Section 3: Matrices and simultaneous equations

Exercise level 3

1. Consider the following simultaneous equations:

$$ax + by = 1$$
$$bx + ay = b$$

(i) Find conditions on a and b for which the simultaneous equations have a unique solution.

(ii) Solve the simultaneous equations for $a = 2$ giving the solution in terms of b. For which values of b will the solution lie on the line $y = x$?

2. (i) Find the value of λ given that

$$\mathbf{T} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \text{ and } \mathbf{T}^{-1} = \frac{1}{2}\begin{pmatrix} \lambda & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & \lambda \end{pmatrix}$$

(ii) Solve

$$\begin{aligned} Z + Y &= a \\ Z + X &= b \\ Y + X &= c \end{aligned}$$

for Z, Y and X.

(iii) If $a + b - c > 0$, $b + c - a > 0$ and $c + a - b > 0$, solve

$$\begin{aligned} xy + xz &= a \\ xy + yz &= b \\ xz + yz &= c \end{aligned}$$

for x, y and z.

3. Consider the following simultaneous equations

$$x\cos\theta - y\sin\theta = 3$$
$$x\sin\theta + y\cos\theta = 4$$

(i) Show that the simultaneous equations have a unique solution for each and every value of θ.

(ii) Solve the simultaneous equations for $\theta = 30°$.

(iii) Find the Cartesian equation of the locus of points that satisfy the simultaneous equations as the value of θ varies.

4. The matrix $\mathbf{A} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & a \\ 1 & 0 & b \end{pmatrix}$.

(i) Find $\mathbf{A}^{-1}$.

Edexcel AS FM Inverse matrices 3 Exercise

(ii)For what values of a, b and c does $\mathbf{A}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} c \\ 1 \\ 1 \end{pmatrix}$ have a unique solution?

(iii)You are told that this equation has solutions, but not a unique one. What can you say about a, b and c now?

5. You are given the matrix equation $\begin{pmatrix} 2 & a & b \\ 4 & c & d \\ 8 & 4 & 12 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} p \\ q \\ 20 \end{pmatrix}$.

This represents the equations for three planes.
Find values for a, b, c, d, p and q that give the following situations:
(i) the three planes meet at a point
(ii)the three planes form a triangular prism
(iii)two of the planes are parallel, the third is not
(iv)all three planes are parallel and distinct
(v) the three planes share a line (sheaf of planes)