

Section 1: Polar coordinates and curves

Exercise level 1 solutions

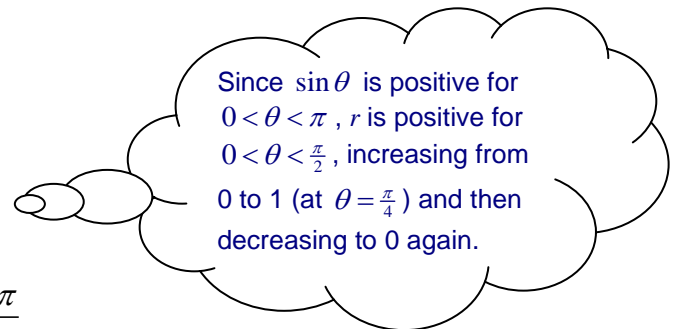
1. (i) $x = 4, y = 4$

$$r = \sqrt{x^2 + y^2} = \sqrt{16 + 16} = 4\sqrt{2}$$

$$\tan \theta = \frac{4}{4} = 1$$

Since point is in first quadrant, $\theta = \frac{\pi}{4}$

Polar coordinates are $\left(4\sqrt{2}, \frac{\pi}{4}\right)$



(ii) $x = 1, y = \sqrt{3}$

$$r = \sqrt{x^2 + y^2} = \sqrt{1 + 3} = 2$$

$$\tan \theta = \frac{\sqrt{3}}{1} = \sqrt{3}$$

Since point is in first quadrant, $\theta = \frac{\pi}{3}$

Polar coordinates are $\left(2, \frac{\pi}{3}\right)$

(iii) $x = -3, y = 4$

$$r = \sqrt{x^2 + y^2} = \sqrt{9 + 16} = 5$$

$$\tan \theta = -\frac{4}{3}$$

Since point is in second quadrant, $\theta = \pi - \arctan \frac{4}{3} = 2.21$ (3 s.f.)

Polar coordinates are $(5, 2.21^\circ)$ (3 s.f.)

(iv) $x = -5, y = -12$

$$r = \sqrt{x^2 + y^2} = \sqrt{25 + 144} = 13$$

$$\tan \theta = \frac{12}{5}$$

Since point is in third quadrant, $\theta = \pi + \arctan \frac{5}{12} = 4.32$

Polar coordinates are $(13, 4.32^\circ)$

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2. (i) $r = 4, \theta = \frac{\pi}{3}$

$$x = r \cos \theta = 4 \cos \frac{\pi}{3} = 4 \times \frac{1}{2} = 2$$

$$y = r \sin \theta = 4 \sin \frac{\pi}{3} = 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}$$

Cartesian coordinates are $(2, 2\sqrt{3})$

(ii) $r = 5, \theta = \frac{\pi}{2}$

$$x = r \cos \theta = 5 \cos \frac{\pi}{2} = 5 \times 0 = 0$$

$$y = r \sin \theta = 5 \sin \frac{\pi}{2} = 5 \times 1 = 5$$

Cartesian coordinates are $(0, 5)$

(iii) $r = 8, \theta = \frac{5\pi}{4}$

$$x = r \cos \theta = 8 \cos \frac{5\pi}{4} = 8 \times -\frac{1}{\sqrt{2}} = -4\sqrt{2}$$

$$y = r \sin \theta = 8 \sin \frac{5\pi}{4} = 8 \times -\frac{1}{\sqrt{2}} = -4\sqrt{2}$$

Cartesian coordinates are $(-4\sqrt{2}, -4\sqrt{2})$

(iv) $r = 6, \theta = \frac{11\pi}{6}$

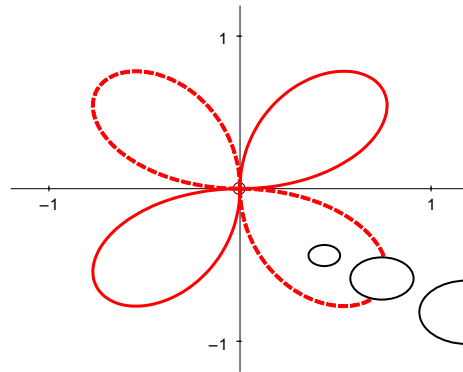
$$x = r \cos \theta = 6 \cos \frac{11\pi}{6} = 6 \times \frac{\sqrt{3}}{2} = 3\sqrt{3}$$

$$y = r \sin \theta = 6 \sin \frac{11\pi}{6} = 6 \times -\frac{1}{2} = -3$$

Cartesian coordinates are $(3\sqrt{3}, -3)$

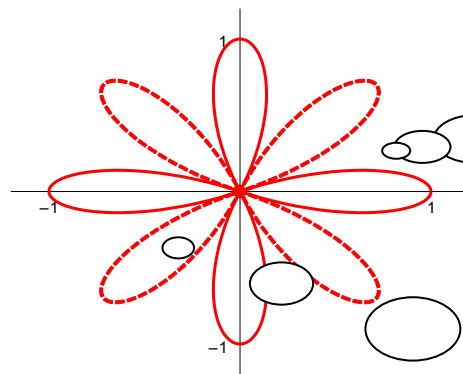
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3. (i) $r = \sin 2\theta$



Since $\sin \theta$ is negative for $\pi < \theta < 2\pi$, r is negative for $\frac{\pi}{2} < \theta < \pi$, decreasing from 0 to -1 (at $\theta = \frac{3\pi}{4}$ and then increasing back to 0. The pattern then continues.

(i) $r = \cos 4\theta$

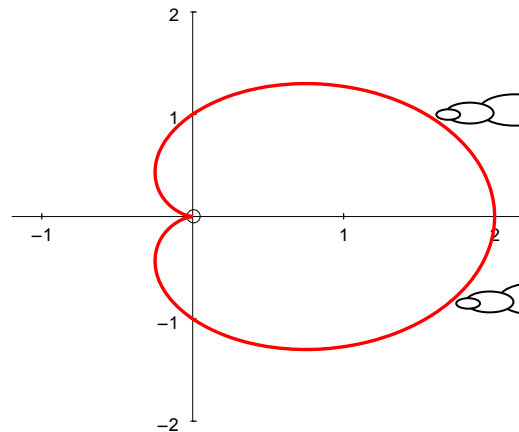


Since $\cos \theta$ is positive for $0 < \theta < \frac{\pi}{2}$, r is positive for $0 < \theta < \frac{\pi}{8}$, decreasing from 1 to 0.

Since $\cos \theta$ is negative for $\frac{\pi}{2} < \theta < \frac{3\pi}{2}$, r is negative for $\frac{3\pi}{8} < \theta < \frac{5\pi}{8}$, decreasing from 0 to -1 (at $\theta = \frac{\pi}{4}$ and then increasing back to 0. The pattern then continues.

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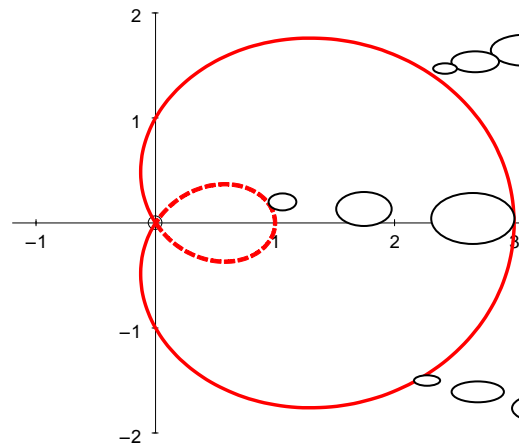
(iii) $r = 1 + \cos \theta$



As θ increases from 0 to π , r decreases from 2 to 0.

As θ increases from π to 2π , r increases from 0 to 2.

(iv) $r = 1 + 2 \cos \theta$

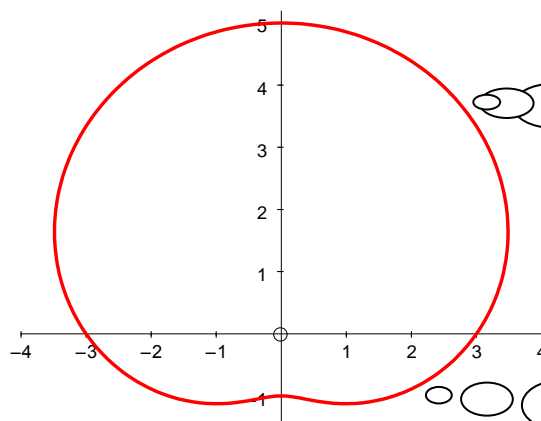


As θ increases from 0 to $\frac{2\pi}{3}$, r decreases from 3 to 0.

As θ increases from $\frac{2\pi}{3}$ to $\frac{4\pi}{3}$, r becomes negative, decreasing from 0 to -1 (at $\theta = \pi$) and then increasing to 0.

As θ increases from $\frac{4\pi}{3}$ to 2π , r increases from 0 to 3.

(v) $r = 3 + 2 \sin \theta$

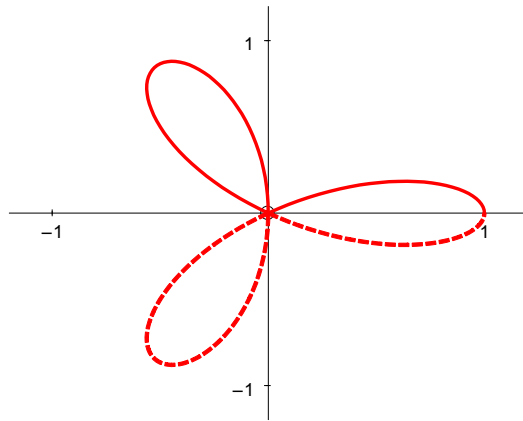


As θ increases from 0 to π , r increases from 3 to 5 (at $\theta = \frac{\pi}{2}$) and then decreases back to 3.

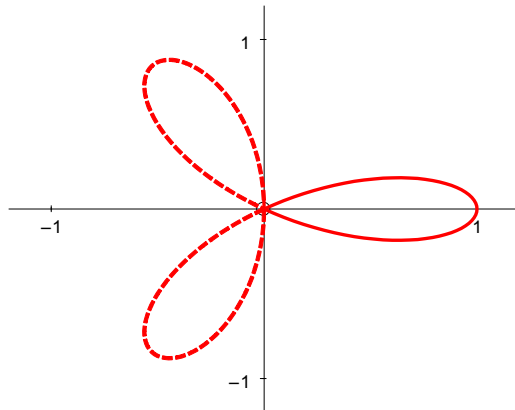
As θ increases from π to 2π , r decreases from 3 to 1 (at $\theta = \frac{3\pi}{2}$) and then increases back to 3.

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4. (i) $r = \cos 3\theta$ for $0 \leq \theta \leq \pi$



(ii) $r = \cos 3\theta$ for $-\frac{1}{2}\pi \leq \theta \leq \frac{1}{2}\pi$



In the first graph:

For $\frac{\pi}{2} < \theta < \frac{5\pi}{6}$, $\frac{3\pi}{2} < 3\theta < \frac{5\pi}{2}$ so $\cos 3\theta > 0$

This is shown by a solid line in the 2nd quadrant.

For $\frac{5\pi}{6} < \theta < \pi$, $\frac{5\pi}{2} < 3\theta < 3\pi$ so $\cos 3\theta < 0$.

This is shown by a broken line in the 4th quadrant.

In the second graph:

For $-\frac{\pi}{2} < \theta < -\frac{\pi}{6}$, $-\frac{3\pi}{2} < 3\theta < -\frac{\pi}{2}$ so $\cos 3\theta < 0$

This is shown by a broken line in the 2nd quadrant.

For $-\frac{\pi}{6} < \theta < 0$, $-\frac{\pi}{2} < 3\theta < 0$ so $\cos 3\theta > 0$.

This is shown by a solid line in the 4th quadrant.

Both graphs are the same for $0 < \theta < \frac{\pi}{2}$:

For $0 < \theta < \frac{\pi}{6}$, $0 < 3\theta < \frac{\pi}{2}$ so $\cos 3\theta > 0$

This is shown by a solid line in the 1st quadrant.

For $\frac{\pi}{6} < \theta < \frac{\pi}{2}$, $\frac{\pi}{2} < 3\theta < \frac{3\pi}{2}$ so $\cos 3\theta < 0$.

This is shown by a broken line in the 2nd quadrant.

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5. (i) $r = \cos \theta$
 $r^2 = r \cos \theta$
 $x^2 + y^2 = x$

(ii) $r = \sin 2\theta$
 $r = 2 \sin \theta \cos \theta$
 $r^3 = 2 \times r \sin \theta \times r \cos \theta$
 $(x^2 + y^2)^{\frac{3}{2}} = 2xy$
 $(x^2 + y^2)^3 = 4x^2y^2$

(iii) $r = 1 + \cos \theta$
 $r^2 = r + r \cos \theta$
 $x^2 + y^2 = \sqrt{x^2 + y^2} + x$

(iv) $r = \sec(\theta - \frac{\pi}{6})$
 $r \cos(\theta - \frac{\pi}{6}) = 1$
 $r \cos \theta \cos \frac{\pi}{6} + r \sin \theta \sin \frac{\pi}{6} = 1$
 $x \times \frac{1}{2} \sqrt{3} + y \times \frac{1}{2} = 1$
 $x\sqrt{3} + y = 2$

6. (i) $y = x^2$
 $r \sin \theta = (r \cos \theta)^2$
 $r \sin \theta = r^2 \cos^2 \theta$
 $r \cos^2 \theta = \sin \theta$
 $r = \frac{\sin \theta}{\cos^2 \theta} = \tan \theta \sec \theta$

(ii) $(x-1)^2 + y^2 = 5$
 $(r \cos \theta - 1)^2 + (r \sin \theta)^2 = 5$
 $r^2 \cos^2 \theta - 2r \cos \theta + 1 + r^2 \sin^2 \theta = 5$
 $r^2 - 2r \cos \theta = 4$

(iii) $xy = 1$
 $r \cos \theta \times r \sin \theta = 1$
 $r^2 \times 2 \sin \theta \cos \theta = 2$
 $r^2 \sin 2\theta = 2$

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$$\begin{aligned} \text{(iv)} \quad (x^2 + y^2)^2 &= x^2 - y^2 \\ (r^2)^2 &= r^2 \cos^2 \theta - r^2 \sin^2 \theta \\ r^4 &= r^2 (\cos^2 \theta - \sin^2 \theta) \\ r^2 &= \cos 2\theta \end{aligned}$$