Section 1: Introduction to matrices

Exercise level 3

1. Find the values of a, b and c such that

$$\begin{pmatrix} a & 4 \\ 5 & 1 \end{pmatrix} + c\begin{pmatrix} 1 & b \\ -2 & 0 \end{pmatrix} = \begin{pmatrix} 5 & 1 \\ -1 & 1 \end{pmatrix}.$$

2. A 2x2 matrix of the form $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$ is called a diagonal matrix.

   (i) Show that the product of two diagonal matrices is also diagonal.

   (ii) Let $\mathbf{A} = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$. Find an expression for $\mathbf{A}^n$ in terms of a and b.

3. Let $\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$.

   (i) Find $\mathbf{A}^2$ and $\mathbf{A}^3$.

   (ii) From your results, express the general matrix $\mathbf{A}^k$ in terms of k.

   (iii) By multiplying your $\mathbf{A}^k$ by $\mathbf{A}$, find $\mathbf{A}^{k+1}$. Explain how this supports your expression for $\mathbf{A}^k$.

   (iv) Find the values of a and b in terms of n such that $\mathbf{A}^n + a\mathbf{A} + b\mathbf{I} = 0$, where $\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is the 2x2 identity matrix.