Section 1: Polar coordinates and curves

Exercise level 1 solutions

Since $\sin\theta$ is positive for $0<\theta<\pi$, r is positive for $0<\theta<\frac{\pi}{2}$, increasing from 0 to 1 (at $\theta=\frac{\pi}{4}$) and then decreasing to 0 again.

1. (i) $x=4, y=4$

$r=\sqrt{x^2+y^2}=\sqrt{16+16}=4\sqrt{2}$

$\tan\theta=\dfrac{4}{4}=1$

Since point is in first quadrant, $\theta=\dfrac{\pi}{4}$

Polar coordinates are $\left(4\sqrt{2},\dfrac{\pi}{4}\right)$

(ii) $x=1, y=\sqrt{3}$

$r=\sqrt{x^2+y^2}=\sqrt{1+3}=2$

$\tan\theta=\dfrac{\sqrt{3}}{1}=\sqrt{3}$

Since point is in first quadrant, $\theta=\dfrac{\pi}{3}$

Polar coordinates are $\left(2,\dfrac{\pi}{3}\right)$

(iii) $x=-3, y=4$

$r=\sqrt{x^2+y^2}=\sqrt{9+16}=5$

$\tan\theta=-\dfrac{4}{3}$

Since point is in second quadrant, $\theta=\pi-\arctan\frac{4}{3}=2.21$ (3 s.f.)

Polar coordinates are $\left(5, 2.21^c\right)$ (3 s.f.)

(iv) $x=-5, y=-12$

$r=\sqrt{x^2+y^2}=\sqrt{25+144}=13$

$\tan\theta=\dfrac{12}{5}$

Since point is in third quadrant, $\theta=\pi+\arctan\frac{5}{12}=4.32$

Polar coordinates are $\left(13, 4.32^c\right)$

Edexcel FM Polar coordinates 1 Exercise solutions

2. (i) $r = 4, \theta = \dfrac{\pi}{3}$

$$x = r\cos\theta = 4\cos\frac{\pi}{3} = 4 \times \frac{1}{2} = 2$$

$$y = r\sin\theta = 4\sin\frac{\pi}{3} = 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}$$

Cartesian coordinates are $\left(2, 2\sqrt{3}\right)$

(ii) $r = 5, \theta = \dfrac{\pi}{2}$

$$x = r\cos\theta = 5\cos\frac{\pi}{2} = 5 \times 0 = 0$$

$$y = r\sin\theta = 5\sin\frac{\pi}{2} = 5 \times 1 = 5$$

Cartesian coordinates are $(0, 5)$

(iii) $r = 8, \theta = \dfrac{5\pi}{4}$

$$x = r\cos\theta = 8\cos\frac{5\pi}{4} = 8 \times -\frac{1}{\sqrt{2}} = -4\sqrt{2}$$

$$y = r\sin\theta = 8\sin\frac{5\pi}{4} = 8 \times -\frac{1}{\sqrt{2}} = -4\sqrt{2}$$

Cartesian coordinates are $\left(-4\sqrt{2}, -4\sqrt{2}\right)$

(iv) $r = 6, \theta = \dfrac{11\pi}{6}$

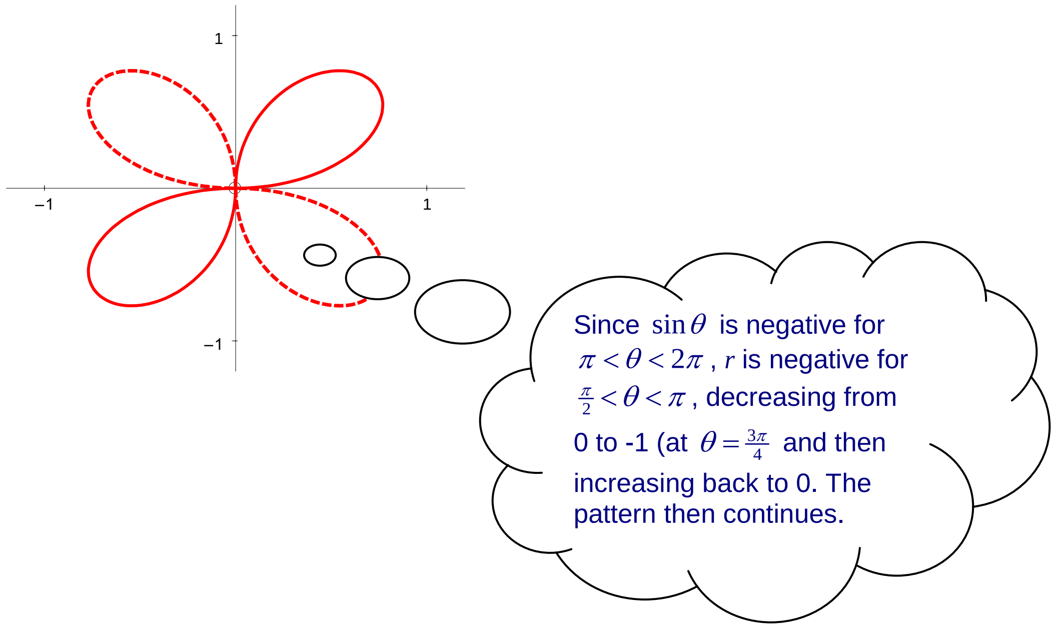
$$x = r\cos\theta = 6\cos\frac{11\pi}{6} = 6 \times \frac{\sqrt{3}}{2} = 3\sqrt{3}$$

$$y = r\sin\theta = 6\sin\frac{11\pi}{6} = 6 \times -\frac{1}{2} = -3$$

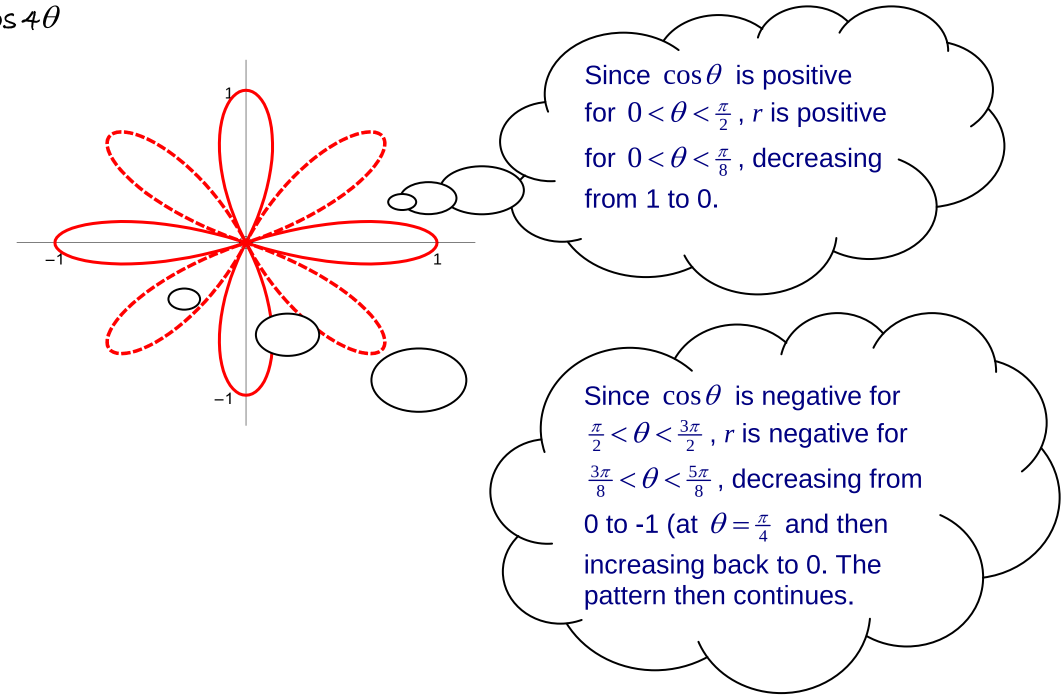
Cartesian coordinates are $\left(3\sqrt{3}, -3\right)$

Edexcel FM Polar coordinates 1 Exercise solutions

3. (i) $r = \sin 2\theta$

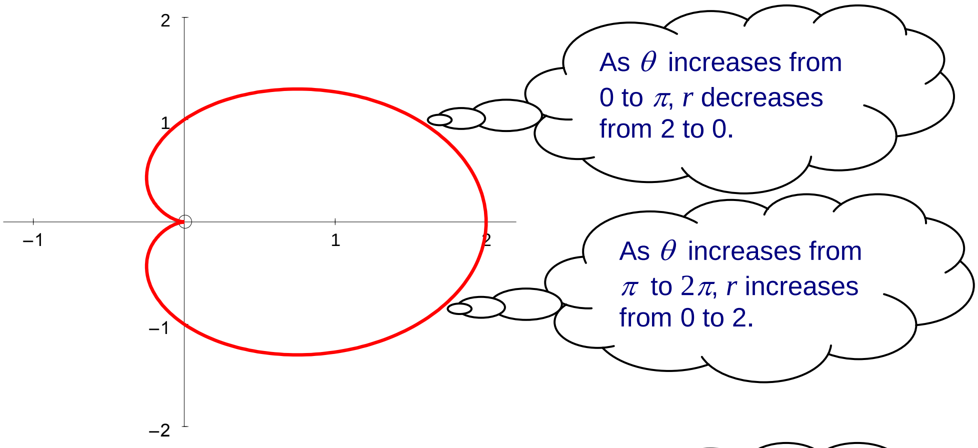


(i) $r = \cos 4\theta$

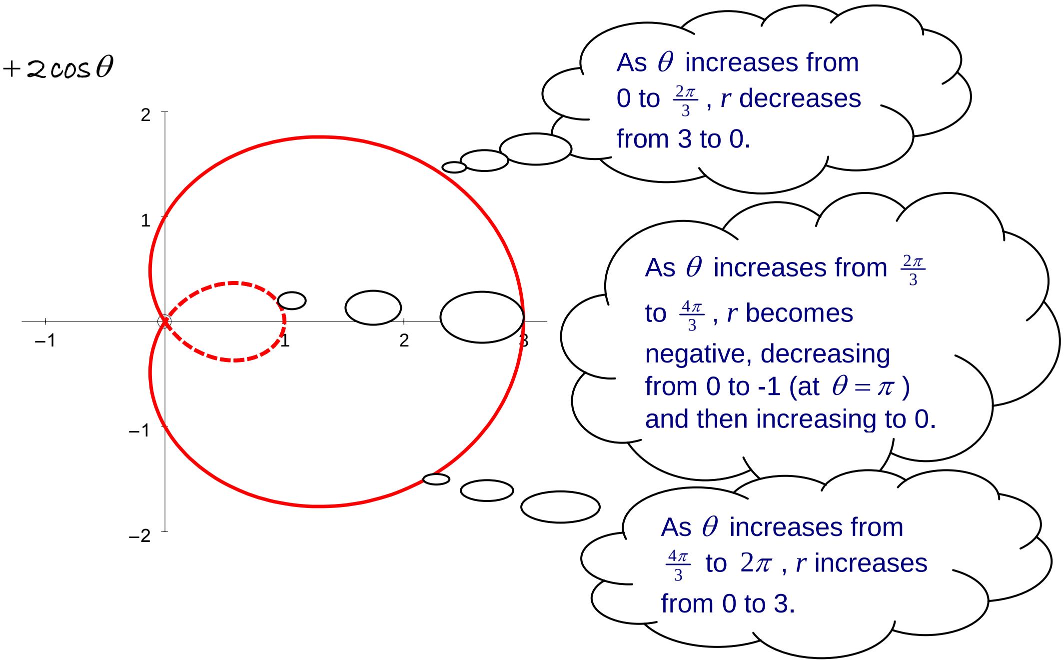


Edexcel FM Polar coordinates 1 Exercise solutions

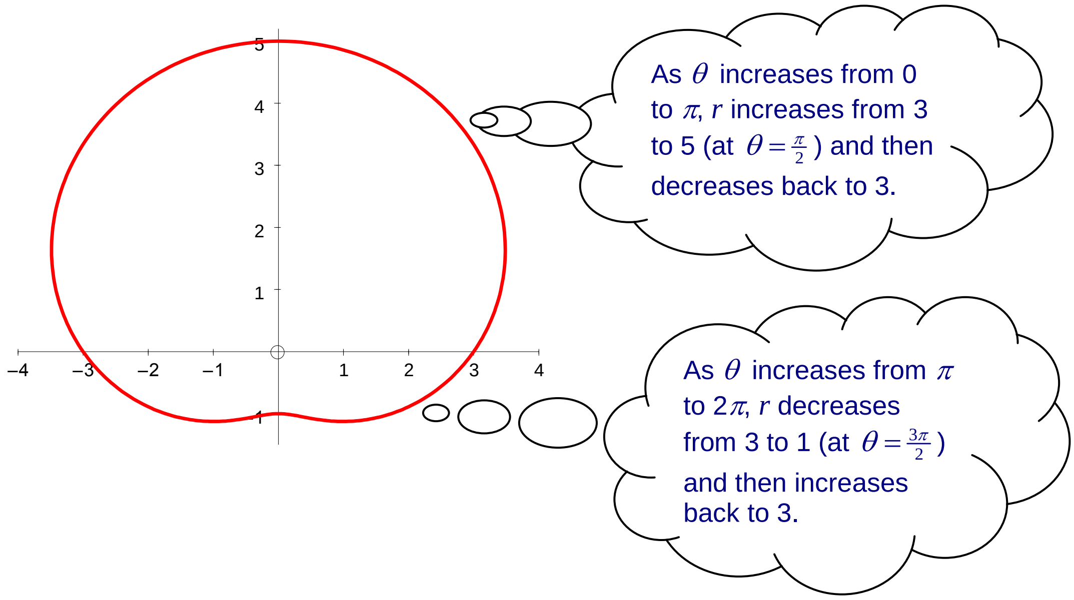
(iii) $r = 1 + \cos\theta$



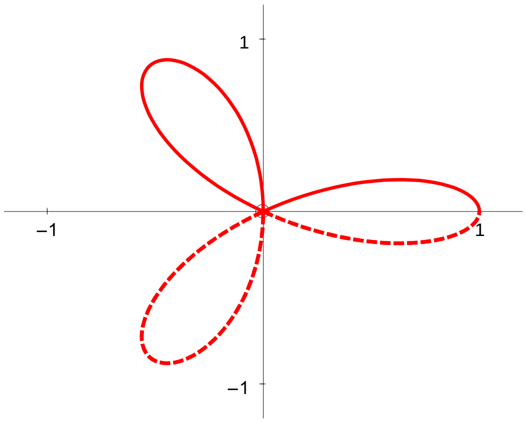
(iv) $r = 1 + 2\cos\theta$



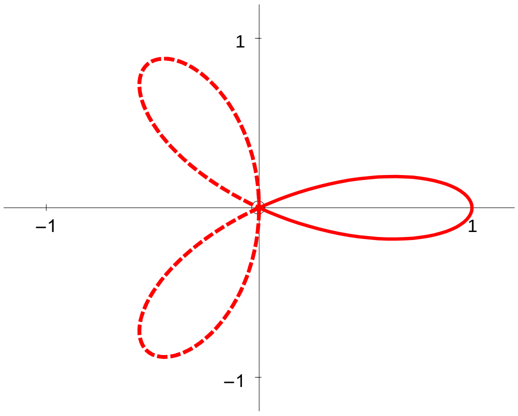
(v) $r = 3 + 2\sin\theta$



4. (i) $r = \cos 3\theta$ for $0 \le \theta \le \pi$



(ii) $r = \cos 3\theta$ for $-\frac{1}{2}\pi \le \theta \le \frac{1}{2}\pi$



In the first graph:

For $\frac{\pi}{2} < \theta < \frac{5\pi}{6}$, $\frac{3\pi}{2} < 3\theta < \frac{5\pi}{2}$ so $\cos 3\theta > 0$

This is shown by a solid line in the 2nd quadrant.

For $\frac{5\pi}{6} < \theta < \pi$, $\frac{5\pi}{2} < \theta < 3\pi$ so $\cos 3\theta < 0$.

This is shown by a broken line in the 4th quadrant.

In the second graph:

For $-\frac{\pi}{2} < \theta < -\frac{\pi}{6}$, $-\frac{3\pi}{2} < 3\theta < -\frac{\pi}{2}$ so $\cos 3\theta < 0$

This is shown by a broken line in the 2nd quadrant.

For $-\frac{\pi}{6} < \theta < 0$, $-\frac{\pi}{2} < 3\theta < 0$ so $\cos 3\theta > 0$.

This is shown by a solid line in the 4th quadrant.

Both graphs are the same for $0 < \theta < \frac{\pi}{2}$:

For $0 < \theta < \frac{\pi}{6}$, $0 < 3\theta < \frac{\pi}{2}$ so $\cos 3\theta > 0$

This is shown by a solid line in the 1st quadrant.

For $\frac{\pi}{6} < \theta < \frac{\pi}{2}$, $\frac{\pi}{2} < \theta < \frac{3\pi}{2}$ so $\cos 3\theta < 0$.

This is shown by a broken line in the 2nd quadrant.

Edexcel FM Polar coordinates 1 Exercise solutions

5. (i) $r = \cos\theta$

$r^2 = r\cos\theta$

$x^2 + y^2 = x$

(ii) $r = \sin 2\theta$

$r = 2\sin\theta\cos\theta$

$r^3 = 2 \times r\sin\theta \times r\cos\theta$

$(x^2 + y^2)^{\frac{3}{2}} = 2xy$

$(x^2 + y^2)^3 = 4x^2y^2$

(iii) $r = 1 + \cos\theta$

$r^2 = r + r\cos\theta$

$x^2 + y^2 = \sqrt{x^2 + y^2} + x$

(iv) $r = \sec(\theta - \frac{\pi}{6})$

$r\cos(\theta - \frac{\pi}{6}) = 1$

$r\cos\theta\cos\frac{\pi}{6} + r\sin\theta\sin\frac{\pi}{6} = 1$

$x \times \frac{1}{2}\sqrt{3} + y \times \frac{1}{2} = 1$

$x\sqrt{3} + y = 2$

6. (i) $y = x^2$

$r\sin\theta = (r\cos\theta)^2$

$r\sin\theta = r^2\cos^2\theta$

$r\cos^2\theta = \sin\theta$

$r = \dfrac{\sin\theta}{\cos^2\theta} = \tan\theta\sec\theta$

(ii) $(x-1)^2 + y^2 = 5$

$(r\cos\theta - 1)^2 + (r\sin\theta)^2 = 5$

$r^2\cos^2\theta - 2r\cos\theta + 1 + r^2\sin^2\theta = 5$

$r^2 - 2r\cos\theta = 4$

(iii) $xy = 1$

$r\cos\theta \times r\sin\theta = 1$

$r^2 \times 2\sin\theta\cos\theta = 2$

$r^2\sin 2\theta = 2$

(iv) $(x^2 + y^2)^2 = x^2 - y^2$

$(r^2)^2 = r^2 \cos^2\theta - r^2 \sin^2\theta$

$r^4 = r^2(\cos^2\theta - \sin^2\theta)$

$r^2 = \cos 2\theta$