

Section 1: Further volumes of revolution

Solutions to Exercise

1. $y = e^x$

$$\begin{aligned} \text{volume} &= \int_0^1 \pi y^2 dx \\ &= \pi \int_0^1 (e^x)^2 dx \\ &= \pi \int_0^1 e^{2x} dx \\ &= \pi \left[\frac{1}{2} e^{2x} \right]_0^1 \\ &= \frac{1}{2} \pi (e^2 - 1) \end{aligned}$$

2. $y = \ln(1+x) \Rightarrow e^y = 1+x \Rightarrow x = e^y - 1$

When $x = 0$, $y = \ln 1 = 0$

When $x = 1$, $y = \ln 2$

$$\begin{aligned} \text{volume } V &= \int_{\frac{1}{2}}^1 \pi x^2 dy \\ &= \pi \int_0^{\ln 2} (e^y - 1)^2 dy \\ &= \pi \int_0^{\ln 2} (e^{2y} - 2e^y + 1) dy \end{aligned}$$

$$\begin{aligned} V &= \pi \left[\frac{1}{2} e^{2y} - 2e^y + y \right]_0^{\ln 2} \\ &= \pi \left(\frac{1}{2} e^{2 \ln 2} - 2e^{\ln 2} + \ln 2 \right) - \pi \left(\frac{1}{2} - 2 + 0 \right) \\ &= \pi \left(\frac{1}{2} \times 4 - 2 \times 2 + \ln 2 - \frac{1}{2} + 2 \right) \\ &= \pi \left(2 - 4 - \frac{1}{2} + 2 + \ln 2 \right) \\ &= \pi \left(\ln 2 - \frac{1}{2} \right) \end{aligned}$$

3. $x = t + t^2$, $y = t - t^2$

When the curve crosses the x -axis, $y = 0$, so $t - t^2 = 0$

$$t(1-t) = 0$$

$$t = 0 \text{ or } 1$$

$$x = t + t^2 \Rightarrow \frac{dx}{dt} = 1 + 2t$$

$$\text{volume} = \int_0^1 \pi y^2 \frac{dx}{dt} dt$$

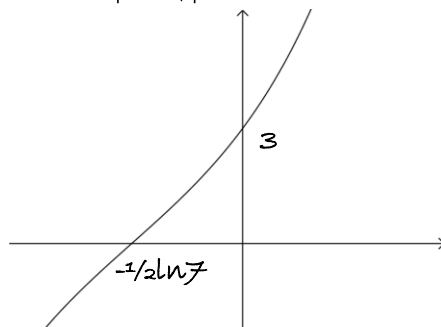
Edexcel FM Applications of integration 1 Exercise solns

$$\begin{aligned}
 &= \pi \int_0^1 (t - t^2)^2 (1 + 2t) dt \\
 &= \pi \int_0^1 (t^2 - 2t^3 + t^4) (1 + 2t) dt \\
 &= \pi \int_0^1 (t^2 - 2t^3 + t^4 + 2t^3 - 4t^4 + 2t^5) dt \\
 &= \pi \int_0^1 (t^2 - 3t^4 + 2t^5) dt \\
 &= \pi \left[\frac{1}{3} t^3 - \frac{3}{5} t^5 + \frac{1}{3} t^6 \right]_0^1 \\
 &= \pi \left(\frac{1}{3} - \frac{3}{5} + \frac{1}{3} - 0 \right) \\
 &= \frac{1}{15} \pi
 \end{aligned}$$

4. (i) When $x = 0, y = 3$

When $y = 0, \tanh x = -\frac{3}{4}$

$$x = \frac{1}{2} \ln \left| \frac{1 - \frac{3}{4}}{1 + \frac{3}{4}} \right| = \frac{1}{2} \ln \frac{1}{7} = -\frac{1}{2} \ln 7$$



$$\begin{aligned}
 \text{(ii)} \quad (4 \sinh x + 3 \cosh x)^2 &= 16 \sinh^2 x + 24 \sinh x \cosh x + 9 \cosh^2 x \\
 &= 8(\cosh 2x - 1) + 12 \sinh 2x + \frac{9}{2}(\cosh 2x + 1) \\
 &= \frac{25}{2} \cosh 2x + 12 \sinh 2x - \frac{7}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii) Volume} &= \pi \int_{-\frac{1}{2} \ln 7}^0 y^2 dx \\
 &= \pi \int_{-\frac{1}{2} \ln 7}^0 \left(\frac{25}{2} \cosh 2x + 12 \sinh 2x - \frac{7}{2} \right) dx \\
 &= \pi \left[\frac{25}{4} \sinh 2x + 6 \cosh 2x - \frac{7}{2} x \right]_{-\frac{1}{2} \ln 7}^0 \\
 &= \pi \left(6 - \frac{25}{4} \sinh(-\ln 7) - 6 \cosh(-\ln 7) - \frac{7}{4} \ln 7 \right) \\
 &= \pi \left(6 + \frac{25(7 - \frac{1}{7})}{8} - \frac{6(7 + \frac{1}{7})}{2} - \frac{7}{4} \ln 7 \right) \\
 &= \pi \left(6 - \frac{7}{4} \ln 7 \right)
 \end{aligned}$$