Section 1: Introducing the hyperbolic functions

Exercise level 1 solutions

1. $$\tanh 3x = \frac{\sinh 3x}{\cosh 3x} = \frac{\frac{1}{2}(e^{3x} - e^{-3x})}{\frac{1}{2}(e^{3x} + e^{-3x})} = \frac{e^{6x} - 1}{e^{6x} + 1} \qquad \left(\text{or } \frac{e^{3x} - e^{-3x}}{e^{3x} + e^{-3x}}\right)$$

2. (i) $$2\sinh x \cosh x = 2\left(\frac{e^{x} - e^{-x}}{2}\right)\left(\frac{e^{x} + e^{-x}}{2}\right) = 2\left(\frac{e^{2x} - 1 + 1 - e^{-2x}}{4}\right) = \frac{e^{2x} - e^{-2x}}{2} = \sinh 2x$$

(ii) $$1 + 2\sinh^2 x = 1 + 2\left(\frac{e^{x} - e^{-x}}{2}\right)^2 = 1 + 2\left(\frac{e^{2x} - 2 + e^{-2x}}{4}\right) = 1 + \frac{e^{2x} + e^{-2x}}{2} - 1 = \cosh 2x$$

(ii) $$\begin{aligned}\cosh x \sinh 4x &= \cosh x \times 2\sinh 2x \cosh 2x \\ &= \cosh x \times 4\sinh x \cosh x(1 + 2\sinh^2 x) \\ &= 4\sinh x \cosh^2 x(1 + 2\sinh^2 x) \\ &= 4\sinh x(1 + \sinh^2 x)(1 + 2\sinh^2 x) \\ &= 4\sinh x + 12\sinh^3 x + 8\sinh^5 x\end{aligned}$$

3. $$\begin{aligned}\frac{d}{dx}\left(x^3 \cosh^2 4x\right) &= 3x^2 \cosh^2 4x + x^3 \times 2 \times 4\sinh 4x \cosh 4x. \\ &= 3x^2 \cosh^2 4x + 8x^3 \sinh 4x \cosh 4x\end{aligned}$$

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4. Using $\cosh 2x = 1 + 2\sinh^2 x$

$$\sinh^2 x = \tfrac{1}{2}(\cosh 2x - 1)$$

$$\int \sinh^2 x \, dx = \int \tfrac{1}{2}(\cosh 2x - 1)\, dx$$

$$= \tfrac{1}{4}\sinh 2x - \tfrac{1}{2}x + c$$

5. $4\cosh x + 5\sinh x = 6$

$$4 \times \tfrac{1}{2}\left(e^x + e^{-x}\right) + 5 \times \tfrac{1}{2}\left(e^x - e^{-x}\right) = 6$$

$$9e^x - e^{-x} = 12$$

$$9e^{2x} - 12e^x - 1 = 0$$

$$e^x = \frac{12 \pm \sqrt{144 + 36}}{18} = \frac{12 \pm 6\sqrt{5}}{18} = \frac{2 \pm \sqrt{5}}{3}$$

e^x cannot be negative, so $e^x \neq \dfrac{2 - \sqrt{5}}{3}$

Hence $e^x = \dfrac{2 + \sqrt{5}}{3}$, i.e. $x = \ln\left(\dfrac{2 + \sqrt{5}}{3}\right)$