Section 1: Modulus and argument

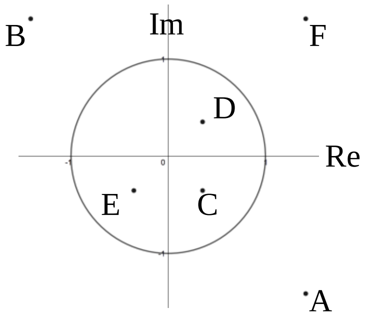
Exercise level 3

1. (i) Show that $|z-w|^2 = zz^* - zw^* - wz^* + ww^*$.

   (ii) Given that $|z+w| = 3$ and $|z| = |w| = 2$, find $|z-w|$.

2. (i) Given a complex number $z = a + ib$, find $\frac{1}{z}$.

   (ii) Several numbers are shown in the Argand diagram below. The circle has radius 1 and centre at the origin.
   Which of these numbers is the reciprocal of F?



3. (i) Show that $\left(\frac{z}{w}\right)^* = \frac{z^*}{w^*}$.

   (ii) Given that $z, w \neq 0$ and $|z+w| = |z-w|$, show that $\frac{z}{w} + \frac{z^*}{w^*} = 0$ and hence find all possible values of $\arg\left(\frac{z}{w}\right)$.

4. (i) $z_1 = \frac{1}{2}(\cos\frac{\pi}{2} + \mathrm{i}\sin\frac{\pi}{2})$, $z_2 = 2(\cos\frac{\pi}{6} + \mathrm{i}\sin\frac{\pi}{6})$
   The triangle with vertices O, z_1 and z_2 has area A. Find A.

   (ii) Show that the triangle with vertices O, z_1^2 and z_2^2 also has area A.

   (iii) Suppose $z_1 = r(\cos a + \mathrm{i}\sin a)$ and $z_2 = s(\cos b + \mathrm{i}\sin b)$, where $0 \le a < b \le \frac{\pi}{2}$, and the two triangles with vertices O, z_1 and z_2 and O, z_1^2 and z_2^2 have equal areas. Show that $\cos(b-a) = \frac{1}{2rs}$.

   (iv) Show that if $s = \frac{1}{r}$, then the areas of the two triangles are equal if and only if $b - a = \frac{\pi}{3}$.