GCE A level Mathematics (9AM0) – Paper 1
Pure Mathematics 1

October 2020 student-friendly mark scheme

Please note that this mark scheme is not the one used by examiners for making scripts. It is intended more as a guide to good practice, indicating where marks are given for correct answers. As such, it doesn't show follow-through marks (marks that are awarded despite errors being made) or special cases.

It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here – they will be covered in the formal mark scheme.

This document is intended for guidance only and may differ significantly from the final mark scheme published in December 2020.

Guidance on the use of codes within this document
M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect. A1 – accuracy mark. This mark is generally given for a correct answer following correct working. B1 – working mark. This mark is usually given when working and the answer cannot easily be separated. Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer).

Question 1 (Total 5 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$(1+8x)^{\frac{1}{2}} =$ $1 + \frac{1}{2} \times 8x + \frac{\frac{1}{2} \times -\frac{1}{2}}{2!} \times (8x)^2 + \frac{\frac{1}{2} \times -\frac{1}{2} \times -\frac{3}{2}}{3!} \times (8x)^3$	M1 1.1b	This mark is given for a method to find the binomial expansion including the 3rd and 4th terms
		A1 1.1b	This mark is given for a correct (unsimplified) expression
	$= 1 + 4x - 8x^2 + 32x^3 + \ldots$	A1 1.1b	This mark is given for the correct answer only
(b)	When $x = \frac{1}{32}$, $(1+8x)^{\frac{1}{2}} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$	M1 1.1b	This mark is given for substituting $x = \frac{1}{32}$ into $(1+8x)^{\frac{1}{2}}$
	When $x = \frac{1}{32}$, $2(1 + 4x - 8x^2 + 32x^3 + \ldots)$ $= 2(1 + \frac{1}{8} - \frac{1}{128} + \frac{1}{1024}) \approx \sqrt{5}$	A1 2.4	This mark is given for a full and correct explanation how the expansion can be used to estimate $\sqrt{5}$

Question 2 (Total 4 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$4^{3p-1} = 5^{210} \Rightarrow (3p - 1) \log 4 = 210 \log 5$	M1 1.1b	This mark is given for taking logs and using the power law on each side of the equation
	$3p = \frac{210 \log 5}{\log 4} + 1$	M1 2.1	This mark is given for a method for to find the value of p
	$p = 81.6$	A1 1.1b	This mark is given for the correct answer only

Question 3 (Total 4 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\overrightarrow{AB} = (3\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}) - (2\mathbf{i} + 5\mathbf{j} - 6\mathbf{k})$	M1 1.1b	This mark is given for a method to find $\overrightarrow{AB}$
	$\overrightarrow{AB} = \mathbf{i} - 8\mathbf{j} + 2\mathbf{k}$	A1 1.1b	This mark is given for the correct answer only
(b)	$\overrightarrow{OC} = 2\overrightarrow{AB}$, so $\overrightarrow{OC}$ is parallel to $\overrightarrow{AB}$	M1 1.1b	This mark is given for showing that $\overrightarrow{OC}$ is parallel to $\overrightarrow{AB}$
	$\lvert\overrightarrow{OC}\rvert \neq \lvert\overrightarrow{AB}\rvert$, so *OABC* is a trapezium	A1 2.4	This mark is given for showing that the length of $\overrightarrow{OC}$ is not the same as the length $\overrightarrow{AB}$, leading to a correct conclusion

Question 4 (Total 5 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\frac{3x-7}{x-2} = 7$ $3x - 7 = 7(x - 2)$ $3x = 7x - 7$	M1 3.1a	This mark is given for a method to find a solution for f(x) = 7
	$x = \frac{7}{4}$	A1 1.1b	This mark is given for the correct answer only
(b)	$\mathrm{ff}(x) = \frac{3\left(\frac{3x-7}{x-2}\right)-7}{\left(\frac{3x-7}{x-2}\right)-2} =$	M1 1.1b	This mark is given for substituting $\frac{3x-7}{x-2}$ into f(x)
	$= \frac{3(3x-7)-7(x-2)}{3x-7-2(x-2)}$	M1 1.1b	This mark is given for multiplying all terms on the numerator and denominator by (x – 2)
	$= \frac{2x-7}{x-3}$	A1 2.1	This mark is given for the correct answer only

Question 5 (Total 6 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$115 = 28 + 5d$ $d = 17.4$	M1 3.1b	This mark is given for translating the problem mathematically using the nth term $a + (n - 1)d$
	$28 + 2 \times 17.4 =$	M1 3.4	This mark is given for a method to find the fastest speed the car can go in 3rd gear
	62.8 km/h	A1 1.1b	This mark is given for the correct answer only
(b)	$115 = 28r^5$ $r = 1.3265$	M1 3.1b	This mark is given for translating the problem mathematically using the nth term ar^{n-1}
	$28 \times (1.3265)^4 =$	M1 3.4	This mark is given for a method to find the fastest speed the car can go in 5th gear
	86.7 km/h	A1 1.1b	This mark is given for the correct answer only

Question 6 (Total 7 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$R = \sqrt{5}$	B1 1.1b	This mark is given for the correct answer only
	$\tan \alpha = 2$	M1 1.1b	This mark is given for a method to find a value for $\tan \alpha$
	$\sin x + 2 \cos x = \sqrt{5} \sin (x + 1.107)$	A1 1.1b	This mark is given for the correct answer only
(b)	$\theta = 5 + \sqrt{5} \sin \left(\frac{\pi t}{12} + 1.107 - 3\right)$ Maximum temperature at $(5 + \sqrt{5})$ °C (or 7.24 °C)	B1 2.2a	This mark is given for deducing the maximum temperature
(c)	$\frac{\pi t}{12} + 1.107 - 3 = \frac{\pi}{2}$	M1 3.1b	This mark is given for a method to find the time when the maximum temperature occurs
	$t = 13.23$	A1 1.1b	This mark is given for finding a correct value for t
	13:14 **or** 1:14 p.m. **or** 13 hours 14 minutes after midnight	A1 3.2a	This mark is given for a correct time only

Question 7 (Total 5 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$y = mx + 25$ When $x = -2$, $y = 13$ $\Rightarrow m = 6$	M1 1.1b	This mark is given for a method to find the equation of the line l
	An equation for the line l is $y = 6x + 25$	A1 1.1b	This mark is given for a correct equation for the line l
	$y = a(x + 2)^2 + 13$ When $x = 0$, $y = 25$ $\Rightarrow a = 3$	M1 3.1a	This mark is given for a method to find the equation of the curve C
	An equation for the curve C is $y = 3(x + 2)^2 + 13$	A1 1.1b	This mark is given for a correct equation for the curve C
	The region R is defined by $3(x + 2)^2 + 13 < y < 6x + 25$	B1 2.5	This mark is given for correct mathematical language to define R

Question 8 (Total 2 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$n = Ae^{kt}$ (where A and k are positive constants)	M1 3.1b	This mark is given for any equation involving an exponential expression
		A1 1.1b	This mark is given for a correct exponential equation (there is no requirement to state that A and k are positive constants)

Question 9 (Total 9 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$f(x) = 4(x^2 - 2)e^{-2x}$	M1 1.1b	This mark is given for an attempt to differentiate $f(x)$
	$f'(x) = 8xe^{-2x} - 8(x^2 - 2)e^{-2x}$	A1 1.1b	This mark is given for a correct (unsimplified) answer
	$f'(x) = 8e^{-2x}(x - (x^2 - 2)) = 8(2 + x - x^2)e^{-2x}$	A1 2.1	This mark is given for a fully correct explanation
(b)	Roots of $f'(x) = 0$ are -1 and 2	B1 1.1b	This mark is given for stating the roots of $f'(x)$
	Stationary points are $(-1, -4e^2)$ and $(2, 8e^{-4})$	M1 1.1b	This mark is given for substituting one value of x to find a value of y
		A1 1.1b	This mark is given for both stationary points found
(c)	Range of g is $[-8e^2, \infty)$ **or** $g(x) \geq -8e^2$	B1 2.5	This mark is given for a correct range stated and using correct notation
	$2f(0) - 3 = 2 \times -8 - 3$	M1 3.1a	This mark is given for a method to find the lower bound
	Range of h is $[-19, 16e^{-4} - 3]$	A1 1.1b	This mark is given for the correct answer only

Question 10 (Total 10 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$x = u^2 + 1 \Rightarrow \mathrm{d}x = 2u\ \mathrm{d}u$	B1 1.1b	This mark is given for finding $\mathrm{d}x = 2u\ \mathrm{d}u$
	$\int \frac{3\,\mathrm{d}x}{(x-1)(3+2\sqrt{(x-1)})} =$ $\int \frac{3\times 2u\,\mathrm{d}u}{(u^2+1-1)(3+2u)}$	M1 1.1b	This mark is given for a full substitution to form an integral in terms of u
	Limits are $p = 2$, $q = 3$	B1 1.1b	This mark is given for finding the correct limits
	$\int \frac{3\times 2u\,\mathrm{d}u}{(u^2+1-1)(3+2u)} = \int_2^3 \frac{6\,\mathrm{d}u}{u(3+2u)}$	A1 2.1	Clear reasoning including one correct intermediate line leading to the given answer.
(b)	$\frac{6}{u(3+2u)} = \frac{A}{u} + \frac{B}{3+2u}$	M1 1.1b	This mark is given for a method to find the correct form of partial fraction
	$\frac{6}{u(3+2u)} = \frac{2}{u} + \frac{4}{3+2u}$	A1 1.1b	This mark is given for finding the correct values of A and B
	$\int \frac{6\,\mathrm{d}u}{u(3+2u)} = 2 \ln u - 2 \ln (3 + 2u) + c$	M1 3.1a	This mark is given for a method to use partial fractions and to integrate using lns.
		A1 1.1b	This mark is given for a correct integration
	$\int_2^3 \frac{6\,\mathrm{d}u}{u(3+2u)}$ $= (2 \ln 3 - 2 \ln 9) - (2 \ln 2 - 2 \ln 7)$ $= \ln a$	M1 2.1	This mark is given for a method to use limits 2 and 3 with correct work leading to ln a.
	$\ln a = \frac{49}{36}$	A1 1.1b	This mark is given for a correct value of ln a only

Question 11 (Total 8 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$x^2 + y^2 = 100$ $(x^2 - 30x + 225 + y^2 = 40$ $325 - 30x = 40$	M1 3.1a	This mark is given for solving two simultaneous equations to find either coordinate for where the two circles meet
	$x = 9.5$ $(y = \pm 3.12)$	A1 1.1b	This mark is given for a correct value for x
	$\cos\theta = \frac{9.5}{10}$	M1 3.1a	This mark is given for using the radius of the circle and correct trigonometry in an attempt to find the angle subtended by chord AB in circle C_1
	Angle $AOB = 2 \times \cos^{-1}\left(\frac{9.5}{10}\right) = 0.635$	A1 2.1	This mark is given for full and correct work proceeding to the given answer
(b)	$10 \times (2\pi - 0.635) = 56.48$	M1 1.1b	This mark is given for a method to use the formula with and $s = r\theta$ with $r = 10$ and $\theta = 2\pi - 0.635$ to find the perimeter of C_1 from A to B
	$\cos\beta = \frac{15 - 9.5}{\sqrt{40}}$ $2 \times \cos^{-1}\left(\frac{5.5}{\sqrt{40}}\right) = 1.03$	M1 3.1a	This mark is given for a method to find the angle subtended by chord AB in circle C_2
	$\sqrt{40} \times (2\pi - 1.03) = 33.22$	M1 2.1	This mark is given for a method to use the formula with and $s = r\theta$ with $r = \sqrt{40}$ and $\theta = 2\pi - 1.03$ to find the perimeter of C_1 from A to B
	$56.48 + 33.22 = 89.7$	A1 1.1b	This mark is given for the correct answer only

Question 12 (Total 8 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	cosec $\theta = \dfrac{1}{\sin\theta}$	B1 1.2	This mark is given for stating the relationship cosec $\theta = \dfrac{1}{\sin\theta}$
	cosec $\theta -$ sin θ = $\dfrac{1}{\sin\theta} - \sin\theta = \dfrac{1-\sin^2\theta}{\sin\theta}$	M1 2.1	This mark is given for a method to form a single fraction
	$= \dfrac{\cos^2\theta}{\sin\theta} = \cos\theta \times \dfrac{\cos\theta}{\sin\theta} = \cos\theta\cot\theta$	A1 2.1	This mark is given for working with all steps shown leading to given answer
(b)	cosec x sin x = cos x cot $(3x - 50°)$ $\Rightarrow$ cos x cot x = cos x cot $(3x - 50°)$ $\Rightarrow$ cot x = cot $(3x - 50°)$	M1 3.1a	This mark is given for a method to cancel the term in cos x to find one solution
	$x = 25°$	A1 1.1b	This mark is given for a correct answer only
	cot x = cot $(3x - 50°)$ $\Rightarrow x + 180° = 3x - 50°$	M1 2.1	This mark is given for a method to show that cot x has a period of 180 °and a second solution can be found
	$x = 115°$	A1 1.1b	This mark is given for a correct answer only
	cos $x = 0 \Rightarrow x = 90°$	B1 2.2a	This mark is given for a deduction that a third solution can be found from cos x = 0.

Question 13 (Total 7 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$a_1 = 2$, $a_2 = 2k$	M1 1.1b	This mark is given for applying $a_1 = 2$ to find an expression for a_2
	$a_3 = k + 1$, $a_4 = \dfrac{k(k+3)}{k+1}$	M1 3.1a	This mark is given for finding expressions for a_3 and a_4
	$\dfrac{k(k+3)}{k+1} = 2$ $\Rightarrow k^2 + 3k = 2k + 2$ $\Rightarrow k^2 + k - 2 = 0$	A1 2.1	This mark is given for setting a1 = a4 and proceeding to the given answer with accurate working.
(b)	For $k = 1$, all the terms would be the same and so the sequence would not have period of order 3	B1 2.3	This mark is given for a correct statement
(c)	Repeating terms are $a1$ and $a_5 = 2$, a_2 and $a_6 = –4$, a_3 and $a_7 = –1$	B1 2.2a	This mark is given for deducing the repeating terms
	$\sum_{n=1}^{80} a_k = 26 \times (2 + –4 + –1) + 2 + –4$	M1 3.1a	This mark is given for a method to find the sum to 80 terms.
	$= –80$	A1 1.1b	This mark is given for the correct answer only

Question 14 (Total 10 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\frac{\mathrm{d}V}{\mathrm{d}t} = -c$	B1 3.1b	This mark is given for using the model to state $\frac{\mathrm{d}V}{\mathrm{d}t} = -c$
	$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}r} \times \frac{\mathrm{d}r}{\mathrm{d}t}$ and $\frac{\mathrm{d}V}{\mathrm{d}r} = 4\pi r^2$	M1 2.1	This mark is given for a method to use expressions for $\frac{\mathrm{d}V}{\mathrm{d}t}$ and $\frac{\mathrm{d}V}{\mathrm{d}r}$
	$-c = 4\pi r^2 \times \frac{\mathrm{d}r}{\mathrm{d}t} \Rightarrow \frac{\mathrm{d}r}{\mathrm{d}t} = -\frac{c}{4\pi r^2} = -\frac{k}{r^2}$	A1 2.2a	This mark is given for a full and correct process to reach the given answer
(b)	$\frac{\mathrm{d}r}{\mathrm{d}t} = -\frac{k}{r^2} \Rightarrow \int r^2 \; \mathrm{d}r = \int k \; \mathrm{d}t$	M1 2.1	This mark is given for separating the variables and integrating (with at least one index correct)
	$\frac{r^3}{3} = -kt + \alpha$	A1 1.1b	This mark is given for correct integration
	$t = 0$ and $r = 40 \Rightarrow \alpha = \frac{64000}{3}$	M1 1.1b	This mark is given for a method to use the initial conditions to find a value for α
	$t = 5$, $r = 20$ and $\alpha = \frac{64000}{3} \Rightarrow k = \frac{11200}{3}$	M1 3.4	This mark is given for a method to use the second set of conditions to find k
	$r^3 = 64\,000 - 11\,200t$	A1 3.3	This mark is given for obtaining a correct equation for the model
(c)	The model is only valid when $64\,000 - 11\,200t \geq 0$	M1 3.4	This mark is given for a statement that the model is only valid when $64\,000 - 11\,200t \geq 0$
	$t \leq \frac{40}{7}$ seconds	A1 3.5b	This mark is given for the correct answer only

Question 15 (Total 7 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$x^2 \tan y = 9 \Rightarrow$ $2x \tan y + x^2 \sec^2 y \dfrac{dy}{dx} = 0$	M1 3.1a	This mark is given for a method to differentiate implicitly
		A1 1.1b	This mark is given for a correct answer only
	$\sec^2 y = 1 + \tan^2 y$	M1 1.1b	This mark is given for a method to find $\dfrac{dy}{dx}$ in terms of x
	$\dfrac{dy}{dx} = \dfrac{-2x \times \dfrac{9}{x^2}}{x^2\left(1 + \dfrac{81}{x^4}\right)} = \dfrac{-18x}{x^4 + 81}$	A1 2.1	This mark is given for fully correct working to lead to the given answer
(b)	$\dfrac{d^2y}{dx^2} = \dfrac{-18x(x^4+81) - (-18x)(4x^3)}{(x+81)^2}$	M1 1.1b	This mark is given for a method to differentiate using the quotient rule.
	$= \dfrac{54(x^2 - 27)}{(x^4+81)^2}$	A1 1.1b	This mark is given for a correct answer only
	$x < \sqrt[4]{27} \Rightarrow \dfrac{d^2y}{dx^2} < 0$ and $x > \sqrt[4]{27} \Rightarrow \dfrac{d^2y}{dx^2} > 0$ gives a point of inflection when $x = \sqrt[4]{27}$	A1 2.4	This mark is given for a correct explanation

Question 16 (Total 4 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	There are positive integers p and q such that $(2p + q)(2p - q) = 25$	M1 2.1	This mark is given for setting up the contradiction and factorising
	If true then $2p + q = 25$ and $2p - q = 1$ or $2p + q = 5$ and $2p - q = 5$	M1 2.2a	This mark is given for deducing that for p and q to be integers then either one or the other of the stated pairs of equations must be true
	Solutions are $p = 6.5$, $q = 12$ or $p = 2.5$, $q = 0$	A1 1.1b	This mark is given for solving the two pairs of simultaneous equations
	This is a contradiction as there are no integer solutions; hence there are no positive integers p and q such that $4p^2 - q = 25$	A1 2.1	This mark is given for a complete and rigorous argument with both possibilities and a correct conclusion