

Section 2: Inverse trigonometric functions

Exercise level 2 solutions

$$1. \quad (i) \quad \frac{d}{dx}(\arctan x^2) = 2x \times \frac{1}{1+(x^2)^2}$$

$$= \frac{2x}{1+x^4}$$

$$(ii) \quad \frac{d}{dx}\left(\arcsin \frac{1}{x}\right) = -\frac{1}{x^2} \times \frac{1}{\sqrt{1-\left(\frac{1}{x}\right)^2}}$$

$$= -\frac{1}{x^2 \sqrt{1-\frac{1}{x^2}}}$$

$$= -\frac{1}{x^2 \sqrt{\frac{x^2-1}{x^2}}}$$

$$= -\frac{1}{x\sqrt{x^2-1}}$$

$$(iii) \quad \frac{d}{dx}(\arccos(1-\sqrt{x})) = -\frac{1}{2}x^{-\frac{1}{2}} \times -\frac{1}{\sqrt{1-(1-\sqrt{x})^2}}$$

$$= \frac{1}{2\sqrt{x}\sqrt{1-(1-2\sqrt{x}+x)}}$$

$$= \frac{1}{2\sqrt{x}\sqrt{2\sqrt{x}-x}}$$

$$(iv) \quad \frac{d}{dx}(\arcsin(\arctan x)) = \frac{1}{\sqrt{1-(\arctan x)^2}} \times \frac{d}{dx}(\arctan x)$$

$$= \frac{1}{\sqrt{1-(\arctan x)^2}} \times \frac{1}{1+x^2}$$

$$= \frac{1}{(1+x^2)\sqrt{1-(\arctan x)^2}}$$

$$2. \quad (a) \quad \int_0^d \frac{9}{1+4x^2} dx = \frac{9}{4} \int_0^d \frac{9}{\frac{1}{4}+x^2} dx = \frac{9}{4} \left[\frac{1}{\frac{1}{2}} \arctan\left(\frac{x}{\frac{1}{2}}\right) \right]_0^d = \frac{9}{2} [\arctan(2x)]_0^d$$

$$(i) \quad \int_0^{1/2} \frac{9}{1+4x^2} dx = \frac{9}{2} [\arctan(2x)]_0^{1/2} = \frac{9}{2} \arctan 1 - 0$$

$$= \frac{9\pi}{8}$$

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$$\begin{aligned}
 \text{(ii)} \quad \int_{\sqrt{3}/2}^{\infty} \frac{9}{1+4x^2} dx &= \frac{9}{2} [\arctan(2x)]_{\sqrt{3}/2}^{\infty} \\
 &= \frac{9}{2} (\arctan \infty - \arctan \sqrt{3}) = \frac{9}{2} \left(\frac{\pi}{2} - \frac{\pi}{3} \right) \\
 &= \frac{3\pi}{4}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad \int_0^d \frac{9}{\sqrt{1-4x^2}} dx &= \frac{1}{2} \int_0^d \frac{9}{\sqrt{\frac{1}{4}-x^2}} dx = \left[\frac{9}{2} \arcsin \left(\frac{x}{\frac{1}{2}} \right) \right]_0^d \\
 &= \left[\frac{9}{2} \arcsin(2x) \right]_0^d
 \end{aligned}$$

$$\begin{aligned}
 \text{(i)} \quad \int_0^{1/2} \frac{9}{\sqrt{1-4x^2}} dx &= \left[\frac{9}{2} \arcsin(2x) \right]_0^{1/2} = \frac{9}{2} \arcsin 1 - 0 \\
 &= \frac{9\pi}{4}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad \int_{1/2\sqrt{2}}^{\sqrt{3}/4} \frac{9}{\sqrt{1-4x^2}} dx &= \left[\frac{9}{2} \arcsin(2x) \right]_{1/2\sqrt{2}}^{\sqrt{3}/4} \\
 &= \frac{9}{2} \left(\arcsin \frac{\sqrt{3}}{2} - \arcsin \frac{1}{\sqrt{2}} \right) = \frac{9}{2} \left(\frac{\pi}{3} - \frac{\pi}{4} \right) \\
 &= \frac{3\pi}{8}
 \end{aligned}$$

$$3. \quad \text{(i)} \quad \int \arcsin 2x \, dx = \int 1 \times \arcsin 2x \, dx$$

$$\text{Let } u = \arcsin 2x \Rightarrow \frac{du}{dx} = \frac{2}{\sqrt{1-(2x)^2}} = \frac{2}{\sqrt{1-4x^2}}$$

$$\frac{dv}{dx} = 1 \Rightarrow v = x$$

$$\text{using integration by parts, } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

$$\begin{aligned}
 \int 1 \times \arcsin 2x \, dx &= x \arcsin 2x - \int x \times \frac{2}{\sqrt{1-4x^2}} dx \\
 &= x \arcsin 2x - \int \frac{2x}{\sqrt{1-4x^2}} dx \\
 &= x \arcsin 2x + \frac{1}{4} \int -8x(1-4x^2)^{-\frac{1}{2}} dx \\
 &= x \arcsin 2x + \frac{1}{4} \times 2(1-4x^2)^{\frac{1}{2}} + c \\
 &= x \arcsin 2x + \frac{1}{2} \sqrt{1-4x^2} + c
 \end{aligned}$$

$$\text{(ii)} \quad \int \arctan 3x \, dx = \int 1 \times \arctan 3x \, dx$$

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$$\text{Let } u = \arctan 3x \Rightarrow \frac{du}{dx} = \frac{3}{1+(3x)^2} = \frac{3}{1+9x^2}$$

$$\frac{dv}{dx} = 1 \Rightarrow v = x$$

$$\text{using integration by parts, } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

$$\begin{aligned} \int \arctan 3x dx &= x \arctan 3x - \int \frac{3x}{1+9x^2} dx \\ &= x \arctan 3x - \frac{1}{6} \ln(1+9x^2) + c \end{aligned}$$

$$\text{(iii) } \int 1 \times (\arcsin x + \arctan x) dx$$

$$\begin{aligned} &= x(\arcsin x + \arctan x) - \int \left(\frac{x}{\sqrt{1-x^2}} + \frac{x}{1+x^2} \right) dx \\ &= x(\arcsin x + \arctan x) + (1-x^2)^{\frac{1}{2}} - \frac{1}{2} \ln(1+x^2) + c \end{aligned}$$