

## Section 2: The inverse of a 3×3 matrix

### Exercise level 3

$$1. \mathbf{P} = \begin{pmatrix} a & 1 & 1 \\ 0 & b & 1 \\ -3 & 0 & -1 \end{pmatrix}, \mathbf{Q} = \mathbf{P} + \mathbf{I}, \mathbf{R} = \mathbf{P} + 2\mathbf{I}.$$

When any shape is mapped by any of the transformations represented by  $\mathbf{P}$ ,  $\mathbf{Q}$  or  $\mathbf{R}$ , the volume scale factor is the same each time. Find all possibilities for  $a$  and  $b$ .

2. (i) A 3×3 matrix has a 1 in its top row, a 2 in its second row, and a 3 in its third row, and all other entries are zero.  
The matrix is non-singular.  
How many such matrices are possible?
- (ii) For each of these matrices, find its inverse.
- (iii) If  $\mathbf{A}$  is the matrix formed by adding all the possible inverses together, show that  $\det \mathbf{A} = 0$ .