Binomial Expansion (Year 2)

In Year 1 you found the Binomial expansion of $(a+b)^n$ where n was a positive integer. This chapter allows you to extend this to when n is any rational number, i.e. could be negative or fractional.

1:: Binomial Expansion for negative/fractional powers.

"Expand $\sqrt{1+x}$ in ascending powers of x up to the x^2 term."

2:: Constant is not 1.

The same, but where the term preceding the x is not 1, e.g.

"Expand $(8+5x)^{-\frac{1}{3}}$ in ascending powers of x up to the x^3 term."

3:: Using Partial Fractions

"Show that the cubic approximation of $\frac{4-5x}{(1+x)(2-x)}$ is $2-\frac{7}{2}x+\frac{11}{4}x^2-\frac{25}{8}x^3$"

Pure Year 1 Recap

Remember that for small integer n you could use a row of Pascal Triangle for the Binomial coefficients, descending powers of the first term and ascending powers of the second. If the first term is 1, we can ignore the powers of 1.

$$(1+x)^5 = 1+5x+10x^2+10x^3+5x^4+x^5$$

$$(1+2x)^4 = 1+8x+24x^2+32x^3+16x^4$$

$$(1-3x)^3 = 1-9x+27x^2-27x^3$$

Binomial Coefficients - recap

Do you remember the simple way to find your Binomial coefficients?

$\binom{n}{1} = n$ $\binom{n}{2} = \frac{n(n-1)}{2!}$ $\binom{n}{3} = \frac{n(n-1)(n-2)}{3!}$ $\binom{n}{4} = \frac{n(n-1)(n-2)(n-3)}{4!}$

$\binom{10}{3} = \frac{10\times9\times8}{3!} = 120$ $\binom{-1}{2} = \frac{-1\times-2}{2!} = 1$ $\binom{-2}{3} = \frac{-2\times-3\times-4}{3!} = -4$

$\binom{0.5}{2} = \frac{0.5\times-0.5}{2!} = -\frac{1}{8}$

Binomial Expansion – Year 2

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots + {}^nC_r\, x^n$$

Binomial expansions, when n is either negative or fractions, are infinitely long.

Use the binomial expansion to find the first four terms of $\frac{1}{1+x} = (1+x)^{-1}$

$x = x, \quad n = -1$

$1 - x + \frac{(-1)(-2)}{2!}x^2 + \frac{(-1)(-2)(-3)}{3!}x^3$

$1 - x + x^2 - x^3$

And the first four terms of $\sqrt{1-3x} = (1-3x)^{1/2}$ $x = -3x$, $n = 1/2$

$1 + \frac{1}{2}(-3x) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2}(-3x)^2 + \frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})}{3!}(-3x)^3$

$1 - \frac{3}{2}x - \frac{9}{8}x^2 - \frac{27}{16}x^3$

When are infinite expansions valid?

Our expansion might be an infinite number of terms. If so, the result must diverge

$$\frac{1}{1+x} = (1+x)^{-1}$$
$$= 1 + (-1)x + \frac{-1 \times -2}{2!}x^2 + \frac{-1 \times -2 \times -3}{3!}x^3 + \cdots$$
$$= \mathbf{1 - x + x^2 - x^3 + \cdots}$$

What would happen in the expansion if:

✗ a) $x > 1$ If $x=2$ $\frac{1}{3} = 1-2+4-8+16-32+64, \ldots$ Diverges
$1-2+2^2-2^3+2^4-2^5+2^6 \ldots$

✓ b) $0 < x < 1$ $x=0.5$ $\frac{2}{3} = 1-0.5+0.5^2-0.5^3+0.5^2 = \frac{21}{32} \approx 0.66$

✓ c) $-1 < x < 0$ $x=-0.5$ The higher powers will get smaller. So it converges

✗ d) $x = 1$ $\frac{1}{2} = 1-1+1^2-1^3+1^4 = 1-1+1-1+1-1$, No, it doesn't converges.

Therefore requirement on x:

$-1 < x < 1$

$|x| < 1$

Expansions are allowed to be infinite. However, the result must converge

$$\sqrt{1-3x} = (1-3x)^{\frac{1}{2}}$$
$$= 1 + \frac{1}{2}(-3x) + \frac{\frac{1}{2} \times -\frac{1}{2}}{2!}(-3x)^2 + \frac{\frac{1}{2} \times -\frac{1}{2} \times -\frac{3}{2}}{3!}(-3x)^3 + \cdots$$
$$= \mathbf{1 - \frac{3}{2}x - \frac{9}{8}x^2 - \frac{27}{16}x^3 - \cdots}$$

This time, what do you think needs to be between -1 and 1 for the expansion to be valid?

$|-3x| < 1$

$|3x| < 1$

$x < \frac{1}{3}$

✎ An infinite expansion $(1+x)^n$ is valid if $|x| < 1$

Quickfire Examples:

Expansion of $(1+2x)^{-1}$ valid if: $|2x| < 1 \rightarrow |x| < \frac{1}{2}$

Expansion of $(1-x)^{-2}$ valid if: $|-x| < 1 \rightarrow |x| < 1$

Expansion of $\left(1+\frac{1}{4}x\right)^{\frac{1}{2}}$ valid if: $|\frac{1}{4}x| < 1 \rightarrow |x| < 4$

Expansion of $\left(1-\frac{2}{3}x\right)^{-1}$ valid if:

$|-\frac{2}{3}x| < 1$

$|\frac{2}{3}x| < 1$

$|x| < \frac{3}{2}$

Combining Expansions

Edexcel C4 June 2013 Q2

(*a*) Use the binomial expansion to show that

$$\sqrt{\left(\frac{1+x}{1-x}\right)} \approx 1+x+\frac{1}{2}x^2, \qquad |x|<1$$

(6)

Firstly express as a product:

$$(1+x)^{1/2}(1-x)^{-1/2}$$

How many terms do we need in each expansion?

$$(1+x)^{1/2} = 1+\frac{1}{2}x + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}x^2 = 1+\frac{1}{2}x-\frac{1}{8}x^2$$

$$(1-x)^{-1/2} = 1-\frac{1}{2}(-x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}(-x)^2 = 1+\frac{1}{2}x+\frac{3}{8}x^2$$

$$(1+x)^{1/2}(1-x)^{-1/2} = \left(1+\frac{1}{2}x-\frac{1}{8}x^2\right)\left(1+\frac{1}{2}x+\frac{3}{8}x^2\right)$$

$$= 1+\frac{1}{2}x+\frac{3}{8}x^2+\frac{1}{2}x+\frac{1}{4}x^2-\frac{1}{8}x^2$$

$$= 1+x+\frac{1}{2}x^2$$

11. (a) Use binomial expansions to show that $\sqrt{\dfrac{1+4x}{1-x}} \approx 1 + \dfrac{5}{2}x - \dfrac{5}{8}x^2$

(6)

A student substitutes $x = \dfrac{1}{2}$ into both sides of the approximation shown in part (a) in an attempt to find an approximation to $\sqrt{6}$

(b) Give a reason why the student **should not** use $x = \dfrac{1}{2}$

(1)

(c) Substitute $x = \dfrac{1}{11}$ into

$$\sqrt{\frac{1+4x}{1-x}} = 1 + \frac{5}{2}x - \frac{5}{8}x^2$$

to obtain an approximation to $\sqrt{6}$. Give your answer as a fraction in its simplest form.

(3)

11a) $(1+4x)^{1/2}(1-x)^{-1/2}$

$(1+4x)^{1/2} = 1 + \frac{1}{2}(4x) + \frac{\frac{1}{2}(-\frac{1}{2})}{2!}(4x)^2 = 1 + 2x - 2x^2$

$(1-x)^{-1/2} = 1 + \frac{1}{2}x + \frac{3}{8}x^2$

$(1 + 2x - 2x^2)(1 + \frac{1}{2}x + \frac{3}{8}x^2) = 1 + \frac{1}{2}x + \frac{3}{8}x^2 + 2x + x^2 - 2x^2$

$= 1 + \frac{5}{2}x - \frac{5}{8}x^2$

b) Student shouldn't use ½ as $(1+4x)^{1/2}$ is only valid between $-\frac{1}{4} < x < \frac{1}{4}$

c) $\sqrt{\dfrac{1+4(\frac{1}{11})}{1-\frac{1}{11}}} = \dfrac{\sqrt{6}}{2}$

$1 + \frac{5}{2}(\frac{1}{11}) - \frac{5}{8}(\frac{1}{11})^2 = \dfrac{1183}{968}$

$\dfrac{\sqrt{6}}{2} = \dfrac{1183}{968}$

$\sqrt{6} = \dfrac{1183}{484}$

Your Turn

Find the binomial expansion of $\frac{1}{(1+4x)^2}$ up to an including the term in x^3.
State the values of x for which the expansion is valid.

$$(1+4x)^{-2} = 1 - 2(4x) + \frac{-2\times-3}{2!}(4x)^2 + \frac{-2\times-3\times-4}{3!}(4x)^3$$

$$= 1 - 8x + 48x^2 - 256x^3$$

$|4x| < 1$

$|x| < 1/4$

Accuracy of an approximation

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - \cdots$$

If $x = 0.01$, how accurate would the approximation $1 - x + x^2$ by for the value of $\frac{1}{1+x}$?

$$\frac{1}{1+0.01} = 0.99009900\ldots \qquad 1 - 0.01 + 0.01^2 = 0.9901$$

Because x is very small, the approximation will be good

Smaller values of x will provide better approximations.

Common Errors

$$\sqrt{1-3x} = (1-3x)^{\frac{1}{2}}$$
$$= 1 + \frac{1}{2}(-3x) + \frac{\frac{1}{2}\times -\frac{1}{2}}{2!}(-3x)^2 + \frac{\frac{1}{2}\times -\frac{1}{2}\times -\frac{3}{2}}{3!}(-3x)^3 + \cdots$$
$$= \mathbf{1 - \frac{3}{2}x - \frac{9}{8}x^2 - \frac{27}{16}x^3 - \cdots}$$

What errors do you think are easy to make?

- Sign errors, e.g. $(-3x)^2 = -9x^2$
- Not putting brackets around the $-3x$, e.g. $-3x^2$ instead of $(-3x)^2$
- Dividing by say 3 instead of 3!

Ex 4A

C4 Edexcel Jan 2010

1. (*a*) Find the binomial expansion of

$\sqrt{(1-8x)}$, $\quad |x| < \frac{1}{8}$, $\quad (1-8x)^{1/2}$ $\quad n = 1/2$, $x = -8x$

in ascending powers of x up to and including the term in x^3, simplifying each term. **(6)**

(*b*) Show that, when $x = \frac{1}{100}$, the exact value of $\sqrt{(1-8x)}$ is $\frac{\sqrt{23}}{5}$. **(2)**

(*c*) Substitute $x = \frac{1}{100}$ into the binomial expansion in part (*a*) and hence obtain an approximation to $\sqrt{23}$. Give your answer to 5 decimal places. **(3)**

a) $1 + \frac{1}{2}(-8x) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(-8x)^2 + \frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})}{3!}(-8x)^3 + \ldots$

(a) $(1-8x)^{\frac{1}{2}} = 1 - 4x - 8x^2; -32x^3 - \ldots$

(b) $\sqrt{(1-8x)} = \sqrt{\frac{92}{100}} = \sqrt{\frac{23}{25}} = \frac{\sqrt{23}}{5}$

(c) $= 1 - 4(0.01) - 8(0.01)^2 - 32(0.01)^3$

$= 1 - 0.04 - 0.0008 - 0.000\,032 = 0.959\,168$

$\sqrt{23} = 5 \times 0.959\,168$

$= 4.795\,84$

Dealing with $(a + bx)^n$

Find first four terms in the binomial expansion of $\sqrt{4 + x}$
State the values of x for which the expansion is valid.

$$(4+x)^{1/2} = \left[4\left(1+\frac{x}{4}\right)\right]^{1/2}$$

$$= 4^{1/2}\left(1+\frac{x}{4}\right)^{1/2}$$

$$= 2\left(1+\frac{x}{4}\right)^{1/2}$$

We need it in the form $(1 + x)^n$. So factorise the 4 out.

$$2\left(1+\frac{x}{4}\right)^{1/2} = 2\left(1 + \frac{1}{2}\left(\frac{x}{4}\right) + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2}\left(\frac{x}{4}\right)^2 + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}\left(\frac{x}{4}\right)^3\right)$$

$$= 2\left(1 + \frac{1}{8}x - \frac{1}{128}x^2 + \frac{1}{1024}x^3\right)$$

$$= 2 + \frac{1}{4}x - \frac{1}{64}x^2 + \frac{1}{512}x^3$$

$$\left|\frac{x}{4}\right| < 1$$

Valid for, $|x| < 4$

Just the First Step

What would be the first step in finding the Binomial expansion of each of these?

Binomial expansion valid if:

$(2 + x)^{-3}$ $\quad 2^{-3}\left(1+\frac{x}{2}\right)^{-3} = \frac{1}{8}\left(1+\frac{x}{2}\right)^{-3}$ $\quad \left|\frac{x}{2}\right| < 1 \rightarrow |x| < 2$

$(9 + 2x)^{\frac{1}{2}}$ $\quad 9^{1/2}\left(1+\frac{2}{9}x\right)^{1/2} = 3\left(1+\frac{2}{9}x\right)^{1/2}$ $\quad \left|\frac{2}{9}x\right| < 1 \rightarrow |x| < \frac{9}{2}$

$(8 - x)^{\frac{1}{3}}$ $\quad 8^{1/3}\left(1-\frac{x}{8}\right)^{1/3} = 2\left(1-\frac{x}{8}\right)^{1/3}$ $\quad \left|\frac{x}{8}\right| < 1 \rightarrow |x| < 8$

$(5 - 2x)^{-3}$ $\quad 5^{-3}\left(1-\frac{2}{5}x\right)^{-3} = \frac{1}{125}\left(1-\frac{2}{5}x\right)^{-3}$ $\quad \left|-\frac{2}{5}x\right| < 1 \rightarrow |x| < \frac{5}{2}$

$(16 + 3x)^{-\frac{1}{2}}$ $\quad 16^{-1/2}\left(1+\frac{3}{16}x\right)^{-1/2} = \frac{1}{4}\left(1+\frac{3}{16}x\right)^{-1/2}$ $\quad \left|\frac{3}{16}x\right| < 1 \rightarrow |x| < \frac{16}{3}$

7. (a) Use the binomial expansion, in ascending powers of x, to show that

$$\sqrt{(4-x)} = 2 - \frac{1}{4}x + kx^2 + \ldots$$

where k is a rational constant to be found.

(4)

Question	Scheme	Marks	AOs
7(a)	$\sqrt{(4-x)} = 2\left(1-\frac{1}{4}x\right)^{\frac{1}{2}}$	M1	2.1
	$\left(1-\frac{1}{4}x\right)^{\frac{1}{2}} = 1+\frac{1}{2}\left(-\frac{1}{4}x\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(-\frac{1}{4}x\right)^2+\ldots$	M1	1.1b
	$\sqrt{(4-x)} = 2\left(1-\frac{1}{8}x-\frac{1}{128}x^2+\ldots\right)$	A1	1.1b
	$\sqrt{(4-x)} = 2-\frac{1}{4}x-\frac{1}{64}x^2+\ldots$ and $k=-\frac{1}{64}$	A1	1.1b
		(4)	
(b)	The expansion is valid for $\lvert x \rvert < 4$, so $x=1$ can be used	B1	2.4
		(1)	
		(5 marks)	

A student attempts to substitute $x = 1$ into both sides of this equation to find an approximate value for $\sqrt{3}$.

(b) State, giving a reason, if the expansion is valid for this value of x.

(1)

a) $(4-x)^{1/2} = 2\left(1-\frac{x}{4}\right)^{1/2}$ $\quad n = \frac{1}{2} \quad x = \frac{-x}{4}$

$= 2\left(1 + \frac{1}{2}\left(-\frac{x}{4}\right) + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2!}\left(-\frac{x}{4}\right)^2\right)$

$= 2\left(1 - \frac{1}{8}x - \frac{1}{128}x^2\right)$

$= 2 - \frac{1}{4}x - \frac{1}{64}x^2$

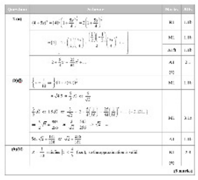
$k = -\frac{1}{64}$

b) Valid for $\left|\frac{x}{4}\right| < 1$

$|x| < 4$

$1 < 4$ so it will be valid.

Ex 4B



2. (a) Show that the binomial expansion of

$$(4+5x)^{\frac{1}{2}}$$

in ascending powers of x, up to and including the term in x^2 is

$$2+\frac{5}{4}x+kx^2$$

giving the value of the constant k as a simplified fraction.

(4)

(b) (i) Use the expansion from part (a), with $x=\frac{1}{10}$, to find an approximate value for $\sqrt{2}$

Give your answer in the form $\frac{p}{q}$ where p and q are integers.

(ii) Explain why substituting $x=\frac{1}{10}$ into this binomial expansion leads to a valid approximation.

(4)

2a) $(4+5x)^{1/2} = 2\left(1+\frac{5}{4}x\right)^{1/2}$ $\quad n = 1/2 \quad x = \frac{5}{4}x$

$= 2\left(1+\frac{1}{2}\left(\frac{5}{4}x\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(\frac{5}{4}x\right)^2\right)$

$= 2\left(1+\frac{5}{8}x-\frac{25}{128}x^2\right)$

$= 2+\frac{5}{4}x-\frac{25}{64}x^2$

bi) $x = 1/10$ $\quad \left(4+(5)\tfrac{1}{10}\right)^{1/2} = \sqrt{\frac{9}{2}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$

$2+\frac{5}{4}\left(\tfrac{1}{10}\right)-\frac{25}{64}\left(\frac{1}{10}\right)^2 = \frac{543}{256}$

$\frac{3\sqrt{2}}{2} \approx 2.1213\ldots$

$\frac{543}{256} = 2.1211\ldots$ $\quad \frac{543}{256} = \frac{181}{128} \Rightarrow$ $\quad P = 181$, $q = 128$

ii) Valid if $\left|\frac{5}{4}x\right| < 1$ $\quad \frac{1}{10} < \frac{4}{5}$, so it's valid

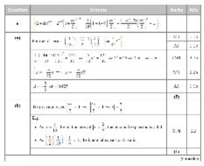
$|x| < \frac{4}{5}$

6. $\mathrm{f}(x) = (2 + kx)^{-4}$ where k is a positive constant

The binomial expansion of f(x), in ascending powers of x, up to and including the term in x^2, is

$$\frac{1}{16} + Ax + \frac{125}{32}x^2$$

where A is a constant.



(a) Find the value of A, giving your answer in simplest form.

(5)

(b) Determine, giving a reason for your answer, whether the binomial expansion for f(x) is valid when $x = \dfrac{1}{10}$

(1)

a) $(2+kx)^{-4} = \frac{1}{16}\left(1+\frac{kx}{2}\right)^{-4}$ $\quad n=-4 \quad x = \frac{kx}{2}$

$= \frac{1}{16}\left(1 + (-4)\left(\frac{kx}{2}\right) + \frac{(-4)(-5)}{2}\left(\frac{kx}{2}\right)^2\right)$

$= \frac{1}{16}\left(1 - 2kx + \frac{5}{2}k^2x^2\right)$

$= \frac{1}{16} - \frac{k}{8}x + \frac{5}{32}k^2x^2$

$\frac{5}{32}k^2 = \frac{125}{32}$

$k^2 = 25$

$k = \pm 5 \quad k = 5$

$A = -\frac{k}{8} = -\frac{5}{8}$

b) $\left(1+\frac{5}{2}x\right)^{-4}$ $\quad \left|\frac{5}{2}x\right| < 1$

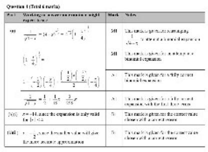
$|x| < \frac{2}{5}$

$\frac{1}{10} < \frac{2}{5}$ so expansion is valid for $x = \frac{1}{10}$

4. (a) Find the first three terms, in ascending powers of x, of the binomial expansion of

$$\frac{1}{\sqrt{4-x}}$$

giving each coefficient in its simplest form.



(4)

The expansion can be used to find an approximation to $\sqrt{2}$

Possible values of x that could be substituted into this expansion are:

- $x = -14$ because $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{\sqrt{18}} = \dfrac{\sqrt{2}}{6}$
- $x = 2$ because $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$
- $x = -\dfrac{1}{2}$ because $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{\sqrt{\dfrac{9}{2}}} = \dfrac{\sqrt{2}}{3}$

Only valid $\left|\frac{x}{4}\right| < 1$

$|x| < 4$

(b) Without evaluating your expansion,

(i) state, giving a reason, which of the three values of x should not be used

(1)

(ii) state, giving a reason, which of the three values of x would lead to the most accurate approximation to $\sqrt{2}$

(1)

4a) $(4-x)^{-1/2} = \frac{1}{2}\left(1-\frac{x}{4}\right)^{-1/2}$

$= \frac{1}{2}\left(1 + \left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}\left(-\frac{x}{4}\right)\right)$

$= \frac{1}{2} + \frac{1}{16}x + \frac{3}{256}x^2$

bi) $x=-14$ is not valid as $|x|<4$

ii) $x=-\frac{1}{2}$ as its the smallest value.

Using Partial Fractions

Partial fractions allows us to split up a fraction into ones we can then find the binomial expansion of.

a) Express $\frac{4-5x}{(1+x)(2-x)}$ as partial fractions.

b) Hence show that the cubic approximation of $\frac{4-5x}{(1+x)(2-x)}$ is $2-\frac{7}{2}x+\frac{11}{4}x^2-\frac{25}{8}x^3$

c) State the range of values of x for which the expansion is valid.

a) $\frac{4-5x}{(1+x)(2-x)} = \frac{A}{1+x} + \frac{B}{2-x}$

$4-5x = A(2-x) + B(1+x)$

$x=2$
$-6 = 3B$
$-2 = B$

$x=-1$
$9 = 3A$
$3 = A$

$\frac{3}{1+x} - \frac{2}{2-x}$

b) $3(1+x)^{-1} = 3\left(1-x+\frac{(-1)(-2)}{2!}(x)^2 + \frac{(-1)(-2)(-3)}{3!}(x)^3\right)$

$= 3(1-x+x^2-x^3)$

$= 3-3x+3x^2-3x^3$

$|x|<1$

$2(2-x)^{-1} = 2\times 2^{-1}\left(1-\frac{x}{2}\right)^{-1}$

$=\left(1-\frac{x}{2}\right)^{-1}$

$= 1+\frac{x}{2}+\frac{x^2}{4}+\frac{x^3}{8}$

$\left|\frac{x}{2}\right|<1 \Rightarrow |x|<2$

$\frac{4-5x}{(1+x)(2-x)} = 3-3x+3x^2-3x^3-1-\frac{x}{2}-\frac{x^2}{4}-\frac{x^3}{8}$

$= 2-\frac{7}{2}x+\frac{11}{4}x^2-\frac{25}{8}x^3$

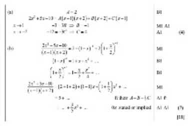
c) Valid for $|x|<1$

[C4 June 2010 Q5]

10.
$$\frac{2x^2+5x-10}{(x-1)(x+2)} \equiv A + \frac{B}{x-1} + \frac{C}{x+2}.$$

(*a*) Find the values of the constants *A*, *B* and *C*. **(4)**

(*b*) Hence, or otherwise, expand $\frac{2x^2+5x-10}{(x-1)(x+2)}$ in ascending powers of *x*, as far as the term in x^2. Give each coefficient as a simplified fraction. **(7)**



a) $2x^2+5x-10 = A(x-1)(x+2) + B(x+2) + C(x-1)$

$x=1$
$-3 = 3B$
$-1 = B$

$x=-2$
$-12 = -3C$
$4 = C$

$x=0$
$-10 = -2A + 2B - C$
$-10 = -2A - 6$
$A = 2$

b) $2 + \frac{-1}{x-1} + \frac{4}{x+2} = 2 + (1-x)^{-1} + 2\left(1+\frac{x}{2}\right)^{-1}$

$(1-x)^{-1} = 1 + x + x^2$

$2\left(1+\frac{x}{2}\right)^{-1} = 2\left(1 - \frac{x}{2} + \frac{x^2}{4}\right) = 2 - x + \frac{x^2}{2}$

Ex 4C

$\frac{2x^2+5x-10}{(x-1)(x+2)} = 2+1+2+x-x+x^2+\frac{x^2}{2}$

$= 5 + \frac{3}{2}x^2$