Topic assessment

1. The points A, B, C and D have coordinates (2, 1, 3), (4, 1, 5), (2, 5, p) and (q, r, 1) respectively. If $\overrightarrow{AB} = \overrightarrow{CD}$ what are the values of p, q and r?

[3]

2. Points A and B have coordinates (2, 1, 1) and (20, -5, 13) respectively.
If point C is such that $2\overrightarrow{AC} = \overrightarrow{CB}$, what are the coordinates of C?

[5]

3. The point P has coordinates (-2, 4, 0).
The point Q is such that $\overrightarrow{PQ} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$.
The point R has coordinates (-1, 1, r).
For which value of r is PQR an equilateral triangle?

[5]

4. Point A has coordinates (2, 3, 6). Point B has coordinates (8, 6, 8). Find the point C so that $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are in the same direction and $|\overrightarrow{AC}| = 77$.

[6]

5. Forces $\mathbf{F}_1 = \lambda(3\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ N and $\mathbf{F}_2 = \mu(\mathbf{i} + \mathbf{j} + 3\mathbf{k})$ N, where λ and μ are scalars, act on a box.
Prove that it is not possible for their resultant force to act in the direction of $\mathbf{k}$.

[6]

Total 25 marks

Edexcel A level Maths Vectors Assessment solutions

Solutions to topic assessment

1. $$\overrightarrow{AB} = \begin{pmatrix} 4 \\ 1 \\ 5 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$$

$$\overrightarrow{CD} = \begin{pmatrix} q \\ r \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 5 \\ p \end{pmatrix} = \begin{pmatrix} q-2 \\ r-5 \\ 1-p \end{pmatrix}$$

$$\begin{pmatrix} q-2 \\ r-5 \\ 1-p \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$$

$p = -1, q = 4, r = 5$

2. Let C have position vector $\begin{pmatrix} p \\ q \\ r \end{pmatrix}$

$$\overrightarrow{AC} = \begin{pmatrix} p \\ q \\ r \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} p-2 \\ q-1 \\ r-1 \end{pmatrix}$$

$$\overrightarrow{CB} = \begin{pmatrix} 20 \\ -5 \\ 13 \end{pmatrix} - \begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} 20-p \\ -5-q \\ 13-r \end{pmatrix}$$

$2\overrightarrow{AC} = \overrightarrow{CB}$

$$2\begin{pmatrix} p-2 \\ q-1 \\ r-1 \end{pmatrix} = \begin{pmatrix} 20-p \\ -5-q \\ 13-r \end{pmatrix}$$

$2(p-2) = 20-p \quad \Rightarrow 3p = 24 \quad \Rightarrow p = 8$

$2(q-1) = -5-q \quad \Rightarrow 3q = -3 \quad \Rightarrow q = -1$

$2(r-1) = 13-r \quad \Rightarrow 3r = 15 \quad \Rightarrow r = 5$

The coordinates of C are (8, -1, 5)

3. $\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ} = \begin{pmatrix} -2 \\ 4 \\ 0 \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

$\overrightarrow{PR} = \begin{pmatrix} -1 \\ 1 \\ r \end{pmatrix} - \begin{pmatrix} -2 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ r \end{pmatrix}$

$\overrightarrow{QR} = \begin{pmatrix} -1 \\ 1 \\ r \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ -1 \\ r-1 \end{pmatrix}$

$\left|\overrightarrow{PQ}\right|^2 = 3^2 + (-2)^2 + 1^2 = 14$

$\left|\overrightarrow{PR}\right|^2 = 1^2 + (-3)^2 + r^2 = 10 + r^2$

$\left|\overrightarrow{QR}\right|^2 = (-2)^2 + (-1)^2 + (r-1)^2 = r^2 - 2r + 6$

$\left|\overrightarrow{PQ}\right|^2 = \left|\overrightarrow{PR}\right|^2 \quad \Rightarrow 10 + r^2 = 14 \quad \Rightarrow r^2 = 4 \quad \Rightarrow r = \pm 2$

$\left|\overrightarrow{PR}\right|^2 = \left|\overrightarrow{QR}\right|^2 \quad \Rightarrow 10 + r^2 = r^2 - 2r + 6 \quad \Rightarrow 2r = -4 \quad \Rightarrow r = -2$

So PQR is an equilateral triangle for $r = -2$

4. $\overrightarrow{AB} = \begin{pmatrix} 8 \\ 6 \\ 8 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix}$

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are in the same direction $\Rightarrow \quad \overrightarrow{AC} = k\begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix}$ where k is positive

$\left|\overrightarrow{AC}\right| = 77 \qquad \Rightarrow k^2(6^2 + 3^2 + 2^2) = 77^2$

$\Rightarrow 49k^2 = 77^2$

$\Rightarrow k^2 = 121$

$\Rightarrow k = \pm 11$

k must be positive for same direction, so $k = 11$

$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC} = \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} + 11\begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 68 \\ 36 \\ 28 \end{pmatrix}$

So C is (68, 36, 28)

5. Resultant force is $\lambda\begin{pmatrix}3\\-2\\1\end{pmatrix}+\mu\begin{pmatrix}1\\1\\3\end{pmatrix}=\begin{pmatrix}3\lambda+\mu\\-2\lambda+\mu\\\lambda+3\mu\end{pmatrix}$

If resultant force is in the direction of $\underline{k}$, it must be $\begin{pmatrix}0\\0\\p\end{pmatrix}$ for some $p \neq 0$

$3\lambda+\mu=0 \quad \Rightarrow \mu=-3\lambda$

$-2\lambda+\mu=0 \quad \Rightarrow \mu=2\lambda$

$-3\lambda=2\lambda \quad \Rightarrow \lambda=0 \quad \Rightarrow \mu=0$

$p=\lambda+3\mu=0$ so $p=0$ which contradicts the assumption that $p \neq 0$
so it is not possible for the resultant force to be in the direction of $\underline{k}$.