:** |
| **B1:** Identifies the complex conjugate as another root**M1:** Uses the conjugate pair and a correct method to find a quadratic factor**A1:** Correct quadratic**M1:** Uses the given quartic and their quadratic to identify the value of *c***A1:** Correct 3TQ**M1:** Solves their second quadratic**A1:** Correct second conjugate pair**B1:** First conjugate pair plotted correctly and labelled**B1ft:** Second conjugate pair plotted correctly and labelled (Follow through their second  conjugate pair) |

|  |  |  |  |
| --- | --- | --- | --- |
| **Question** | **Scheme** | **Marks** | **AOs** |
| **4** |  | M1 | 3.1a |
|   | A1 | 1.1b |
|  | M1 | 3.1a |
|  | M1 | 3.1a |
|  | A1 | 1.1b |
|  | M1 | 1.1b |
| Area of triangle =  | M1 | 3.1a |
| Area of *R* =  | M1 | 1.1b |
|   | A1 | 1.1b |
| **(9 marks)** |
| **Notes:** |
| **M1:** Realises the angle for *A* is required and attempts to find it**A1:** Correct angle**M1:** Uses a correct area formula and squares *r* to achieve a 3TQ integrand in cos 2*θ***M1:** Use of the correct double angle identity on the integrand to achieve a suitable form for  integration**A1:** Correct integration**M1:** Correct use of limits**M1:** Identifies the need to subtract the area of a triangle and so finds the area of the triangle**M1:** Complete method for the area of *R***A1:** Correct final answer |

|  |  |  |  |
| --- | --- | --- | --- |
| **Question** | **Scheme** | **Marks** | **AOs** |
| **5(a)** | Pond contains 1000 + 5*t* litres after *t* days | M1 | 3.3 |
| If *x* is the amount of pollutant in the pond after *t* daysRate of pollutant out = g per day | M1 | 3.3 |
| Rate of pollutant in = 25  2 g = 50g per day | B1 | 2.2a |
|  | A1\* | 1.1b |
|  | **(4)** |  |
| **(b)** |  | M1 | 3.1b |
|  | A1 | 1.1b |
|  | M1 | 3.4 |
|  | M1 | 1.1b |
|  = 370g | A1 | 2.2b |
|  | **(5)** |  |
| **(c)** | e.g.* The model should take into account the fact that the pollutant does not dissolve throughout the pond upon entry
* The rate of leaking could be made to vary with the volume of water in the pond
 | B1 | 3.5c |
|  | **(1)** |  |
| **(10 marks)** |
| **Notes:** |
| **(a)****M1:** Forms an expression of the form 1000 + *kt* for the volume of water in the pond at time *t***M1:** Expresses the amount of pollutant out in terms of *x* and *t***B1:** Correct interpretation for pollutant entering the pond**A1\*:** Puts all the components together to form the correct differential equation |
| **(b)****M1:** Uses the model to find the integrating factor and attempts solution of their differential  equation**A1:** Correct solution**M1:** Interprets the initial conditions to find the constant of integration**M1:** Uses their solution to the problem to find the amount of pollutant after 8 days**A1:** Correct number of grams |
| **(c)****B1:** Suggests a suitable refinement to the model |

|  |  |  |  |
| --- | --- | --- | --- |
| **Question** | **Scheme** | **Marks** | **AOs** |
| **6(a)** |  | B1 | 3.1a |
|   | M1 | 1.1b |
|   | M1 | 1.1b |
|   | A1 | 1.1b |
|  | **(4)** |  |
| **(b)** |  | M1 | 1.1b |
| Mean value =  | M1 | 2.1 |
|  | A1\* | 2.2a |
|  | **(3)** |  |
| **(c)** |  | M1 | 2.2a |
|   | A1 | 1.1b |
|  | **(2)** |  |
| **(9 marks)** |
| **Notes:** |
| **(a)****B1:** Splits the fraction into two correct separate expressions**M1:** Recognises the required form for the first integration**M1:** Recognises the required form for the second integration**A1:** Both expressions integrated correctly and added together with constant of integration  included |
| **(b)****M1:** Uses limits correctly and combines logarithmic terms**M1:** Correctly applies the method for the mean value for their integration**A1\*:** Correct work leading to the given answer |
| **(c)****M1:** Realises thatthe effect of the transformation is to increase the mean value by ln *k***A1:** Combines ln’s correctly to obtain the correct expression |

|  |  |  |  |
| --- | --- | --- | --- |
| **Question** | **Scheme** | **Marks** | **AOs** |
| **7(a)** |  | M1 | 2.1 |
|  | M1 | 2.1 |
|  | M1 | 2.1 |
|  | A1\* | 1.1b |
|  | **(4)** |  |
| **(b)** |  | M1 | 3.4 |
|   | A1 | 1.1b |
|   | M1 | 3.4 |
|   **or** awrt 5.03  | A1 | 1.1b |
|  | **(4)** |  |
| **(8 marks)** |
| **Notes:** |
| **(a)** **M1:** Obtains *x* in terms of *y* and cos *θ***M1:** Obtains an equation connecting *y* and sin *θ***M1:** Uses Pythagoras to obtain an equation in *x* and *y* only**A1\*:** Obtains printed answer |
|  **(b)** **M1:** Uses the correct volume of revolution formula with the given expression**A1:** Correct integration**M1:** Correct use of correct limits**A1:** Correct volume |

|  |  |  |  |
| --- | --- | --- | --- |
| **Question** | **Scheme** | **Marks** | **AOs** |
| **8** |  | M1 | 1.1b |
|  | A1 | 1.1b |
|  | M1 | 3.1a |
|  | M1 | 3.1a |
|  | A1 | 1.1b |
|  | M1 | 3.1a |
| e.g.  | A1 | 2.5 |
|  | **(7)** |  |
| **(7 marks)** |
| **Notes:** |
| **M1:** Substitutes the parametric equation of the line into the equation of the plane and solves for  *λ***A1:** Obtains the correct coordinates of the intersection of the line and the plane**M1:** Substitutes the parametric form of the line perpendicular to the plane passing through  (2, 4, 6) into the equation of the plane to find *t***M1:** Find the reflection of (2, 4, 6) in the plane**A1:** Correct coordinates**M1:** Determines the direction of *l* by subtracting the appropriate vectors**A1:** Correct vector equation using the correct notationπ(8,8, 0)(10, 0,4)(2, 4,6) |

|  |  |  |  |
| --- | --- | --- | --- |
| **Question** | **Scheme** | **Marks** | **AOs** |
| **9(a)(i)** | Weight = massgBut mass is in thousands of kg, so *m* = 3 | M1 | 3.3 |
| **(ii)** |  | M1 | 1.1b |
|   | M1 | 1.1b |
|  = 200 cos *t* so PI is *x* = 40 sin *t* – 20 cos  | A1\* | 2.1 |
| **or** |  |  |
| Let  | M1 | 1.1b |
|   | M1 | 2.1 |
|  | A1\* | 1.1b |
| **(iii)** |  | M1 | 1.1b |
|   | A1 | 1.1b |
|   | M1 | 1.1b |
|   | A1 | 1.1b |
|  | **(8)** |  |
| **(b)** |  | M1 | 3.4 |
|   | M1 | 3.4 |
|   | A1 | 1.1b |
|   | A1 | 3.4 |
|  | (4) |  |
| **(12 marks)** |

|  |
| --- |
| **Question 9 notes:** |
| **(a)(i)****M1:** Correct explanation that in the model, *m* = 3 |
| **(ii)****M1:** Differentiates the given PI twice**M1:** Substitutes into the given differential equation**A1\*:** Reaches 200cos*t* and makes a conclusion**or****M1:** Uses the correct form for the PI and differentiates twice**M1:** Substitutes into the given differential equation and attempts to solve**A1\*:** Correct PI |
| **(iii)****M1:** Uses the model to form and solve the auxiliary equation**A1:** Correct complementary function**M1:** Uses the correct notation for the general solution by combining PI and CF**A1:** Correct General Solution for the model |
| **(b)****M1:** Uses the initial conditions of the model, *t* = 0 at *x* = 0, to form an equation in *A* and *B***M1:** Uses  0 at *x* = 0 in the model to form an equation in *A* and *B***A1:** Correct PS**A1:** Obtains 33m using the assumptions made in the model |