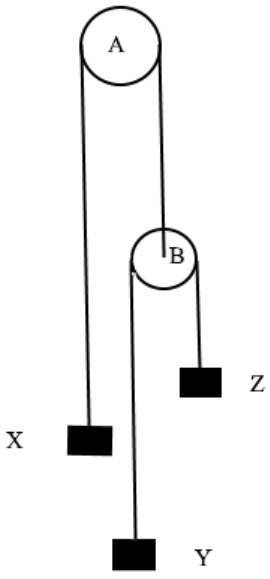
Section 3: Connected objects

Exercise level 3 (Extension)

In all questions leave your answers in terms of g.

1. A particle X on a plane horizontal table is attached to the end of long, light, inextensible string at right angles to the edge of the table. The string passes over a smooth pulley at the edge of the table to hang with a second particle Y attached to the end. Assume that at no stage during motion does X reach the edge of the table or collide with the pulley. Do not substitute a value for g.
   (i) If X has mass 3 kg and Y has mass 4 kg, find the acceleration of the system on the assumption that the table is smooth.
   (ii) Suppose now that the table is not smooth, and the acceleration is found to be $\frac{g}{2}$ ms^{-2}. Find the friction force.
   (iii) Assuming that X has mass 3 kg still and the friction force is unchanged, what is the minimum mass for Y sufficient for motion to take place?
   For the rest of the question assume once more that the table is smooth.
   (iv) Now X has mass m kg and Y has mass n kg. Find the acceleration b ms^{-2}.
   (v) When X has mass m kg what mass must Y have (in terms of m and n) to reduce the acceleration to $\frac{b}{2}$ ms^{-2}?
   (vi) When Y has mass n kg what must the mass of X be (in terms of m and n) to make the acceleration $\frac{b}{2}$ ms^{-2}?

2. A pulley system of strings, pulley wheels and suspended masses is shown in the diagram. The pulleys are light and free-running. Pulley A is fixed while pulley B is suspended from the end of one string is such a way that the plane of the pulley wheel stays vertical. Each pulley runs smoothly. The strings are light, inextensible, and hang vertically. The masses at X, Y, and Z are 4, 3 and 5 kg respectively.



   (i) Draw separate diagrams to show the forces which act on the pulley B and on each of the masses X, Y, and Z.
   (ii) What is the relationship between the accelerations of mass X and pulley B?
   (iii) If the acceleration of pulley B and of masses Y and Z are b, c and d respectively (in each case positive is downwards) explain why $2b = c + d$.
   (iv) Find the acceleration of each of the masses X and Y and the tension in the string which joins pulleys A and B. (Assume that at no stage do the masses collide!)

3. A single, free-running light pulley has its axle fixed horizontally to the wall of a lift. The pulley has a long, light, inextensible string running over it, with a 1 kg particle suspended by from one side and 2 kg suspended from the other.
   (i) The system is held still and then released when the lift sets off. What is the

apparent acceleration of each weight when the lift is accelerating upwards at $0.05g$ ms^{-2}?

(ii) If instead each mass is suspended from its part of the string by a Newton metre of mass 0.25 kg what weight (force) will the Newton metre register in each case?