

Section 1: Introducing the binomial distribution

Exercise level 3 (Extension)

- It is known that, on average, one matchbox in ten contains fewer than 42 matches. Eight boxes are selected at random from a large batch, and the number of boxes that contain fewer than 42 matches is denoted by Y .
 - State two conditions needed to model Y by a binomial distribution.
 - Assume now that a binomial model is valid.

On Wednesday eight boxes are selected and on Thursday another eight boxes are selected. Find the probability that on one of these days there are no boxes containing fewer than 42 matches and on the other day there are 2 or more boxes that contain fewer than 42 matches.
- Experience shows that when seeds of a certain type are planted, on average 90% of the seeds will germinate. A gardener plants ten of these seeds in a tray and waits to see how many will germinate.
 - Find the probability that fewer seeds germinate than expected.
 - The gardener then plants 20 trays of seeds, each with ten seeds. Find the probability that there are at least 19 trays in each of which at least 8 seeds germinate.
- The random variable W has a binomial distribution with parameters n and p . If $p = 0.27$, find the smallest value of n such that $P(W \geq 1) > 0.95$.
- A variable T has the distribution $B(11, p)$. Given that $\text{Var}(T) = 1.76$, find two possible values of p .
- Claire travels to and from college by bus on the five days of the week that she attends college. She believes that the probability of having to wait more than six minutes to catch a bus is 0.4 and is independent of the time of day and the direction of travel.
 - Assuming Claire's beliefs are correct, find the probability that, during a particular week, the number of times she has to wait more than six minutes is
 - 3 or fewer
 - more than 4.
 - Assuming Claire's beliefs are correct, calculate values of the mean and standard deviation of the number of times she has to wait more than six minutes to catch a bus in a week.
 - During a thirteen-week period, the number of times (per week) Claire had to wait more than six minutes to catch a bus were as follows:

4	8	8	9	3	2	2
7	0	1	5	2	0	

 - Calculate the mean and standard deviation of this data.
 - State, with reasons, whether your answers to (iii) (a) support Claire's belief that the probability of having to wait more than six minutes to catch a bus is 0.4 and is independent of the time of day and direction of travel.