

## Section 2: The area of a sector

## Exercise level 3 solutions

$$1. (a) r = \frac{1}{\sin \theta - \cos \theta}$$

$$r \sin \theta - r \cos \theta = 1$$

$$y - x = 1$$

$$r = \frac{1}{a \sin \theta - b \cos \theta}$$

$$ar \sin \theta - br \cos \theta = 1$$

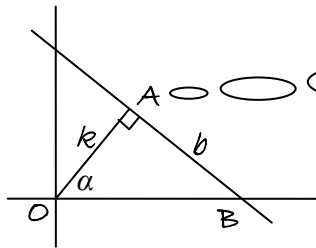
$$ay - bx = 1$$

$$(b) r = k \sec(\theta - \alpha)$$

$$r \cos(\theta - \alpha) = k$$

$$r(\cos \theta \cos \alpha + \sin \theta \sin \alpha) = k$$

$$x \cos \alpha + y \sin \alpha = k$$



A is the closest point on the line to O, so it is the point where  $\sec(\theta - \alpha)$  takes its minimum value, so  $\theta = \alpha$

$$(i) \text{ using basic trigonometry gives } b = k \tan \alpha$$

$$\text{Area} = \frac{1}{2} kb$$

$$= \frac{1}{2} k \times k \tan \alpha$$

$$= \frac{1}{2} k^2 \tan \alpha$$

$$(ii) \text{ Area} = \int_0^\alpha \frac{1}{2} r^2 d\theta$$

$$= \int_0^\alpha \frac{1}{2} k^2 \sec^2(\theta - \alpha) d\theta$$

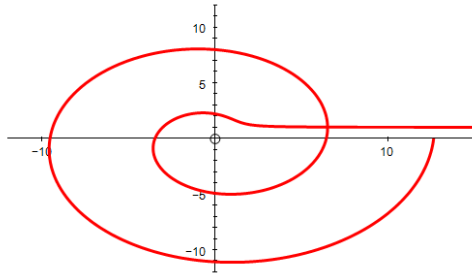
$$= \frac{1}{2} k^2 [\tan(\theta - \alpha)]_0^\alpha$$

$$= 0 - \frac{1}{2} k^2 \tan(-\alpha)$$

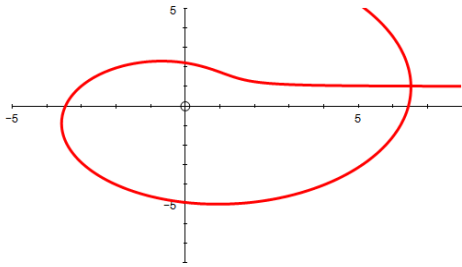
$$= \frac{1}{2} k^2 \tan \alpha$$

## Edexcel FM Polar coordinates 2 Exercise solutions

2.



The central loop looks like this:



$$\begin{aligned}
 \text{At the intersection point, } \alpha + \frac{1}{\alpha} &= 2\pi + \alpha + \frac{1}{2\pi + \alpha} \\
 \Rightarrow \frac{1}{\alpha} &= 2\pi + \frac{1}{2\pi + \alpha} \\
 \Rightarrow 2\pi + \alpha &= 2\pi\alpha(2\pi + \alpha) + \alpha \\
 \Rightarrow 1 &= \alpha(2\pi + \alpha) \\
 \Rightarrow \alpha^2 + 2\pi\alpha - 1 &= 0 \\
 \Rightarrow \alpha &= \frac{-2\pi + \sqrt{4\pi^2 + 4}}{2} = -\pi + \sqrt{\pi^2 + 1}
 \end{aligned}$$

$$\text{Let } \beta = \sqrt{\pi^2 + 1}$$

So limits of integration are  $-\pi + \beta$  and  $2\pi - \pi + \beta = \pi + \beta$

$$\begin{aligned}
 \text{Area} &= \int_{\beta-\pi}^{\beta+\pi} \frac{1}{2} \left( \theta + \frac{1}{\theta} \right)^2 d\theta \\
 &= \frac{1}{2} \int_{\beta-\pi}^{\beta+\pi} \left( \theta^2 + 2 + \frac{1}{\theta^2} \right) d\theta \\
 &= \frac{1}{2} \left[ \frac{1}{3} \theta^3 + 2\theta - \frac{1}{\theta} \right]_{\beta-\pi}^{\beta+\pi} \\
 &= \frac{1}{6} \left[ (\beta + \pi)^3 - (\beta - \pi)^3 \right] + (\beta + \pi) - (\beta - \pi) - \frac{1}{2} \left[ \frac{1}{\beta + \pi} - \frac{1}{\beta - \pi} \right] \\
 &= \frac{1}{6} (\beta^3 + 3\beta^2\pi + 3\beta\pi^2 + \pi^3 - \beta^3 + 3\beta^2\pi - 3\beta\pi^2 + \pi^3) \\
 &\quad + 2\pi - \frac{\beta - \pi - (\beta + \pi)}{2(\beta + \pi)(\beta - \pi)}
 \end{aligned}$$

## Edexcel FM Polar coordinates 2 Exercise solutions

$$\begin{aligned} &= \beta^2 \pi + \frac{1}{3} \pi^3 + 2\pi + \frac{\pi}{\beta^2 - \pi^2} \\ &= (\pi^2 + 1)\pi + \frac{1}{3} \pi^3 + 2\pi + \frac{\pi}{\pi^2 + 1 - \pi^2} \\ &= (\pi^2 + 1)\pi + \frac{1}{3} \pi^3 + 2\pi + \pi \\ &= \frac{4}{3} \pi^3 + 4\pi \end{aligned}$$