

Section 1: Homogeneous differential equations

Solutions to Exercise level 1

$$1. \text{ (i) } \frac{d^2 y}{dx^2} + \frac{dy}{dx} - 6y = 0$$

Auxiliary equation is $k^2 + k - 6 = 0$

$$(k+3)(k-2) = 0$$

$$k = -3 \text{ or } 2$$

General solution is $y = Ae^{-3x} + Be^{2x}$

$$\text{(ii) } \frac{d^2 y}{dx^2} + 6\frac{dy}{dx} + 9y = 0$$

Auxiliary equation is $k^2 + 6k + 9 = 0$

$$(k+3)^2 = 0$$

$$k = -3$$

General solution is $y = (Ax + B)e^{-3x}$

$$\text{(iii) } \frac{d^2 y}{dx^2} + 16y = 0$$

Auxiliary equation is $k^2 + 16 = 0$

$$k^2 = -16$$

$$k = \pm 4i$$

General solution is $y = A \cos 4x + B \sin 4x$

$$\text{(iv) } 3\frac{d^2 y}{dx^2} - 2\frac{dy}{dx} = 0$$

Auxiliary equation is $3k^2 - 2k = 0$

$$k(3k - 2) = 0$$

$$k = 0 \text{ or } k = \frac{2}{3}$$

General solution is $y = A + Be^{\frac{2}{3}x}$

$$\text{(v) } \frac{d^2 y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$

Edexcel FM Second order DEs 1 Exercise solutions

Auxiliary equation is $k^2 - 2k + 2 = 0$

$$k = \frac{2 \pm \sqrt{(-2)^2 - 4 \times 1 \times 2}}{2} = \frac{2 \pm \sqrt{-4}}{2} = 1 \pm i$$

General solution is $y = e^x(A \cos x + B \sin x)$

$$(vi) \quad 3 \frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} - y = 0$$

Auxiliary equation is $3k^2 - 2k - 1 = 0$

$$(3k+1)(k-1) = 0$$

$$k = -\frac{1}{3} \text{ or } k = 1$$

General solution is $y = Ae^{-\frac{1}{3}x} + Be^x$

$$2. (i) \quad \frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} - 3y = 0$$

Auxiliary equation is $k^2 - 2k - 3 = 0$

$$(k-3)(k+1) = 0$$

$$k = 3 \text{ or } k = -1$$

General solution is $y = Ae^{3x} + Be^{-x}$

When $x=0, y=1$: $A+B=1$

$$\frac{dy}{dx} = 3Ae^{3x} - Be^{-x}$$

When $x=0, \frac{dy}{dx}=0$: $3A - B = 0 \Rightarrow B = 3A$

$$A + 3A = 1$$

$$A = \frac{1}{4}, B = \frac{3}{4}$$

Particular solution is $y = \frac{1}{4}e^{3x} + \frac{3}{4}e^{-x}$

$$(ii) \quad 4 \frac{d^2 y}{dx^2} + y = 0$$

Auxiliary equation is $4k^2 + 1 = 0$

$$k^2 = -\frac{1}{4}$$

$$k = \pm \frac{1}{2}i$$

General solution is $y = A \cos \frac{1}{2}x + B \sin \frac{1}{2}x$

When $x=0, y=1$: $1 = A$

Edexcel FM Second order DEs 1 Exercise solutions

$$\frac{dy}{dx} = -\frac{1}{2}A \sin \frac{1}{2}x + \frac{1}{2}B \cos \frac{1}{2}x$$

$$\text{When } x=0, \frac{dy}{dx} = 1: \quad 1 = \frac{1}{2}B \Rightarrow B = 2$$

$$\text{Particular solution is } y = \cos \frac{1}{2}x + 2 \sin \frac{1}{2}x$$

$$(iii) \quad 2 \frac{d^2y}{dx^2} - \frac{dy}{dx} - 3y = 0$$

$$\text{Auxiliary equation is } 2k^2 - k - 3 = 0$$

$$(2k-3)(k+1) = 0$$

$$k = \frac{3}{2} \text{ or } k = -1$$

$$\text{General solution is } y = Ae^{\frac{3}{2}x} + Be^{-x}$$

$$\text{When } x=0, y=0: \quad 0 = A+B \Rightarrow B = -A$$

$$\frac{dy}{dx} = \frac{3}{2}Ae^{\frac{3}{2}x} - Be^{-x}$$

$$\text{When } x=0, \frac{dy}{dx} = 1: \quad \frac{3}{2}A - B = 1$$

$$\frac{3}{2}A + A = 1$$

$$\frac{5}{2}A = 1$$

$$A = \frac{2}{5}, B = -\frac{2}{5}$$

$$\text{Particular solution is } y = \frac{2}{5}e^{\frac{3}{2}x} - \frac{2}{5}e^{-x}$$

$$(iv) \quad \frac{d^2y}{dx^2} + 4 \frac{dy}{dx} + 5y = 0$$

$$\text{Auxiliary equation is } k^2 + 4k + 5 = 0$$

$$k = \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times 5}}{2} = \frac{-4 \pm \sqrt{-4}}{2} = -2 \pm i$$

$$\text{General solution is } y = e^{-2x}(A \cos x + B \sin x)$$

$$\text{When } x=0, y=1: \quad 1 = A$$

$$\frac{dy}{dx} = -2e^{-2x}(A \cos x + B \sin x) + e^{-2x}(-A \sin x + B \cos x)$$

$$\text{When } x=0, \frac{dy}{dx} = 2: \quad 2 = -2A + B \Rightarrow B = 2 + 2A = 4$$

$$\text{Particular solution is } y = e^{-2x}(\cos x + 4 \sin x)$$

Edexcel FM Second order DEs 1 Exercise solutions

$$(v) \quad 4 \frac{d^2 y}{dx^2} - 4 \frac{dy}{dx} + y = 0$$

Auxiliary equation is $4k^2 - 4k + 1 = 0$

$$(2k - 1)^2 = 0$$

$$k = \frac{1}{2}$$

General solution is $y = (Ax + B)e^{\frac{1}{2}x}$

$$\text{When } x = 0, y = 2: \quad 2 = B$$

$$\frac{dy}{dx} = Ae^{\frac{1}{2}x} + (Ax + B) \times \frac{1}{2}e^{\frac{1}{2}x}$$

$$\text{When } x = 0, \frac{dy}{dx} = 0: \quad 0 = A + \frac{1}{2}B \Rightarrow A = -\frac{1}{2}B = -1$$

Particular solution is $y = (x - 2)e^{\frac{1}{2}x}$