Section 2: Mean values and general integration

Exercise level 1

1. (i) $\text{Mean value} = \frac{1}{1-0}\int_0^1 x^3\,dx$

$= \left[\frac{1}{4}x^4\right]_0^1$

$= \frac{1}{4}(1-0)$

$= \frac{1}{4}$

(ii) $\text{Mean value} = \frac{1}{4-0}\int_0^4 x^{\frac{1}{2}}\,dx$

$= \frac{1}{4}\left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^4$

$= \frac{1}{6}(8-0)$

$= \frac{4}{3}$

(iii) $\text{Mean value} = \frac{1}{1-(-1)}\int_{-1}^1 (1-x^2)\,dx$

$= \frac{1}{2}\left[x-\frac{1}{3}x^3\right]_{-1}^1$

$= \frac{1}{2}\left(1-\frac{1}{3}-\left(-1+\frac{1}{3}\right)\right)$

$= \frac{1}{2}\times\frac{4}{3}$

$= \frac{2}{3}$

(iv) $\text{Mean value} = \frac{1}{2-1}\int_1^2 \left(1-\frac{1}{x^2}\right)dx$

$= \left[x+\frac{1}{x}\right]_1^2$

$= 2+\frac{1}{2}-(1+1)$

$= \frac{1}{2}$

(v) $\text{Mean value} = \frac{1}{2-(-1)}\int_{-1}^2 (x^2+1)\,dx$

$= \frac{1}{3}\left[\frac{1}{3}x^3+x\right]_{-1}^2$

$= \frac{1}{3}\left(\frac{8}{3}+2-\left(-\frac{1}{3}-1\right)\right)$

$= 2$

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2. (i) $\int \frac{1}{\sqrt{4-x^2}}\,dx = \arcsin\left(\frac{x}{2}\right)+c$

(ii) $\int \frac{1}{\sqrt{1-4x^2}}\,dx = \int \frac{1}{2\sqrt{\frac{1}{4}-x^2}}\,dx = \frac{1}{2}\arcsin\left(\frac{x}{\frac{1}{2}}\right)+c = \frac{1}{2}\arcsin(2x)+c$

(iii) $\int \frac{1}{\sqrt{x^2-4}}\,dx = \operatorname{arcosh}\left(\frac{x}{2}\right)+c$

(iv) $\int \frac{1}{\sqrt{4x^2-1}}\,dx = \int \frac{1}{2\sqrt{x^2-\frac{1}{4}}}\,dx = \frac{1}{2}\operatorname{arcosh}\left(\frac{x}{\frac{1}{2}}\right)+c = \frac{1}{2}\operatorname{arcosh}(2x)+c$

(v) $\int \frac{1}{\sqrt{4+x^2}}\,dx = \operatorname{arsinh}\left(\frac{x}{2}\right)+c$

(vi) $\int \frac{1}{\sqrt{1+4x^2}}\,dx = \int \frac{1}{2\sqrt{\frac{1}{4}+x^2}}\,dx = \frac{1}{2}\operatorname{arsinh}\left(\frac{x}{\frac{1}{2}}\right)+c = \frac{1}{2}\operatorname{arsinh}(2x)+c$

(vii) $\int \frac{1}{4+x^2}\,dx = \frac{1}{2}\arctan\left(\frac{x}{2}\right)+c$

(viii) $\int \frac{1}{1+4x^2}\,dx = \frac{1}{4}\int \frac{1}{\frac{1}{4}+x^2}\,dx = \frac{1}{4}\times\frac{1}{\frac{1}{2}}\arctan\left(\frac{x}{\frac{1}{2}}\right)+c = \frac{1}{2}\arctan(2x)+c$

(ix) $\int \frac{1}{4-x^2}\,dx = \frac{1}{2}\operatorname{artanh}\left(\frac{x}{2}\right)+c$

(x) $\int \frac{1}{1-4x^2}\,dx = \frac{1}{4}\int \frac{1}{\frac{1}{4}-x^2}\,dx = \frac{1}{4}\times\frac{1}{\frac{1}{2}}\operatorname{artanh}\left(\frac{x}{\frac{1}{2}}\right)+c = \frac{1}{2}\operatorname{artanh}(2x)+c$