

Section 1: The method of differences

Solutions to Exercise level 1

$$\begin{aligned}
 1. \quad \frac{1}{r+1} - \frac{1}{r+2} &= \frac{r+2}{(r+1)(r+2)} - \frac{r+1}{(r+1)(r+2)} \\
 &= \frac{r+2-r-1}{(r+1)(r+2)} \\
 &= \frac{1}{(r+1)(r+2)}
 \end{aligned}$$

$$\begin{aligned}
 \sum_{r=1}^n \frac{1}{(r+1)(r+2)} &= \sum_{r=1}^n \left(\frac{1}{r+1} - \frac{1}{r+2} \right) \\
 &= \frac{1}{2} - \cancel{\frac{1}{3}} \\
 &\quad + \cancel{\frac{1}{3}} - \cancel{\frac{1}{4}} \\
 &\quad + \dots \\
 &\quad + \cancel{\frac{1}{n}} - \cancel{\frac{1}{n+1}} \\
 &\quad + \cancel{\frac{1}{n+1}} - \frac{1}{n+2} \\
 &= \frac{1}{2} - \frac{1}{n+2}
 \end{aligned}$$

Edexcel Sequences and series 1 Exercise solutions

$$\begin{aligned}
 2. \quad \frac{1}{3r} - \frac{1}{3(r+1)} &= \frac{r+1}{3r(r+1)} - \frac{r}{3r(r+1)} \\
 &= \frac{r+1-r}{3r(r+1)} \\
 &= \frac{1}{3r(r+1)}
 \end{aligned}$$

$$\begin{aligned}
 \sum_{r=1}^n \frac{1}{3r(r+1)} &= \sum_{r=1}^n \left(\frac{1}{3r} - \frac{1}{3(r+1)} \right) \\
 &= \frac{1}{3} \cancel{\frac{1}{6}} \\
 &\quad + \cancel{\frac{1}{3}} \cancel{\frac{1}{9}} \\
 &\quad + \dots \\
 &\quad + \frac{1}{3(n-1)} \cancel{\frac{1}{3n}} \\
 &\quad + \cancel{\frac{1}{3n}} - \frac{1}{3(n+1)} \\
 &= \frac{1}{3} - \frac{1}{3(n+1)} = \frac{n}{3(n+1)}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad (i) \quad (r+1)! - r! &= (r+1)r! - r! \\
 &= r!(r+1-1) \\
 &= r \times r!
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad \sum_{r=1}^n r \times r! &= \sum_{r=1}^n ((r+1)! - r!) \\
 &= \cancel{2!} - 1! \\
 &\quad + \cancel{3!} - \cancel{2!} \\
 &\quad + \dots \\
 &\quad + \cancel{n!} - \cancel{(n-1)!} \\
 &\quad + (n+1)! - \cancel{n!} \\
 &= (n+1)! - 1! \\
 &= (n+1)! - 1
 \end{aligned}$$