Section 2: Matrices and transformations

Exercise level 3

1. Let $\mathbf{A} = \begin{pmatrix} -1 & 5 \\ -2 & 10 \end{pmatrix}$.

   (i)   Find $\mathbf{A}\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ and $\mathbf{A}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

   (ii) Are the vectors $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ parallel?

   (iii) Are the vectors given by $\mathbf{A}\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ and $\mathbf{A}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ parallel?

2. A sequence of vectors $\begin{pmatrix} a_1 \\ b_1 \end{pmatrix}, \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}, \begin{pmatrix} a_3 \\ b_3 \end{pmatrix}, \ldots$ satisfies

$$\begin{pmatrix} a_{n+1} \\ b_{n+1} \end{pmatrix} = \begin{pmatrix} a_n - \sqrt{3}b_n \\ \sqrt{3}a_n + b_n \end{pmatrix}$$

   for $n = 1, 2, 3, \ldots$.

   Given that $\begin{pmatrix} a_7 \\ b_7 \end{pmatrix} = \begin{pmatrix} 8 \\ 32 \end{pmatrix}$, find $\begin{pmatrix} a_1 \\ b_1 \end{pmatrix}$.