# Section 1: Finding and using Maclaurin series

## Solutions to Exercise level 3

1. (i) $f(x) = \cos(x + \frac{\pi}{3})$

$f'(x) = -\sin(x + \frac{\pi}{3})$

$f''(x) = -\cos(x + \frac{\pi}{3})$

$f(0) = \frac{1}{2}$

$f'(x) = -\frac{1}{2}\sqrt{3}$

$f''(x) = -\frac{1}{2}$

$$\cos x = \frac{1}{2} - \frac{x\sqrt{3}}{2} - \frac{x^2}{4} + \ldots$$

(ii) $f(x) = \sec(x + \frac{\pi}{3})$

$f'(x) = \sec(x + \frac{\pi}{3})\tan(x + \frac{\pi}{3})$

$f''(x) = (\sec(x + \frac{\pi}{3})\tan(x + \frac{\pi}{3}))\tan(x + \frac{\pi}{3}) + \sec x(\sec^2(x + \frac{\pi}{3}))$

$= \sec(x + \frac{\pi}{3})\tan^2(x + \frac{\pi}{3}) + \sec^3(x + \frac{\pi}{3})$

$f(0) = 2$

$f'(0) = 2\sqrt{3}$

$f''(0) = 6 + 8 = 14$

$$\sec x = 2 + 2x\sqrt{3} + 7x^2 + \ldots$$

(iii) $$\cos x \sec x = \left(\frac{1}{2} - \frac{x\sqrt{3}}{2} - \frac{x^2}{4} + \ldots\right)\left(2 + 2x\sqrt{3} + 7x^2 + \ldots\right)$$

$$= 1 - x\sqrt{3} - \frac{x^2}{2} + \ldots + x\sqrt{3} - 3x^2 + \ldots + \frac{7}{2}x^2 + \ldots$$

$$= 1 + 0 + 0 + \ldots$$

as expected since $\cos x \sec x = 1$.