Section 1: Finding and using Maclaurin series

Solutions to Exercise level 1

1. (i) $$e^{-3x} = 1 + (-3x) + \frac{(-3x)^2}{2!} + \frac{(-3x)^3}{3!} + \dots$$
$$= 1 - 3x + \frac{9x^2}{2} - \frac{27x^3}{6} + \dots$$
$$= 1 - 3x + \frac{9x^2}{2} - \frac{9x^3}{2} + \dots$$
Valid for all values of x.

(ii) $$\sin 2x = 2x - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} - \frac{(2x)^7}{7!} + \dots$$
$$= 2x - \frac{8x^3}{6} + \frac{32x^5}{120} - \frac{128x^7}{5040} + \dots$$
$$= 2x - \frac{4x^3}{3} + \frac{4x^5}{15} - \frac{8x^7}{315} + \dots$$
Valid for all values of x.

(iii) $$\ln(1-2x) = (-2x) - \frac{(-2x)^2}{2} + \frac{(-2x)^3}{3} - \frac{(-2x)^4}{4} + \dots$$
$$= -2x - \frac{4x^2}{2} - \frac{8x^3}{3} - \frac{16x^4}{4} + \dots$$
$$= -2x - 2x^2 - \frac{8x^3}{3} - 4x^4 + \dots$$
Valid for $-1 < -2x \le 1$
$$-\tfrac{1}{2} \le x < \tfrac{1}{2}$$

(iv) $$\cos \tfrac{1}{2}x = 1 - \frac{(\frac{1}{2}x)^2}{2!} + \frac{(\frac{1}{2}x)^4}{4!} - \frac{(\frac{1}{2}x)^6}{6!} + \dots$$
$$= 1 - \frac{\frac{1}{4}x^2}{2} + \frac{\frac{1}{16}x^4}{24} - \frac{\frac{1}{64}x^6}{720} + \dots$$
$$= 1 - \frac{x^2}{8} + \frac{x^4}{384} - \frac{x^6}{46080} + \dots$$
Valid for all x.

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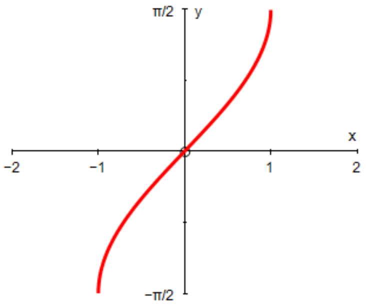
2. (i)



(ii) $y = \arcsin x$

$\sin y = x$

$\cos y \dfrac{dy}{dx} = 1$

$\dfrac{dy}{dx} = \dfrac{1}{\cos y} = \dfrac{1}{\sqrt{1-\sin^2 y}} = \dfrac{1}{\sqrt{1-x^2}}$

(iii) $y' = (1-x^2)^{-1/2}$

$y'' = -\frac{1}{2}(1-x^2)^{-3/2} \times -2x = x(1-x^2)^{-3/2}$

$y''' = (1-x^2)^{-3/2} - \frac{3}{2}x(1-x^2)^{-5/2} \times -2x$

$= (1-x^2)^{-3/2} + 3x^2(1-x^2)^{-5/2}$

$y(0) = 0$

$y'(0) = 1$

$y''(0) = 0$

$y'''(0) = 1$

so $\arcsin x = x + \dfrac{x^3}{3!} + \ldots$

$= x + \dfrac{x^3}{6} + \ldots$

3. $\sin x = x - \dfrac{x^3}{6} + \dfrac{x^5}{120} - \ldots$

$10° = \dfrac{\pi}{18}$ radians

$\sin x = \dfrac{\pi}{18} - \dfrac{(\pi/18)^3}{6} + \dfrac{(\pi/18)^5}{120} - \ldots$

$= 0.174566 - 0.0008861 + 0.0000001$

$= 0.173648$

This and subsequent terms do not affect the 6th decimal place

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4. $y = \ln x$ is not defined at $x = 0$, so the Maclaurin series does not exist.

$$y = \ln(x+2) \quad \Rightarrow y(0) = \ln 2$$

$$y' = \frac{1}{x+2} \quad \Rightarrow y'(1) = \frac{1}{2}$$

$$y'' = -\frac{1}{(x+2)^2} \quad \Rightarrow y''(0) = -\frac{1}{4}$$

$$y''' = \frac{2}{(x+2)^3} \quad \Rightarrow y'''(0) = \frac{1}{4}$$

$$\ln(x+2) = \ln 2 + \frac{1}{2}x - \frac{1}{4}\times\frac{x^2}{2} + \frac{1}{4}\times\frac{x^3}{6}$$

$$= \ln 2 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{24}x^3$$

5. (i) $(1+x)^{-2} = 1 - 2x + \dfrac{-2\times -3}{1\times 2}x^2 + \dfrac{-2\times -3\times -4}{1\times 2\times 3}x^3 + \ldots$

$$= 1 - 2x + 3x^2 - 4x^3 + \ldots$$

Coefficient of x^n is $(-1)^n(n+1)$

(ii) $y = (1+x)^{-2} \quad \Rightarrow y(0) = 1$

$$y' = -2(1+x)^{-3} \Rightarrow y'(0) = -2$$

$$y'' = -2\times -3(1+x)^{-4} \Rightarrow y''(0) = -2\times -3$$

$$y^{(n)}(0) = (-1)^n(n+1)!$$

Coefficient of x^n in Maclaurin expansion is $\dfrac{(-1)^n(n+1)!}{n!} = (-1)^n(n+1)$