Section 1: Introduction

Solutions to Exercise level 2

1. (i) $\frac{dy}{dx} = x^2y + 2y = y(x^2+2)$

$\int \frac{1}{y}dy = \int (x^2+2)dx$

$\ln y = \frac{1}{3}x^3 + 2x + c$

$y = e^{\frac{1}{3}x^3+2x+c}$

$y = Ae^{\frac{1}{3}x^3+2x}$

Replacing e^c by A

(ii) $x\frac{dy}{dx} = \cos^2 y$

$\int \sec^2 y\, dy = \int \frac{1}{x}dx$

$\tan y = \ln x + c = \ln(Ax)$

Replacing c by $\ln A$

$y = \arctan[\ln(Ax)]$

(iii) $\frac{dy}{dx} = xe^{x+y}$

$\int e^{-y}dy = \int xe^x dx$

$-e^{-y} = xe^x - \int e^x dx$

$e^{-y} = -xe^x + \int e^x dx = -xe^x + e^x + c = (1-x)e^x + c$

$-y = \ln[(1-x)e^x + c]$

$y = -\ln[(1-x)e^x + c]$

(iv) $\frac{dy}{dx} = x(1-y^2)$

$\int \frac{1}{1-y^2}dy = \int x\,dx$

$\frac{1}{2}\int\left(\frac{1}{1-y} + \frac{1}{1+y}\right)dy = \int x\,dx$

$\int\left(\frac{1}{1-y} + \frac{1}{1+y}\right)dy = \int 2x\,dx$

$-\ln(1-y) + \ln(1+y) = x^2 + c$

$$\ln\left(\frac{1+y}{1-y}\right) = x^2 + c$$

$$\frac{1+y}{1-y} = Ae^{x^2}$$

$$1 + y = Ae^{x^2} - Ae^{x^2}y$$

$$y + Ae^{x^2}y = Ae^{x^2} - 1$$

$$y = \frac{Ae^{x^2} - 1}{Ae^{x^2} + 1}$$

(v) $\dfrac{dy}{dx} = \dfrac{2y}{x(x-2)}$

$$\int \frac{1}{y}\,dy = \int \frac{2}{x(x-2)}\,dx$$

Using partial fractions: $\dfrac{2}{x(x-2)} = \dfrac{A}{x} + \dfrac{B}{x-2}$

$$2 = A(x-2) + Bx$$

$$x = 0 \Rightarrow A = -1$$

$$x = 2 \Rightarrow B = 1$$

$$\int \frac{1}{y}\,dy = \int \left(\frac{1}{x-2} - \frac{1}{x}\right)dx$$

$$\ln y = \ln(x-2) - \ln x + \ln c$$

$$\ln y = \ln \frac{c(x-2)}{x}$$

$$y = \frac{c(x-2)}{x}$$

$$y = c\left(1 - \frac{2}{x}\right)$$

2. (i) $x\dfrac{dy}{dx} = y + 1$

$$\int \frac{1}{y+1}\,dy = \int \frac{1}{x}\,dx$$

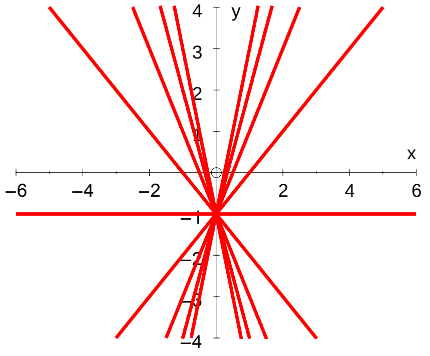
$$\ln(y+1) = \ln x + c = \ln(Ax)$$

Replacing c by $\ln A$

$$y + 1 = Ax$$

$$y = Ax - 1$$

(ii)



(iii) When $x=1,\ y=1$: $1=A-1 \quad \Rightarrow A=2$

Particular solution is $y=2x-1$

3. (i) $\dfrac{dy}{dx}=y$

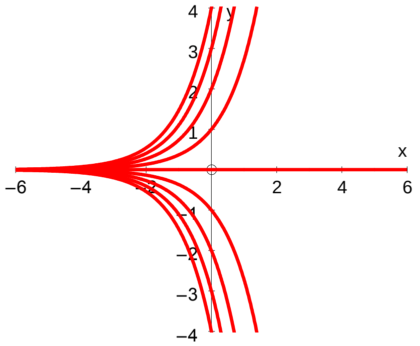
$\int \frac{1}{y}\,dy=\int dx$

$\ln y = x + c$

$y = e^{x+c} = Ae^{x}$

Replacing e^c by A

(ii)



(iii) When $x=1,\ y=2$: $2=Ae \quad \Rightarrow A=2e^{-1}$

Particular solution is $y=2e^{x-1}$.

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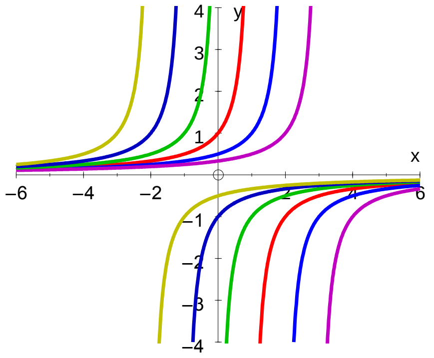
4. (i) $\frac{dy}{dx} = y^2$

$\int \frac{1}{y^2}\,dy = \int dx$

$-\frac{1}{y} = x + c$

$y = \frac{-1}{x+c}$ or $y = \frac{1}{A-x}$

(ii)



(iii) When $x = 0, y = 2$: $2 = \frac{1}{A} \Rightarrow A = \frac{1}{2}$

Particular solution is $y = \frac{1}{\frac{1}{2} - x} = \frac{2}{1-2x}$