Section 1: The method of differences

Solutions to Exercise level 3

1. (i) $\dfrac{1}{\sqrt{3}+\sqrt{2}} = \dfrac{\sqrt{3}-\sqrt{2}}{\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}-\sqrt{2}\right)} = \dfrac{\sqrt{3}-\sqrt{2}}{3-2} = \sqrt{3}-\sqrt{2}$

(ii) $\dfrac{1}{\sqrt{r+1}+\sqrt{r}} = \dfrac{\sqrt{r+1}-\sqrt{r}}{\left(\sqrt{r+1}+\sqrt{r}\right)\left(\sqrt{r+1}-\sqrt{r}\right)} = \dfrac{\sqrt{r+1}-\sqrt{r}}{r+1-r} = \sqrt{r+1}-\sqrt{r}$

$$\sum_{r=0}^{n} \frac{1}{\sqrt{r+1}+\sqrt{r}} = \sum_{r=0}^{n} \sqrt{r+1}-\sqrt{r}$$

$$= \cancel{\sqrt{1}} - \sqrt{0}$$
$$+\cancel{\sqrt{2}} - \cancel{\sqrt{1}}$$
$$+\ldots$$
$$= \cancel{\sqrt{n}} - \cancel{\sqrt{n-1}}$$
$$+\sqrt{n+1} - \cancel{\sqrt{n}}$$
$$= \sqrt{n+1}$$