Section 3: Invariance

Exercise level 3

Note: a translation is not a linear transformation (since the origin is not mapped to itself), so it cannot be represented by a 2x2 matrix. However a translation can be represented by a 3x3 matrix, as shown in the questions below.

1. T is a translation of the plane by the vector $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$.

   (i) Show that the matrix $\begin{pmatrix} 1 & 0 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ maps the point $\begin{pmatrix} p \\ q \\ 1 \end{pmatrix}$ and $\begin{pmatrix} p-3 \\ q+1 \\ 1 \end{pmatrix}$.

   (ii) Point (X, Y) is the image of (x, y) under a combined transformation TM such that $\begin{pmatrix} X \\ Y \\ 1 \end{pmatrix} = \begin{pmatrix} 0 & 2 & -3 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$.

   Find any invariant points and invariant lines of the combined transformation.

2. S is a translation of the plane by the vector $\begin{pmatrix} 3 \\ -3 \end{pmatrix}$.

   The image of (x, y) under a combined transformation SN is such that
   $\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} 2 & -1 & 3 \\ 3 & 4 & -3 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$.

   Find any invariant points of this combined transformation and show that there are no invariant lines.